Efficient Planning of Specialized Diagnostic Services in a Segmented Healthcare System

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Abstract

In this paper, a problem related to a specialized diagnostic service that requires costly equipment in a segmented public healthcare system such as the one in Mexico is addressed. The aim is to determine which hospitals could provide the service and their capacity levels, the allocation of demand in each institution, and the reallocation of uncovered demand to other institutions or private sector providers while minimizing total annual cost of investment and operating cost required to satisfy all demand. A mixed integer linear programming model taking into account different characteristics such as patient acuity level, types of equipment, and demand variation through time is introduced. The model was empirically assessed to evaluate its impact in the decision-making process. Good quality solutions were found for instances up to 90 facilities. A sensitive analysis was performed to evaluate solution behavior for variations of critical parameters. It was found that some values could generate an effect in the total costs for the service coverage and in the efficiency of the service.

Keywords: Operations research in healthcare services; location-allocation model; public healthcare planning; diagnostic services.

1 Introduction

Developing countries such as Mexico face important problems in public healthcare along with economic and demographic issues. A segmented healthcare system such as the one prevailing in Mexico causes unequal quality service and access to medical services. Under this scheme, each institution decides how to distribute funds received from government, affiliated workers, donations, or revenues (Gómez et al., 2011). This implies that the number of general and specialized hospitals, workforce, technological resources, hospital beds, operating rooms, and medical services vary widely from one institution to another (Secretaria de Salud de México, 2013). Moreover, economic barriers keep off improvements in quality and coverage in public healthcare institutions; but also, for a large portion of the population, it prevents the opportunity to receive healthcare from the private sector. For these type of healthcare systems, it is desirable to establish a unified system, with the aim that healthcare must be completely guaranteed and equitably distributed through a more efficient use of resources (Chertorivski and Fajardo, 2012).

An important motivation for this research is the fact that Mexico will require, in the short-term, a coordinated planning of resources that includes all public healthcare institutions and the integration of the private sector to ensure healthcare coverage. In the Mexican National Development Plan 2013-2018, for instance, one of the strategies about ensuring effective access to healthcare services is the promotion of an inter-institutional healthcare service network to improve early detection, diagnosis and treatment of diseases. These strategies try to avoid inequality in access due to social or employment status. In fact, this type of strategies have been already implemented in some states, such as Sinaloa, Baja California Sur, and Tabasco, in which the three main public healthcare institutions have established collaboration to provide service coverage when one of the institutions does not have enough capacity to provide a service by itself.

Diagnostic medical services, such as radiology and imaging, are commonly required by many areas in hospitals as part of preventive, curative, and rehabilitative treatments. Specialized equipment such as magnetic resonance imaging (MRI), computed tomography (CT), positron emission tomographycomputed tomography (PET/CT), or digital mammography are very scarce in the public sector, where in some cases only highly specialized hospitals have availability of these types of technology. Access to these services is very complex and restricted, causing that a large number of patients do not receive a timely and accurate diagnosis. Moreover, the increasing rate of cardiovascular diseases, cancer, diabetes, and chronic lung diseases in Mexico and worldwide requires a better decision making in the usage of resources. Because of this, an efficient infrastructure planning of specialized healthcare service considering an inter-institutional coordination is required in order to improve the ability to respond to future challenges. In this context, the Operation Research (OR) field provides effective analytical methods to help make better decisions to these types of problems.

Although optimization problems in healthcare have received considerable attention in the last few years, many issues are becoming much more important and relevant now because of the growth in ageing population and decreasing birth rates in nearly all of developing countries, and increasing longevity globally (Rais and Viana, 2011). OR is not only used for equipment/hospital location problems, but also, it is being employed much more in day-to-day problems of hospital management, resource-constrained operations, and treatment planning activities (Royston, 2009). New problem formulations take into account some issues, such as service planning, resource and staff scheduling, logistics in emergency services, medical therapeutics, disease diagnostics, and treatment or preventive care (Brandeau et al., 2004). A challenge in this investigation is to introduce a novel model that could be used for a high variety of specialized healthcare services or even for other type of problems that share similar characteristics.

The problem addressed in this paper is aimed to ensure the coverage of a specialized healthcare service for all the demand of public healthcare institutions in a particular system, minimizing the total investment and operating cost in an annual planning horizon. The decision is to determine in which hospitals to install or to increase capacity, and it is directly associated with the number and type of diagnostic equipment. Patients can be transferred to hospitals of other institutions, or to a private provider of an specific network of suppliers when internal capacity of his/her institution is not enough to provide the service.

In this paper, a deterministic optimization model that takes into account different levels of patient acuity, types of equipment to provide the service, and the allocation of demand in periods is proposed. An empirical evaluation of the model is performed with some test instances. The branch and bound algorithm (B&B) is used to solve the problems. It is determined if the size of the instance (number of facilities), level of patient acuity, and type of equipment produce a significant change in the quality of solutions. A sensitivity analysis is performed to evaluate the behavior of solutions and the performance of the B&B when the values of different parameters change.

The paper is organized as follows. Relevant literature review is highlighted in Section 2. The description of the problem and its mixed-integer programming model are presented in Section 3. The full experiments are presented in Section 4. Conlusions and directions for future work are discussed in Section 5.

2 Literature Review

Location, allocation and capacity planning issues have been studied for decades for a large amount of application areas. The literature of location models is extensive, in Owen and Daskin (1998), an overview of facility location literature for complex time and uncertainty characteristics of real-world problem instances is presented. In ReVelle and Eiselt (2005) a synthesis and survey of important problems in facility location is presented. In ReVelle et al. (2008), a bibliography of recent papers on discrete location theory and modeling is provided for the median and plant locations models, and center and covering models. In Klose and Drexl (2005), facility location models from the perspective of the distribution system design are presented. Another review is presented in Melo et al. (2009) but in the context of supply chain management, and a more recent literature review is presented in Arabani and Farahani (2012), but considering the involved dynamics of facility location problems. A review of recent trends specifically in the context of public facility location modeling is given by Marianov and Serra (2002).

The state of the art in healthcare facility location is vast. Papageorgiou (1978) presented one of the earliest surveys of OR applied to problems in healthcare. A decade later, a literature review was presented by Smith-Daniels et al. (1988) in which a classification and analysis of the literature in capacity management in healthcare, facility acquisition, facility allocation, work-force acquisition and allocation was made. In Daskin and Dean (2004) some models that can be considered the heart of the models used in location planning in healthcare are presented. In Rahman and Smith (2000), some location-allocation models for health service development planning in the developing nations are presented. A recent literature review for research work in OR applied exclusively to problems in healthcare is presented by Rais and Viana (2011).

One of the first models applied to location of hospital services was due to Ruth (1981), who proposed a quantitative model to aid in planning hospital inpatient service among a hospital network in a region. In a later work, McLafferty and Broe (1990) evaluated critical care services considering two attributes: the geographical accessibility of services and the number of patients served by each facility. In Stummer et al. (2004), a multi-objective combinatorial problem to determine the location of medical departments within a hospital network was proposed. The planning of hospital capacity taking into account multiple types of patients was proposed by Ayvaz and Huh (2010).

In Mahar et al. (2011), a nonlinear optimization model was proposed in order to prove how hospital networks with multiple locations can leverage pooling benefits when deciding where to locate specialized services (e.g., MRI, CT scans, transplant, neonatal intensive care, etc.). The model takes into account not only financial considerations but also patient service levels and determines how many and which hospitals in a network should be set up with specialized capacity, the levels of capacity to provide and a guidelines for which locations should serve the demand in the network.

In Coté et al. (2007), a location-allocation model for specialized traumatic brain injury treatment services for the Department of Veterans Affairs (VA) was given. The model aimed at determining the best location for treatment units between the VA medical centers and the allocation of admissions to these units, while minimizing the sum of admission treatment cost, admission travel cost, and the penalty cost associated with foregone treatment revenue and excess capacity utilization. Later on, in Syam and Coté (2010) the same problem from a perspective of non-profit service organization was modeled. In Syam and Coté (2012), an extension of the model that minimizes the total cost borne by the health system and its patients incorporating admission acuity levels, service proportion requirements, and admissions retention rates was proposed. Important findings suggest that a decentralized system is costlier than a centralized one but also serves a higher proportion of admissions. More recently, Mestre et al. (2015) dealt with uncertainty in location-allocation models in the strategic planning of hospital networks. The presence of uncertainty is associated with the demand and supply of hospital services considering several features relevant to many real-world applications.

The characteristics and considerations of each model depends on the type of problem to be addressed and the real system in which the model is based on. In our case, not all considerations of previous problems are required since some of them are not applicable to our problem. For example, the evaluation of lost demand and penalty cost are not considered because the objective of the public healthcare institutions is to guarantee the full coverage of the services. The sharing of capacity among institutions and the private providers are used to maximize the responsiveness and to reduce costs. The evaluation of patient's length stay was considered in many works, but in our case, the provision of diagnostic services does not require the use of equipment for long periods, regardless of the type of customer inpatient or outpatient. Novel features are considered in our model that were not previously considered in similar works: the evaluation of different diagnostic equipment for the same service, the coordination among different hospital networks (institutions) to share the service, the incorporation of private entities to improve the capacity, and the transferring of patient among hospitals to provide the service.

3 Problem Description and Model Formulation

In this section, an mixed-integer linear programming (MILP) model for location-allocation of a specialized diagnostic service across a segmented hospital network in an annual planning horizon is introduced. The aim is to minimize the total annual cost required to guarantee the service coverage for a given demand. The hospital network is integrated by public institutions that can share capacity among them or to request the service to a specific network of providers. The first decision is to determine the location and capacity of the service in each institution and the second decision is to allocate all demand. The demand must be allocated at a first stage within the hospitals of each institution, and in a second stage the uncovered demand allocated in hospitals with capacity of each institution could be reallocated to hospitals of other institutions with idle capacity or outsourced to private providers. In Figure 1, an example of the allocation strategies among hospitals and institutions is presented in order to illustrate the problem. The demand is classified in levels of patient acuity, this is a measure of the intensity of care required for a patient accomplished by a nurse, and it is related to the degree of severity of an illness. The model considers the variation of demand in different periods within the annual planning horizon, different levels of patient acuity are served with the same equipment and different types of equipment that only change capacity and cost. The evaluation of the service in periods is required, when demand variation is significant among periods or presents trends or seasonality. The incorporation of types of equipment is designed for services that present different alternatives of equipment among brands while servicing the same type of demand.

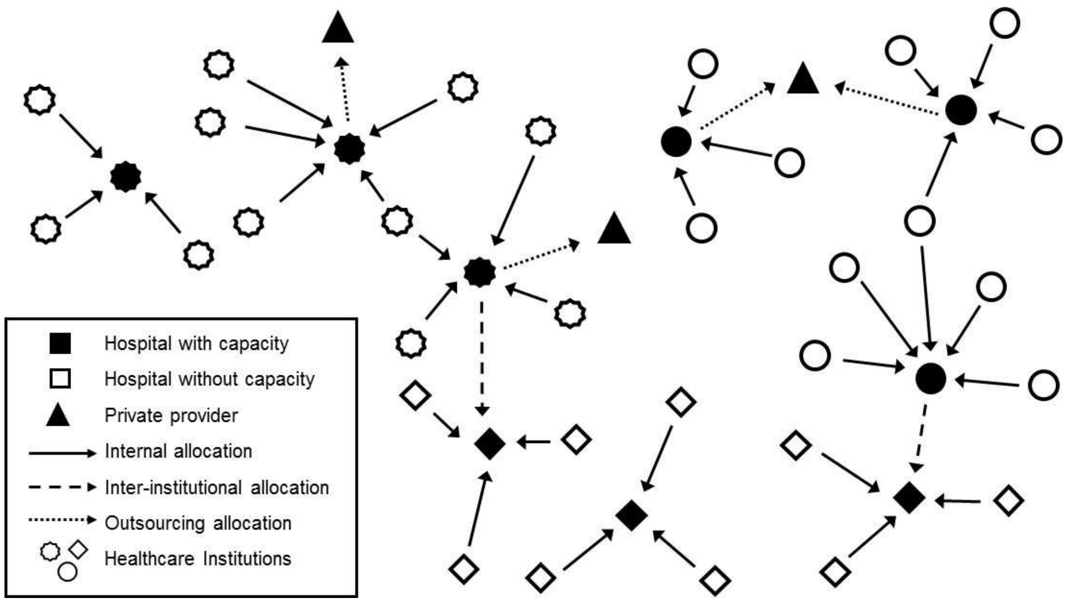

Figure 1: Example of the allocation problem.

The objective function evaluates investment and operational costs required to satisfy all demand of the service in an annual planning horizon. To evaluate both costs in the same scale the equivalent annual cost over the equipment lifespan is used. The investment cost considers a fixed initial cost that does not depend on the type and amount of the equipment in each hospital; and the variable initial cost is related to the type and amount of this equipment. This cost must include all infrastructure, fixed-term staff, supplies and periodic maintenance required to provide the service. To evaluate operational costs, the unit cost per service is considered. This cost could be integrated by additional staff, supplies, materials, and other expenses required to provide the service, and depends on the level of patient acuity or specialization of the service according to the case. In the evaluation of operational costs, the cost of transferring patients from a hospital to another in order to receive service is also considered. An additional administrative cost is incurred when an institution provides the service to patients from another institution. Each private service supplier has its own fees and capacities designated to provide the service to public institutions. To establish a minimum level of coverage, the problem considers a minimum percentage of capacity that each institution has to ensure. To prevent saturation in hospitals, a policy to limit the number of uncovered patients allocated at each hospital according to its capacity is incorporated. This avoids to concentrate in few hospitals all internal unmet demand to be reallocated to other institutions or private services. Finally, in order to control the percentage of demand reallocated to outsourced services, each institution determines the maximum annual percentage of outsourced demand. This policy helps institutions to reach their goal of annual internal service coverage. Although the capacity is considered the same in each period, if it is required to specify the capacity at each period, the parameter could be considered with an additional index for each period.

The sets, parameters, and variables of the model are the following:

Sets

$k \in K$	Set of institutions in the network.
$u \in U$	Set of levels of patient acuity.
$l \in L$	Set of types of equipment.
$n \in N$	Set of time periods.
$i, j \in I$	Set of hospitals in the network.
$G \subset I$	Set of public hospitals in the network.
$G^k \subset G$	Set of public hospitals of institution k.
$G_i \subset G$	Set of public hospitals of institution to which hospital i belongs to.
$P \subset I$	Set of private service providers.
$k_j \in K$	Institution to which hospital j belongs to.

Parameters

FC_j	Fixed annual service setup cost in hospital j; $j \in G$.
VC_l	Variable annual setup cost for equipment of type l; $l \in L$.
OC^u	Operational cost for providing a service with patient acuity level u; $u \in U$.
TC^u_{ij}	Transfer cost for sending a patient that requires a service for a patient acuity level u from hospital i to hospital j; $i \in G$, $j \in I$, $u \in U$.
AC^u_k	Additional charge that institution k requests to provide a service for a patient acuity level u of other institutions; $u \in U$.
PC^u_j	Cost of provider j for a service for a patient acuity level u; $j \in P$, $u \in U$.
D^u_{in}	Demand (number of patients) with acuity level u in hospital i in period n; $i \in G$, $n \in N$, $u \in U$.
EC_l	Maximum capacity of number of services of equipment of type l in each period; $l \in L$.
CP_{jn}	Maximum capacity of number of services of provider j in period n; $j \in P$, $n \in N$.
H_{jl}	Minimum number of required equipment of type l in hospital j; $j \in G$, $l \in L$.
δ_k	Minimum percentage of annual demand to be internally covered by institution k; $k \in K$.
σ_k	Maximum demand in proportion to the capacity that each hospital of institution k can allocate; $k \in K$.

ω_k	Maximum percentage of annual demand that institution k is allowed to allocate to outsourcing; $k \in K$.
M	A very large positive value.

Variables

x_{ijn}^u	Number of patients with acuity level u from hospital i allocated to hospital j in period n; $i \in G,\ j \in I,\ u \in U,\ n \in N$.
α_{jn}^u	Number of patients with acuity level u allocated in hospital j in period n unserved by any hospital of institution to which the hospital j belongs; $j \in G,\ u \in U,\ n \in N,\ l \in L$.
β_{jn}	Capacity available in hospital j in period n unused by any hospital of institution to which hospital j belongs; $j \in G,\ n \in N$.
s_{jn}^u	Service level for patient acuity level u in hospital j in period n; $j \in G,\ u \in U,\ n \in N,\ l \in L$.
t_{jl}	Number of equipment units of type l that are allocated to hospital j; $j \in G,\ l \in L$.
y_j	Binary variable equal to 1 if any service is set up in hospital j, and 0 otherwise; $j \in G$.

The corresponding MILP model is given by:

$$
\begin{aligned}
\text{Minimize} \quad & \sum_{j \in G} FC_j \cdot y_j + \sum_{j \in G} \sum_{l \in L} VC_l \cdot t_{jl} + \sum_{j \in G} \sum_{u \in U} \sum_{n \in N} OC^u \cdot s_{jn}^u \\
& + \sum_{k \in K} \sum_{u \in U} AC_k^u \cdot \sum_{n \in N} \sum_{i \in G \setminus G^k} \sum_{j \in G^k} x_{ijn}^u + \sum_{i \in G} \sum_{j \in P} \sum_{u \in U} \sum_{n \in N} PC_j^u \cdot x_{ijn}^u \\
& + \sum_{i \in I^*} \sum_{j \in I} \sum_{u \in U} \sum_{n \in N} TC_{ij}^u \cdot x_{ijn}^u
\end{aligned} \tag{1}
$$

$$\text{subject to:} \quad \sum_{j \in G_i} x_{ijn}^u = D_{in}^u \qquad i \in G,\ u \in U,\ n \in N \tag{2}$$

$$\sum_{i \in G_j} \sum_{u \in U} x_{ijn}^u - \sum_{u \in U} \alpha_{jn}^u + \beta_{jn} = \sum_{l \in L} EC_l \cdot t_{jl} \qquad j \in G,\ n \in N \tag{3}$$

$$\sum_{i \in G} \sum_{u \in U} x_{ijn}^u \leq M \cdot y_j \qquad j \in G,\ n \in N \tag{4}$$

$$\alpha_{jn}^u \leq \sum_{i \in G_j} x_{ijn}^u \qquad j \in G,\ u \in U,\ n \in N \tag{5}$$

$$\beta_{jn} \leq \sum_{l \in L} EC_l \cdot t_{jl} \qquad j \in G,\ n \in N \tag{6}$$

$$\sum_{j\in G^k}\sum_{l\in L}|N|\cdot EC_l\cdot t_{jl}\geq \delta_k\cdot\sum_{i\in G^{k*}}\sum_{u\in U}\sum_{n\in N}D^u_{in} \qquad k\in K \tag{7}$$

$$\sum_{i\in G}\sum_{u\in U}x^u_{ijn}\leq \sigma_{k_j}\cdot\sum_{l\in L}EC_l\cdot t_{jl} \qquad j\in G,\ n\in N \tag{8}$$

$$\sum_{i\in G^k}\sum_{j\in P}\sum_{u\in U}\sum_{n\in N}x^u_{ijn}\leq \omega_k\cdot\sum_{i\in G^{k*}}\sum_{u\in U}\sum_{n\in N}D^u_{in} \qquad k\in K \tag{9}$$

$$\sum_{j\in I\setminus G_i}x^u_{ijn}=\alpha^u_{in} \qquad i\in G,\ u\in U,\ n\in N \tag{10}$$

$$\sum_{i\in G\setminus G_j}\sum_{u\in U}x^u_{ijn}\leq \beta_{jn} \qquad j\in G,\ n\in N \tag{11}$$

$$\sum_{i\in G}x^u_{ijn}-\alpha^u_{jn}=s^u_{jn} \qquad j\in G,\ u\in U,\ n\in N \tag{12}$$

$$\sum_{i\in G}\sum_{u\in U}x^u_{ijn}\leq CP_{jn} \qquad j\in P,\ n\in N \tag{13}$$

$$t_{jl}\leq M\cdot y_j \qquad j\in G,\ l\in L \tag{14}$$

$$y_j\leq\sum_{l\in L}t_{jl} \qquad j\in G \tag{15}$$

$$t_{jl}\geq H_{jl} \qquad j\in G,\ l\in L \tag{16}$$

$$x^u_{ijn}\in\mathbb{N}\cup\{0\} \qquad i\in G,\ j\in I,\ u\in U,\ n\in N \tag{17}$$

$$\alpha^u_{jn},\ s^u_{jn}\in\mathbb{N}\cup\{0\} \qquad j\in G,\ u\in U,\ n\in N,\ l\in L \tag{18}$$

$$\beta_{jn}\in\mathbb{N}\cup\{0\} \qquad j\in G,\ n\in N \tag{19}$$

$$t_{jl}\in\mathbb{N}\cup\{0\} \qquad j\in G,\ l\in L \tag{20}$$

$$y_j\in\{0,1\} \qquad j\in G \tag{21}$$

The objective function (1) minimizes the total equivalent annual cost to provide the service for all the demand of the public hospitals in the network. The first and second terms represent fixed and variable annual investment costs, respectively. The third term represents the total operational costs of all provided services, the fourth term represents the inter-institutional fee for all services sent to other institutions. The fifth term corresponds to the total outsourcing costs and the last term represents the total transport cost for all types of patient acuity levels.

Constraints (2) ensure that all demand of each hospital is allocated within its own institution in each period. Constraints (3) determine the demand from the same institution allocated to a hospital according to its capacity in each period. If there is idle capacity, the variable β_{jn} will have a positive value, and if there is uncovered demand, some variables α^u_{jn} will take positive values. Constraints (4) prevent allocating demand to a hospital if there is not enough capacity. Constraints (5) ensure that the variables for uncovered demand of a hospital only take values equal or lower than total internal demand allocated to that hospital in the same period. Constraints (6) ensure that for each period an idle capacity variable takes values lower or equal than the capacity of the hospital which it belongs to. Constraints (7) ensure a minimum percentage of annual capacity according to the total annual demand of each institution defined by $0 \leq \delta_k \leq 1$. Constraints (8) establish an upper bound for demand that can be allocated in a hospital in each period. This limit must not exceed a percentage of its capacity defined by $\sigma_k \geq 1$ by each institution.

Constraints (9) set the maximum percentage of total annual demand of an institution to be reallocated to the private providers and it is defined by $0 \leq \omega_k \leq 1$ by each institution. Constraints (10) ensure that all internal uncovered demand of a hospital inside its institution will be reallocated to another hospital of different institution with idle capacity or to private providers in each period. Constraints (11) allow to allocate uncovered demand of other institutions to a hospital without exceeding its idle capacity in each period. Constraints (12) is used to determine the service level of each hospital for each patient acuity level in each period. Constraints (13) limit the demand reallocated to each private provider according to its capacity in each period. Constraints (14)-(15) relate the integer variables y_j and t_{jl}, respectively. Constraints (16) enforce to set up the service with pre-determinated number of equipment units of each type in a hospital that already have the service or it is mandatory to set up. Finally, the nature of the decision variables are given by (17)-(20).

4 Empirical Assessment and Results

4.1 Design of experiments and instances

Given the inherent complexity of the problem and the exponential solution times of B&B, the purpose of the first experiment is to determine the instance size and conditions for which B&B finds optimal solutions or find out how far the solutions obtained are from optimality. The quality of solutions, computational efforts, the effects of factors such as number of hospitals, patient acuity levels, specialization levels and types of equipment in solutions are presented. A sensitivity analysis was also performed in order to provide an insight of the behavior of solutions when some parameters are subject to change.

The proposed scheme of joint resource planning is not yet implemented in Mexico due to the lack of coordination among institutions and the lack of regulations. This is precisely one of the motivations of the present work, that is, to present a model that can aid the decision makers make a better resource planning based on scientific grounds; however, due to this, there is no sufficient data instances publicly available for assessing the proposed model. To cope with this, random instances were generated based on real-world data or reasonable distributions. Some decisions such as instances size, the number of institutions, and private provider were fixed. Some parameters related to the costs and capacities were randomly generated between reasonable current real-world values.

The instance size or network size (NS) was defined as the total number of hospitals and private providers in the network. For this evaluation, instances of 30, 60, 90, 120, 180, 240, and 300 facilities were generated; 30 samples for each instance size were created. The number of institutions was set to 5 for all instances, the same number of hospitals were set for each institution and in the same proportion for private providers. For example, for instances of 120 hospitals, there are 20 private providers and for each institution there are 20 hospitals. The number of patient acuity levels, specialization levels and types of equipment in each instance size were proportionally selected from values of 1, 2 or 3. The equipment capacity, defined as the maximum number of services by period, was randomly selected from some possible values.

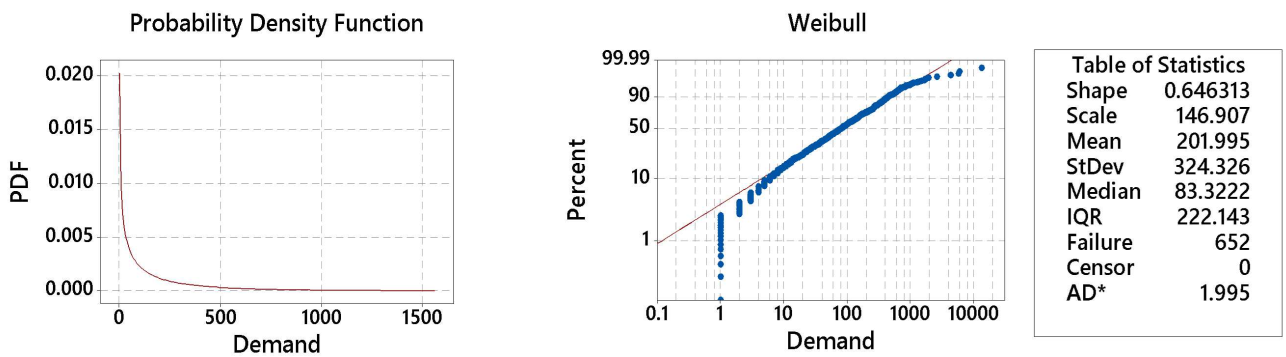

Figure 2: Demand distribution of MRI service in hospitals of the SSA in Mexico 2012.

The demand of each hospital in each period was randomly generated according to a Weibull probability distribution obtained from data of Mexico National Health Information System (SINAIS) for MRI services in public hospitals in 2012, in Figure 2 the annual demand distribution of 652 health units of the Ministry of Health of Mexico (SSA) is illustrated. A Weibull distribution is used to simulate demand behavior, this is a versatile distribution that can be used to model a wide range of applications in engineering, medical research, quality control, finance, and climatology.

The experiments were performed in an Intel Core i7-2620M processor at 2.70 GHz with 16 GB of RAM. Each instance was solved by the B&B method setting a CPU time limit at one hour. In this case, CPLEX 12.6 was used.

4.2 Model evaluation with B&B

In Table 1 the relatives optimality gaps, CPU running times and number of optimal solutions (out of 30 instances tested in each row) for each network size are presented. The relative gap represents the percentage of the relative difference between the best integer solution found and the best lower bound found by the B&B. The mean, standard deviation, the minimum, and maximum values of the relative gaps are shown for each network size. For the CPU time, a value of 3600 seconds was used when an optimal solution was not found after 1 hour of computing.

NS	Relative gap (%)				CPU time (s)	No. of optimal
	Mean	St. Dev.	Min	Max	Mean	solutions (out of 30)
30	0.25	0.87	0.00	4.34	609	26
60	0.37	0.52	0.00	2.31	2,505	11
90	0.60	0.77	0.00	2.88	3,323	3
120	5.32	4.90	0.00	19.48	3,522	1
180	21.20	17.64	1.85	65.17	3,600	0
240	42.34	19.01	5.65	72.69	3,600	0
300	56.51	13.07	33.54	81.71	3,600	0

Table 1: Model assessment when applying B&B to instances of different size.

As can be seen, the results indicate that the model was difficult to solve for large instances, while near optimal results for instances of 30, 60, and 90 facilities were found. In terms of finding true optimal solutions (with a relative gap tolerance of 1×10^{-4}), it was observed that only a few were found within 1 hr of CPU time. Optimal solutions were found for almost all instances of 30 facilities. B&B observed a success rate of 87.7%, 36.7%, 10.0%, and 3.3% for instances of 30, 60, 90, and 120 facilities, respectively. For instances of 180, 240, and 300 facilities no optimal solutions were found. Overall, only 69 of 210 instances were optimally solved. In conclusion, the time limit of one hour was not enough to find good quality solutions for large instances. The high relative gaps for large instances suggest that even with an additional CPU time limit, good result are not be expected. Nonetheless, the instances of size 30, 60, and 90 are still large enough to draw some meaningful results as it will be shown in the following subsection.

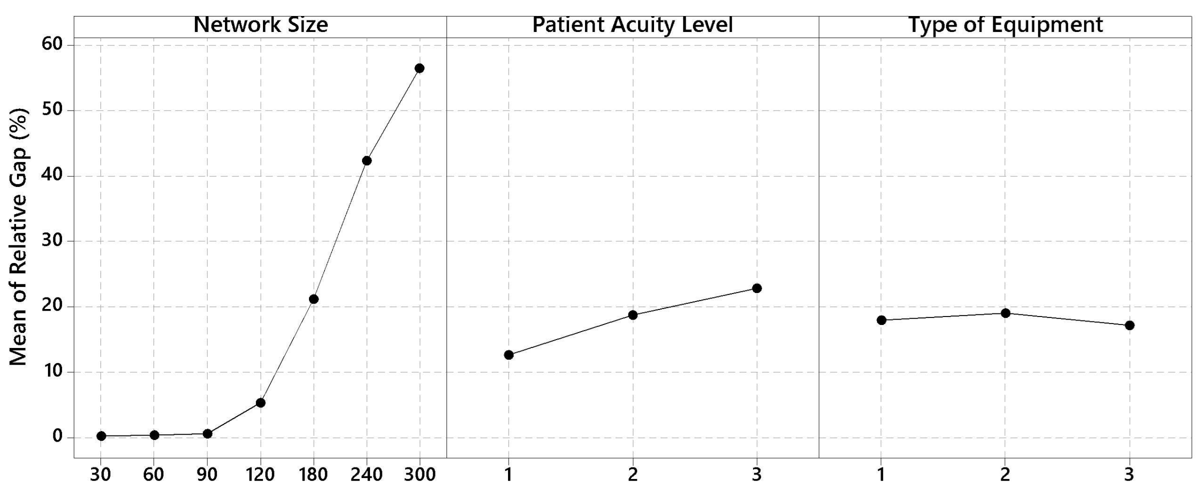

Figure 3: Main effects plot for each factor.

The effect that instances size, patient acuity levels, and types of equipment have in solution quality (measured by the relative optimality gap) is shown in Figure 3, the average value for each instance size is presented. To verify the results, all factor were evaluated with Mood's median test in order to identify significant statistical differences, these results are shown in Table 2. It can be easily seen that the factor with the greatest impact was the instances size. There was no significant difference in relative gaps between instances of 30, 60, 90, and 120 facilities. And there was a significant difference in relative gaps from these instances and instances of 180, 240, and 300 facilities. The patient acuity levels only produced a statistically difference in the relative gaps between instances of levels 1 and 3. The types of equipment did not produce any statistical difference in the relative gap between 1, 2 or 3 types of equipment.

Group	Network size							Patient acuity level			Type of equipment		
	30	60	90	120	180	240	300	1	2	3	1	2	3
I	1	1	1	1				1	1		1	1	1
II					1				1	1			
III						1							
IV							1						

Table 2: Mood's median test result for each factor of the model.

4.3 Sensitivity analysis

There are some important parameters that may affect the objective function values, namely, the way in which demand is allocated, the number and type of equipment in the system, and the complexity to solve the problem. Some of them must be decided by decision makers while others depend on the type of medical diagnostic service. The purpose of this section is to present how the variation of some parameters affect some characteristic in the solutions. In the present subsection some sensitive analysis for the model using instances size of 30, 60, 90, and 120 facilities are presented in detail (30 of each size, 120 in total).

Equipment capacity

Equipment capacity depends on the type of service and medical technology advances. The maximum number of services by period must be determined considering the required time to carry out each procedure, the preparation time before and after providing the service, the availability of staff, and all the inputs required. In the following experiment we considered different values of the equipment capacity for each instance. The experiment was carried out by considering just one type of equipment in all instances varying the equipment capacity to 100, 250, 500, 750, and 1000 services per period. The results are shown in Table 3. The second column corresponds to the average number of equipment units required in the system. As expected, it was observed that with equipment of high capacity a fewer number of units are required. In the third column, the averages of the number of hospitals by each capacity level are presented. Results indicate that with a high number of equipment units more hospitals are required to set up the service. The total cost displayed in the fourth column is expressed as a percentage of variation with respect to the capacity of 100 services per period. We can see that when there are fewer hospitals that provide the service the total cost is reduced. A decentralized system is more expensive than a centralized one. The numbers of optimal solutions are presented in the fifth column (out of 120), equipment of high capacity reach more optimal solutions in 1 hour of computing. This is because fewer number of equipment units in the system are required and therefore the integer variables take a reduced number of possible values reducing the number of branches in the algorithm. Finally, the capacity utilization rates (CU Rate) are shown for each network size in the last four columns. This rate is the percentage of the total capacity in public hospitals that is used. In the results, the equipment of low capacity was found to have a higher utilization rate regardless the size of the network, but equipment with high capacity is highly employed when the size of the network is greater.

Equipment	Number of	Number of	Total cost	Optimal solutions	CU Rate by NS (%)			
Capacity	equipment units	hospitals	variation (%)	(out of 120)	30	60	90	120
100	65	15.3	-	16	96.7	99.0	99.3	99.1
250	27	12.3	-43.6	32	87.5	97.5	98.4	98.4
500	15	9.6	-57.6	68	65.6	88.3	93.2	93.3
750	12	8.6	-61.1	85	48.8	75.5	80.9	85.0
1000	12	8.8	-61.4	93	37.7	61.3	67.7	71.3

Table 3: Results for different capacity values.

Reallocation of uncovered services in each institution

The inter-institutional and outsourcing allocation are strategies to improve the service efficiency, reducing the total number of equipment required in the system and the total cost of the service. In some periods, some institution could present high levels of demand and the internal capacity may not be enough. Constraints (8) are used in the model to limit the percentage of patients that will be reallocated in each hospital according to its capacity. This proportion is defined by the parameter σ_k for each institution, which takes values starting from 1 (not additional demand beyond the capacity is allowed in each hospital and no reallocation strategies are available) to infinite (the allocation of additional demand is not constrained and reallocation strategies are permitted).

An experiment was conducted in order to identify how the variation in this parameter impacts the solution. For these experiments, the value of σ_k was considered the same for all institutions fixing its value at 1, 1.25, 1.5, 2, and infinite (that is, eliminating the constraint). For example, a value of 1.25 represents that an additional 25% of services in relation to the capacity of a hospital are allowed to be reallocated to other institutions or to outsourcing. The results are presented in Table 4. All columns except CU Rate are compared with respect to σ_k equal to 1. When no reallocation strategies are permitted ($\sigma_k = 1$), the CU Rate has an average of 86.7% and when they are permitted the CU Rate increases between 7.9% up to 9.3% in average. A reduction up to 10.8% in public capacity and 5.5% in the total cost of the system can be reached considering inter-institutional allocation and outsourcing for these test instances. The optimal percentage of reallocation of services was 7.4%, 4.3% for inter-institutional allocation and 3.1 % for outsourcing. This occurs when there was no constraint to control the percentage of reallocation (σ_k = infinite). Since all the variations of σ_k in this analysis were higher than 1.074 (given the optimal percentage of reallocations was 7.4%), the reduction in capacity and cost were significant. In conclusion, there is less efficiency in the system and higher costs when the values of σ_k are closer or lower than the optimal percentage of reallocation of services (this values are determined when the constraints (8) are not used).

σ_k	CU rate (%)	Total capacity (%)	Total cost variation (%)	Demand allocation (%)		
				Internal	I-I	Outsourcing
1.00	86.7	-	-	100.0	0.0	0.0
1.25	94.6	-9.6	-5.0	95.5	2.2	2.4
1.50	95.7	-10.9	-5.3	94.0	3.2	2.8
2.00	96.2	-11.6	-5.4	93.2	3.8	3.0
Inf	96.0	-10.8	-5.5	92.6	4.3	3.1

Table 4: Experimental results for variation of parameter σ_k.

Allocation of private services

The use of outsourcing to assure coverage of some services is very frequent in public healthcare institutions. In this model we make the assumption that a network of private service providers with different cost and capacities was available. The allocation of this capacity was limited by constraint (9) where parameter ω_k controls the maximum proportion of internal demand that each institution can send to outsourcing. For this analysis, different values of this parameter were evaluated: 0 (no outsourcing allowed), 0.05, 0.25, 0.75, and 1 (infinite outsourcing allowed limited only by the capacity of providers). This values were tested in all instances considering the same values for all institutions. Results are displayed in In Table 5. The cells show the values with respect to the case $\omega_k = 0$. It can be observed that a reduction in total cost of 2.45% in average was reached when a 5% of demand was outsourced and for the rest of values of ω_k the increment was less significant. This occurs because, as it can be seen in demand allocation, the optimal value of outsourced demand (ω_k = 1) is 3.25% and therefore higher values of ω_k will not generate significant changes in total cost. We conclude with this analysis that values of ω_k higher than the optimal percentage of outsourced demand (this value is determined when constraints (9) are not considered) could reduce the total cost when the outsourcing is used.

ω_k	Total cost variation (%)	Demand allocation (%)	
		Own capacity	Outsourcing
0.00	-	100.00	0.00
0.05	-2.45	97.51	2.49
0.25	-2.55	96.78	3.22
0.75	-2.56	96.77	3.23
1.00	-2.56	96.75	3.25

Table 5: Experimental results for variation of parameter ω_k.

Minimum internal capacity

In this section, the effects in the solutions for different values of parameter δ_k are presented. This parameter is used in constraints (7) to establish a minimum level of capacity that each institution requires to cover its own demand. The result of this analysis are presented in Table 6. The values of parameter δ_k were fixed at 0 (unrestricted minimum capacity), 0.25, 0.5, 0.75, and 1.0 (required enough capacity in the institution to satisfy all demand); the same value for all institutions in all test instances is considered. In the second column of the table, it is observed that more instances were optimally solved when the values of δ_k tended to 1. This was because the inter-institutional and outsourcing allocations were less required and the number of variables and branches used in the B&B were reduced. The results for CU rate and total capacity in the third and fourth column of the table were compared with respect to values of δ_k equal to 0. The percentage of services allocated for internal capacity (own capacity of each institution), inter-institutional allocation (I-I) and outsourcing allocation are presented in the last three columns. When the capacity of each institution was forced to cover all internal demand (δ_k=1), the total capacity increased up to 8.6% and the total cost up to 4.2% for this test instances. The optimal percentage of services allocated internally was 93.1% when δ_k was equal to zero. Therefore values of δ_k lower than 0.931 presented less variation in the results. Even though, there was enough capacity to cover all demand in each institution when δ_k was equal to 1, the inter-institutional allocation and outsourcing could still be more convenient in some cases. In conclusion, when δ_k values are closer or greater than the optimal percentage of services covered internally, the capacity and total cost could be increased significantly.

δ_k	Optimal solutions (out of 120)	CU rate (%)	Total capacity (%)	Total cost variation (%)	Services allocation (%)		
					Internal	I-I	Outsourcing
0.00	35	96.8	-	-	93.1	3.6	3.3
0.25	35	96.8	0.0	0.0	93.1	3.6	3.3
0.50	37	96.7	0.1	0.0	93.2	3.5	3.2
0.75	44	96.1	1.0	0.3	94.3	2.9	2.8
1.00	94	91.0	8.6	4.2	99.0	0.4	0.6

Table 6: Experimental results for variation of parameter δ_k.

5 Conclusions and Future Work

In this research, we contribute to healthcare location literature by addressing a capacity planning problem for medical diagnostic services in a public healthcare system. We introduced a new scheme of joint resource planning in public healthcare institutions together with a network of private providers. The aim is to satisfy all demand without limited budget, in this sense, the problem trying to minimize the annual required investment and operative cost to provide the service. There are three alternatives to allocate demand: (1) internal allocation (inside a hospital of an institution), (2) inter-institutional allocation and (3) allocation to the private providers. A MILP model incorporating patient acuity levels, different types of equipment and the evaluation of demand in periods was introduced.

The test instances created for empirical evaluation offered good insights about the scope of the B&B to solve the problems. Although, instances up to 90 facilities were solved with good relative gaps, there was no significant difference in relative gaps for instances up to 120 facilities. The B&B was not efficient for instances of 180, 240 and 300 facilities, which presented high relative gaps. Overall, instances of 30, 60, and 90 facilities presented in average a relative gap of 0.41%, instances of 120 facilities a relative gap of 5.32%, and instances of 180, 240, 300, and 300 facilities a relative gap of 46.48%.

A sensitivity analysis was performed to understand the behavior of some important parameters. We found out that the capacity of the equipment could affect the complexity of the problem. The higher the number of equipment units are required in the system, the more complex to solve the problem is. A similar effect is presented with the policy of minimum internal capacity in each institution, when the policy is more flexible the problem is more difficult to solve. The values of σ_k, ω_k, and δ_k could affect the total cost and the efficiency of the service. The values of σ_k and ω_k must be high to avoid increasing the costs, and the values of δ_k must be low. For this reason, it is required to identify the corresponding values of each parameter when the related constraints of each one are not considered in the model, values outside of these boundaries will produce significant changes in the solutions.

The presented problem minimize the total cost of providing the total coverage of a diagnostic service. This assumption, in some cases, could not be very possible to consider because in some countries or institution there is a limitation with the available budget in the planning of resources. For this type of situations, it is required to modify the current formulation of the problem. Instead of considering the minimization of the total cost, this could be considered as a constraint and in the objective function could be considered the maximization of the coverage for instance. As an alternative, the use of multi-objective optimization could be used to minimize operative costs and maximize the coverage. Some additional opportunities are available to extend the research adding new characteristics evaluated in recent researches like the uncertainty in demand or supply of services, the use of hierarchical hospital structure, multiple services evaluation, evaluation of lost demand or patient dissatisfaction. Furthermore, the problem formulated is focused to help decision makers to take strategic decisions about the infrastructure planning, but a second problem, that is required to solve, is the one associated with the operative decisions. In this sense, it is required a methodology to evaluate the best strategies for the programming of services which requires a coordination between departments, staff and institutions.

As it was observed, solutions to large scale instances were far from optimal. The optimality gaps found by B&B were very high. Therefore, this opens up the opportunity for developing specialized heuristics that can find good quality solutions for larger scales instances. The structure of the problem can be exploited to develop efficient heuristics by solving an appropriate easy associated subproblem in an iterative procedure.

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