# Multiobjective Scatter Search for a Commercial Territory Design Problem

M. Angélica Salazar-Aguilar
Graduate Program in Systems Engineering
Universidad Autónoma de Nuevo León, Mexico
*e.mail: angy@yalma.fime.uanl.mx*

Roger Z. Ríos-Mercado
Graduate Program in Systems Engineering
Universidad Autónoma de Nuevo León, Mexico
*e.mail: roger@yalma.fime.uanl.mx*

José Luis González-Velarde
Center for Quality and Manufacturing
Tecnológico de Monterrey, Mexico
*e.mail: gonzalez.velarde@itesm.mx*

Julián Molina
Department of Applied Economics
Universidad de Málaga, Spain
*e.mail: julian.molina@uma.es*

September 2010

**Abstract**

In this paper, a multiobjective scatter search (SS) procedure for a bi-objective territory design problem is proposed. A territory design problem consist of partitioning a set of basic units into larger groups that are suitable with respect to some specific planning criteria. These groups must be compact, connected, and balanced with respect to the number of customers and sales volume. The bi-objective commercial territory design problem belongs to the class of NP-hard problems. Previous work showed that large instances of the problem addressed in this work are practically intractable even for the single-objective version. Therefore, the use of heuristic methods is the best alternative for obtaining approximate efficient solutions for relatively large instances. The proposed SSMTDP (Scatter Search for a Multiobjective Territory Design Problem) method contains a diversification generation method based on a GRASP framework. The improvement method is based on a relinked local search strategy (RLS). The combination method is a two-phase method that takes as input two given $p$-partitions with their corresponding set of territory centers, and solves an assignment problem to find new territory centers. A partial solution is created by keeping on each territory those nodes that belong to the same territory in both solutions. Three new solutions are obtained from this partial solution. Each of these new solutions is generated using a GRASP procedure with an independent merit function. For instance, the first new solution is generated by assigning those unassigned nodes to the partial solution such that a dispersion measure is minimized. The second solution is created by considering a merit function related to the deviation from the target number of customers, and the third one is created by taking into account the infeasibility with respect to the allowed quantity of sales volume. The proposed SSMTDP is evaluated over a variety of instances taken from literature. This includes a comparison with two of the most successful multiobjective heuristics from literature such as SSPMP (a scatter search metaheuristic) and NSGA-II (a genetic algorithm). Experimental work reveals that the proposed SSMTDP consistently outperforms both SSPMP and NSGA-II on all instances tested.

*Keywords:* Territory design; bi-objective programming; Pareto front; Scatter search, GRASP; SSPMO; NSGA-II.

# 1 Introduction

Commercial territory design is a recent districting application. It consists of partitioning a set of basic units (BUs) into larger groups according to some specific planning criteria. In the problem addressed in this work, these groups must be compact, connected, and balanced with respect to the number of customers and sales volume. The single objective version of this problem was introduced by Ríos-Mercado and Fernández [15]. Due to the complexity of the problem, they developed a reactive GRASP procedure to solve it. Their proposed procedure outperformed the company method in both solution quality and degree of infeasibility with respect to the balancing requirements. Different versions of this problem have been studied as well. Segura-Ramiro et al. [19] use another dispersion measure that is very common in facility location. It is the objective function of the $p$-Median Problem ($p$MP). Balancing requirements are considered as constraints. They solved the problem by an implementation of a well-known heuristic technique called location-allocation. The results showed good heuristic performance. Caballero-Hernández et al. [4] developed a GRASP for a commercial territory design problem with joint assignemnt constraints with relatively good results. The bi-objective version of this problem was introduced by [16]. In that work, an improved implementation of the $\varepsilon$-constraint method for solving instances of small to medium size is proposed. To the best of our knowledge there is no previous work that address the bi-objective commercial territory design problem from the heuristic point of view.

In this work, the well-known framework of Scatter Search (SS) is used to develop a heuristic that allows to obtain approximate efficient solutions to the bi-objective commercial territory design problem. Five key components were derived and developed within the SS framework: (i) a diversification generation method based on GRASP, (ii) an improvement method based on a novel relinked search strategy, (iii) a solution combination method based on a hybrid scheme; (iv) a reference set update method, and (v) a subset generation method. As usual in SS, the first three methods were specifically tailored to attempt to exploit the problem structure.

The proposed SSMTDP was evaluated over a set of large instances. The results indicate that the SSMTDP is able to find good solutions that are very well distributed along the efficient frontier. Even though the initial solutions have a poor evaluation in the objective functions, the proposed combination method has the ability of exploring new regions in the search space and the improvement method allows to obtain better solutions that are very far from the initial set. When compared to state-of-the-art multi-objective methods such as SSPMO and NSGA-II, it was observed that these procedures struggled in generating feasible solutions to the problem. A few instances could be solved by these procedures. In contrast, the SSMTDP reported efficient solutions for all instances tested. Furthermore, SSMTDP reported significantly better solutions for those instances that were solved for both NSGA-II and SSPMO.

The paper is organized as follows. Section 3 provides a description of the problem. Section 2 discusses relevant work developed in multiobjective territory design. Section 4 describes the SSMTDP procedure. Experimental work is discussed in Section 5 and finally in Section 6 some conclusions are presented.

## 2 Previous Work on Multiobjective Territory Design

The multiobjective nature of the territory design problems can be found in diverse fields such as sales districting, political districting, and school districting. However, most of the works transform the multiobjective problem into a single objective problem. Few works develop procedures for obtaining approximate efficient solutions.

Tavares et al. [21] study a multiobjective public service districting problem. They considered multiple criteria such as location of the zone with respect to the network, mobility structure within a zone, zone corresponding to administrative structures, centers of attraction in the zone, social nature, and geographical nature. They proposed an evolutionary algorithm with local search and applied it to a real-world case of the Paris region public transportation. They discussed results for bi-objective cases considering different criteria combination.

Guo, Trinidad, and Smith [8] propose a multiobjective zoning and aggregation tool (MOZART). MOZART is an integration of a graph partitioning engine with a Geographic Information System (GIS) through a graphical user interface. They illustrated the performance of MOZART by solving two zoning problems from three government local areas in Victoria: Kingston, Bayside, and Glen Eira. The first part of their experimental work was done by taking into account a single objective of equality in population size. In contrast, in the second part of their experimental work, both equity in population and compactness were treated as objective functions. They report a case with 577 census collection districts and 20 zones. The inclusion of compactness as the second zoning objective yields zones with better shapes.

Bong and Wang [1] present a multiobjective hybrid metaheuristic approach for a GIS-based spatial zoning model. Their heuristic procedure is a combination of tabu search and scatter search. They show the procedure performance by solving a political districting problem with 55 basic units and 3 districts. Equity in population, compactness, and socio-economic homogeneity are treated as objectives.

Ricca and Simeone [14] address a multiple criteria political districting problem. Such criteria were connectivity, population equality, compactness, and conformity to administrative boundaries. They transformed the multiobjective model into a single-objective model, where the objective function is a convex combination of three objective functions (inequality, noncompactness, and nonconformity to administrative boundaries), and connectivity is considered as a constraint. They compared the behavior of four local search metaheuristics (descent, tabu search, simulated annealing, and old bachelor acceptance) over a sample of five Italian regions. The old bachelor acceptance produced the best results in most of the cases.

Bowerman, Hall, and Calamai [2] present a multiobjective approach for solving a school bus routing problem. They proposed a heuristic technique that firsts groups students into clusters using a multiobjective districting algorithm. After that, a school bus route and the bus stops for each cluster are generated by using a combination of a set covering procedure and a traveling salesman problem procedure. They report experimental results for a real-world instance in Wellington County, Ontario. The districting algorithm considers four objectives: minimizing the number of routes, minimizing the length of the routes, load balancing, and compactness of the routes. The last three criteria are placed in a weighted objective function where the number of routes is the dominant objective, i.e., a solution with fewer routes is always favored over a solution with more. Different plans were designed using different sets of weights over the optimization criteria.

Scott, Cromley, and Cromley [18] make a multiobjective analysis of school districting in a case study from Connecticut, USA. They propose a mixed-integer goal programming model where the goal constraints are to minimize disparities in: minority enrollments, grand-list/student ratios, student-teacher ratios, and overall enrollment. The number of districts is not fixed and the contiguity criterion is not formulated in an explicit way. Experimental work using different weighting scenarios reveals that the traditional distance-minimizing or transportation-minimizing objectives are in conflict with all other aims of equity and quality of educational opportunities.

Ricca [13] addresses a territory aggregation problem in Rome. A heuristic procedure based on an old bachelor acceptance is implemented. Compactness, population equality, and inner variance are the optimization criteria. Inner variance is used to guarantee homogeneous zones according to some socio-economic factors such as the population, number of schools, hospitals, and shopping centers. The objective function used in this work is a convex combination of the optimization criteria. Different sets of weights were used to obtain approximate efficient solutions. The heuristic technique reported better designs than the existing one.

To the best of our knowledge the only work on multiobjective commercial territory design is the one by Salazar-Aguilar, Ríos-Mercado, and González-Velarde [16]. In that work, the bi-objective model is introduced and an improved $\varepsilon$-constraint method is proposed for finding optimal Pareto fronts. One of the limitations of that work is of course the size of the instances that could be solved exactly. The largest tractable instance has 150 BUs and 6 territories. Therefore, the motivation of the present work is to develop an effective method for tackling larger instances of this commercial territory design problem (TDP). For a survey on single-objective TDP applications, the reader is referred to the work of Kalcsics, Nickel, and Schröder [9].

# 3 Problem Description

Given a set $V$ of city blocks (basic units, BUs), the firm wishes to partition this set into a fixed number ($p$) of disjoint territories that are suitable according to some planning criteria. The territories need to be balanced with respect to each of two different activity measures (number of customers and sales volume). Additionally, each territory has to be connected, so that each basic unit can be reached from any other without leaving the territory. Territory compactness is required to guarantee that customers within a territory are relatively close to each other. Compactness and balance with respect to the number of customers are the most important criteria identified by the firm. Therefore in this work these criteria are considered as objective functions and the remaining criteria are treated as constraints.

Let $G = (V, E)$, where $E$ is the set of edges that represents adjacency between BUs. An edge connecting nodes $i$ and $j$ exists if $i$ and $j$ are adjacent BUs. Multiple attributes such as geographical coordinates $(c_j^x, c_j^y)$, number of customers and sales volume are associated to each node $j \in V$. In particular, the firm wishes perfect balance among territories, that is, each territory needs to have the same number of customers and sales volume. Let $A = \{1, 2\}$ be the set of node activities, where 1 refers to the number of customers and 2 refers to sales volume. We define the size of territory $B_k$ with respect to activity $a$ as $w^{(a)}(B_k) = \sum_{i \in B_k} w_i^{(a)}$, where $w_i^{(a)}$ is the value associated to activity $a \in A$ in node $i \in V$. Hence, the target value is given by $\mu^{(a)} = \sum_{j \in V} w_j^{(a)}/p$. Due to the discrete nature of this problem, it is practically impossible to have perfectly balanced territories. Thus, a tolerance parameter $\tau^{(2)}$ is introduced to allow a relative deviation from the average sales volume.

Let $\Pi$ be the set of all possible $p$-partitions of $V$. For a particular territory $B_k$, $c(k)$ is a territory center and $d_{ij}$ is the Euclidian distance between nodes $i$ and $j$, $i, j \in B_k$. A territory center is computed as

$$c(k) = \arg\min_{j \in B_k} \sum_{i \in B_k} d_{ij}$$

Under the previous assumptions, the bi-objective combinatorial model can be written as follows.

$$\begin{aligned}
&\min_{B \in \Pi} & f_1(B) &= \sum_{k=1,\ldots p} \sum_{i \in B_k} d_{ic(k)} & (1)\\
&\min_{B \in \Pi} & f_2(B) &= \max_{k=1,\ldots p} \frac{1}{\mu^{(1)}} \left[ \max \left\{ w^{(1)}(B_k) - \mu^{(1)}, w^{(1)}(B_k) - \mu^{(1)} \right\} \right] & (2)\\
&\text{Subject to :} & & & \\
& & w^{(2)}(B_k) &\in \left[ (1 - \tau^{(2)})\mu^{(2)}, (1 + \tau^{(2)})\mu^{(2)} \right], k = 1, \ldots, p & (3)\\
& & G &= (B_k, E(B_k)) \quad \text{is connected} \quad \forall k = 1, \ldots, p & (4)
\end{aligned}$$

The goal is to find a $p$-partition of V, such that both the dispersion (1) on each territory $B_k$ and the maximum relative deviation with respect to the number of customers in each territory (2) are simultaneously minimized. Constraints (3) establish that the territory size (sales volume) should be between the range allowed by the tolerance parameter $\tau^{(2)}$. In addition, each territory should induce a connected subgraph (4).

This is an NP-hard problem and previous work [16] reveals that large instances are intractable by applying the existing exact solution procedures. In this paper we introduce a heuristic procedure for obtaining approximate efficient solutions to large instances.

# 4 The SSMTDP Procedure

The evolutionary approach called Scatter Search (SS) was first introduced in [7] as a metaheuristic for integer programming. It is based on diversifying the search through the solution space. It operates on a set of solutions, named the reference set (PR), formed by good and diverse solutions of the main population (P). These solutions are combined with the aim of generating new solutions with better fitness, while maintaining diversity. Furthermore, an improvement phase using local search is applied. As detailed in [11], the basic structure of SS is formed by five main methods. SS is a very flexible technique, since some modules of its structure can be defined according to the problem at hand. For instance, the diversification, the improvement, and the combination methods are commonly tailored to the specific problem.

The components of the proposed SSMTDP procedure are described next.

- A *diversification generation method* that generates a set of initial solutions. It is based on the proposed GRASP procedures developed by [17]. Specifically, we use the procedure called BGRASP-I. This procedure uses a merit function based on two components: dispersion and maximum deviation with respect to the target value in the number of customers. This method keeps connectivity as a hard constraint. The post-processing phase of BGRASP-I is carried out by the improvement method described below.

- An *improvement method* that transforms a trial solution into one or more trial solutions. This method is an implementation of a relinked local search (RLS) strategy and is applied to each solution obtained by either the diversification generation or the combination method.

  As mentioned in [12], most local search applications to multiobjective optimization use multiple runs to approximate the Pareto front. This technique is usually based on a weighted aggregation of the objective functions where each run consists of solving the single-objective optimization problem that results from applying a given set of weights. To obtain an approximation of the Pareto front the procedure must be run as many times as the desired number of points, using different weight values. The performance of implementations based on multiple runs deteriorates as the need for generating more efficient solutions increases, since this is directly proportional to the number of times that the procedure must be executed.

  On the other hand, [12] propose the use of relinked local searches, where linked means that the last point of one search becomes the initial point of the next search and where each point visited at any iteration could be included in the final approximation. This method is based on the very well known Fritz-John optimality principle for multiobjetive optimization which has been empirically demonstrated to provide a dense and diverse initial set of efficient points.

  Our improvement method consists of optimizing three objective functions: (i) dispersion measure

$$z_1(S) = \sum_{j \in V_t, t \in T} d_{jc(t)}, \tag{5}$$

  (ii) maximum deviation with respect to the number of customers

$$z_2(S) = \frac{1}{\mu^{(1)}} \max_{t \in T} \left\{ \max\{w^{(1)}(V_t) - \mu^{(1)}, \mu^{(1)} - w^{(1)}(V_t)\} \right\}, \tag{6}$$

  and (iii) total infeasibility

$$z_3(S) = \frac{1}{\mu^{(2)}} \sum_{t \in T} \max \left\{ w^{(2)}(V_t) - (1 + \tau^{(2)})\mu^{(2)}, (1 - \tau^{(2)})\mu^{(2)} - w^{(2)}(V_t), 0 \right\} \tag{7}$$

  related to the balancing of sales volume. Note that $c(t)$ is the center of territory $V_t$. Then, the post-processing phase consists of systematically applying the local search sequentially to each of the three objectives individually. That is, first local search is applied using $z_1$ as the merit function in a single-objective manner. After a local optimum is found, the local search is continued with $z_2$ as merit function, and then $z_3$. Finally, the initial objective $z_1$ is used after the local optimum is obtained for the last objective. During the search, the set of nondominated solutions is updated at every solution.

- A *reference set update method* that maintains a portion of the best solutions of the reference set. In this case, the reference set is formed by efficient solutions according to the Pareto sense. When an efficient solution is found, this enters the reference set and those solutions that are dominated by the added solution are deleted from the reference set.

- A *subset generation method* that operates in the reference set in such a way so as to select some solutions to be combined. All possible pairs of solutions from the reference set are selected. During each SSMTDP iteration, a temporal memory is used to avoid those combinations that were done in the previous iteration. In other words, for a specific iteration, the combination process is applied just to those pairs of solutions that were not combined in the previous iteration.

- A *solution combination method* that transforms the solution sets formed by the subset generation method into one or more combined solution. In this work, three solutions are generated (see Algorithm 1) from each pair of solutions. There are many ways of combining a pair of solutions. In the proposed SSMTDP procedure, this component is developed by attempting to keep good features present in the current solutions. Then, given a pair of solutions $S_1$ and $S_2$, these are combined by identifying the best matching between territories. An exhaustive evaluation of the possible matchings requires a high computational effort. Therefore, the method attempts to find the best territory matching based on their corresponding territory centers only. This is done by solving an associated assignment problem. The assignment problem used in this method minimizes the sum of distances between the territory centers identified on these solutions. For instance, suppose that solutions $S_1$ and $S_2$, with corresponding center sets $C_1$ and $C_2$, are to be combined. The assignment problem is solved between the center sets $C_1$ and $C_2$, and after that, the resulting assignment is used to determine which territories are matched (see Algorithm 2). Each matching pair $(i, j)$ of this assignment yields a territory in the combined solution by assigning to this territory all those nodes that are common to both territory with center in $i$ in $S_1$ and territory with center in $j$ in $S_2$ (see Algorithm 3). Let $S^p$ be the partial territory design obtained this way. Finally, this partial solution $S^p$ is used as a starting solution for generating three different trajectories, each of them guided by a different objective function, namely: i) dispersion, ii) maximum deviation with respect to the number of customers, and iii) total infeasibility with respect to the sales volume (see Algorithm 4). This, of course, generates three solutions called $S_{z_1}, S_{z_2}, S_{z_3}$, respectively.

The SSMTDP stops by iteration limit or by convergence, that is, when the reference set does not change (see Algorithm 1). Note that the updating of the reference set takes place after a potential set of nondominated solutions is obtained by applying the improvement method over all trial solutions ($S(z_1), S(z_2)$, and $S(z_3)$) generated by the combination method. This strategy was adopted given that the computational effort increases considerably when the typical strategy (i.e., updating after each new feasible solution is generated) is performed.

**Algorithm 1** General scheme of SSMTDP

**Output:** *REFSet* Set of efficient solutions (reference set)
- *flag* $\leftarrow$ 1 := 1 if the solution *REFSet* changes, 0 in otherwise
- $iter = 0, NS = \emptyset, E = \emptyset$, *REFSet* $= \emptyset$
- *REFSet* $\leftarrow$ DiverseSolutions {*use any of the proposed GRASP procedures*}
- **while** ((*flag*) **and** ($iter < max_{\text{iter}}$)) **do**
  - *COM* $\leftarrow$ SubsetGeneration(*REFSet*) {*pairs of solutions to be combined*}
  - *NS* $\leftarrow \emptyset$
  - **for** $(S_1, S_2) \in COM$ **do**
    - $(S_{z_1}, S_{z_2}, S_{z_3}) \leftarrow$ CombinationMethod$(S_1, S_2)$
    - $NS \leftarrow NS \cup \{S_{z_1}, S_{z_2}, S_{z_3}\}$
  - **end for**
  - $E \leftarrow$ Improvement(*NS*) {calls to post-processing phase of the GRASP procedures by minimizing of $(z_1, z_2, z_3, z_4)$ }
  - *flag* $\leftarrow$ UpdateRefSet(*E*,*REFSet*)
  - *iter*+1
- **end while**
- **return** *REFSet*

**Algorithm 2** CombinationMethod$(S_1, S_2)$

**Input:**
- $(S_1, S_2)$:= Pair of solutions to be combined

**Output:** $(S_{z_1}, S_{z_2}, S_{z_3})$ Three new solutions obtained by combining $S_1$ and $S_2$
- $C_1 \leftarrow \bigcup_{t\in\{1,\ldots,p\}} c(t) \in V_t,\ v_t \in S_1$
- $C_2 \leftarrow \bigcup_{t\in\{1,\ldots,p\}} c(t) \in V_t,\ v_t \in S_2$
- $M \leftarrow \emptyset$ := Matching $\{(i_1, j_1), (i_2, j_2), ..., (i_p, j_p)\}$ between elements from $C_1$ and $C_2$, where $i_t \in C_1$ and $j_t \in C_2, t \in \{1, ..., p\}$
- $M \leftarrow$ SolveAssignmentProblem$(C_1,C_2)$
- $S^p \leftarrow$ BuildPartialSolution$(S_1, S_2, M)$
- Compute $I' \subset V$ such that $I'$ contains those nodes that have not been assigned in the partial solution $S^p$
- $(S_{z_1}, S_{z_2}, S_{z_3}) \leftarrow$ GenerateNewSolutions$(S^p, I')$
- **return** $(S_{z_1}, S_{z_2}, S_{z_3})$

Figure 1 illustrates the process of generating a partial solution by combining a pair of trial solutions $S_1$ and $S_2$. In this figure, the black nodes represent the territory centers. Suppose that after solving the assignment problem, the resulting assignment is represented by territories enclosed by dotted lines in $S_1$ and $S_2$. The partial solution is the basis for generating three new solutions $S_{z_1}, S_{z_2}$, and $S_{z_3}$. These new solutions are obtained by adding the unassigned nodes to the partial

**Algorithm 3** BuildPartialSolution($(S_1, S_2), M$)

**Input:**
$(S1, S_2)$:= Pair of solutions to be combined
$M$:= Matching $\{(i_1, j_1), (i_2, j_2), ..., (i_p, j_p)\}$ between territory centers from $S_1$ and $S_2$
**Output:** $(S^p = (V_1, V_2, ..., V_p))$ Partial assignment of nodes to territories
  **for** $(t = 1, ..., p)$ **do**
    $V_t \leftarrow \emptyset$
    $V_t \leftarrow \cup V_{i_t} \cap V_{j_t}$, where $V_{i_t} \in S_1$ and $V_{j_t} \in S_2$
    **if** $(V_t == \emptyset)$ **then**
      $V_t \leftarrow j_t$
    **end if**
  **end for**
  **return** $(S^p = (V_1, V_2, ..., V_p))$

**Algorithm 4** GenerateNewSolutions($S^p$, $I'$)

**Input:**
$S^p$ := Partial solution
$I'$ := Unassigned nodes
**Output:** $(S_{z_1}, S_{z_2}, S_{z_3})$ Three new solutions obtained since $S^p$
  $S_{z_1} \leftarrow$ AssignmentGRASP($S^p, I', z_1$) {Merit function for minimizing $z_1(S)$}
  $S_{z_2} \leftarrow$ AssignmentGRASP($S^p, I', z_2$) {Merit function for minimizing $z_2(S)$}
  $S_{z_3} \leftarrow$ AssignmentGRASP($S^p, I', z_3$) {Merit function for minimizing $z_3(S)$}
  **return** $(S_{z_1}, S_{z_2}, S_{z_3})$

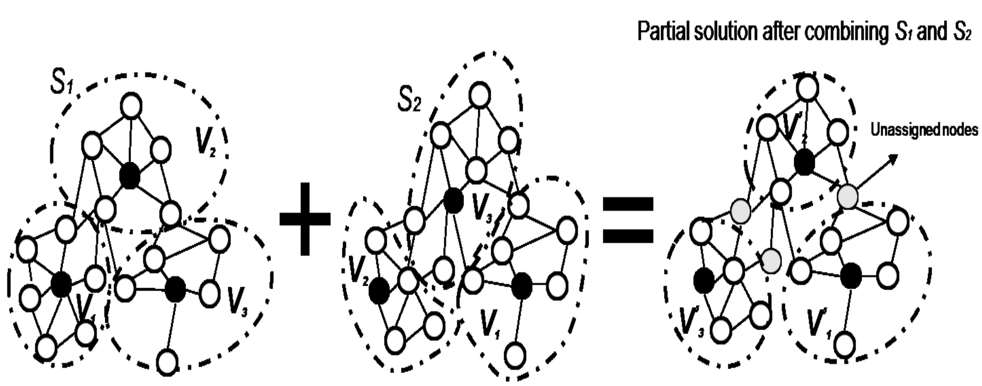


Figure 1: Combination of territories between a pair of solutions.

territories, carrying out three independent applications of the diversification method. That is, for generating $S_{z_1}$, the unassigned nodes are assigned to the partial territories through a call to the diversification method whose merit function is given by the dispersion measure. Then, for generating $S_{z_2}$ the merit function is given by the maximum deviation with respect to the number of customers. Finally, for generating $S_{z_3}$ a merit function that computes the total infeasibility with respect to the balancing of sales volume is used.

When all trial solutions are generated (i.e., when all pairs of solutions are combined), this set of solutions is improved by using the improvement method previously described. At the end, the improvement process reports a potential set of nondominated solutions that can be included in the current reference set. Thus, each solution from the potential set enters the reference set if it is an efficient solution with respect to the current set of solutions belonging to the reference set. Those solutions that are dominated by the new solution are removed from the current reference set. The SSMTDP stops when there are no new solutions included in the reference set.

# 5 Experimental Work

The procedure was coded in C++, and compiled with with the Sun C++ compiler workshop 8.0 under the Solaris 9 operating system and run on a SunFire V440. The data sets were taken from the library developed by [15]. These data set contains randomly generated instances based on real-world data provided by the firm. The SSMTDP was applied over two instance sets with $(n, p) \in \{(500,20),(1000,50)\}$. In all instances, a tolerance parameter $\tau^{(2)} = 0.05$ was used. Two stopping criteria were used in the SSMTDP, iteration limit and convergence. In this experiment, the maximum number of iterations was set to 10.

## 5.1 Assessing the Performance of SSMTDP

During the experimental work, it was observed that SSMTDP converged without reaching the iteration limit over all instances tested. That is, in all cases the SSMTDP stopped when there were no new solutions to be added to the reference set. Figure 2 shows the behavior exhibited by an instance with 500 BUs and 20 territories. The first front (BGRASP-I) is the initial solution set generated by the diversification method (BGRASP-I). The following fronts show the solutions that belong to the reference set on each SSMTDP iteration. Recall that SSMTDP starts with an efficient solution set that is obtained by the diversification method. These solutions are assigned to the initial reference set. After that, each pair of solutions in the reference set is combined to generate three different solutions. The new generated solutions are improved through the RLS and then, the updating of the reference set is done for obtaining a new reference set. When the reference set does not change, the SSMTDP stops. In the case illustrated in Figure 2, the SSMTDP converged in iteration 9. That is, in this iteration, the combination of solutions from the reference set did not yield potential nondominated solutions to be added to the reference set. Thus, SSMTDP reports as efficient solutions set those solutions belonging to the reference set in the last iteration.

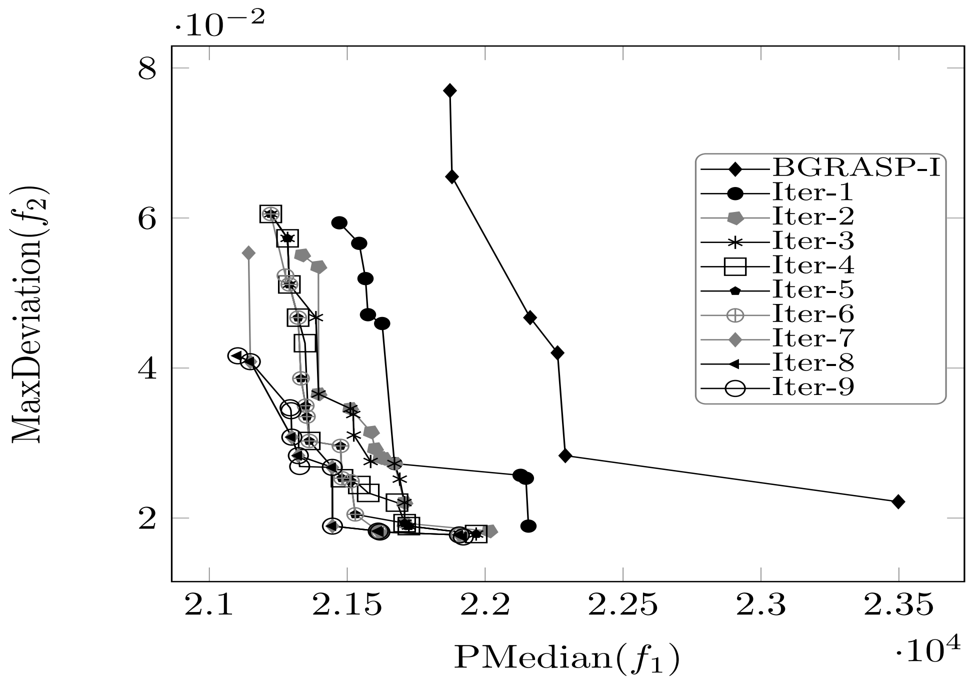


Figure 2: SSMTDP performance for an instance with 500 BUs and 20 territories.

To illustrate the behavior of SSMTDP by using instances from (1000,50), Figure 3 shows the SSMTDP iterations over an instance with 1000 BUs and 50 territories. In this case the SSMTDP stopped in iteration 8. In summary, the efficient fronts obtained by SSMTDP represent a significant improvement with respect to the initial fronts provided by BGRASP-I. It was observed that in all instances tested (20 instances), the SSMTDP method stopped by convergence. These results are used in Section 5.2 for comparing SSMTDP with another SS heuristic called SSPMO.

In the following section, a comparison between the proposed SSMTDP and SSPMPO, a state-of-the-art SS heuristic is done.

## 5.2 Comparison with Existing Multiobjective SS Procedure

### Description of SSPMO

SSPMO is a metaheuristic introduced by Molina et al. [12] initially developed for solving non-linear multiobjective optimization problems; however, it has been adapted for multiobjective clustering problems as well. It consists of a scatter/tabu search hybrid procedure that includes two different phases: i) generation of an initial set of efficient points through various tabu searches (MOAMP), and ii) combination of solutions and updating of efficient set via scatter search.

The generation of the initial set is based on the MOAMP method proposed by Caballero et al. [3]. To build the initial set of efficient points, MOAMP carries out a series of relinked tabu searches where each visited point could be included in the final efficient set. The second phase of MOAMP consists of an intensification search around the initial set of efficient points. For more details see

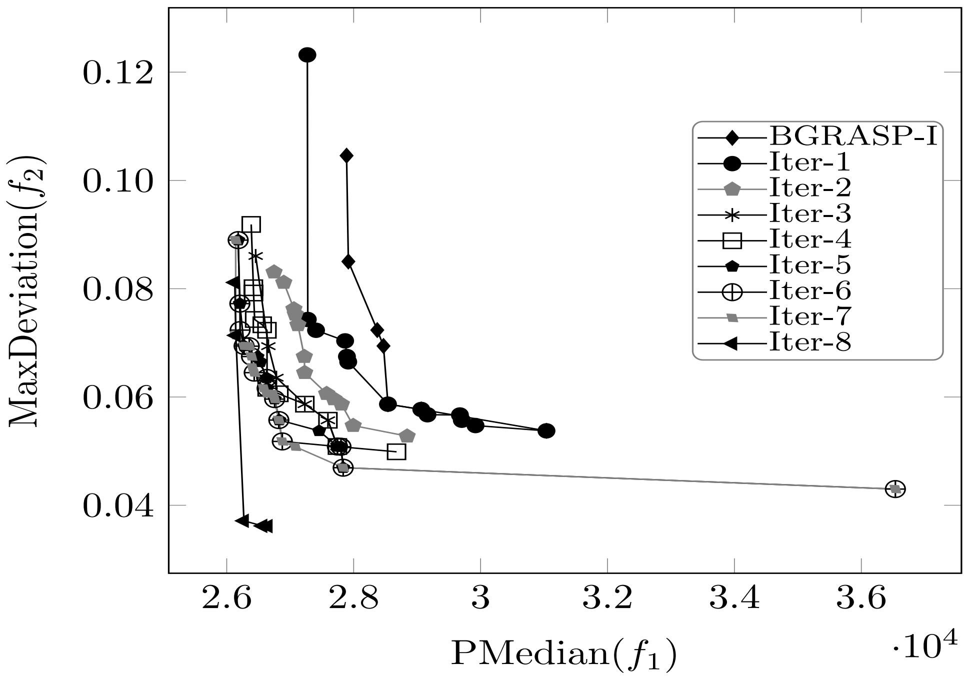


Figure 3: SSMTDP performance for an instance with 1000 BUs and 50 territories.

[3] and [12].

The SSPMO procedure creates a reference set ($E$) using the efficient solutions reported by MOAMP. A list of solutions that have been selected as reference points is kept to prevent the selection of those solutions in future iterations. Then, each solution that is added to the set $E$, is added to a $TE$ (tabu set). A linear-combination method is used to combine reference solutions. All pair of solutions in $E$ are combined and each combination yields four new trial solutions. Each new solution is subject to an improvement method based on MOAMP. Solutions generated after the improvement procedure are tested for possible inclusion in $E$.

Once all pairs of solutions in $E$ are combined and the new trial solutions are improved, SSPMO updates the reference set $E$ and proceeds to the next iteration. The first step in the updating process is to choose the best solutions according to each of the objective functions taken separately. In this selection, those solutions belonging to $TE$ are not considered. The remaining solutions are chosen by using a metric $L_\infty$, that is a generalization of the Euclidean distance. For each $x \in E \setminus TE$ the minimum distance ($L_\infty^{\min}(x)$) from $x$ to $TE$ is computed, and a uniform random number is generated. If it is less than ($L_\infty^{\min}(x)$), then $x$ is declared eligible. Let $y$ be the maximum number of solutions to be combined. Then, $y - g$ solutions with largest minimum distance to $TE$ are selected sequentially. Note that, $TE$ is updated after each selection in order to avoid choosing points that are too close to each other. The updating process continues until the mean value of ($L_\infty^{\min}(x)$) for the set of eligible solutions falls below a pre-specified threshold mean-distance. For a complete description of SSPMO method, see [12].

The SSPMO method was adapted to the multiobjective commercial territory design problem. Four objective functions are minimized: i) dispersion (5), ii) maximum deviation with respect to the average number of customers (6), iii) total infeasibility with respect to the balancing constraints of sales volume (7), and iv) total number of unconnected nodes. The initial solution set fed to MOAMP is generated by choosing $p$ seeds (configuration of centers) and each of the remaining BUs is assigned to its closest center. The maximum number of updates of the reference set was set to 10 (equal to the number of iterations used in SSMTDP), the maximum number of tabu solutions was set to 55, the threshold value was set to 0.05, and the maximum number of efficient solutions included in the reference set was set to 100. The neighborhoods are the same that those defined in the NSGA-II method (following section). For each pair of solutions, four new trial solutions are generated.

At the end, the efficient solutions reported by SSPMO are filtered using only those feasible solutions that are efficient with respect to the dispersion measure and the maximum deviation with respect to the average number of customers.

### Comparing SSPMO and SSMTDP

In this part of the computational work, the SSMTDP procedure is compared with SSPMO. Both SS-based procedures stop by convergence or by iteration limit (10 updates of the reference set). Figures 4 and 5 show a comparison between the Pareto fronts obtained by SSPMO and SSMTDP, respectively. These results correspond to 10 instances with 500 BUs and 20 territories. The maximum number of allowed movements in SSMTDP was set to 800. Graphically, SSMTDP outperforms SSPMO over all instances tested.

There are different performance measures used to evaluate the quality of those approximated efficient solutions obtained by approximation procedures in multiobjective optimization. In the literature of multiobjective optimization, the most used performance measures are the following:

1. *Number of points:* It is an important measure because efficient frontiers that provide more alternatives to the decision maker are preferred than those frontiers with few efficient points.

2. *k-distance:* This density-estimation technique used by Zitzler, Laumanns, and Thiele [22] in connection with the computational testing of SPEA2 is based on the $k$th-nearest neighbor method of Silverman [20]. This metric is simply the distance to the $k$th-nearest efficient point. So, the smaller the $k$-distance the better in terms of the frontier density. We use $k$=4 and calculate both the mean and the max of $k$th-nearest distance values.

3. *Size of space covered (SSC):* This metric was suggested by Zitzler and Thiele [23]. This measure computes the volume of the dominated points. Hence, the larger the value of SSC the better.

4. *C(A,B):* It is known as the coverage of two sets measure [23]. This measure represents the proportion of points in the estimated efficient $B$ that are dominated by the efficient points in the estimated frontier $A$.

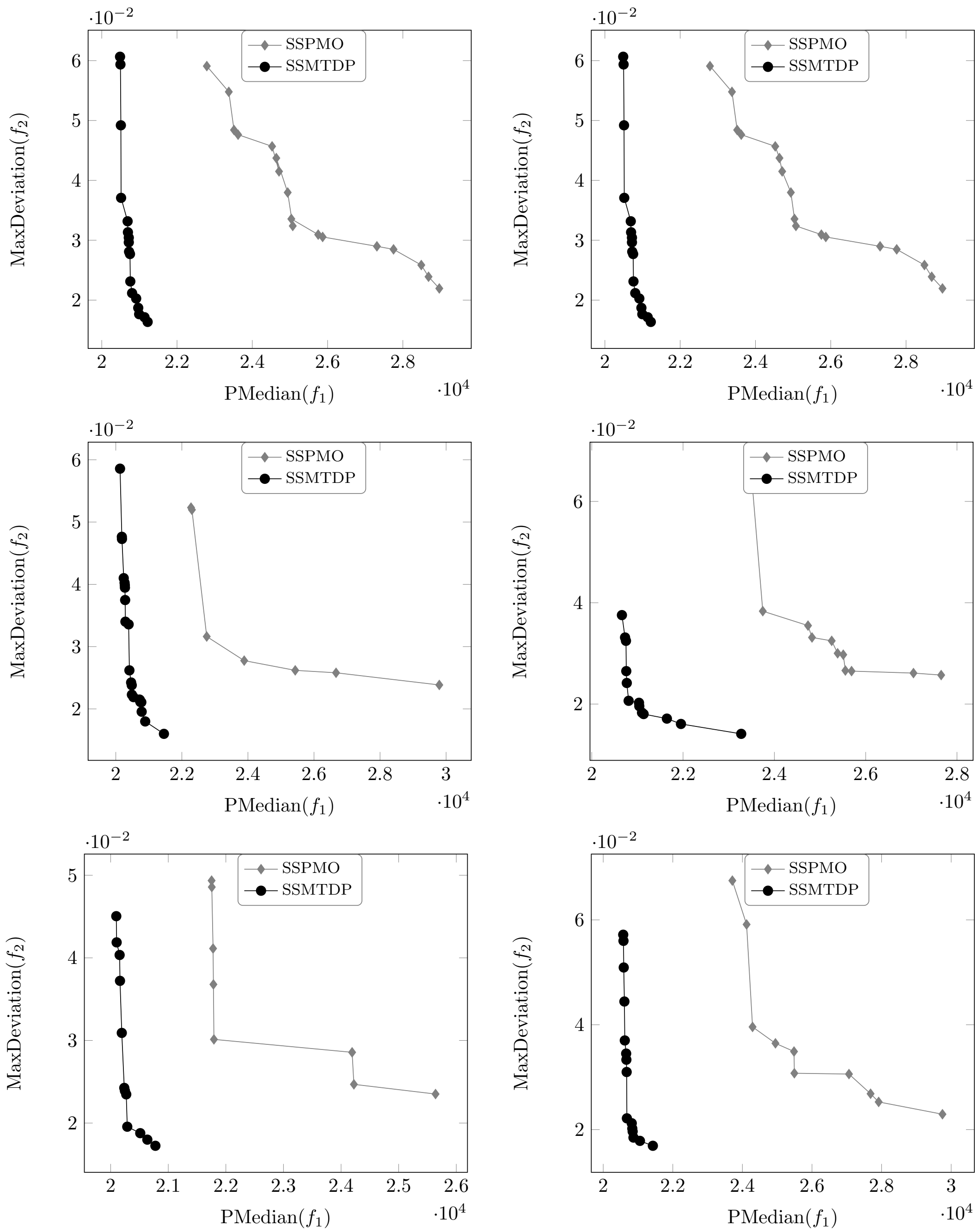


Figure 4: Comparison of Pareto fronts, SSPMO vs. SSMTDP. Instances from (500,20), part 1.

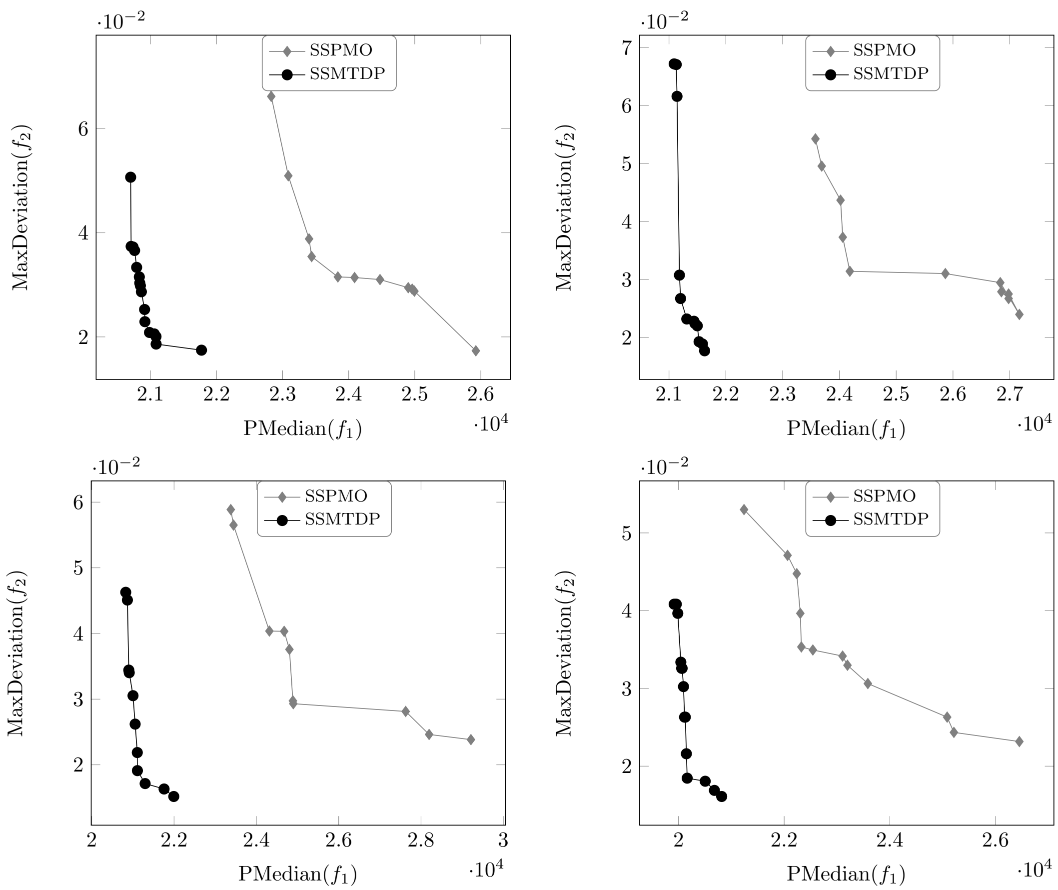


Figure 5: Comparison of Pareto fronts, SSPMO vs. SSMTDP. Instances from (500,20), part 2.

Tables 1, 2, 3, and 4 show a summary of these metrics, these were computed taking into account all instances from set (500, 20). Clearly, SSMTDP outperforms SSPMO in all metrics for all the instances, specially when considering convergence, where the SSC metric is around double the obtained by SSPMO.

Table 1: Summary of minimum values in the set (500, 20).

| Procedure | No. Points | $k$-distance(mean) | $k$-distance(max) | SSC |
|---|---|---|---|---|
| SSPMO | 7.00 | 0.16 | 0.30 | 0.38 |
| SSMTDP | 11.00 | 0.09 | 0.22 | 0.93 |

Table 2: Summary of average values in the set (500, 20).

| Procedure | No. Points | $k$-distance(mean) | $k$-distance(max) | SSC |
|---|---|---|---|---|
| SSPMO | 10.82 | 0.31 | 0.56 | 0.42 |
| SSMTDP | 14.36 | 0.16 | 0.44 | 0.97 |

Table 3: Summary of maximum values in the set (500, 20).

| Procedure | No. Points | $k$-distance(mean) | $k$-distance(max) | SSC |
|---|---|---|---|---|
| SSPMO | 17.00 | 0.58 | 0.81 | 0.54 |
| SSMTDP | 22.00 | 0.26 | 0.83 | 0.99 |

In addition, 10 instances with 1000 BUs and 50 territories were tested by applying both SSPMO and SSMTDP using the same stopping criteria as in the previous cases, SSPMO spent more than 30 days without getting convergence for the first instance tested. Then, the stopping criteria was changed and the iteration limit was set to 2. SSMTDP converged and reported efficient solutions for all instances tested. The maximum number of moves for these cases was set to 2000. Due that the tremendous computational effort required by the SSPMO, the procedure was not applied over all instances with 1000 BUs and 50 territories. Figure 6 shows the performance of SSPMO and SSMTDP. The approximated front reported by SSPMO corresponds to those solutions that belong to the reference set after iteration 2.

## 5.3 Comparison with Existing Evolutionary Algorithm

### Description of NSGA-II

The Nondominated Sorting Genetic Algorithm (NSGA-II) is an evolutionary algorithm that has been successfully applied to many multiobjective combinatorial optimization problems in the literature [5]. It has been empirically shown this method finds significantly better spread of solutions and better convergence near the true Pareto-optimal front compared to Pareto-archived evolutionary strategy (PAES) [10] and Strength Pareto EA (SPEA). The general description of NSGA-II is

Table 4: Coverage of two sets C(A,B) in the set (500, 20).

| C(A,B) | SSPMO | SSMTDP |
|---|---|---|
| SSPMO | 0.00 | 0.00 |
| SSMTDP | 0.90 | 0.00 |

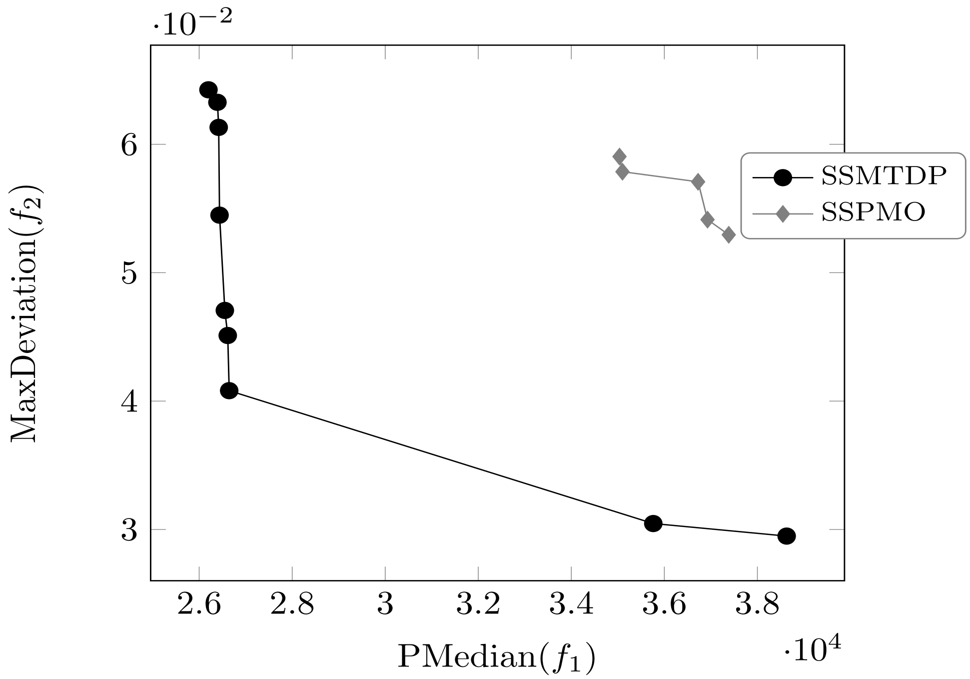


Figure 6: Comparison of Pareto fronts, SSPMO vs. SSMTDP. Instance with 1000 BUs and 50 territories.

given in Deb et al. [6].

In this work, NSGA-II was adapted to the problem. Four objective functions are minimized: i) dispersion (5), ii) maximum deviation with respect to the average number of customers (6), iii) total infeasibility with respect to the balancing constraints of sales volume (7), and iv) total number of unconnected nodes. The main features present in this adaptation of the NSGA-II procedure are the following. The generation of solutions consists of randomly selecting $p$ seeds from the set of nodes ($V$) and assigning the remaining $n - p$ nodes to the closest center. NSGA-II uses different nondomination levels (ranks). In a few words, for each solution $h$ two entities are calculated: 1) domination count $d_h$ which corresponds to the number of solutions that dominate the solution $h$, and 2) a set of solutions $D_h$ that solution $h$ dominates. All solutions in the first nondominated front have their domination count as zero. Then, for each solution $h$ with $d_h = 0$, each member ($g$) from $S_p$ is visited, and its domination count is reduced by one. In doing so, if for any member $g$ the domination count becomes zero, it is put in a separate list $\bar{Q}$. These members belong to the second front. Now, the above procedure is continued with each member of $\bar{Q}$ and the third front is identified. The process continues until all fronts are identified.

In the first iteration, the population is sorted based on the nondomination. Then, the fitness function is defined according to the nondomination level. At first, the binary tournament selection is used to create an offspring population $\bar{Q_0}$ of size $N$. Since elitism is introduced by comparing the current population with previously found best nondominated solutions, the procedure is different after the initial generation. In the following iterations, the selection is based on the crowded operator which combines the rank (nondomination level) and crowded distance. For more details see [6].

For each pair of solutions two new solutions are obtained. Each new solution copies each center from the father or from the mother with the same probability and the assignment process is equal to that of the initial generation. For each generated solution, a random integer number is generated in the range [0,4]. If the random number is equal to 0, then the mutation process is not applied. Otherwise, the mutation process takes place by using the kind of move determined by the generated number. The different neighborhoods are defined by the following moves:

1. Select a center and change it for another randomly selected node. Do a re-assignment of nodes using the new configuration of centers.

2. Select a node in the border of a territory and assign this node to the adjacent territory (keeping connectivity).

3. Select a territory $r$ and assign a randomly selected node from an adjacent territory to $r$.

4. Interchange two nodes between a pair of territories by holding connectivity.

When the convergence criterion is reached, the best nondominated solutions are filtered to obtain those feasible solutions that are efficient with respect to the dispersion measure and the maximum deviation with respect to the average number of customers.

**Comparing NSGA-II, SSMTDP, and SSPMO**

NSGA-II was applied over the two instance sets used in the previous section. The number of generations and the population size was set to 500, respectively. On each generation 250 solutions were combined. NSGA-II reported efficient solutions only for a single instance with 500 BUs and 20 territories. For the other 19 instances tested NSGA-II did not obtain feasible solutions and the SSMTDP procedure reported efficient solutions over all tested instances. It was observed how NSGA-II failed on appropriately handling the connectivity constraints. Most of the solutions generated by NSGA-II are highly infeasible with respect to the connectivity constraints, even though the NSGA-II considers this requirement as objective to be minimized. The selection mechanism and the combining processes are not enough to efficiently handling these very difficult constraints. In contrast, the proposed SSMTDP procedure is specifically designed to take the connectivity into account over all its components. Thus, for this problem, exploiting problem structure definitely pays off. Figure 7 shows the comparison among the SSMTDP, SSPMO, and NSGA-II procedures.

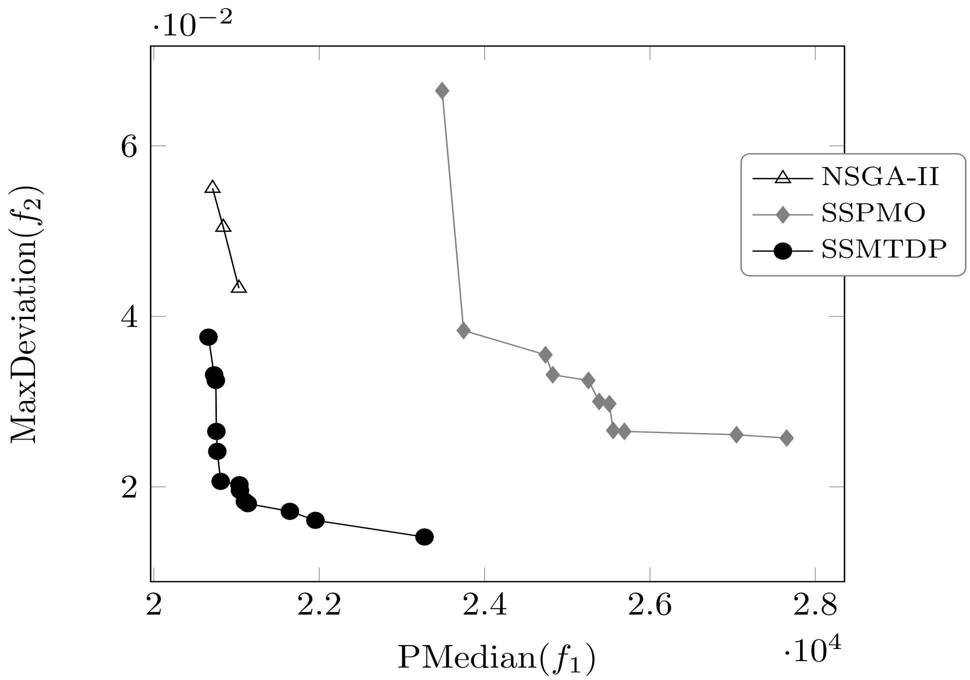


Figure 7: Comparison of Pareto fronts for an instance with 500 BUs and 20 territories, SSMTDP, SSPMO, and NSGA-II.

Note that a few efficient solutions from SSPMO are dominated by the efficient set reported by NSGA-II. In addition, both SSPMO and SSMTDP reported efficient points in a region that is not covered by the Pareto front obtained by NSGA-II.

Table 5: Summary of metrics for an instance from (500, 20).

| Procedure | No. Points | $k$-distance(mean) | $k$-distance(max) | SSC |
|---|---|---|---|---|
| SSPMO | 13.00 | 0.20 | 0.62 | 0.38 |
| NSGA-II | 4.00 | - | - | 0.43 |
| SSMTDP | 13.00 | 0.13 | 0.32 | 0.97 |

Table 5 shows again the superiority of SSMTDP that clearly outperforms both NSGA-II and SSPMO, demonstrating the efficiency of the proposed method. In the $k$-distance (mean and max), the corresponding values for NSGA-II could not be computed given that we used $k = 4$. The coverage of two sets measure C(A,B) is shown in Table 6. Note that the points obtained by NSGA-II dominated some points obtained by SSPMO. Table 6 shows that NSGA-II dominates 15% of the points reported by SSPMO. For this metric, SSMTDP dominates the frontiers reported by NSGA-II and SSPMO (see Figure 7). Moreover, NSGA-II reported feasible solutions just for a single instance out of 20 instances tested, while SSMTDP reported feasible solutions for all instances tested. In summary, SSMTDP outperforms both the NSGA-II and SSPMO procedures.

Table 6: Coverage of two sets C(A,B) for an instance from (500, 20).

| C(A,B) | SSPMO | NSGA-II | SSMTDP |
|---|---|---|---|
| SSPMO | 0.00 | 0.00 | 0.00 |
| NSGA-II | 0.15 | 0.00 | 0.00 |
| SSMTDP | 1.00 | 1.00 | 0.00 |

## 6 Conclusions and Future Work

In this paper we proposed a novel heuristic procedure called SSMTDP which is based on Scatter Search. Each component of the SSMTDP has been intelligently designed attempting to take advantage of the problem structure. For instance, a combination method that consists of a hybrid approach was implemented. It was shown to obtain quality and diversification of solutions. This procedure can be used in other applications such as clustering. Empirical evaluation of this method was carried out over two large instance sets, with 500 BUs and 1000 BUs. The solutions generated by SSMTDP were compared against those solutions obtained by a well-known multiobjective scatter search method called SSPMO. SSMTDP reported better solutions than SSPMO in all instances tested. In addition, the popular evolutionary algorithm NSGA-II was adapted to the problem. Empirical work revealed the proposed SSMTDP significantly outperforms NSGA-II on all instances tested. It was observed NSGA-II struggled on trying to generate feasible solutions on this highly constrained problem.

The SSMTDP have been tested in the bi-objective version of the commercial territory design problem. However, the problem can be addressed by optimizing one more objective such as the balancing with respect to the sales volume. In the other hand, due that the territory design problem takes place in a previous stage of the product routing, the routing cost is another requirement that can be incorporated to the current models. This requirement could be treated as objective or as a constraint. The recently developed procedures can be used as a basis for deriving new solution techniques to tackled this new problems.

*Acknowledgements:* This work was supported by the Mexican National Council for Science and Technology under grants SEP-CONACYT 48499–Y and SEP-CONACYT 61903, Universidad Autónoma de Nuevo León under its Scientific and Technological Research Support, grant UANL-PAICYT CE012–09, and Tecnológico de Monterrey under research grant CAT128. The research of the first author has been funded by a doctoral fellowship from Universidad Autónoma de Nuevo León, grant NL–2006–C09–32652.

## References

[1] C. W. Bong and Y. C. Wang. A multiobjective hybrid metaheuristic approach for GIS-based spatial zone model. *Journal of Mathematical Modelling and Algorithms*, 3(3):245–261, 2004.

[2] R. Bowerman, B. Hall, and P. Calamai. A multi-objective optimization approach to urban school bus routing: Formulation and solution method. *Transportation Research Part A*, 29(2):107–123, 1995.

[3] R. Caballero, X. Gandibleux, and J. Molina. MOAMP: A generic multiobjective metaheuristic using an adaptive memory. Technical report, University of Valenciennes, Valenciennes, France, 2004.

[4] S. I. Caballero-Hernández, R. Z. Ríos-Mercado, F. López, and E. Schaeffer. Empirical evaluation of a metaheuristic for commercial territory design with joint assignment constraints. In J. E. Fernandez, S. Noriega, A. Mital, S. E. Butt, and T. K. Fredericks, editors, *Proceedings of the 12th Annual International Conference on Industrial Engineering Theory, Applications, and Practice (IJIE)*, pages 422–427, Cancun, Mexico, November 2007. ISBN: 978-0-9654506-3-8.

[5] K. Deb, S. Agrawal, A. Pratap, and T. Meyarivan. A fast elistist non-dominated sorting genetic algorithm for multi-objective optimization: NSGA-II. In M. Schoenauer, K. Deb, G. Rudolph, X. Yao, E. Lutton, J. J. Merelo, and H.-P. Schwefel, editors, *Parallel Problem Solving from Nature - PPSN VI*, volume 1917 of *Lecture Notes in Computer Science*, pages 849–858. Springer, Berlin, Germany, 2000.

[6] K. Deb, A. Pratap, S. Agarwal, and T. Meyerivan. A fast elitist multiobjective genetic algorithm: NSGA-II. *IEEE Transactions on Evolutionary Computation*, 6(2):182–197, 2002.

[7] F. Glover. Heuristics for integer programming usign surrogate constraints. *Decision Sciences*, 8(1):156–166, 1977.

[8] J. Guo, G. Trinidad, and N. Smith. MOZART: A multi-objective zoning and aggregation tool. In *Proceedings of the Philippine Computing Science Congress (PCSC)*, pages 197–201, Manila, Philippines, 2000.

[9] J. Kalcsics, S. Nickel, and M. Schröder. Toward a unified territorial design approach: Applications, algorithms, and GIS integration. *Top*, 13(1):1–56, 2005.

[10] J. Knowles and D. Corne. The Pareto archived evolution strategy: A new baseline algorithm for multiobjective optimisation. In *Proceedings of the 1999 Congress on Evolutionary Computation*, pages 98–105, Piscataway, USA, 1999. IEEE Service Center.

[11] R. Martí, M. Laguna, and F. Glover. Principles of scatter search. *European Journal of Operational Research*, 169(2):359–372, 2006.

[12] J. Molina, R. Martí, and R. Caballero. SSPMO: A scatter tabu search procedure for non-linear multiobjective optimization. *INFORMS Journal on Computing*, 19(1):91–100, 2007.

[13] F. Ricca. A multicriteria districting heuristic for the aggregation of zones and its use in computing origin-destination matrices. *Information Systems and Operational Research*, 42(1):61–77, 2004.

[14] F. Ricca and B. Simeone. Local search algorithms for political districting. *European Journal of Operational Research*, 189(3):1409–1426, 2008.

[15] R. Z. Ríos-Mercado and E. A. Fernández. A reactive GRASP for a commercial territory design problem with multiple balancing requirements. *Computers & Operations Research*, 36(3):755–776, 2009.

[16] M. A. Salazar-Aguilar, R. Z. Ríos-Mercado, and J. L. González-Velarde. A bi-objective programming model for designing compact and balanced territories in commercial districting. *Transportation Research Part C*. Forthcoming.

[17] M. A. Salazar-Aguilar, R. Z. Ríos-Mercado, and J. L. González-Velarde. GRASP strategies for a bi-objective commercial territory design problem. Technical Report PISIS-2010-01, Graduate Program in Systems Engineering, UANL, San Nicolás de los Garza, Mexico, January 2010.

[18] M. N. Scott, R. G. Cromley, and E. K. Cromley. Multi-objective analysis of school district regionalization alternatives in Connecticut. *The Professional Geographer*, 48(1):1–14, 1996.

[19] J. A. Segura-Ramiro, R. Z. Ríos-Mercado, A. M. Álvarez-Socarrás, and K. de Alba Romenus. A location-allocation heuristic for a territory design problem in a beverage distribution firm. In J. E. Fernandez, S. Noriega, A. Mital, S. E. Butt, and T. K. Fredericks, editors, *Proceedings of the 12th Annual International Conference on Industrial Engineering Theory, Applications, and Practice (IJIE)*, pages 428–434, Cancun, Mexico, November 2007. ISBN: 978-0-9654506-3-8.

[20] B. W. Silverman. *Density Estimation for Statistics and Data Analysis*. Chapman and Hall, London, UK, 1986.

[21] F. Tavares-Pereira, J. R. Figueira, V. Mousseau, and R. Bernard. Multiple criteria districting problems. The public transportation network pricing system of the Paris region. *Annals of Operations Research*, 154(1):69–92, 2007.

[22] E. Ziztler, M. Laumanns, and L. Thiele. SPEA2: Improving the strenght Pareto evolutionary algorithm. Technical Report 103, Computer Engineering and Networks Laboratory (TIK), Swiss Federal Institute of Technology (ETH), Zurich, Switzerland, 2001.

[23] E. Ziztler and L. Thiele. Multiobjective evolutionary algorithms: A comparative case study and the strength Pareto approach. *IEEE Transactions on Evolutionary Computation*, 3(4):257–271, 1999.