Sixty Years of Research on Districting Problems: Models, Algorithms, and Applications

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Abstract

Districting consists of grouping small geographic areas or units into larger units, known as districts, territories, zones, or clusters, according to relevant planning criteria such as compactness, connectivity, and/or balance requirements, among others. This class of problems arises in many applications, such as political organization, sales territory design, school districting, commercial and distribution districting, service districting, and health care management, among others. In this paper, we provide a comprehensive state-of-the-art review of the districting literature over the last sixty years. First, a discussion of a general modeling structure is presented to identify the main components of the problem. Then, existing works are reviewed according to two distinct classifications: one by application area and another by modeling techniques and solution methodologies. Finally, we conclude with a discussion of the main challenges and future research lines in this exciting area of discrete optimization.

Keywords: Discrete optimization; Territory design; Districting; Zone design; Survey.

1 Introduction and motivation

The Territory Design Problem (TDP), also referred to as the districting problem or zone design problem, has attracted the attention of researchers and practitioners such as city planners, geographers, policy-makers, and architects, among others, over the last 60 years. This goes back to the pioneering works on political districting by Harris (1964), Hess et al. (1965), Thoreson and Liittschwager (1967), and Davis et al. (1968); and school districting by Clarke and Surkis (1968), Koenigsberg (1968), and Heckman and Taylor (1969), back in the sixties. The literature covers different models, algorithms, and applications providing theoretical, methodological, and practical contributions (see, e.g., Duque et al. (2007); Kalcsics and Ríos-Mercado (2019); Butsch (2016), for comprehensive surveys of different TDPs).

Territory design plays a crucial role in various contexts. It is used to secure fair electoral systems (Ricca et al., 2013), including courtroom assistance (Buchanan et al., 2025), to help companies define service protocols and assign customers to salespersons (Ríos-Mercado and Fernández, 2009; Ríos-Mercado and Escalante, 2016), as well as to assign students to schools (Caro et al., 2004) and, in-home health care management, territory design defines districts that fall under the responsibility of each team (Lin et al., 2017, 2020). Similarly, territory design defines police patrol routes (Camacho-Collados and Liberatore, 2015; Camacho-Collados et al., 2015; Liberatore and Camacho-Collados, 2016; Liberatore et al., 2023) or to establish emergency response plans (Ramos and Oliveira, 2011; Cortinhal et al., 2016; Regis-Hernández et al., 2023). In other areas, such as municipal solid waste collection, territory design contributes to the optimization of regular operations (Ramos and Oliveira, 2011; Cortinhal et al., 2016; Mostafayi Darmian et al., 2020).

In each of these application areas, the *division* of a geographical area (e.g., a city) into clusters (i.e., *districts*) of basic functional units (e.g., customers, streets, zip codes, etc.) is an important design decision. Defining these groups corresponds to a *districting* (or *territory design*) problem and usually aims at finding a partition that is optimal according to one or more planning criteria (such as the balance of the number of voters, or a fair assignment of customers) that complies with several functional and topological requirements (such as territory compactness and connectivity).

In this work, we aim to provide a comprehensive overview of the districting literature from an optimization standpoint. We analyzed 322 documents published over the last six decades. We identified *European Journal of Operational Research* (25), *Computers & Operations Research* (24), *Annals of Operations Research* (15), *Operations Research* (14), *Journal of the Operational Research Society* (11), *Socio-Economic Planning Sciences* (11), *Computers & Industrial Engineering* (9), *Management Science* (9), *Omega* (7), and *International Journal of Geographical Information Science* (6), as the journals with the largest number of publications on TDP-related topics. There are also references published in book series such as *International Series in Operations Research and Management Science* (11) and *Lecture Notes in Computer Science* (9), both by Springer. The remaining 171 papers belong to 125 different sources. When applicable, we categorized the work according to the application setting studied and the proposed algorithmic procedure. These applications are classified into nine major areas: political (73), commercial (38), school policy (26), sales (24), health care (24), distribution (23), patrolling (19), emergency response (15), and others (50). The remaining 30 publications provide methodological improvements not geared toward a specific application area.

Regarding the algorithmic strategies used to solve these problems, 92 scientific papers propose or use exact approaches, while 232 papers present heuristic algorithms. Finally, 25 works focused more on providing surveys or methodological improvements applicable to the area rather than actually solving a problem. The fact that most of the works are based on heuristic methodologies is not surprising given the inherent computational complexity of the problem. Practically, all TDPs are $\mathcal{NP}$-hard as they generalize the classical partition problem, including the edge-based version of the problem (Jost et al., 2025).

This review aims to investigate the following issues.

Q.1 What are the areas and applications in which territory design has been most frequently studied?

Q.2 How can we categorize modeling and solution strategies used to solve real-world territory design problems?

Q.3 What are the main research challenges and future research lines in current territory design problems?

We address these questions and organize this paper as follows. First, in Section 2, we provide a generic mathematical model for the TDP to identify its main components. Second, we conduct a comprehensive review of the literature, focusing on the most prevalent application domains of territorial design in Section 3. Third, we show the relationship among models, solution methods, and application areas, highlighting similarities in Section 4. In Section 5, we provide a critical analysis of the state of the art and venues for future work on TDPs. Finally, Section 6 presents the key conclusions and final remarks.

To the best of our knowledge, the most recent reviews on territorial design correspond to Kalcsics et al. (2005) and Duque et al. (2007). Some significant works in the area are also surveyed by Kalcsics and Ríos-Mercado (2019). Our research improves these reviews by incorporating findings from 235 papers published over the last 20 years, using a snowball search methodology (Wohlin, 2014). Additionally, a generic model that considers the diversity of territory design problems is described.

2 A general mathematical modeling framework for TDPs

As pointed out by Kalcsics and Ríos-Mercado (2019, p. 707), it is surprising that only a few authors consider TDPs independently from their practical background. To the best of our knowledge, Kalcsics and Ríos-Mercado (2019) is also the only work that provides some indications on a general TDP model.

In this work, we try to establish a general TDP model based on the family of network design problems, i.e., optimization problems in networks or graphs (Quilliot, 2013). Let $G = (V, E)$ be a discretized representation of a given geographical area (i.e., a country, a province, or a city). In this setting, V corresponds to *basic units* (BU), that is, indivisible spatial entities such as customers, streets, or zip code areas. Likewise, let $E \subseteq V \times V$ be the set of *edges* or *connections* among units. These connections could integrate a street or a highway network, represent geographical contiguity rules, and describe functional linkages, among others. Typically, each unit $i \in V$ associates one or more *activity measures* (e.g., the number of voters, estimated revenues, waste volume, service cost, to name a few); let $\mathbf{w}^k : V \to \mathbb{R}$ be the function corresponding to the k-th attribute (with $k \in \{1, \dots, m\}$). Additionally, let $\mathbf{d} : E \to \mathbb{R}^+$ be the distance function; where d_{ij}, for $(i, j) \in E$ may correspond to the Euclidean or to the road distance between BUs i and j.

Generally, a district D^q corresponds to a component of G, i.e., $D^q = (V^q, E^q)$ such that $V^q \subseteq V$ and $E^q = E(V^q) \subseteq E$. For a given district D^q, let $\mathbf{w}^k(D^q)$ be the value associated with its k-th activity measure. The term $\mathbf{w}^k(D^q)$ is referred to as the *size* of district D^q with respect to activity k, and it is usually computed as the sum of the attribute values of the individual basic units associated with the corresponding district. Similarly, let $\mathbf{d}(D^q)$ denote a distance-based measure associated with district D^q. Depending on the application, $\mathbf{d}(D^q)$ may correspond to the total length of an optimal Hamiltonian tour in D^q, the diameter of D^q, or the value of a minimum spanning tree on D^q. Hereafter, we refer to $\mathbf{d}(D^q)$ as the *length* of district D^q.

2.1 Districting considerations

In a general districting setting, we look for a partition of G into p districts; therefore, let $D = \{D^1, \ldots, D^p\}$ denote a districting solution, and let Ξ denote the collection of all districting solutions fulfilling a certain number of additional constraints or requirements. When the partition is on the vertices (set V), as usually done, for ease of notation, we might identify a partition as $(V^1, \ldots, V^p)$. We now describe some of these constraints.

Exclusive assignment of basic units Each basic unit can not be assigned to two or more districts, i.e., $V^q \cap V^r = \emptyset$, $\forall q, r \in \{1, \ldots, p\}, q \neq r$. This constraint has different meanings for each real-world application. For instance, in political districting, the rationale of this criterion is obvious. Commercial territory planning generates transparent responsibilities for the sales force, avoiding contentions and favoring long-term stability in customer relations. There are some exceptional cases where units might be assigned to more than one district. For instance, in the collection of waste electric and electronic equipment (Fernández et al., 2010), the collection bins may be assigned to multiple companies.

Balance Each district should have approximately the same value $\mathbf{w}^k(\cdot)$ for each activity measure k. For example, in political districting, it aims to ensure the *one-man one-vote* principle. Likewise, in sales territory design, it is intended to balance workload (or travel times) among salespeople, and to provide a similar income opportunity from their corresponding accounts.

Let $B^k : \Xi \to \mathbb{R}$ be the balance function of the k-th measure; then the value $B^k(D)$ measures the balance of the districting plan $D \in \Xi$ is according to the k-th measure. As an example, $B^k(D)$ may correspond to $B(D) = \max_q \mathbf{w}^k(D^q) - \min_q \mathbf{w}^k(D^k)$, i.e., the maximum difference of the values attained for the k-th measure among all districts, which may be assumed to be at most $\mu \geq 0$.

Compactness A district is said to be *compact* if it is nearly round-shaped or square, undistorted, without holes, and has a smooth boundary. Compactness tries to avoid malpractices and helps to reduce travel times within the districts, and can be related to the convexity of the resulting territories (Bozeman et al., 2018). While it is a very intuitive concept, no comprehensive definition exists. Many authors (Horn et al., 1993; Fryer and Holden, 2011; Salazar-Aguilar et al., 2011b; Datta et al., 2012; Xie and Ouyang, 2016) have proposed compactness measures in the literature, but unfortunately, all of them have some issues that avoid their predominant use.

Let $\mathcal{C} : D \to \mathbb{R}^+$ be a compactness function, such that for a given $D \in \Xi$, $\mathcal{C}(D^q)$ represents *how compact* the q-th district is. Sometimes compactness is handled by means of a dispersion measure, understanding that maximizing compactness is equivalent to minimizing dispersion.

Contiguity This requirement means that individual territories must be connected. A territory $D^q = (V^q, E^q)$, $q \in \{1, \ldots, p\}$, is said to be connected if any pair of BUs $i, j \in V^q$ are connected by a path composed solely of elements in E_q. In other words, every subgraph $D^q = (V^q, E^q)$ of G must induce a connected subgraph. The special characteristic of the *contiguity* criteria makes it an unambiguous topological property independent of the application area.

Number of districts In most applications, the number of districts p into which the area will be partitioned is usually established in advance (Kalcsics and Ríos-Mercado, 2019; Ríos-Mercado, 2020). This criterion is crucial since it has an effect on how algorithms will organize or distribute the resources, population, or territories among the required districts.

2.2 Basic formulation

In its most basic form, a districting problem can be formulated by the following mathematical optimization model;

$$
\begin{aligned}
\text{maximize} \quad & \sum_{q=1}^{p} \mathcal{C}(D^q) && && \text{(D.1)}\\
\text{subject to} \quad & \bigcup_{q\in\{1,\dots,p\}} V^q = V \quad \text{and} \quad V^q \cap V^r = \emptyset, && q,r \in \{1,\dots,p\}, q \neq r && \text{(D.2)}\\
& B^k(D) \leq \mu, && k \in \{1,\dots,m\} && \text{(D.3)}\\
& D_q \in \Phi(G), && q \in \{1,\dots,p\} && \text{(D.4)}\\
& D \in \Xi. && && \text{(D.5)}
\end{aligned}
$$

The objective function (D.1) maximizes compactness among all districts. In many applications, this is equivalently handled as minimizing a measure of district dispersion. Constraints (D.2) ensure that each unit is contained in exactly one district, i.e., districts define a partition of V. Constraints (D.3) force that the balance of every district does not exceed μ. The territory balance or size is typically obtained by adding the value of the attribute associated with the individual basic units. In some applications, there might be multiple balancing constraints. The contiguity of the districts is imposed by constraints (D.4), where $\Phi(G)$ denotes the set of all connected components of G. Finally, constraints (D.5) ensure the general additional conditions of the feasibility of the districting plan.

For instance, (D.5) may represent the requirement of the districting plans to be such that the length $\mathbf{d}(\cdot)$ of every cluster is at most $\ell \geq 0$. This can be accomplished by incorporating the following constraints

$$\mathbf{d}(D^q) \leq \ell, \qquad q \in \{1,\dots,p\}, \tag{D.6}$$

into model (D.1)-(D.5).

On the other hand, in some cases, the objective function may seek the simultaneous optimization of the m balance criteria, i.e., as a multi-objective function. Then, in those cases, the objective function (D.1) may be replaced by

$$(B^{1^*},\dots,B^{m*}) = \text{opt}\left(B^1(D),\dots,B^m(D)\right), \tag{D.1a}$$

Moreover, instead of bounding the length of the districts, one could also seek a districting plan where the length of the districts is optimized. Such a model replaces the objective function (D.1a) by

$$(B^{1^*},\dots,B^{m*},B^{\ell^*}) = \text{opt}\left(B^1(D),\dots,B^m(D),B^\ell(D)\right), \tag{D.1b}$$

where $B^\ell(D)$ encodes how similar are the lengths of the districts.

In the previous scenarios, compactness can be imposed as a constraint by means of:

$$\mathcal{C}(D^q) \geq \phi, \qquad q \in \{1,\dots,p\}, \tag{D.7}$$

where ϕ defines the minimum required level of compactness.

The elements presented above allow the definition of different TDPs (with different objectives and requirements) by considering additional side constraints. The proposed modeling framework can be adapted to the requirements of many of the application areas presented in the following section. As a result, models and algorithmic components from different applications can be expressed within a unified framework.

3 TDP Applications

This section reviews the literature according to the application area, highlighting their specific requirements and the corresponding modeling decisions considered in the literature.

Figure 1 provides a chart reporting the number of publications per application area among the reviewed works. We can observe that 84% of the reviewed works focus on nine application areas, which are discussed in detail from Sections 3.1 to 3.8. The remaining publications are grouped into a single "Others" category, which will be the focus of Section 3.9.

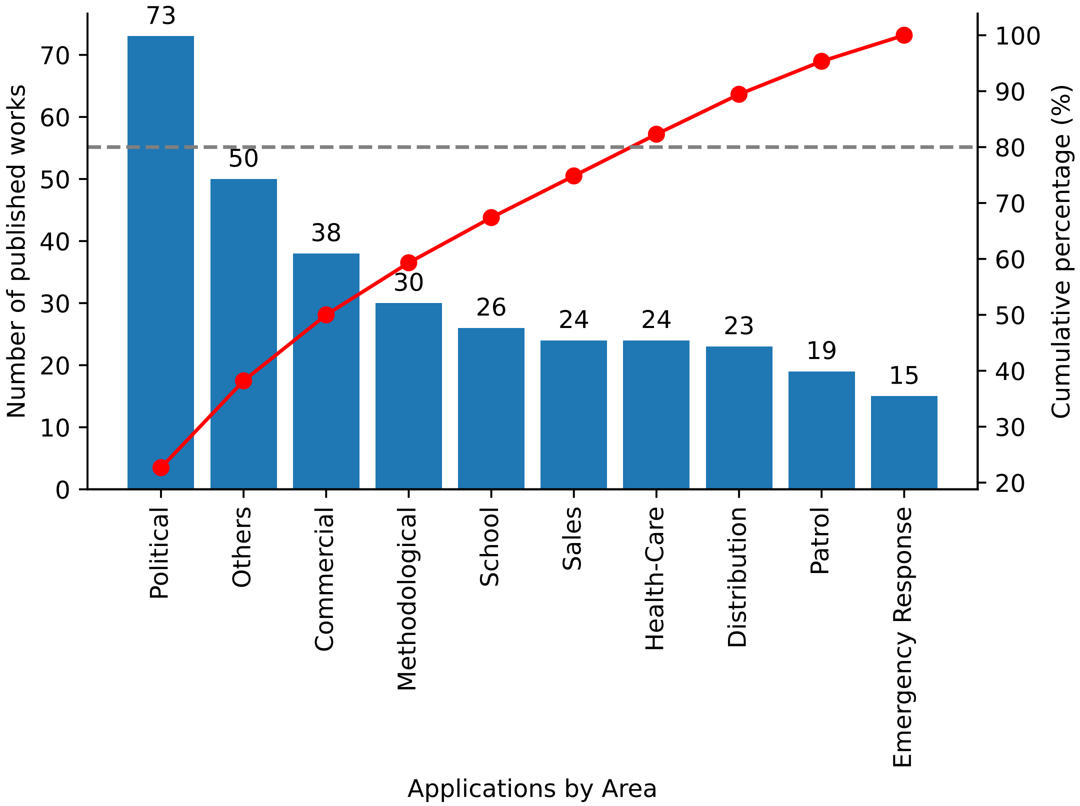

Figure 1: Chart of application areas of TDPs.

Figure 2 provides a histogram grouping the literature according to the year of publication using a different fill pattern for each application area. While the increase of publications in each area is significant, the increase is more significant on some areas like distribution districting, police patrol design, or emergency response districting.

The above table and results show a positive trend in the number of scientific publications dealing with different TDP applications. Additionally, the range of applications has significantly increased, as well as the list of major contributors to the area.

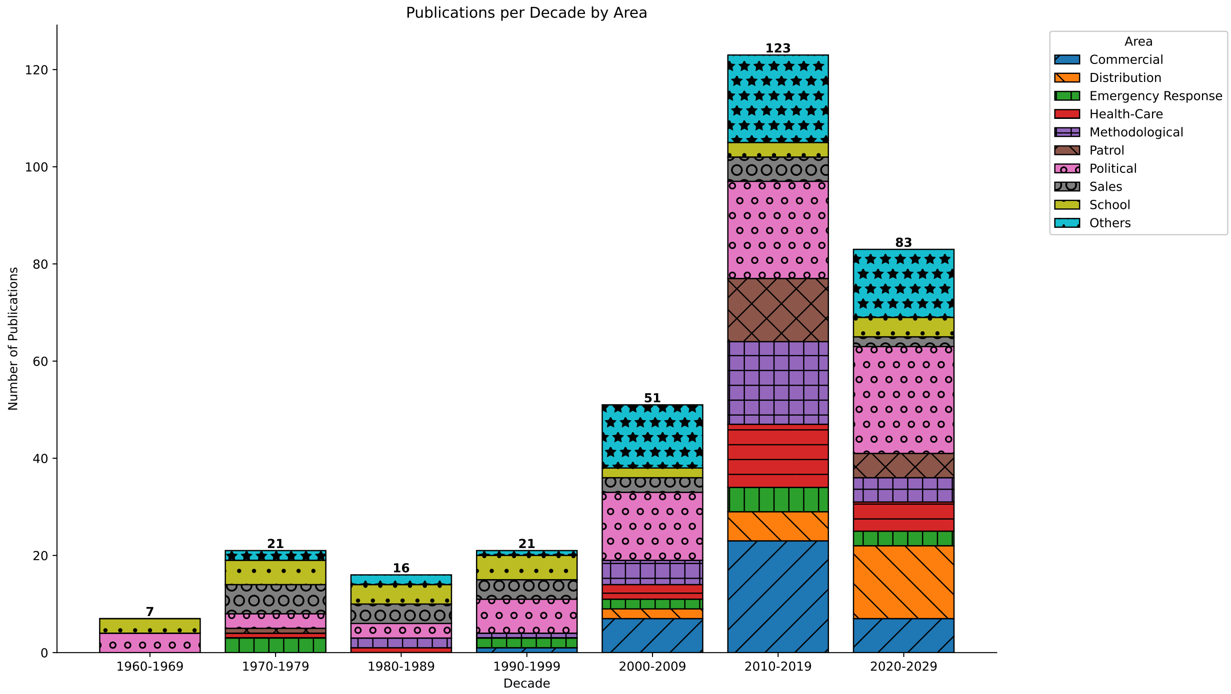

Figure 2: Histogram of publications per year by application area.

3.1 Political districting

The aim of the *Political Districting problem* (PDP) is to draw district boundaries to prevent political interference and to maximize electoral fairness. Political interference deals with the use of districting decisions to benefit specific candidates or parties; in this sense, non-compact districts are particularly vulnerable to this practice (Puppe and Tasnádi, 2008). Consequently, decision makers impose different conditions on the composition and shape of the districts. These conditions are explained below (we refer the reader to Webster (2013) for an extensive analysis of PDP criteria).

- *Integrity*, which corresponds to (D.2). This criterion mitigates *gerrymandering* by ensuring unique allocations of BUs to districts (Ricca and Simeone, 2008; Fryer and Holden, 2011).
- *Compactness*, which corresponds to (D.7). A common strategy for modeling compactness is the use of the concept of *moment of inertia*, defined as the product of the sum of the total population of each BU and the squared distance between the unit and the center of its district (Hess et al., 1965; Horn et al., 1993; Hojati, 1996). Other common strategies for ensuring compactness are to impose limits on the *diameter* of the district (Mehrotra et al., 1998), to evaluate the convexity of the district boundaries (Bozeman et al., 2018), or to minimize the number of edges to cut from an underlying graph representation of contiguity among BUs in order to generate the districting plan (Validi and Buchanan, 2022). More recently, Almeida and Manquinho (2022) present a new compact formulation for political districting. Moreover, the authors propose a new compactness measure that does not depend on geographic centers. Belotti et al. (2025) use the Polsby-Popper score to measure compactness. The reader is referred to Fryer and Holden (2011) for a detailed review of compactness measures in the context of PDP (most of which can be extended to other applications of TDP).
- *Contiguity* or *connectivity*, which corresponds to (D.4). Since the very first references of political

districting (Garfinkel and Nemhauser, 1970), connectivity has been imposed on any feasible solution. Several strategies for ensuring connectivity have been used over the decades, depending mainly on the used algorithmic strategy; ad-hoc algorithms (Bodin, 1973; King et al., 2012), metaheuristics (Bozkaya et al., 2003; Chou and Li, 2006), or MIP-based approaches (Shirabe, 2009; Validi et al., 2022), use different ways of modeling and imposing contiguity.

- *Fairness.* Electoral fairness aims at avoiding *malapportionment*, i.e., the definition of electoral districts with a different ratio of voters to representatives. An alternative for ensuring such fairness is to seek *population equality* among districts (balance criterion (D.3)), i.e., all districts should have approximately the same number of voters in order to respect the "one-man, one-vote" principle (Bozkaya et al., 2003; Wang et al., 2009; Kong et al., 2019; Dugošija et al., 2020; Swamy et al., 2022).

The above conditions are common in most of works (Gopalan et al., 2013); however, there are specific PDPs that seek additional conditions, such as *geographical considerations* (George et al., 1997; Ricca et al., 2008; Gopalan et al., 2013; Ludden et al., 2023), *socio-economic homogeneity* (Bozkaya et al., 2011), *compliance with existing administrative subdivisions* (Cirincione et al., 2000; Chou and Li, 2007; Shahmizad and Buchanan, 2025), *representation of ethnic minorities* (Mehrotra et al., 1998; Rincón García et al., 2015; Arredondo et al., 2021; Cannon et al., 2023), *similarity to old electoral districts* (Bozkaya et al., 2003; Altman and McDonald, 2011), or *competitiveness* (Swamy et al., 2022).

In some formulations, several of these conditions are simultaneously optimized. For instance, Bozkaya et al. (2003), Bozkaya et al. (2011), Fragoso et al. (2016), Kong et al. (2019), and Hirose et al. (2020) consider weighted objective functions that combine multiple criteria such as *population equality*, *compactness*, *socio-economic homogeneity*, *similarity to existing plan*, or *integrity of communities*, while Swamy et al. (2024) tackle *contiguity*, *representation*, *compactness*, and *competitiveness* in multiple separate stages.

In addition, there are some authors who consider constrained clustering (Brieden et al., 2017), physical or computational geometry models (Chou and Li, 2006; Chou et al., 2007; Ricca et al., 2008; Karimi et al., 2010; Chou, 2011; Kong et al., 2019), or even Markov chains for the PDP (Cannon et al., 2023). These models generate solutions that indirectly fulfill some conditions required of a districting plan.

Fleiner et al. (2017) provide a polynomial-time algorithm for determining an optimal partisan districting for a simplified version of the problem. The authors considered criteria such as equal representation, contiguity, and a fixed number of districts. Buchanan (2023) proposes a multilevel approach composed of three phases to solve a PDP with four objectives (compactness and three political fairness metrics). In the first phase, the instance graph is reduced (contracted) in size by merging pairs of matched units. Then, in the second phase, the reduced graph is used as an input for a Mixed Integer Linear Programming model (MILP), which is solved to optimality or near-optimality to find a Pareto-optimal solution using an ϵ-constraint method. Finally, in the third phase, the optimal solution is decontracted to analyze the trade-off among the objectives.

Liu et al. (2020) present two integer programming models considering political criteria such as fairness and competitiveness. Fairness aims to ensure fair allocation of seats to political parties, considering the distribution of voters. Competitiveness aims to maximize the number of competitive districts to prevent districting solutions that provide a clear advantage to one political party. Both models are implemented with a case study of South Carolina, USA.

When districts already exist, and some political issues are identified, a *redraw* of district boundaries should be performed to ensure a fair representation of voters. This variant is usually called *the Political Redistricting Problem (PRP)*. Some works have been developed to study the PRP (Altman and McDonald, 2011; Ludden et al., 2023; Tomczyk and Kadziński, 2024), where Altman and McDonald (2011) present an open-source software package for General Political Redistricting Analysis (BARD). This work focuses on the problem of legislative redistricting. The considered criteria were: population equality, contiguity, and compactness, together with other political and demographic criteria. More recently, Ludden et al. (2023) introduced *the bisection protocol*, which divides a region into two balanced sub-regions (those that minimize an efficiency gap), bisecting each new region, until the desired number of districts is reached, ensuring criteria such as population equality and compactness. Tomczyk and Kadziński (2024) proposed evolutionary algorithms for solving single/multi-objective PRP variants, in which criteria such as population equality, compactness, and deviation from district borders are considered as objectives to optimize.

A special case of redistricting arises as follows. In the context of general welfare reforms in Western economies, a recent trend has concerned the rationalization of administrative structures of local authorities, including a reduction in the number of administrative levels and units (through mergers and amalgamation processes) and the subsequent rearrangement of their boundaries. Along these lines, Bruno et al. (2017b) develop mathematical models to analyze the amalgamation and redistricting policy decisions implemented in Italy. Results provided by such models can provide valuable support to stakeholders and policy makers.

Dobbs et al. (2024a) propose a framework to facilitate compromise between two redistricting stakeholders by identifying a midpoint between their plans with respect to a distance metric, thereby enabling the visualization of compromise solutions that preserve common district structures. They first consider multiple distance metrics and evaluate whether midpoints with respect to these metrics are achievable and align with redistricting requirements. Then, they formulate and solve an integer program to find a midpoint (or any fractional point) between two given plans with respect to the transfer distance. They present experiments on the grid and Missouri's congressional redistricting instance to demonstrate how this method can quickly generate compromise options that align with redistricting requirements. Following up, Dobbs et al. (2024b) address a PRP and apply a local search optimization framework to analyze the interplay between political geography, constitutional requirements, and political fairness in Missouri. They use this framework to produce district plans that satisfy the new criteria and prioritize different aspects of fairness.

For further analysis of the PDP and its variants, we refer the reader to Kalcsics et al. (2005) and Ricca et al. (2013); additionally, literature reviews on the PDP are presented by Williams (1995), Arrington (2010), Tasnádi (2011), Ricca et al. (2013), and more recently Ricca and Scozzari (2020) and Buchanan (2023).

3.2 Sales districting

The *sales districting problem*, or *sales territory alignment problem*(SDP), aims to partition a commercial area (typically comprising customers or accounts) into territories, i.e., districts, each served by one salesperson or a team of salespersons. To ensure an adequate service, sales districts are expected not to overlap among each other (D.2) and to be *connected* (or contiguous, (D.4)). The goal is to design *balanced* districts (D.1) so that salespersons face a similar workload (and/or travel time) and each has a similar income opportunity from the corresponding accounts (Zoltners and Sinha, 2005).

For the SDP, *compactness* (D.3) plays a role in the efficiency of the districts since compact districts ensure that territories or districts are geographically clustered. This minimizes travel distances, reduces transportation costs, and increases the overall efficiency of routing. When districts are compact, service providers can cover areas with fewer resources and time, improving operational efficiency (Li et al., 2025).

Shirabe (2009) further mentions that the relevance of *contiguity* is minor, as an efficient business environment does not correlate to its shape. Another SDP criterion with a different interpretation is the *balance* among territories, as the size of a district is quantified using different attributes (like salesperson workloads or potential sales). While initial works such as Hess and Samuels (1971) consider squared Euclidean distances to evaluate the workload of a district as in other TDPs, the use of squared distances may not be ideal for the SDP as service costs increase proportionally to travelled distance (Marlin, 1981), and models in which the distance is measured according to road networks (Ronen, 1983; Zoltners and Sinha, 1983) are to be preferred. Alternatively, the balance is measured according to some score related to the workload of the BUs (Fleischmann and Paraschis, 1988). Drexl and Haase (1999) studied a more general context of sales force deployment involving the simultaneous resolution of four interrelated subproblems: sales force sizing, salesman location, sales territory alignment, and sales resource allocation. The first subproblem deals with selecting the appropriate number of salespersons. The salesperson location aspect of the problem involves determining the location of each salesperson in one sales coverage unit (the BUs of the problem). Sales territory alignment may be viewed as the problem of grouping sales coverage units into larger geographic clusters called sales territories. Sales resource allocation refers to the problem of allocating scarce salesperson time to the aligned sales coverage units. All four subproblems have to be resolved in order to maximize the profit of the selling organization. In their work, the authors present a nonlinear mixed-integer programming model that covers all four subproblems simultaneously. For the solution of the model, they present heuristic methods capable of solving large-scale, real-world instances. Later, Haase and Müller (2014) propose an improved method based on branch-and-price for the same sales force deployment problem, yielding better results.

In addition, alternative objectives may better portray the goals of the decision-maker. For instance, several articles consider profit-maximization (Lodish, 1975; Shanker et al., 1975; Zoltners, 1976; Beswick, 1977). Skiera and Albers (1998) emphasize that profit-maximization should be preferred to workload balancing. The reasoning stems from the fact that many factors influence sales, making it very difficult to generate territories with equal sales. Similarly, it is also difficult to assess workload due to uncertainty. However, Zoltners and Sinha (2005) state that such a workload-balancing/profit-maximization distinction may have very little practical value. There are also approaches that try to cover the multi-objective nature of the problem at hand (Deckro, 1977).

Other issues considered by previous works include multi-period models (Bender et al., 2018) and models including scheduling and routing decisions (Hervert-Escobar and Alexandrov, 2017, 2018; Bender and Kalcsics, 2020). Moya-García and Salazar-Aguilar (2020) present a case study of an SDP arising in a Mexican company. They propose a simple heuristic for this problem and analyze its performance in two real cases from the company.

Chen et al. (2025) studied an SDP arising in the operational management of online-to-offline processes. They propose an efficient districting system comprising a hierarchy-aware map segmentation module and a block aggregation module. The map segmentation module partitions large geographic areas into small blocks using hierarchical road networks and other physical boundaries. The block aggregation module introduces a novel mixed-integer programming model, incorporating a global Polsby-Popper score to enhance district compactness.

The literature also provides specific surveys for the SDP, such as the works by Zoltners and Sinha (1983), Howick and Pidd (1990), and Zoltners and Sinha (2005).

3.3 School districting

A *School Districting problem* (ScDP) consists of assigning city blocks (corresponding to census tracks, neighborhoods, or grid blocks) to schools according to a set of criteria defined by the school system.

The most common objective is to minimize the total distance traveled by the students, an objective related to the *compactness* criterion, condition (D.1). Common conditions include: (i) *balancing* some attributes of the students among the districts, such as race (Clarke and Surkis, 1968; Heckman and Taylor, 1969; Liggett, 1973; Holloway et al., 1975; McKeown and Workman, 1976; Lemberg and Church, 2000; Caro et al., 2004), gender, academic performance, socio-economic profile, or transportation difficulties (Bouzarth et al., 2018); (ii) assigning each block to exactly one school unless the school does not offer the grade, condition (D.2) (Liggett, 1973; Holloway et al., 1975; McKeown and Workman, 1976; Ferland and Guénette, 1990; Lemberg and Church, 2000; Shirabe, 2012); (iii) complying with school capacity limits, condition (D.5) (Ferland and Guénette, 1990; Caro et al., 2004; Shirabe, 2012; Bouzarth et al., 2018); and (iv) *Contiguity* (D.4), although a desirable property was not considered explicitly within the problem formulation until Caro et al. (2004).

Another feature that many ScDP studies incorporate is geographical factors. For instance, models may seek to ensure that the resulting districts do not traverse land obstacles such as railroads, rivers, or streets with heavy traffic (Holloway et al., 1975; Jennergren and Obel, 1980), the model may evaluate the difficulty of the trips of the students (Sutcliffe et al., 1984), or the model may impose a distance limit for the student trips (D.5) (Clarke and Surkis, 1968; Belford and Ratliff, 1972; Caro et al., 2004; Ashlagi and Shi, 2015).

From a practical standpoint, it is important to seek a districting plan that maintains a certain similarity with the plan in use (Holloway et al., 1975; Lemberg and Church, 2000) while considering future school consolidation (Church and Murray, 1993) plans. School consolidation tries to reduce education costs by closing one or more schools and assigning (consolidating) their students to some of the remaining schools. Such a procedure is common in developed countries to mitigate the declining number of students in the educational system.

Sutcliffe et al. (1984) present a goal programming model to analyze the problem of proposing catchment areas for the secondary schools in the Reading (UK) area. Six goals are identified: distance, difficulty of the journey, racial balance, reading-age retarded balance, sex balance, and capacity utilization. They solve a relatively small case study, obtaining results that dominate current practice.

Taking a school districting problem as an example, Shirabe (2012) introduces a systematic approach to designing a map algebraic procedure for a cartographic allocation problem with capacity constraints. *Map algebra* is a methodology for organizing and processing digital cartographic data in a geographic information system (GIS). While its capabilities to describe the existence of (hidden) patterns in data have been well studied, its capabilities to prescribe new patterns in response to specific requirements have not been much explored.

More recently, Ozel et al. (2025) revisited the school district design problem with a focus on codesigning with community partners. They introduce a new compact formulation that incorporates multiple decisions simultaneously by assigning students in each geographic unit to a set of schools (e.g., elementary, middle, high schools, and schools with specialized programming) with a single composite variable, referred to as a *stream*. This formulation is computationally efficient and easily reconfigurable for evolving problem specifications that are endemic to community codesign. These features were essential in the district redesign process described in that paper, allowing the community to iteratively develop proposals to address inequities in access to education and improve the student assignment process.

Guan et al. (2025) present an algorithmic school rezoning framework and apply it to a large-scale rezoning effort impacting over 50,000 students through an ongoing researcher-school district partnership. The framework is designed to incorporate feedback from community members and policymakers, both by deciding which goals are optimized, and also by placing different levels of importance on goals through weights from community surveys. Empirical results reveal the framework's ability to surface school redistricting plans that simultaneously advance a number of objectives often thought to be in competition with one another, including socioeconomic integration, transportation efficiency, and stable feeder patterns (transitions) between elementary, middle, and high schools.

The variety of constraints and objectives can also yield multi-objective optimization formulations. For instance, Diamond and Wright (1987) consider distance, geographical, and school utilization objectives within a parametric programming model. November et al. (1996) try to minimize social, racial, economic, and resource differences between enrolled students, while Lemberg and Church (2000) consider both cost and similarity among districting plans across different time periods.

Comprehensive surveys for the ScDP are available in Sutcliffe et al. (1984) and Caro et al. (2004).

3.4 Commercial districting

Distribution firms design pickup and delivery areas to lower response times, divide the workload among workers, familiarize truckers with their assigned area, and ensure efficient use of resources (e.g., fuel, trucks). In this context, a TDP provides support either as a standalone problem, referred to as the *Commercial Districting Problem* (CDP), or as part of a two-stage framework in which the TDP constitutes a first-stage tactical problem whose solution serves as the basis for operational-level vehicle routing decisions within each district. We refer to the latter as *distribution or routing districting*, which is discussed in the next subsection.

We can view the CDP as an SDP (Section 3.2) in which the objective is to generate districts for the distribution of goods. As in other TDPs, delivery districts are expected to be (optimally) balanced regarding different activity measures (i.e., product demand, number of customers, or the workload) (D.1) and should present good topological features such as compactness (D.3) and contiguity (D.4) to support the effective delivery operations. As in the SDP, some works show that some of these characteristics may not apply to aspecific situation. For example, in Aly et al. (2024), the authors highlight that contiguity requirements were not seen as an essential issue by their industrial partner as long as there exists a road between the delivery points assigned to a driver and thus their solution approach does not consider contiguity.

Ríos-Mercado and Fernández (2009) formalize a CDP in which a p-center objective (minimization of the maximum distance between BUs and their district center) is used to define a *dispersion* measure. Additionally, balance requirements for each activity measure and connectivity conditions are imposed.

CDPs usually resort to p-center or p-median *dispersion* metrics. Salazar-Aguilar et al. (2011a) present linear and quadratic integer programming models for these two types of problems, one under the p-center metric for minimizing dispersion, and the other based on the p-median metric. These models also consider balancing and connectivity constraints. The authors propose a cut-generation exact optimization framework for solving these models. One interesting finding was that the p-median-based models are more LP-friendly providing tighter LP relaxations than those based on p-center metrics.

In a more recent study, Sandoval-Esquivel et al. (2019) compared both models in terms of their robustness. The results indicate that the p-center-based model was more robust than the p-median-based model. Díaz et al. (2020) and Sandoval et al. (2020) present other exact algorithms for the p-median and p-center based CDPs, respectively.

Given the inherent computational complexity of CDPs, most of the work done has been on heuristics. Some heuristics are applied to p-center based models, such as the reactive GRASP of Ríos-Mercado and Fernández (2009). Others are applied to p-median-based models such as the location-allocation heuristic of Ríos-Mercado et al. (2021).

A key observation is that these p-center and p-median metrics may be too costly to evaluate because they depend on a particular centroid (center or median), which is solution-dependent. As a result, other non-centroid metrics, such as the diameter, have also been considered, for example, the GRASP-path relinking approach of Ríos-Mercado and Escalante (2016), the Variable Neighborhood Search approach of Aly et al. (2024) or the GRASP of Ríos-Mercado and Salazar-Acosta (2011) for a CDP that also considers routing.

Other works consider alternative objectives such as cost minimization (Novaes et al., 2000; Konur and Geunes, 2019), waiting times during deliveries (Camacho-Vallejo et al., 2019), customer service satisfaction (Sandoval et al., 2022), the minimization of late deliveries (Álvarez-Miranda and Pereira, 2022), or a penalized violation of the balancing constraints (Vargas-Suárez et al., 2005).

In addition to metaheuristics, computational geometry approaches (Galvão et al., 2006) have also been studied. These are particularly useful when the underlying network is not equivalent to a Euclidean, rectangular, or ring-radial metric.

Furthermore, there have been models that consider other additional requirements, such as similarity with existing plan (López-Pérez and Ríos-Mercado, 2013; Ríos-Mercado and López-Pérez, 2013) where the goal is to find a districting plan that is similar to the current plan, or joint assignment (Caballero-Hernández et al., 2007), in which a given set of BUs must all belong to the same territory.

There are also some models that, rather than seeking to minimize dispersion, aim to find the minimum number of territories to service customers within a specified coverage radius (Jarrah and Bard, 2012; Moreno et al., 2020).

In terms of developing lower bounds, Elizondo-Amaya et al. (2014) present a lower bounding scheme for a TDP with p-center-based minimization and no connectivity constraints. The results indicate this bound is tighter than the LP relaxation.

CDPs within multi-objective programming frameworks have also been studied. These include exact optimization algorithms based on ε-constraint methods (Salazar-Aguilar et al., 2011b; de Abreu et al., 2026) and heuristic approaches (Salazar-Aguilar et al., 2012b, 2013; Álvarez-Miranda et al., 2025).

3.5 Distribution districting

While districting mainly involves tactical decisions, the nature of distribution problems usually involves a routing step within each district. Sometimes this issue is addressed separately, that is, find a districting plan first, and then solve the routing operational problem in a later stage. However, there are a few works in the literature that have tried to address these two decisions simultaneously in districting-routing frameworks, or considering districting as an initial subproblem to ease the computational burden of the last-mile routing problem (Ramírez-Villamil et al., 2023). This is typically studied in a two-stage approach, where the first stage chooses the distribution areas, and the second stage evaluates routing decisions. This can be done iteratively as proposed by Zhou et al. (2021). A similar work is proposed by Zhen et al. (2023), where the authors study a TDP considering the demand frequency of customers over a time horizon. A set partitioning model and a solution algorithm based on column generation were proposed, in which each district is solved as a pricing problem using dynamic programming. The proposed model considers a criterion of workload through operational constraints (each district has a maximum capacity and each route has a maximum time limit). The authors recognize the importance of compactness and contiguity in territorial design, although they do not specify whether the latter are directly included in the formulation of the model.

Although there exist a few deterministic approaches (Schneider et al., 2015; Bender et al., 2020; Sandoval et al., 2022), this two-stage approach enables decision makers to incorporate the inherent uncertainty associated with daily operations leading to integer stochastic programming models with recourse (Haugland et al., 2007; Zhong et al., 2007; Lei et al., 2015, 2016; Sudtachat, 2016; Bruni et al., 2026).

In stochastic CDPs, we usually assume that demand is uncertain (Haugland et al., 2007; Diglio et al., 2021). For example, Diglio et al. (2021) address a distribution territorial design problem, which considers four different probability distributions for demand (Uniform, Lognormal, Poisson, and Exponential). The aim is to minimize the total cost of assigning basic units to districts, which can be seen as a compactness criterion, subject to contiguity and chance-constrained balancing requirements. On the other hand, uncertainty can also be applied to the location of the customers when the CDP is solved and is only revealed during the routing stage (Carlsson and Devulapalli, 2012; Lei et al., 2012).

Alternatively, Carlsson and Delage (2013) present a robust optimization approach where the location of the demand is not known. Lei et al. (2015) propose a model with multi-periods and multi-depots, which includes the variation of the customers of territory over the periods of the planning horizon, while Bender et al. (2016) also provide a multi-period model. More recently, Carlsson et al. (2024, 2025) propose a data-driven zoning optimization model that integrates the additively weighted Voronoi diagram and the vehicle routing problem through a subgradient algorithm. This is the first zoning optimization model that considers general multi-vehicle zones with uncertain demand. Their motivation stems from a real-world application on last-mile delivery at an Asian company.

While there are no specific reviews for commercial or distribution districting, there are some articles with extensive descriptions of the state-of-the-art of the problem (Kalcsics and Ríos-Mercado, 2019; Moreno et al., 2020).

3.6 Police patrol districting

Police patrolling plays several roles, including crime prevention, criminal apprehension, law enforcement, order maintenance and traffic enforcement, among others (Zhang and Brown, 2013; Bucarey et al., 2015).

Its operation involves the division of the city into police command areas or patrol areas assigned to one or several police units. Such a division corresponds to the Police Patrol Districting Problem (PPDP).

The main criteria considered in a PPDP are: *workload* balance, measured in terms of the crime rate of the districts, condition (D.1); *response time* to crimes, both to combat and to prevent crime, condition (D.5); *compactness*; condition (D.3); and *contiguity*, condition (D.4), to facilitate patrolling operations and jurisdiction administration. The assignment of a BU to a single district is not always required. For example, Curtin et al. (2010) introduce the concept of backup coverage to assign high-priority BUs to more than one patrolling district.

An initial trend in PPDP to evaluate workload is to focus on traveling and/or service times. Mitchell (1972) considers a PPDP model that minimizes travel distances to accidents while balancing the workload of each district. A similar model is proposed in Wheeler (2019). An alternative metric for the workload is given in Curtin et al. (2010), where the workload is measured according to the number of calls (or incidents) of the district.

Camacho-Collados et al. (2015) focus on crime prevention. To increase agents' presence as a crime deterrent, agents are concentrated in areas with a higher risk of crime. The proposed model considers different attributes of the districts, like the risk of a crime, their *compactness*, and the possibility of *mutual support*, a generalization of the backup coverage proposal of Curtin et al. (2010). More recently, Liberatore et al. (2023) proposed a multi-objective mixed-integer program to distribute patrol agents to districts considering territorial (low/high-risk areas), proportioned police exposure, and workload fairness (balanced exposition to the risk) criteria. The aim is to maximize the total fairness value while ensuring compactness, integrity, contiguity, and balance.

Note that the stochastic behavior of the crime activity is very important for the PPDP (as the patrolling plan affects crime patterns). The issue has been added to PPDPs through simulation (Chen and Yum, 2010; Zhang and Brown, 2013, 2014) and forecasting-based methodologies (Camacho-Collados and Liberatore, 2015).

Camacho-Collados and Liberatore (2015) propose a decision support system (DSS) with a methodology for the temporal and spatial description of uncertain crime events, and a tool to compute the performance of the proposed district.

Zhu et al. (2022) present a data-driven optimization framework for redesigning police patrol zones in an urban environment. The objectives are to rebalance police workload along geographical areas and to reduce response time to emergency calls. They develop a stochastic model for police emergency response by integrating multiple data sources, including police incident reports, demographic surveys, and traffic data. Using this stochastic model, they optimize zone-redesign plans using a mixed-integer linear programming model. Their proposed design was implemented by the Atlanta Police Department in March 2019. By analyzing data before and after the zone redesign, they showed that the new design has reduced the response time to high-priority 911 calls by 5.8% and the imbalance of police workload among Atlanta's zones by 43%.

Multi-objective models for PPDP are common. These models aim to depict the PPDP more realistically by including the conflicting goals involved in police patrolling. Zhang and Brown (2013, 2014) propose a bi-objective model that minimizes both average response time and workload variability, while Camacho-Collados et al. (2015) and Liberatore and Camacho-Collados (2016) develop a bi-criteria model to minimize both the average and the maximum workload among districts within a single weighted objective function. Chen et al. (2018) minimize a weighted function considering the risk, area, and diameter of districts.

A variant of the PPDP is the *Districting and Routing problem for Security Control* (DRPSC) (Prischink et al., 2016). This problem focuses on security controls geared towards the prevention of theft and vandalism in business buildings. The DRPSC splits the problem into a districting part, where they assign BUs to districts, and a routing part, where routes are defined while ensuring workload limits and time windows. The problemis later approached by Kim et al. (2017), where the authors soften some constraints and seek a solution in which the unfulfillment of these soft constraints is minimized.

Vlćek et al. (2024) present a strategic decision support system to assess different districting layouts, department locations, staffing decisions, and dispatching strategies for police patrol districting. They present an integer programming model based on the p-median problem, considering contiguity and compactness of district layouts. Then, they use a discrete event simulation that accounts for the variability of spatial and temporal incident patterns and the driving times to evaluate the district layouts according to several criteria. Their simulation results demonstrate that their proposed district layouts can lead to a reduction of the response time by up to 14.52% while also lowering the dispatch time, the overall driving time, and the number of unanswered calls for service.

Liberatore et al. (2020) present a systematic review of the literature related to the PPDP. Contributions are categorized in terms of attributes and the solution methodology adopted. Other papers where the reader can find models and solution schemes for PPDPs can be found in Zhang and Brown (2013) and Camacho-Collados and Liberatore (2015), while Bucarey et al. (2015) include an extensive description of the problem.

3.7 Health care districting

The *health care districting problem* (HCDP) aims to partition an area into districts, each being assigned to the responsibility of one team (physicians, nurses, physiotherapists, pharmacists, and more) or hospital. The districting plan should enforce a *balanced* workload (D.3) and territorial *integrity* (D.2) among the districts. Balancing workload tries to ensure that each district covers a similar number of patients, while territorial integrity tries to avoid interference between the responsibilities of different healthcare teams.

Other topological requirements, such as *compactness* (D.1) and *contiguity* (D.4), are used to simplify mobility within a district and must account for mobility limitations arising from major barriers such as railway lines or motorways (Blais et al., 2003; Benzarti et al., 2013).

To the best of our knowledge, the first paper on HCDP corresponds to Ghiggi et al. (1976). The model considers the following conditions: (i) The region is made up of a certain number of indivisible communities; (ii) each community must be univocally assigned to a district, i.e., no overlap among districts is allowed, condition (D.2); (iii) there is a hard limit on the capacity (number of beds of the hospital) of every facility, condition (D.5); (iv) each district must be connected, condition (D.4); and (v) the population of each district must lie within some bounds, condition (D.1). The model optimizes a distance-based metric.

Distance optimization metrics are common (Pezzella et al., 1981; Benzarti et al., 2013), but alternatives exist. Minciardi et al. (1981) try to optimize the quality of service through a weighted rating function, Blais et al. (2003) aim at optimizing the ease of the visiting personnel to travel through their district while ensuring *workload balance*. Steiner et al. (2015) try to optimize the homogeneity of the districts while increasing the decentralization of medical services within the area. Lin et al. (2020) consider a multi-objective problem that simultaneously optimizes hiring costs, district workload balance, and district compactness. Yanık et al. (2019) optimize a weighted function for the design and management of primary health care services in a multi-period planning horizon. Other works have addressed problems where the demand (Mostafayi Darmian et al., 2021) or the travel and service times (Nikzad et al., 2021) are uncertain. Alternative computational geometry methods have also been proposed (Furuta et al., 2005).

Another interesting application of districting models is the one related to organ procurement. For instance, severe geographic disparities existing in liver transplantation have been successfully addressed through redistricting models (Gentry et al., 2013, 2015a,b,c).

Emiliano et al. (2017) provide a review on home health care planning in which the HCDP plays an important role. Then, in a follow-up work, Emiliano et al. (2025) developed a decision support system (DSS) that integrates districting and fleet management for the HCDP. Both problems are formulated using a multi-objective programming model and solved via the augmented weighted Tchebycheff method. The DSS was applied to a Brazilian city, resulting in significant improvements in districting and fleet management.

Yanık and Bozkaya (2020) review the literature in the health care domain. They discuss the most relevant studies in the literature as well as a direction for future research. The authors classify the health care districting problems into three main areas: home care services, primary and secondary health care services, and emergency health care services. Other works where the reader may find additional information on literature for HCDPs are Steiner et al. (2015) and Lin et al. (2017).

3.8 Emergency response districting

The *Emergency Response Districting problem* (ERDP) covers a group of spatially oriented resource allocation problems arising in different areas such as, electricity service, mobile repair units, police, fire, and medical emergency services. The main conditions considered in an ERDP are workload *balance* (in other words, a homogeneous distribution of demand across districts), condition (D.1); *compactness*, condition (D.3); and integrity, condition (D.2), which serve as constraints while aiming to improve operational efficiency.

Given the nature of emergencies, it is common to use queueing or simulation-based models for the problem. Larson (1974, 1975) developed the *hypercube queuing model* (HQM), which is often used in these models (Iannoni et al., 2008; Geroliminis et al., 2009, 2011; Mayorga et al., 2013). Berman and Larson (1985) consider a special case of the districting problem for a demand-responsive service system in which queuing is allowed, and two districts are sought. Each service territory, with its response unit, behaves as an independently operating $M/G/l$ queuing system. The authors seek to determine the optimal service territories, given fixed home locations for each of the service units, to minimize the average response time (queuing delay plus travel time) to a random customer. The authors develop some heuristics for arbitrary demand rates.

When considering emergency medical systems (EMS), the main districting criteria are *workload* balance, condition (D.1); *mean user response time*, the *loss probability* (the ratio of calls that are not serviced by the EMS) and the *inability to fulfill on threshold* (ratio of calls with response times exceeding a specific period) (Bandara and Mayorga, 2011). Iannoni et al. (2008, 2011) propose a method based on HQM that supports two combined configuration decisions: the location of ambulance bases along the highway and the districting of the territory based on the *mean user response time*, the *balance of ambulance workloads*, or the *ratio of calls with response times exceeding a predetermined threshold.* Iannoni et al. (2009) consider a multi-objective version of the problem in which all of the objectives are jointly optimized.

Geroliminis et al. (2009, 2011) proposed the spatial queueing model (SQM) that seeks to locate servers and assign their areas of responsibility. This model considers different service rates that depend on the characteristics of the emergency. Mohammadi et al. (2014) improved the SQM with the incorporation of the deployment cost of the emergency vehicles.

An extension to the work by Geroliminis et al. (2011) is considered in Akdoğan et al. (2018), where a location-emergency-vehicles model to minimize the *mean response time of service* is put forward. The model is based on the approximate queuing model (AQM), requiring simulation to evaluate the performance of a solution.

Bertolazzi et al. (1977) suggest a model to define the response area of firefighting units. The model considers that each unit answers all of the alarms coming from its area, unless it is already busy, and travel time is used to measure compactness.

Zografos et al. (1992, 1998) propose an iterative methodological framework to reduce the response time of emergency repair trucks (ERTs) by combining the optimization model with subsequent simulations of the emergency repair operations for each territory. The main issue considered in this model is the generation of homogeneous service areas that minimizes the weighted travel times according to both the *workload* of the ERT and the area *covered* by each unit.

Ansari et al. (2017) propose a mixed integer linear programming model that determines how to locate and dispatch ambulances through district design. The model allows for uncertainty in both ambulance travel times and ambulance availability, and it maximizes the coverage level, i.e., the fraction of high-priority calls that can be responded to within a fixed-time threshold. Their proposed MILP model determines the stations where ambulances should be located and assigns each call location to the open ambulance stations according to a preference list. The preference list is a rank ordering of the ambulances to assign to patients at a call location, from the most preferred to the least preferred. The preference lists partition the region into a series of response districts that depend on ambulance availability, and the model balances the workload among the servers while maintaining contiguity in the first-priority response districts. Their results suggest that the reduction in coverage to maintain contiguity and balanced workloads among the ambulances is small.

Sudtachat et al. (2020) propose a modeling approach for joint relocation and districting strategies in EMS systems. This combined approach aims to increase the efficiency of EMS systems. They analyze the decision regarding how to partition the service area into smaller sub-areas in which each sub-area operates independently under separate relocation strategies. Once the district solution is determined, a tabu search metaheuristic is used to allocate stations, and zones to districts and the optimal nested-compliance model solution is applied to each sub-area. The objective is to maximize the overall realized expected coverage. Their numerical results show the benefits of their model over the adjusted maximum expected covering location problem (AMEXCLP) based on average realized coverage and the fraction of covered calls responded to by the first closest ambulance in the dispatching rank list.

Enayati et al. (2020) introduce a two-stage stochastic mixed-integer programming model for an ambulance location problem under uncertainty. The proposed model recommends how to locate ambulances at the waiting sites in the service area, and how to assign a set of demand zones to each ambulance at different backup levels. The objective function is to maximize the expected number of covered calls while restricting the workload of each ambulance. A different ERDP considers the location of shelters to earthquakes. Hu et al. (2014) developed a bi-objective programming model minimizing costs and evacuation distance while ensuring *capacity* (D.5) and *contiguity* (D.4) conditions.

Baghersad et al. (2023) present a districting approach to the problem of designing contiguous policy zones for pandemic response. The goal is to find the optimal set of locations or jurisdictions to create policy coordination zones that would help develop appropriate response policies guided by geospatial approaches when addressing this disaster. They propose optimization models and algorithms to identify coordination communities based on the natural movement of people.

Recently, Regis-Hernández et al. (2023) addressed a districting problem for emergency medical services, which involves dividing a territory into districts and determining the location and allocation of ambulances, creating compact and balanced districts to minimize response times.

Given the variability among the areas associated with ERDPs, there are no previous literature reviews for this area of application; however, Yanık and Bozkaya (2020) present a section dedicated to reviewing literature on emergency health care services.

3.9 Other districting applications

Other areas of application have been explored in the TDP literature. Some of these works suggest alternative criteria, graph representations, or solution approaches that are relevant to mention. Among these, we mention the *Solid Waste Collection Districting problem* (SWCDP) (Male and Liebman, 1978; Hanafi et al., 1999; Lin and Kao, 2008; Mourão et al., 2009; Ramos and Oliveira, 2011; Cortinhal et al., 2016; Mostafayi Darmian et al., 2020), urban planning (Tavares Pereira et al., 2009), schedule of inspection activities (Kallioras et al., 2015), and reliable public census data (Liu et al., 2024), among others.

The earliest approach of SWCDP is due to Male and Liebman (1978). Their work aims to construct sectors for household waste collection, where the objective is to minimize the total deadheading time. Districts should be *contiguous* and *balanced* in terms of collected waste. This study considers the service area as an undirected graph. A single vehicle trip per sector must be performed to collect waste using a capacitated vehicle. Mostafayi Darmian et al. (2020) propose a multi-objective location-districting SWCDP optimization model, which aims to minimize the integration of economic, environmental, and social objectives while achieving criteria such as exclusive assignment, contiguity, balance, and compactness.

Tavares Pereira et al. (2009) consider a districting problem in urban planning. They introduce three new indices to compare territory partitions: compatibility, inclusion, and distance, for connected, undirected, and planar graphs, all of which have importance for real-world planning situations.

Kallioras et al. (2015) proposed a two-stage optimization approach to model the problem of scheduling transit stop inspection and maintenance activities. In particular, the first stage involves the districting of the bus stop locations into areas of responsibility for different inspection and maintenance crews, while the second stage determines the schedule of inspection activities. This model is then applied to the Athens bus system in Greece.

Royuela and Duque (2013) and De Fréminville et al. (2015) consider the districting of areas to define homogeneous price areas, as Deng and Pantuso (2026) do, where the focus is to define such pricing zones for carsharing. Yanık et al. (2016) propose a p-median-based formulation with special criteria for obtaining districts such that the energy requirement of each one matches the available green energy potential in the same cluster in a real-world instance in Turkey. In this model, the objective is to optimize the compactness to reduce energy transmission costs. However, the study also considers the maximum unsatisfied demand and the total deviation (i.e., the sum of unsatisfied demand values for all of the territories). Gliesch et al. (2017) use a TDP scheme to support the distribution of land tracts among farmers for the Brazilian National Institute of Colonization and Agrarian Reform (INCRA). The objective is to balance land productivity among tracts.

The work by Lum et al. (2017) is a clear example of how districting models can be helpful in related contexts. In their paper, the authors address a windy rural postman problem, in which the objective is to route a homogeneous fleet of vehicles such that the cost of the longest route is minimized. Although the core decision-making process is not districting, the authors use a districting model to achieve their goal of finding compact and balanced routes in street networks.

A districting problem arising in meter reading in power distribution networks is studied by de Assis et al. (2014). They consider two optimization objectives: compactness and homogeneity of districts. They develop a GRASP metaheuristic embedded in a scalarization technique for handling the two objectives. They applied their method to a case study in São Paulo, Brazil.

Another interesting application lies in the collection of waste electrical and electronic equipment, where the goal is to assign collection bins (BUs) to companies (districts) according to certain market share requirements. What makes this problem different is that the objective is to avoid regional monopolies which is achieved by maximizing a dispersion function (Fernández et al., 2010; Ríos-Mercado and Bard, 2019; Ríos-Mercado et al., 2023).

There have also been studies addressing districting decisions on planning models for microfinance institutions, introducing risk factors. In this class of models, the locations of the bank branches must be determined (seen as territory centers) and customers must be allocated to these branches, considering planning criteria such as total workload, monetary amount of loans, and profit allocation, while balancing the territory risk (López et al., 2015; López Pérez et al., 2020; Salazar Treviño, 2024).

A special class of districting problems is the so-called family of max-p-regions problems. The p-regions problem (Duque et al., 2011; Kim et al., 2013) involves the aggregation or clustering of n small areas into p spatially contiguous regions while optimizing some criteria. A variation of this problem is the max-p-regions problem (Duque et al., 2012), a spatially constrained clustering or districting problem that involves the districting of a set of geographic areas into the maximum number of homogeneous regions such that the value of a spatially extensive regional attribute is above a predefined threshold value. One of the best uses of these types of models is the definition of study regions. For example, in the statistical analysis of rates for small area estimation (i.e., crime rates, disease rates, unemployment rates) the precision with which the underlying rate can be measured is inversely related to the size of the population within the district. It is often desirable to combine small contiguous units so as to increase the precision of the estimation. In these cases, for instance, the max-p-regions model can be used to design new study regions where (a) the loss of observations is minimized, as the approach seeks to perform the minimum number of spatial aggregations; (b) the degree of aggregation bias is minimized because intraregional homogeneity is maximized; the degree of aggregation bias is minimized because intraregional homogeneity is maximized; and (c) the new regions ensure valid statistical inference. There are special cases of each of the above problems, named the p-compact-regions problem (Li et al., 2014b,a) and the max-p-compact-regions problem (Feng et al., 2022), where one seeks compact districts as an added feature.

Kim and Kim (2020) study a districting problem arising when determining the location of polling facilities and polling stations tailored to the regulations of the voting process of South Korea. They present a case study of an area with several precincts in Seoul, South Korea. The results indicate the need to relocate the

Table 1: Summary of papers addressing other TDP applications (discussed in Section 3.9)

Author (year)	**Application**	**Instance description**	**Objective**	**Criteria**
Segal and Weinberger (1977)	Turfing		Minimize workload differences among districts	(D.1),(D.2)
Male and Liebman (1978)	Solid waste collection		Minimize number of districts	(D.1), (D.3), (D.4), load capacity
Marlin (1981)	Logistics	Wisconsin, USA	Minimize total distance	(D.1), (D.2), (D.3)
Hanafi et al. (1999)	Solid waste collection	Quito, Ecuador; Echirolles and Saint-Martin d'Heres, France	Minimize differences among districts (workload time, variance workload time) and number of BU per sector	(D.1),(D.2) and demand
Muyldermans et al. (2002)	Salt spreading	Antwerp, Belgium	Minimize total distance (and dead mileage) and the number of vehicles	(D.1), (D.3), (D.4), capacity
Bergey et al. (2003a)	Electrical power		Minimize deviation of a districting plan and total distance	(D.1),(D.2), (D.3), (D.4)
Bergey et al. (2003b)	Electrical power		Minimize deviation of a districting plan and total distance	(D.1), (D.2), (D.3), (D.4)
Muyldermans et al. (2003)	Road network design	Road network of Flanders, Belgium	Minimize total deadheading distance and the number of vehicles	(D.1), (D.2), (D.3), (D.4)
Yun and Yang (2003)	Road network design		Minimize total distance	(D.1), (D.2)
Ricca (2004)	Territorial aggregation	Rome, Italy	Minimize inertia moment, population differences among districts and maximize compactness	(D.1), (D.2), (D.3) and predetermined number of the districts to be located
Marianov and Fresard (2005)	Jail assignment	Chile	Minimize cost of the system (new and expansion facilities)	(D.2) facilities to be located and capacity
Muyldermans (2006)	Road network design	Road network of Flanders, Belgium	Minimize total deadheading distance	(D.1),(D.2), (D.4)
Haugland et al. (2007)	Node-based districting	Adapted VRP instances	Minimize expected routing cost	(D.2),(D.3), (D.4), (D.5)
Tavares Pereira et al. (2007)	Product pricing	Paris, France	Minimize differences among districts (surface, population, active population, rail station) and within districts (by number of cars)	(D.1),(D.2), (D.3), (D.4), similarity to the old plan
Lin and Kao (2008)	Solid waste collection	Taiwan	Minimize the difference of MSW quantity per road length among districts	(D.1), (D.2), (D.3), (D.4)
Mourão et al. (2009)	Solid waste collection		Minimize total distance	(D.1), (D.2), (D.3), (D.4)
Fernández et al. (2010)	WEEE collection	Germany and Spain	Maximize territory dispersion	(D.1), (D.2), (D.3)
Duque et al. (2011)	Regionalization	Synthetic spatial dataset	Minimize the total intraregional dissimilarity	(D.2), (D.4)
Ramos and Oliveira (2011)	Solid waste collection	Alentejo, Portugal	Minimize total distance and workload differences among districts	(D.1),(D.2),(D.3),
Datta et al. (2012)	Delineating census tracts	Census Metropolitan area of London, Ontario, Canada	Minimize compactness deviation and worload per population and zone	(D.1), (D.2),(D.3), (D.4), (D.5) and range of zones
Duque et al. (2012)	Regionalization	Synthetic spatial dataset	Minimize the total intraregional dissimilarity	(D.2)
Kim et al. (2013)	Spatial community detection and regionalization	Synthetic spatially embedded graphs and a real-world spatial network	Minimize cut edges and maximize compactness	(D.1), (D.2), (D.4), sink constraints, and flow feasibility.
Royuela and Duque (2013)	Housing sub-markets	Barcelona, Spain	Minimize total travel cost	(D.1), (D.2), (D.3), (D.4)
de Assis et al. (2014)	Electrical power	Sao Pãulo, Brazil	Multi-objective: Achive maximum territory compactness and balance	(D.1), (D.2), (D.3), (D.4)
Li et al. (2014a)	Regionalization	Southern California	Maximize region compactness	(D.1), (D.2), (D.3)
Li et al. (2014b)	Regionalization		Maximize region compactness	(D.1), (D.2), (D.3)
De Fréminville et al. (2015)	Product pricing		Minimize cost price differences within districts	(D.1), (D.3), boundaries for number of districts, splitting are allowed
Kallioras et al. (2015)	Transit inspection	Athens, Greece	Minimize work-time differences among districts	(D.1), (D.2),(D.3), (D.4)
López et al. (2015)	Finance		Minimize territory dispersion	(D.1), (D.2), (D.3), (D.4)
Cortinhal et al. (2016)	Solid waste collection		Minimize the routing time	(D.1), (D.2),(D.3), (D.4) and capacity
Yanık et al. (2016)	Electrical power	Turkey	Minimize distances	(D.1), (D.2), (D.3), (D.4)
Gliesch et al. (2017)	Agrarian reform		Minimize differences of land quality among districts	(D.1), (D.2), (D.3)
Ríos-Mercado and Bard (2019)	WEEE collection	Germany and Spain	Maximize territory dispersion	(D.1), (D.2), (D.3)
Diglio et al. (2020)	General	Novara, Italy	Minimize the total expected cost	(D.2), (D.3), (D.7), reallocation constraints, and similarity between districting plans
Kim and Kim (2020)	Polling facility location	Seoul, South Korea	Minimize utility cost	(D.1), (D.2), (D.3)
López Pérez et al. (2020)	Finance		Minimize territory dispersion	(D.1), (D.2), (D.3), (D.4)
Mostafayi Darmian et al. (2020)	Solid waste collection	Iran	Minimize the cost of establishing collection centers and collecting waste, destructive environmental consequences, and social dissatisfaction.	(D.2), (D.3), (D.7), (D.4)
Wang et al. (2021)	Railway dispatching	Real-world railway networks from Chinese HSR companies	Minimize workload deviation and inter-district interaction	(D.2), (D.3), (D.4), and symmetry-breaking constraints
Feng et al. (2022)	Regionalization	Synthetic and real-world dataset of Southern California	Maximize number of p regions and the compactness of all regions	(D.1), (D.2), (D.3), (D.4)
Baldassarre et al. (2023)	General	Diglio et al. (2023)	Minimize the total assignment costs of BUs to districts	(D.1), (D.2), (D.3), (D.4)
Diglio et al. (2023)	General		Minimize the total assignment costs of BUs to districts	(D.1), (D.2), (D.3), (D.4)
Ríos-Mercado et al. (2023)	WEEE collection	Germany and Spain	Maximize territory dispersion	(D.1), (D.2), (D.3)
Liu et al. (2024)	Regionalization	New York City	Minimize intrazonal heterogeneity	(D.1), (D.2), (D.4)
Salazar Treviño (2024)	Finance		Minimize territory dispersion	(D.1), (D.2), (D.3)
Calvete et al. (2025)	Logistics	Artificial	Minimize the variance of the number of customers in the clusters and total distance from all reassignments	(D.1), (D.2), (D.3)
Deng and Pantuso (2026)	Carsharing/shared mobility	GreenMobility carsharing system in Denmark	Maximize operating profit	(D.1), (D.2), pricing citeria

existing polling facilities, merge certain precincts, and adjust the existing boundaries of precincts to enhance the efficiency of the administration of the voting process.

Applications related to the management of transportation systems have also been studied. Wang et al. (2021) consider such a case in the context of a railway transportation network, where the goal is to partition its basic elements (stations, intersections and rail segments) into districts to be managed by independent controllers.

Recently, Liu et al. (2024) addressed a TDP to improve the reliability of census data in order to support decision-making in the transportation sector. The objective is to minimize total intrazonal heterogeneity and maximize the number of zones (balance). Also, other districting considerations, such as contiguity is taken into account. They proposed a set-partitioning formulation and a two-phase heuristic to solve two real-world case studies.

In recent years, there has been growing interest in developing decision support systems (DSS) to address complex sectorization problems from several contexts. For instance, Öztürk et al. (2024) proposed a web-based DSS, called D3S (a comprehensive solver-oriented tool), designed to solve TDPs such as school districting, sales territory planning, and political districting. D3S uses two metaheuristics and considers objectives such as balance, compactness, and contiguity, taking into account constraints such as capacity, boundaries, and neighborhood constraints.

Table 1 provides a comprehensive summary of most of the applications categorized as **Others**, including specific applications (e.g., solid waste collection, turfing, electrical power distribution system, and product pricing, among others), their objective and main criteria.

4 Computational methods for TDPs

The development of solution methods for TDPs has evolved throughout the years. While the focus of early TDP studies considered the relationship with other classical combinatorial optimization problems (Hess et al., 1965; Davis et al., 1968; Garfinkel and Nemhauser, 1970), recent work has leaned towards specific models focused on the functional (e.g., *balance among districts*) and topological (e.g., *connectivity*, *compactness*) requirements of some application settings. These models are then solved through some ad hoc method (either of exact or heuristic nature), an adaptation of a metaheuristic, or some geometry-based partition approach.

This section classifies the literature according to the proposed solution methods. Section 4.1 reviews the methods in which the major focus corresponds to an exact solution approach. Section 4.2 reviews heuristic methods that provide a feasible, and probably good, solution. Finally, Section 4.3 reviews spatially based methods such as Voronoi Diagrams.

The proposed division is unclear and contains some fuzzy areas (see Figure 3). For example, given the complexity of some proposed exact methods, there have been several works in which the authors use some simplifying assumption that ultimately leads to the development of a heuristic. Similarly, some geometry-based methods provide optimal solutions under specific conditions. Consequently, the division among these three categories corresponds to what we consider to be the major focus, or contribution, of each of the reviewed works, and not as a statement of their exact or heuristic nature. As such, within the category of exact methods, we are considering those works in which a mathematical model solved using a commercial optimization solver is proposed.

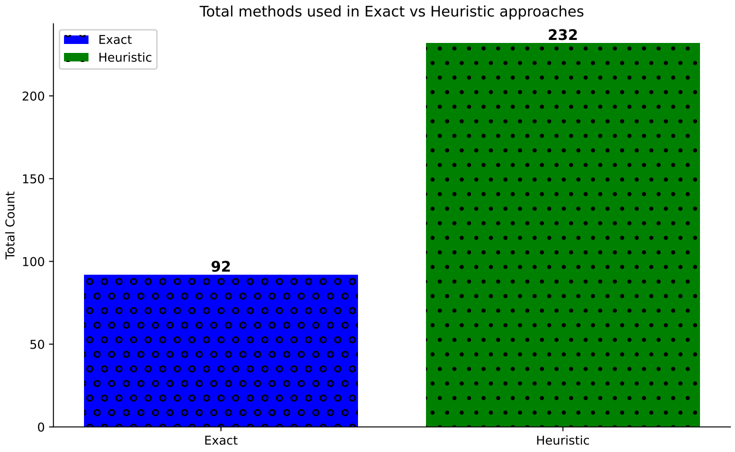

Figure 3: Comparison on the number of works focused on exact or heuristic methods found in the literature

4.1 Exact optimization methods

There are two main approaches used by authors when attempting to find exact optimal solutions to districting problems. On one hand, some authors propose integer programming models and use off-the-shelf general-purpose commercial solvers (Clarke and Surkis, 1968; Heckman and Taylor, 1969; McKeown and Workman, 1976; Jennergren and Obel, 1980; Diamond and Wright, 1987; Church and Murray, 1993; Lemberg and Church, 2000; Caro et al., 2004; Marianov and Fresard, 2005; Bandara and Mayorga, 2011; Benzarti et al., 2013; Bruno et al., 2017a,b). Others propose network-flow-based models (Belford and Ratliff, 1972; Marlin, 1981; Schoepfle and Church, 1991) or covering-based models (Curtin et al., 2010). Given the complexity of districting problems, the size of the instances that can be solved to optimality tends to be small when compared to heuristic approaches, but large-scale instances with specific objectives can be solved optimally with specific formulations (Validi and Buchanan, 2022; Validi et al., 2022). On the other hand, there have been several attempts to develop special exact optimization algorithms to tackle some districting problems with particular properties. For instance, districting problems with connectivity requirements are hard to tackle with general-purpose solvers because of the exponential number of constraints required to ensure connectivity, making these models impossible to write explicitly. In the remainder of this section, we review the most relevant exact optimization approaches that have been used for districting problems.

The first exact methods for TDPs were set-partitioning-based algorithms (Garfinkel and Nemhauser, 1970). This can be attributed to the fact that TDPs are essentially partitioning problems. Typically, these methods work in an iterative way where, in a first step, balanced, connected, and compact territories are generated using a heuristic or exact method, and then, in a second step, territories are chosen from a set of candidates to optimize the overall balance. Unfortunately, only relatively small instances can be solved optimally with these methods.

A major issue of the set-based formulation is the cardinality of the set of districts, as it may be a very large set. Several works impose limits on the generation of districts (Shanker et al., 1975; Ghiggi et al., 1976; Nygreen, 1988; Mehrotra et al., 1998) or generate them heuristically (Fiorucci et al., 2006; Jarrah and Bard, 2012; De Fréminville et al., 2015; Konur and Geunes, 2019) or use a set-based model as a post-optimization step for a multi-start greedy districting plan generation algorithm (Bard and Jarrah, 2009; Moreno et al., 2020). Either approach leads to a heuristic method and may fail to ensure the optimality of a solution. Yet, the heuristic generation of districts may filter candidate districts and impose desired topological properties that would be difficult to ensure in exact solution approaches (Nygreen, 1988). Note that the resulting set partition (covering) problem can be solved using different methods, such as branch-and-bound (Garfinkel and Nemhauser, 1970), a general-purpose IP solver (Mehrotra et al., 1998), a branch-and-price method (Clautiaux et al., 2019), or some ad hoc method (Shanker et al., 1975).

The set-partitioning model idea has also been extended and used more recently in more sophisticated schemes to address other districting problems in a heuristic way (Jarrah and Bard, 2012; Moreno et al., 2020; Wang et al., 2021; Liu et al., 2024). These are further discussed in the following section.

Other earlier exact approaches involve implicit enumeration algorithms for special cases of school districting problems (Liggett, 1973; De Giorgi et al., 1976). Naturally, these were limited to relatively small instances and rely heavily on developing good lower bounds, which, incidentally, is an area that has remained unexplored in districting problems. A particular case seeking to find a 2-connected partition of a given undirected graph is studied by Christofides and Brooker (1976).

Location-allocation techniques have also been used with some success. The location-allocation scheme, originally proposed by Hess et al. (1965) for political districting, consists of the following. The main idea is to model the problem as a capacitated p-median problem. The facilities to be located are the district centers, and the capacity of the facilities is chosen in such a way that the districts obtained by solving the problem are well-balanced. The location-allocation scheme is an iterative procedure where, in the first stage, the p medians are computed (from the previous iteration) by solving a 1-median problem for each territory. Then, in the second stage, the allocation of basic units is optimally found for the given set of fixed medians. The second step can be efficiently solved by solving a related transportation problem. Again, only relatively small instances could be solved under this approach.

Other authors have used this technique and adapted it to a heuristic way to solve larger instances, (Fleischmann and Paraschis, 1988; Hojati, 1996; George et al., 1997). It is important to note that this technique was not particularly tailored for handling connectivity constraints; however, some authors have adapted the main idea of this technique for handling connectivity constraints in a heuristic manner (Ríos-Mercado et al., 2021). See Section 4.2 for reviewing location-allocation heuristics.

One particular exact optimization algorithm for handling TDPs with connectivity constraints is due to Salazar-Aguilar et al. (2011b). The authors introduce an exact solution framework based on a branch-and-bound algorithm combined with a cut-generation strategy. First, the (exponentially many) connectivity constraints are relaxed, and then the integer relaxation is solved by branch-and-bound. Afterward, a separation problem is solved to find unconnected districts. The corresponding violated constraints are then added to the formulation, and the iterative process starts again. When no more violated cuts are found, the algorithm stops with an optimal solution. This algorithm is applied to two classes of districting problems: One is based on a p-center dispersion measure, and the other is based on a p-median dispersion measure. The algorithm was successful in handling instances of several hundred of basic units. The results also indicated that the p-median-based model turned out to have a stronger linear programming relaxation, resulting in fewer violated connectivity constraints and faster convergence. This cut-generation idea has also been used in other TDPs to handle larger instances in a heuristic way (López-Pérez and Ríos-Mercado, 2013; Ríos-Mercado and López-Pérez, 2013).

Sandoval et al. (2020) developed an exact algorithm that also makes use of this cut-generation idea, but in a different manner to solve a CDP with a p-center dispersion objective function. They proposed an algorithm that uses auxiliary sub-problems to obtain lower bounds on the distance value that minimizes the dispersion objective. The method is an iterative algorithm inspired by the procedure developed by Özsoy and Pınar (2006) for the capacitated p-center problem. The algorithm exploits the similarities between both problems and the fact that there is a finite list of candidate values for the dispersion objective, which makes it suitable for inspection. The main idea of this iterative algorithm is to use different subproblems to validate if, for given values of the objective function of the original problem, it is possible to find feasible solutions with at most p territories. This change allows trying several candidate distance values as lower bounds on the optimal solution of the original problem. These lower bounds are iteratively improved until an optimal solution is reached.

Another exact algorithm that uses the cut-generation strategy of Salazar-Aguilar et al. (2011b) was presented by Díaz et al. (2020) for a districting problem with p-median dispersion objective function. The authors use different relaxations of integer linear programming formulations of the problem. Additionally, two methodologies to obtain upper bounds (feasible solutions) are presented. The first one uses the relaxation of an integer quadratic programming formulation. The second methodology obtains feasible solutions using a primal heuristic within the framework of a subgradient optimization algorithm to solve a Lagrangian dual that also provides lower bounds for the optimal solution.

Oehrlein and Haunert (2017) present a cutting-plane algorithm for a districting problem where the number of territories is not fixed but a variable. In their model, territories whose size exceeds a given bound are sought. The model also considers connectivity constraints following the model by Shirabe (2009, 2012).

Other exact methods, based on cutting plane approaches, have been developed by Validi and Buchanan (2022) and Validi et al. (2022) to solve political districting problems with dispersion as well as compactness objectives. The authors impose connectivity constraints through a set of cutting planes and provides heuristic to generate incumbents as well as methods for variable fixing and symmetry breaking.

Bender et al. (2018) present an exact branch-and-price algorithm for scheduling customer visits in a multi-period service territory design context. The authors propose specialized acceleration techniques, particularly a fast pricing heuristic, and techniques to reduce the symmetry inherent to the problem. Experiments on real-world data sets show that instances with up to 55 customers and a planning horizon of four weeks with five days per week can be solved to optimality in reasonable running times.

In a slightly different context, Ríos-Mercado and Bard (2019) present an exact algorithm for a maximum dispersion territory design problem (Fernández et al., 2010) arising in the collection of waste electric and electronic equipment. The exact algorithm intelligently combines the development of tight dual bounds, problem reformulation, and binary search. Results were strong as they were able to optimally solve instances with up to 800 basic units and 12 companies, and instances with up to 1400 basic units and 8 companies.

Kassem and Escobedo (2023) present an exact branch-and-bound algorithm with cut-generation strategies for an edge-based districting problem. The algorithm is very similar to the algorithm developed by Salazar-Aguilar et al. (2011b) for node-based districting.

Regarding multi-objective districting, to the best of our knowledge, the first exact algorithm is due to Salazar-Aguilar et al. (2011b). The authors present an improved ε-constraint method to solve a bi-objective programming model where compactness and balancing concerning the number of customers are used as performance criteria. Constraints such as connectivity and balancing for product demand are also considered in the model. Empirical evidence over a variety of instances shows that the proposed method finds optimal Pareto fronts very efficiently.

Bouzarth et al. (2018) present an ε-constraint algorithm for a school districting problem that seeks to optimize both balancing the socioeconomic compositions of the schools and minimizing total travel distance. They present a case study from the Greenville County School District in Greenville, South Carolina, USA.

Swamy et al. (2022) present a multilevel algorithm for a multi-objective political districting problem. This algorithm coarsens a large graph input by a series of graph contractions and solves an exact bi-objective problem at the coarsest graph using the ϵ-constraint method. A case study on congressional districting in Wisconsin demonstrates that district plans constituting the approximate Pareto-front are geographically compact, as well as efficient (i.e., proportional), symmetric, or competitive.

More recently, de Abreu et al. (2026) developed an augmented ε-constraint method for a bi-objective districting and routing model seeking to minimize the total travel time within a district, and to minimize territory dispersion.

4.2 Heuristic procedures

Given the inherent computational complexity of TDPs and the interest in solving large-scale instances, it is not surprising that most of the methods devised for tackling TDPs are heuristics. This is mostly due to the fact that, except for a very few particular cases, most of the TDPs are $\mathcal{NP}$-hard.

The evolution of the heuristics for the TDP has been significant. Initial heuristic methods included manual approaches (Harris, 1964), sometimes with the help of some partial calculation from a computer (Lodish, 1975). In the past few decades, many interesting ideas have been developed for addressing TDPs of many kinds. Heuristics based on mathematical programming concepts, geometric approaches, metaheuristics (including construction and local search methods), and hybrid heuristics have been proposed with a certain degree of success.

In this section, we discuss some of the most successful heuristics in these categories.

4.2.1 Mathematical programming approaches

One of the most popular techniques for TDPs is the location-allocation heuristic. This heuristic is a two-phase iterative heuristic where centers or medians are determined in the first phase, and then units are allocated to these centers by solving a related or relaxed problem, depending on the specific context. Typically, the allocation subproblem consists of a transportation-like problem that can be solved very efficiently. For relatively small instances, this technique can be used as an exact algorithm, as discussed in the previous section; however, in most applications, the location-allocation algorithm is used as a heuristic. One of the main reasons is that when solving the allocation subproblem, some relaxed integer variables turn out to have fractional values. These are called split variables, and a split resolution heuristic that determines which centroid this basic unit should be assigned to is applied. Originally proposed by Hess et al. (1965), this idea has been successfully applied in a variety of TDPs (Hess and Samuels, 1971; Mitchell, 1972; Segal and Weinberger, 1977; Hardy, 1980; Mulvey and Beck, 1984; Fleischmann and Paraschis, 1988; Zografos et al., 1992; Hojati, 1996; George et al., 1997; Kallioras et al., 2015; Ríos-Mercado et al., 2021). This location-allocation idea has also been applied to edge-based districting problems (Muyldermans, 2006) and to extensions of TDPs with additional complicating features. For instance, Bender et al. (2016) developed a location-allocation heuristic for a multi-period service territory design problem. Their planning problem has two subproblems: a partitioning subproblem, where customers must be grouped into service territories, and a scheduling subproblem, where customer visits must be scheduled throughout the multi-period planning horizon. Ríos-Mercado et al. (2021) extend the location-allocation idea to handle TDPs with connectivity constraints. Their approach is enhanced by adding a local search phase.

Other mathematical programming approaches include Lagrangean relaxation heuristics (Mulvey and Beck, 1984), iterative methods that make use of network-based formulations (Schoepfle and Church, 1989), heuristics based on eigenvector analysis (Thill, 1998), heuristics based on solution to mixed-integer second-order cone programs (Belotti et al., 2025), column-generation-based heuristics (Mehrotra et al., 1998; Haase and Müller, 2014; De Fréminville et al., 2015; Wang et al., 2021; Baghersad et al., 2023), and methods that cluster BUs according to the expected routing plans (Bowerman et al., 1995) or according to balance and total distance criteria (Calvete et al., 2025).

Bender et al. (2020) studied a real-world problem arising in the context of parcel delivery. Given a heterogeneous set of resources, i.e., different drivers and different vehicles, the problem for each day consists of assigning a driver and a vehicle to each customer requiring service. Two conflicting aspects must be taken into account. On the one hand, service consistency is desirable, meaning that a customer should always be served by the same driver. On the other hand, daily demand fluctuations and tight resource constraints prohibit fixed resource assignments. To find a reasonable compromise between these aspects, they propose a novel two-stage districting approach, which establishes delivery districts in the first stage and adapts them to the daily demand realizations in the second stage. For the first stage problem, they propose three integer programming models that differ in the level of detail of their input data, their expected compliance with service consistency, the driver's contractual working times, and their computational effort. Their two-stage approach merges the two dominant approaches in the literature, which either determine a priori routes and then adapt them daily, or derive fixed service regions for drivers. They present a case study based on a real-world data set. The results highlight the differences between the three first-stage models and show that only a few adaptations of the districts are necessary in the second stage to achieve feasible daily delivery tours along with a very good workload balance for drivers.

López Pérez et al. (2020) address a risk-balanced territory planning model for a microfinance institution. They propose an integer programming model that chooses the location of the branches to be designated as territory centers and allocates the customers to these centers concerning planning criteria such as the total workload, monetary amount of loans, and profit allocation, while balancing the territory risk. They implement a hybrid heuristic framework using cut generation within a branch and bound.

Given the similarity with clustering problems, some heuristics attempt to take advantage of this similarity. Both districting and clustering problems try to partition basic units into subsets or clusters. One main difference is that in clustering problems, homogeneity among units in the same cluster is sought, which contrasts with TDPs, where there is a large class of different objective functions used. In addition, clustering problems tend to be unrestricted for the most part, while TDPs typically have many constraints of many kinds. Nevertheless, the similarity between these two problems has led to the development of algorithms based on k-means (a common heuristic for clustering) for addressing districting problems. For instance, Kong et al. (2019),Moreno et al. (2020), and Calvete et al. (2025) have provided adaptations of different k-means clustering methods to different districting problems.

Nikzad et al. (2021) propose a matheuristic method to solve the Stochastic Districting-Staff Dimensioning-Assignment-Routing Problem. The matheuristic is composed of four phases: dimension reduction, initial solution, infeasibility repair, and solution improvement. In all phases, a mathematical model is solved, and computational results based on real-world data from the Austrian Red Cross demonstrate the effectiveness of the proposed matheuristic algorithm. On the other hand, Álvarez-Miranda and Pereira (2022) propose a hybrid approach to address a real-world problem of designing delivery zones for courier companies. In this work, a mathematical model is used to select the best territorial design that minimizes the total number of late deliveries, from a set of multiple candidates for distribution zones obtained by a multi-start construction heuristic.

Alternatively, Swamy et al. (2022) propose a multilevel algorithm to solve a multi-objective districting problem considering political fairness criteria. In the first phase, an aggregation of smaller geographic units into larger ones (*coarsening*) is performed. In the second phase, an MILP is solved to find pareto-optimal solutions that balance compactness and political fairness. Then, in the last phase, the *uncoarsening* is carried out using local search heuristics to improve compactness while maintaining feasibility.

Recently, Regis-Hernández et al. (2023) proposed an integrated matheuristic to address districting and resource allocation decisions simultaneously in order to create compact and balanced districts that minimize response times. In their algorithm, districts are initially created, and a number of available ambulances are assigned. Then, the resource location-allocation decisions are determined for each of them. Subsequently, districts are updated with the previous location-allocation information. An adaptive diversification phase is applied after a given number of iterations to generate a new set of districts to evaluate.

Baldassarre et al. (2023) address a stochastic TDP, based on the work of Diglio et al. (2023), where the demand for each BU is uncertain. The objective is to find a districting plan in which the probability that the districts are balanced is greater than a minimum threshold. The authors propose a matheuristic to solve the problem, which uses a heuristic algorithm based on a location-allocation scheme, where an initial set of district centers is determined and BUs are assigned to those centers. The difference from the method proposed by Diglio et al. (2023) lies in the hybrid component of the proposed matheuristic, which progressively generates balance constraints that dynamically incorporate scenarios until a feasible solution is found. The results show that the algorithm provides solutions of comparable quality to existing methods.

More recently, Pomes et al. (2025) studied a multi-stage stochastic districting problem. The goal is to devise a districting plan considering a few uncertain parameters changing over a discrete multi-period planning horizon. The problem is modeled as a multi-stage stochastic programming problem, where it is assumed that uncertainty can be captured by a finite set of scenarios. A mathematical programming model is proposed that embeds redistricting recourse decisions and other recourse actions to ensure that the districts are balanced regarding their activity. The results demonstrate the relevance of hedging against uncertainty in multi-period districting. Since the model is intractable for larger instances, they propose a heuristic based on a restricted model. The empirical results indicate the heuristic can produce high-quality solutions within reasonable running times.

4.2.2 Constructive heuristics

Constructive greedy heuristics for districting start from an empty solution and build the solution iteratively by assigning BUs to districts either by forming node-based (Thoreson and Liittschwager, 1967; Holloway et al., 1975; Birge, 1983; Ronen, 1983; Wang et al., 2009; Lin et al., 2017) or edge-based districts (Ferland and Guénette, 1990; Muyldermans et al., 2002, 2003; Mourão et al., 2009). These constructive methods can be classified into two main categories: those that build districts sequentially, one at a time (see Arredondo et al., 2021, for example), and those that build all districts simultaneously (Álvarez-Miranda et al., 2025). During this construction, heuristics use greedy functions that take into account both the objective function and sometimes a penalty term for violating some constraints. One of the major limitations of constructive methods is that they may fail to obtain feasible solutions. Therefore, these heuristics are often enhanced by local search heuristics to attempt to recover feasibility. These methods are discussed below.

An alternative to the constructive methods is the use of divisional methods. A divisional method works by dividing the territory into smaller problems, and then recursively solving these smaller problems and aggregating the solution for the original problem. Some of these methods are based on divide-and-conquer (Davis et al., 1968; Male and Liebman, 1978; Salazar-Aguilar et al., 2012a; Levin and Friedler, 2019) and others on min-cut approaches (Bianchi et al., 2016).

In the context of finding pre-determined response areas for EMS systems, effective districting may allow an EMS system to reduce the response time of paramedic support to the incident. Given the stochastic nature of the problem, heuristic procedures embedded within a simulation framework have proven successful. For instance, Mayorga et al. (2013) propose several policies for districting/dispatching, which are provided as inputs to a simulation model that compares the performance of different policies. The response areas, or districts, are designed using a constructive heuristic that considers adjusted expected coverage. Intra-district and inter-district dispatching policies are developed considering the degree of urgency of the call. Computational results show that integrated districting and dispatching policies are vital in increasing patient survivability.

4.2.3 Local search heuristics

Once an initial solution is available, a common step is to improve some or all of the metrics used to evaluate the districting plan through the use of a local search procedure. A local search procedure relies on performing small changes to a solution to reach a different, possibly better, solution. After finding an improved solution, this solution substitutes the initial one, and the process repeats itself until a local optimum is found, i.e., no more improvements are found.

A local search method is defined by the operator used to change the solutions, also named *move*. Two common operators used in local search for TDPs are the *reallocation* (also known as *flip*) operator (Camacho-Collados et al., 2015; She et al., 2017; Ríos-Mercado et al., 2021; Haas et al., 2022) and the *swap* operator. The *reallocation* operator changes the assignment of a BU from a district to a neighboring district, while the *swap* operator exchanges the assignment of two BUs. Note that the *swap* operator can be seen as a composite of two moves, an idea that has been generalized in Zhong et al. (2007).

Given the importance of efficient local search implementations, King et al. (2012, 2014) studied the data structures and algorithms required for their implementation, and King et al. (2018) evaluated their efficiency. Gliesch et al. (2018) also consider the efficiency of local search implementation within a metaheuristic approach that separately optimizes balance and compactness using a *flip* operator. To optimize compactness, they develop an auxiliary objective function that avoids plateaus in the search space.

Besides the neighbor operator, a local search method heavily depends on the representation of the solution. For instance, Bodin (1973), Maravalle and Simeone (1995), and Yamada (2009) represent the districting plan through trees, where the neighborhood involves moving a subtree from a district to a contiguous district.

While these basic neighbor operators may suffice to improve a districting plan, a relatively common alternative but more computationally intensive method consists of re-optimizing portions of the incumbent solution. For example, Kim et al. (2016) and Prischink et al. (2016) eliminate a district and try to reassign the BUs of the said district to different districts, while Berman and Mandowsky (1986), Xie and Ouyang (2016), and Sandoval et al. (2022) consider the redistricting of two neighboring districts. This reoptimization approach of redistricting a smaller problem based on a subset of neighboring districts has led to the development of a spanning tree-based neighborhood structure denoted as ReComb (DeFord et al., 2021). The method takes a subset of neighboring districts (in practical implementations, two districts) and uniformly samples a spanning tree over the edges of the contiguity graph of these BUs. By removing edges from said spanning tree you obtain a new connected partition of the BUs into districts. This approach is fast, allowing for many samples to be drawn and evaluated. Moreover, it offers multiple solutions, which is a requirement in its application area (political redistricting in the US, where the final decision is usually made by a committee). The ReComb neighorhood is used in Dobbs et al. (2024b) to analyze the interplay between political geography, constitutional requirements, and political fairness in Missouri (US) within a political redistricting context and in Dobbs et al. (2023) the authors compare different versions of the Recomb operator to the flip operator evaluating their performance on the design of congressional districts in Illinois, Missouri and Tennessee (US).

An extension to local search dealing with large neighborhoods led to Large Neighborhood Search (Lei et al., 2012) and Adaptive Large Neighborhood Search (Lei et al., 2015; Bruni et al., 2026) methods, both of which have been considered in the literature.

4.2.4 Metaheuristics

Metaheuristics are high-level heuristics that make use of constructive and local search heuristics in creative ways in an attempt to build and/or improve solutions to optimization problems. Metaheuristic frameworks usually have components that must be tailored to the particular problem being addressed by an intelligent exploitation of the problem structure.

In the previous section, we reviewed some heuristics that merely apply a local search phase as an improvement step. In this section, we review heuristics that make use of local search components embedded into these metaheuristic frameworks.

Greedy Randomized Adaptive Search Procedures An alternative to explore multiple solutions and to escape from local optima is to start the search from various starting points. This is the main idea from the Greedy Randomized Adaptive Search Procedure (GRASP) metaheuristic. A GRASP is a multi-start heuristic where each iteration is built upon two components: (1) a randomized construction heuristic to obtain initial solutions and (2) a local search method to attempt to improve this solution. Early applications of GRASP in the districting literature were aimed at the commercial territory design.

Ríos-Mercado and Fernández (2009) present a reactive GRASP approach for a commercial territory design problem motivated by a real-world application in a beverage distribution firm. The mathematical framework includes, as planning criteria, minimizing a measure of territory dispersion, balancing the different node activity measures among territories and territory contiguity. The proposed GRASP approach incorporates several features to improve upon basic GRASP implementations, such as reactivity by allowing self-adjustment of the restricted candidate list quality parameter, and filtering, which avoids executing the local search phase in unpromising bad solutions generated by the construction phase. The algorithm was tested in several data sets. The results show the effectiveness of the proposed approach. It was observed that the reactivity and the filtering proved useful in terms of feasibility concerning the balancing constraints, as well as the ability of the method to find more robust solutions when tested over the basic GRASP. Moreover, the proposed approach obtained solutions of much better quality (both in terms of its dispersion measure and feasibility for the balancing constraints) than those found by the firm method in relatively fast computation times. In follow-up work, Ríos-Mercado (2016) carried out further testing to assess this GRASP in very large-scale instances. The algorithm was found to be very effective in handling these large instances.

Caballero-Hernández et al. (2007) address a commercial design problem motivated by a real-world application in a beverage distribution company with connectivity, balancing, and joint assignment constraints. The authors present a GRASP with a preprocessing phase that ensures meeting the joint assignment constraints. The GRASP greedy function is a convex combination of the original dispersion-based objective function and the relative violation of the balance constraints. The method was tested on a set of randomly generated instances constructed according to the patterns found in real-world data from the industrial partner.

Cano-Belmán et al. (2012) developed four GRASP heuristics for a commercial territory design problem with compactness maximization criterion subject to territory balancing and connectivity. The first three construction heuristics (named GRLH1, GRLH2, and GRDL) build the territories simultaneously. Their construction phase consists of two parts: a location phase where p territory seeds are identified, and an allocation phase where the remaining basic units are iteratively assigned to a territory. In contrast, the other heuristic (named SLA) builds the territories one at a time. Empirical results reveal that GRLH1 and GRLH2 find near-optimal or optimal solutions to relatively small instances in which the optimal solution can be found through an exact approach.

Ríos-Mercado and Salazar-Acosta (2011) propose a GRASP for a commercial districting problem considering both design and routing decisions simultaneously. The problem consists of grouping a set of city blocks into territories to maximize territory compactness. As planning requirements, the grouping seeks to balance both the number of customers and product demand across territories, maintain connectivity of territories, and limit the total cost of routing. Their GRASP incorporates advanced features such as adaptive memory and strategic oscillation. Empirical evidence over a wide set of randomly generated instances based on real-world data showed a very positive impact of these advanced components.

Ríos-Mercado and Escalante (2016) develop a GRASP for a commercial districting problem using a diameter-based dispersion function for maximizing compactness. Their GRASP was enhanced by two variants of forward-backward path relinking, namely static and dynamic. The proposed algorithm and its components were extensively evaluated over a wide set of data instances. Experimental results revealed that the construction mechanism produced feasible solutions of acceptable quality, which were improved by an effective local search procedure. In addition, empirical evidence indicated that the two path relinking strategies had a significant impact on solution quality when incorporated within the GRASP framework.

Fernández et al. (2010) studied an unusual territory design problem motivated by the recycling directive for waste electric and electronic equipment in the European Union. The core of this law is that each company that sells electrical or electronic equipment in a European country must collect and recycle a quantity of returned items proportional to its market share. To assign collection stations to companies, a territory design approach was developed. What made this problem different, in contrast to classical territory design, is that territories should be as geographically dispersed as possible to avoid a company, or its corresponding logistics provider being responsible for the recollection, gaining a monopoly in some regions. The authors develop a GRASP for this problem over several construction strategies. Extensive computational results proved the effectiveness of the heuristic.

Mendes et al. (2022) study the capacitated and economic districting problem, which aims to find the best edge partition defining connected, capacitated, and balanced districts in an undirected connected graph, weighing the economic value of each district. This problem provides a comprehensive description of the decision-making on service network districting, where the order in which the districts are serviced plays a role in the profit. To tackle large instances, the authors propose a reactive GRASP. Their results indicate that GRASP was effective in handling large networks, achieving feasible solutions in almost all instances.

Salazar Treviño (2024) develops a GRASP for a districting problem arising in microfinancial institutions. In this problem, the locations of the bank branches must be determined, and customers must be allocated to these branches considering planning criteria such as total workload, monetary amount of loans, and profit allocation while balancing territory risk.

GRASP variants have also been used for multi-objective districting. For instance, Salazar-Aguilar et al. (2013) developed a GRASP for a multi-objective commercial districting problem seeking to maximize compactness and minimize territory imbalance regarding both product demand and the number of customers, while de Assis et al. (2014) study a districting problem arising in meter reading in power distribution networks. They consider two optimization objectives: compactness and homogeneity of districts. Their propsed GRASP is embedded in a scalarization technique for handling the two objectives.

Simulated Annealing One of the first proposed metaheuristics was the Simulated Annealing method. We can see this method as a local search procedure with a mechanism to escape from local optima through the acceptance of non-improving moves based on a probabilistic scheme. Implementations considering reallocation operators (Ricca, 2004) and both reassignment and swap operators (Browdy, 1990; Hurley and Moutinho, 1996; Hanafi et al., 1999) have been considered. Moreover, similar methods, such as the Old Bachelor Acceptance method proposed in Ricca (2004), have also been used to solve TDPs.

Ricca and Simeone (2008) compare the Old Bachelor Acceptance method with a classical Simulated Annealing implementation, as well as other local search-based heuristics and metaheuristics for political districting. They found that Old Bachelor Acceptance produced the best results in the majority of the cases, especially when the objective function was compactness. Moreover, the district maps generated by this heuristic dominate the institutional district plan with respect to all the districting criteria under consideration. When properly designed, automatic procedures tend to be impartial and yield good districting alternatives while being remarkably fast, hence allowing for the exploration of a large number of scenarios.

Tabu Search Tabu Search (TS) is another local-search-based metaheuristic whose main feature is the use of short- and long-term memory structures to escape from local optima and diversify the search. As in Simulated Annealing, the definition of the neighbor operators plays a fundamental role. For instance, Blais et al. (2003), Bozkaya et al. (2003, 2011), Haugland et al. (2007), Butsch et al. (2014), Cortinhal et al. (2016), and Tran et al. (2017) make use of reallocation and swap moves, Royuela and Duque (2013) and Liu et al. (2024) only consider reallocation moves, while Cortinhal et al. (2016) consider more elaborate neighborhood operators. TS has been very successful in different applications, such as political districting (Bozkaya et al., 2003, 2011), home-care districting problem in urban settings (Blais et al., 2003), nurse districting (Tran et al., 2017), police patrol districting (Chen et al., 2018), districting on emergency medical systems (Sudtachat et al., 2020), arc routing districting problems (Butsch et al., 2014; Cortinhal et al., 2016), and transportation (Liu et al., 2024), to name a few.

Ríos-Mercado et al. (2023) developed a TS with strategic oscillation for a districting problem arising in the collection of waste of electrical and electronic equipment. Given a set of collection bins, where users return end-of-life electronic goods, located across a region or country, the design problem involves assigning these bins to the companies responsible for the collection at a later stage. This assignment must meet specific planning and legal requirements, such as a fair household distribution according to each company's market share and a fair assignment based on the bin infrastructure quality. According to the current law, this assignment must be done in such a way as to avoid, to the best possible extent, regional monopolies. This anti-monopoly requirement is achieved by maximizing a dispersion function. The empirical work shows the tabu search and its components effectiveness over a broad set of instances from the literature. In particular, the strategic oscillation idea turned out to have a very positive impact.

Elizondo-Amaya et al. (2013) present a TS for a stochastic commercial districting problem with uncertainty on the customer demands. The problem consists of finding a compact partition while minimizing the expected territory imbalances for the product demand subject to planning criteria such as territory connectivity, compactness, and balance with respect to the number of customers. The proposed method incorporates advanced techniques such as strategic oscillation and specific neighborhood exploration strategies. Empirical evidence shows the value of using such advanced techniques.

Gliesch and Ritt (2019) present a unified heuristic approach that can handle three of the most common districting objective functions concerning compactness (diameter, p-center, and p-median), as well as a variable number of balancing attributes. They propose a multi-start method that iteratively constructs greedy randomized solutions followed by an improvement phase that alternates between optimizing compactness and satisfying balancing constraints through a series of tabu searches. Experiments show that the proposed method is competitive when compared to approaches in the literature, which are specific to each objective, improving known upper bounds in some cases.

TS has also been applied to multi-objective districting problems, such as land-use spatial districting (Lan and Liu, 2015), police districting (Liberatore and Camacho-Collados, 2016), spatial zoning (Wei and Chai, 2004), and stochastic versions of the problem (Haugland et al., 2007; Elizondo-Amaya et al., 2013).

Population-based evolutionary approaches A relevant segment of metaheuristic procedures falls within the category of population-based evolutionary approaches, sometimes also referred to as nature-inspired optimization methods.

The most representative metaheuristics of this group are the Genetic Algorithms. A Genetic algorithm (GA) is a heuristic that combines solutions to create new ones through the principles of evolution.

A major source of difference among implementations of the GA metaheuristic comes from how the GA encodes the solution. For example, Datta et al. (2012), Ahuja et al. (2015), and Gliesch et al. (2017) identify the partitions that make up each district, while Zhou et al. (2002) represent the districting plan as a tree structure, and Forman and Yue (2003) encode the solution as a list that needs to be decoded into a districting plan through an auxiliary procedure. Some implementations put their focus on finding the territory centers (Bação et al., 2005; Geroliminis et al., 2011; Iannoni et al., 2011; Yanık et al., 2016) by solving the allocation process separately, while others codify a solution through both centers and partitions (Hu et al., 2014).

Besides the representation of the solutions, the method used to ensure the feasibility of the generated solutions also plays a crucial role in distinguishing different evolutionary methods.

Chou et al. (2007) define a special crossover method to combine the borders of different districting plans, and Gliesch et al. (2017) use a modified crossover operator to ensure feasibility. An alternative to ensure feasibility is to include a penalty function in the evaluation of the fitness of a solution that imposes an extra cost if the solution does not satisfy some conditions (Datta et al., 2012).

Note that GAs are not the only population-based evolutionary metaheuristic found in the literature, nor used for districting problems. For example, Xiao et al. (2025) report an evolutionary-based method for a general-purpose districting problem, denoted as *spatial aggregation*, where smaller units are to be aggregated into larger ones while satisfying connectivity and balance conditions. Their proposed method creates an initial population, defines a recombination operator to find new solutions starting from an initial one adding a local search operator to improve their solution. A similar method, under the name of a memetic algorithm (a genetic algorithm with local search) is proposed by Biswas et al. (2023).

Stochastic districting problems have also been addressed by population-based evolutionary methods. Mostafayi Darmian et al. (2021) developed a GA for a healthcare districting problem subject to demand uncertainty.

GAs have been widely used for addressing multi-objective combinatorial optimization problems. In particular, multi-objective evolutionary and genetic algorithms have also been developed for many districting problems (Bergey et al., 2003a; Hu et al., 2014; Steiner et al., 2015; Lei et al., 2016; Neto et al., 2017; Tavares Pereira et al., 2007; Lin et al., 2020; Mostafayi Darmian et al., 2020). For instance, in the latter, Mostafayi Darmian et al. (2020) propose a multi-objective-based population local search heuristic for a solid waste collection problem, which starts by generating an initial set of solutions based on an encode-decode procedure to represent centers and the assignment of BUs to districts. The initial solutions are created randomly, and a breadth-first-search strategy is used to ensure the contiguity of districts. Then, a local search is applied to improve the solution population by creating two new neighboring solutions for each current solution component using a perturbation factor. The current solution can be replaced if it does not dominate the new ones. The improved population is diversified by using the farthest-candidate method, and the best-worst method is used to select the final solution from the Pareto set.

Hybrid Metaheuristics Solution methods combining two or more metaheuristics (hybrid metaheuristics) have also been developed for territory design problems. Bergey et al. (2003b) present a heuristic combining SA and GA for an electrical power districting problem. González-Ramírez et al. (2017) develop a metaheuristic combining GRASP for the constructive phase and TS for the local search phase for a territory design problem in pickup and delivery operations. The TS uses allocation, swap, and combined swap moves (multiple swaps between two districts) as its neighbor operators. Gopalan et al. (2013) describe a solution procedure that combines GA with integer programming in a real-world application of political districting.

Chou (2011) presents a knowledge-based evolution algorithm for political districting. They mapped this political problem onto a q-state Potts model system by using statistical physics methods. The political constraints (such as contiguity, population equality, etc.) are transformed into an energy function with interactions between sites or external fields acting on the system. Their method combines an evolutionary algorithm with SA to improve solutions. They tested their heuristic in two real-world cases (Taipei and Kaohsiung) and simulated cities.

Jarrah and Bard (2012) present a new approach to rationalize the design of work areas for drivers who pick up and deliver hundreds of packages a day. Taking into account the random nature of demand, visit frequency, and service time, the objective is to partition the customers into the minimum number of convex, continuous clusters such that each can be serviced by a single vehicle within the time available in a day. The authors present a novel solution methodology combining implicit generation of clusters, generation of valid inequalities to strengthen the subproblems, and tabu search to limit the number of subproblems solved at each iteration. The heuristic was tested with data provided by an industrial partner.

Moreno et al. (2020) developed a novel hybrid approach for territorial design through combining a K-means-based approach for clustering construction with an optimization framework. The K-means approach incorporates the novelty of using tour length approximation techniques to satisfy the conditions of a pork and poultry distributor based in the region of Valparaíso in Chile. The resulting method proves to be robust in the experiments performed, and the Valparaíso case study shows significant savings when compared to the original solution used by the company.

Zhou et al. (2021) present a heuristic technique that combines GA and SA for real-world territory design and routing problems of a major dairy company that produces and distributes perishable products. The problem calls for grouping customers into geographic districts to minimize the total operational cost, computed as a function of the fixed costs of the districts and the routing costs. Two interconnected decision levels should be tackled: partitioning customers into districts and routing vehicles according to complex operational constraints. To solve the problem, a hybrid multi-population genetic algorithm is designed and enhanced with several evolutionary and local search techniques. The proposed design is extensively tested on instances derived from the literature and on real-world large-scale instances, involving more than 1000 customers. The results show the effectiveness of the different components of the algorithm, and the feedback from the company's planners confirms that it produces high-quality, operational solutions. Additionally, they explore some managerial findings in the adoption of alternative objectives and service requirements.

Combined metaheuristics have also been developed for multi-objective districting problems. Wei and Chai (2004) present a metaheuristic combining TS and scatter search for a multi-objective GIS-based spatial zoning problem. Salazar-Aguilar et al. (2012b) propose a heuristic based on GRASP and scatter search for a multi-objective commercial districting problem. These districts must be compact and balanced with respect to the number of customers and product demand. The proposed scatter search-based framework contains a diversification generation module based on a greedy randomized adaptive search procedure, an improvement module based on a local search strategy, and a combination module based on a solution to an assignment problem. The proposed metaheuristic is evaluated over a variety of instances taken from the literature. This includes a comparison with two of the most successful multiobjective heuristics from the literature, such as the Scatter Tabu Search Procedure for Multiobjective Optimization (SSPMO) (Molina et al., 2007), and the Non-dominated Sorting Genetic Algorithm (NSGA-II) (Deb et al., 2002). Experimental work reveals that the proposed procedure consistently outperforms both heuristics, SSPMO and NSGA-II, in all instances tested.

Vanneschi et al. (2017) combine a genetic algorithm and a variable neighborhood search (VNS) to solve a multi-objective TDP. The VNS uses allocation moves but defines different neighborhoods according to the different objectives to optimize.

4.3 Geometry-based procedures

While most of the previous solution methods focus on discrete optimization problems (i.e., the problem tries to select a solution from a set of candidates), the literature also covers problems in which the solution space is continuous. Under these circumstances, it is common to consider the problem as a computational geometry problem in which we aim to divide the space into areas (districts).

If the centers of these groups are known, and the aim is to assign each point of the space to its nearest center. This problem equates to obtaining a Voronoi diagram. The set of centers defines the Voronoi generators that divide the space according to a distance metric. The distance metric can be unweighted (Furuta et al., 2005), weighted (Galvão et al., 2006) or adaptively weighted (Karimi et al., 2010). The literature covers other generalizations, such as Voronoi diagrams with overlapping regions (Drezner and Drezner, 2013).

Among the methods, we can cite the work of Honiden et al. (2009), who consider the problem of generating a Voronoi partition of a discrete graph to achieve balanced conditions on the region sizes. Through experimentation, they first establish that the region sizes of randomly generated graph Voronoi diagrams vary greatly in practice. Then, they show how to achieve a balanced partition of a graph via Voronoi site resampling.

Geroliminis et al. (2011) present a model and a heuristic solution for the optimal deployment of many emergency response units in an urban transportation network and an application for transit mobile repair units. The proposed model integrates a queuing model (the hypercube model), a location model, and a metaheuristic optimization algorithm (genetic algorithm) for obtaining appropriate unit locations in a two-step approach. In the first step, the service area is partitioned into sub-areas (called superdistricts) while, in parallel, the necessary number of units is determined for each superdistrict. An approximate solution to the symmetric hypercube model with spatially homogeneous demand is developed. A Genetic Algorithm is combined with the approximate hypercube model to obtain the best superdistricts and associated unit numbers. With both of the above requirements defined in step one, the second step proceeds with the optimal deployment of units within each superdistrict. For the partitioning phase, they use a simple algorithm not restricted by a specific shape based on the Voronoi diagrams.

Ricca et al. (2008) propose a class of heuristics based on weighted Voronoi regions for obtaining compact and balanced political districts. These algorithms feature an iterative updating step of the distances to balance district populations as much as possible. Their performance was tested on randomly generated rectangular grids, as well as on real-life instances. For the latter instances, the resulting district maps were compared with the institutional ones adopted in the Italian political elections from 1994 to 2001.

The problem of coordinating a fleet of vehicles so that all demand points on a territory are serviced and that the workload is most evenly distributed among the vehicles is an important one in the vehicle-routing literature. To divide the computational burden, a natural strategy used in Haugland et al. (2007) and in Carlsson and Devulapalli (2012) consists of first dividing the service territory into subregions, requiring that each vehicle is only responsible for the demand occurring in its subregion.

Carlsson and Devulapalli (2012) consider a partitioning problem in which customer locations are distributed according to a known continuous probability density on a planar region, with the objective of dividing the region into subregions to balance the workloads among the vehicles. The work shows that when the objective is to minimize the maximum workload of all facilities, the optimal partition consists of a collection of circular arcs that are induced by a multiplicatively weighted Voronoi diagram.

In a follow-up paper, Carlsson and Delage (2013) consider how to address the problem discussed in Carlsson and Devulapalli (2012) when one cannot identify the exact distribution of future demand points at the time of assigning subregions. The authors describe a procedure for finding a globally optimal convex 2-partitions and make use of this result to propose a partitioning method based on Voronoi diagrams for the n-partition problem.

Carlsson et al. (2016) address a continuous partitioning problem from a theoretical perspective. They solve the problem of optimally partitioning a region efficiently by reducing it to a low-dimensional convex optimization problem. This result generalizes and gives very short and constructive proofs of several existing results in the literature on equitable partitioning for particular forms of the balancing functions.

Novaes and Graciolli (1999) and Novaes et al. (2000) studied the problem of setting the district boundaries while determining an optimal vehicle fleet to minimize total daily transport costs. Both vehicle time and vehicle load are treated probabilistically. Each district is related to a characteristic function that takes into account distribution costs, time, and capacity constraints, distribution effort, and shape considerations (district slenderness). In the former, the region under analysis is represented by a rectangular grid structure. In the latter, a continuous approach is developed. The mathematical model developed to solve this problem is a combination of a gradient method with random perturbations and a hybrid genetic algorithm. Novaes et al. (2009) use Voronoi diagrams in association with a continuous approximation model to solve a location–districting problem with application in transportation and logistics.

Moreno-Regidora et al. (2012) studied a regionalization problem with the requirement that the districts or zones should be built around a given set of seeds. They proposed a method based on a discrete version of the adaptive additively weighted Voronoi diagram that makes it possible to partition a two-dimensional space into zones of specific sizes, taking both the position and the weight of each seed into account. The method consists of repeatedly solving a traditional additively weighted Voronoi diagram, so that the weight of each seed is updated at every iteration. The zones are geographically connected using a metric based on the shortest path. The method is tested on data from three municipalities in Castile-La Mancha, Spain.

More recently, Carlsson et al. (2024, 2025) addressed a stochastic districting and routing problem that arises in last-mile delivery at a leading logistics service provider in Southeast Asia. This is the first zoning optimization model that considers general multi-vehicle zones with uncertain demand. They present a novel zoning optimization framework that determines the assignment of customer locations to last-mile delivery stations to improve operational efficiency and work equity. They propose a novel data-driven zoning optimization model that integrates the additively weighted Voronoi diagram and vehicle routing problem through a subgradient algorithm. The algorithm exploits the primal-dual formulation of the partitioning problem and is flexible enough to handle practical delivery scenarios with varying vehicle capacities and travel speeds.

Behroozi and Carlsson (2020) present a survey on models and methods for solving districting problems using computational geometry.

5 Discussion and venues for future work

In this paper, we provide a compilation of the most relevant literature on applications, mathematical models, solution techniques, and methodological improvements for TDPs over the past six decades from an optimization standpoint. First, we introduce a general model based on a set of conditions and criteria that can be used as a basis to identify the parts of the applications reviewed in the article. Then, the papers are classified according to their application area, and finally according to their solution methodology.

This is a very active research area. We now discuss some of the most interesting research challenges in two parts. First, we discuss areas of opportunity in terms of particular TDP applications, and then we highlight possible research avenues from a more general standpoint.

5.1 Potential research areas by specific TDP application

These are some areas of opportunities for further research by particular TDP area, listed below.

Political districting: An interesting issue to be further investigated is incorporating uncertainty in voting patterns and population demographics to create robust district plans. Analyzing the impact of the coarsening process on demographic representation and fairness is another interesting line of work. It would also be interesting to understand the fundamental tradeoffs between the compactness of the districts in the week and other criteria such as population deviation, minority representation, or fairness of the partisans. From an algorithmic perspective, some of the existing integer programming models could be strengthened by developing valid inequalities (Ludden et al., 2023).

Sales districting: Incorporating service customization features such as inventory management, among others could be worthwhile investigating (Bender et al., 2018).

Commercial and distribution districting: One particular aspect that needs further analysis is the inclusion of other types of services within the districting phase. The methods could be extended to include standard service or pickup operations, such as returns, within service in the same territorial design process. Furthermore, models could be enhanced by considering other side constraints that may impact the operational viability of the territories, such as the physical capacity of the vehicle or the size of the items delivered. Another important issue is to determine the optimal number of districts as part of the decision process. In other words, let the number of territories be considered as a decision variable.

School districting: One first area of opportunity is to carry out a more comprehensive assessment of the different model parameters and their impacts. We believe that this has been done to some extent, but more work is needed in this line. Some preliminary insights are offered by Gillani et al. (2023).

Police patrol districting: According to Liberatore et al. (2023), most existing models do not contain all operational aspects related to police districting and patrolling, such as the deployment of special police forces and interactions with other public security bodies. Including these elements would make the model more realistic and applicable and would benefit the analysis. Another area of opportunity for further research is to extend current models to account for the interests of a larger number of stakeholders. Most models assume that the distribution of the crime risk is deterministic and known a priori. However, issues such as selection biases driven by unequal crime reporting rates across socioeconomic groups and geographical areas can be better studied by considering uncertainty in the risk of crime. This would require addressing the problem from a stochastic programming perspective.

Health care districting: The models could be extended to consider other relevant factors, such as patient preferences, caregiver availability, and traffic conditions. Another area of opportunity is to further study the dynamic nature of the problem. In a dynamic planning setting, models could be adapted to handle dynamic changes in patient needs and caregiver availability over time. Furthermore, the development of a real-time decision support system could help healthcare providers make more agile and responsive decisions. Finally, given the variability of the demand, extending the model to consider multi-period planning horizons and dynamic patient demands constitutes another interesting area of promising research.

5.2 General research areas

An improvement over different areas is the consideration of parameter changes over a dynamic time horizon (Benzarti et al., 2013; Akdoğan et al., 2018). Modeling strategies to take into account the evolution over time, such as a *rolling horizon* method, would be particularly useful from a decision-making point of view.

From a computational standpoint, there are significant challenges in developing methods that can effectively tackle large real-world instances. Among the recent proposals in the reviewed literature, we found improvements in exact algorithms (Bender et al., 2018), modification of classical heuristic strategies (Benzarti et al., 2013; Ríos-Mercado, 2016) and hybrid algorithms (Moreno et al., 2020) as the most promising areas of research.

In terms of exact optimization algorithms, there are a few works that have had a certain degree of success (Salazar-Aguilar et al., 2011a; Bender et al., 2018; Ríos-Mercado and Bard, 2019), thus there is a tremendous area of opportunity in this regard. For instance, to the best of our knowledge, no work on polyhedral theory has been done on TDPs. Undertaking such work could result in significant improvements in cut generation schemes such as branch and cut. In addition, there is practically no work done on developing lower bounding schemes (Elizondo-Amaya et al., 2014). This is also very important because it is well known that having tight bounds may help speed up exact optimization algorithms and help measure the solution quality of heuristic procedures.

In terms of modeling, it is clear that districting problems are tactical problems that involve decisions on plans taken every few months or years. However, in many applications, there may be a second stage of operational decisions that somehow depend on the previous districting plan. This issue has often been handled separately, that is, seen as two separate problems, districting first and routing second. During the past few years, we have seen some work addressing these types of problems simultaneously as districting-routing problems (Haugland et al., 2007; Lei et al., 2012, 2015). In this regard, decomposition-based approaches become another good area of opportunity for handling districting problems that involve operational decisions.

Most of the work on TDP is node-based. That is, basic units are seen as geographic points or nodes in such a way that the districting is based on partitioning the set of nodes. However, there are a few applications where the partitioning is not made over the set of nodes, but the set of edges in the underlying graph (Yun and Yang, 2003; Butsch et al., 2014; García-Ayala et al., 2016; Kassem and Escobedo, 2023; Álvarez-Miranda et al., 2025). The structure of these problems is entirely different from the classical node-based districting problems and requires special treatment. Developing heuristic and exact algorithms for edge-based districting is another important area of future research work.

Two other topics stand out from the other ones for their adaptability to real-world representation; that is, uncertainty management, and multi-objective modeling.

Although the methodological advances for modeling and solving TDP are strongly motivated by complex real-world applications, there are few works in the literature dealing with districting decisions under uncertainty. Furthermore, in almost all of the cases, uncertainty management has been modeled by Stochastic Optimization (SO). Some representative examples correspond to Haugland et al. (2007), Lei et al. (2012, 2016), Diglio et al. (2020), Baldassarre et al. (2023), and Pomes et al. (2025). A crucial element in SO is the characterization of uncertain data by means of probabilistic measures. Although this can be suitable in some cases, there are situations in which parameters cannot be modeled by probability functions. Instead, the values of these parameters belong to known sets (e.g., ellipsoidal sets, polyhedral sets, closed intervals, or sets of discrete scenarios) from which they can determine any element. This type of uncertainty, usually referred to as deterministic uncertainty (Bertsimas et al., 2011), cannot be tackled by SO, and it is incorporated into the decision-making process through Robust Optimization (RO). Therefore, in real-world districting applications, uncertainty modeling might not be possible, which calls for the need to incorporate RO schemes. Despite the aforementioned discussion, the use of RO for designing decision-making tools in districting applications is still an unexplored field, so far only studied by Carlsson and Delage (2013).

In decision contexts such as political organization, sales territory design, or police patrol zone management, several performance measures must be simultaneously optimized (e.g., functional, economic, or topological measures). As a matter of fact, there are several articles presenting multi-objective optimization methodologies applied to districting problems (Salazar-Aguilar et al., 2011b; Camacho-Collados et al., 2015; Fragoso et al., 2016; Lei et al., 2016; Liberatore and Camacho-Collados, 2016), yet the number of works in multi-objective optimization still accounts for a minority of the total literature, and many use weighted formulations (and thus deal with the problem as a mono-objective optimization problem). Moreover, to the best of our knowledge, there are only a few exact algorithms for multi-objective districting (Salazar-Aguilar et al., 2011b; de Abreu et al., 2026), which is an area open for future research.

6 Conclusions

In this paper, we reviewed the literature on territory design for the last sixty years from an optimization standpoint. Our paper updates to the best of our knowledge the last two reviews on TDP by considering more than 200 papers published over the last 20 years, as well as complementing the reviewed literature with a so-called snowball search. In total, 322 articles were reviewed.

To guide this review, these three research questions were addressed.

- *Q.1* was fully addressed in Section 3 by providing a detailed classification of the existing TDP literature in terms of its most widely studied application areas, namely: Political districting, sales districting, commercial and distribution districting, school districting, police patrol districting, and health care and emergency response districting. Other districting applications were also reviewed.
- *Q.2* was addressed in Section 4 by providing an extensive review of the most widely used approaches for solving TDPs, including exact methods and heuristic/metaheuristic approaches, for a variety of different TDP models.
- *Q.3* was covered in 5, by elaborating on the most promising lines for future work in terms of particular TDP application area, and in terms of general modeling and algorithmic developments.

We hope that this work helps motivate more fruitful ideas in this exciting area of discrete optimization and serves as a good reference point for years to come.

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