Maximal Covering Location Models with Partial Coverage for Second-Level Specialized Health Care Services[1]

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Abstract

The lack of access to second-level health care services in developing countries is primarily due to the scarcity of facilities and the limited investment of resources. Access to these services directly relates to the distance that the population travels to these facilities. In that sense, a maximal covering location problem can be helpful to maximize the impact of decisions related to the location of new health care units. In this paper, we develop models to guide the decision of location new second-level services in a network of public hospitals. The partial coverage and variable radius are considered in the models to assess a large territory with different characteristics and populations. A case study is being conducted in the Mexican public health care system to assess five specialized health care services. The obtained results evidence the benefit of using optimization tools in the resource planning of health care services.

Keywords: health care planning; facility location; maximal covering location; partial coverage; integer programming.

1 Introduction

Second-level specialized health care services such as gynecology and pediatrics are essential in society. A large part of the population will require these services in various moments of their lives, and the demand for these services grows year by year [15]. In rural areas, the main problem is access to facilities that offer these services. In contrast, the problem is more related to capacity issues in urban areas. However, distance and time are critical to survival in emergencies in both cases.

The lack of access to second-level health care services in public hospitals is an important issue in developing countries such as Mexico. The investment in the health care infrastructure is limited and insufficient to ensure the total coverage of demand. Hence, each decision to invest new resources in the public sector must be taken, maximizing its impact on society. Like many other countries, Mexico has a segmented health care system [29]. This type of system avoids making global decisions, and efforts are made individually by federal states or institutions. Recently, a change has been promoted to take federal decisions to invest resources to improve health care services in Mexico [43]. This change aims at creating tools for infrastructure planning as a whole system.

The second-level health care services are available in most public hospitals in two schemes: outpatient care and emergency care. In both cases, the travel time/distance is the most critical parameter for measuring health care services access. A hospital not very accessible discourages people from timely attending to their illness when an appointment with a medical specialist is required. The travel time for emergency patients is crucial to safeguard their lives [38]. Together with the capacity of these services, access is the main factor in the decision to select new locations. However, the capacity level can be adjusted according to the demand characteristics, but the location is permanent. Therefore, the location of services can be analyzed as a strategic first-level decision, and capacity planning can be done after the site has been located based on the specific characteristic of each location. However, this decision can be made in single-stage planning based on the needs and context of the situation.

Like many other developing countries, the main problem in Mexico is the geographic distribution of the specialist. For instance, 54.2% of them are located in 3 out of the 32 federal states of the country. The number of specialists in Mexico City was 505.7 specialists per 100 thousand people, while in the federal state with the lowest rate was 35.9 specialists per 100 thousand people. This contrast is because most of the second-and-third level hospitals are located in the biggest cities of the country. However, gynecology and pediatrics services have become more widely needed because the demand is distributed throughout the territory at different levels.

In the Operations Research (OR) field, the maximal covering location problem (MCLP) is typically used in the health care area to locate emergency services such as ambulance stations or emergency centers [27]. However, recent works have extended its use to many other applications, such as the location of primary health care centers or hospitals [1]. In this case, we address the location of second-level specialized health care services. To this end, we use the model proposed by Karasakal and Karasakal [32] to evaluate the location of a new service in a set of candidates sites. Two extensions of this model are proposed to evaluate additional situations of the real application. The first extension considers the existing facilities that provide the services, candidate facilities where the service can be installed, and candidate locations where new facilities can be installed. The second extension is proposed for multiple institutions that collaborate to expand the coverage. This last model is based on a segmented public health care system like the Mexico case. In all cases, partial coverage is considered to avoid an abrupt ending of coverage, and each candidate site has a different coverage critical distance due to the extensively evaluated territory composed of rural and urban areas with different characteristics. For the last two models, the coverage rate of each candidate site must be adjusted considering the interaction with existing facilities.

To solve these models, we used CPLEX branch-and-bound (B&B) solver. All instances were optimally solved, managing to solve large instances up to 188,026 demand points and 1,835 candidate sites for the first model; 179,716 demand points and 1,081 candidate sites for the second model; and 512,666 demand points and 2,710 candidate sites for the multi-institutional version. The presented case study is based on the Mexican health care system for five services: gynecology, pediatric care, internal medicine, trauma care, and orthopedics. The variable distance of coverage is based on the population density of each location. We compare the solution obtained under the first model with the current practice. The goal is to evaluate the impact of using OR tools for improving access to these types of services. Then, we evaluate the benefit of solving the model in a single global instance instead of solving multiple federal state-wide instances or even regional-wide instances. The second model was applied to the orthopedic service, evaluating the objective function's improvement when the new locations are installed in existing hospitals, in new sites, or a combination of both. Finally, the third model (multi-institutional version) is evaluated with the pediatric service for different levels of participation in the coverage between institutions. These results encourage using these types of models as part of the decision-making process in the location of public health care services to optimize the impact of limited resources on society.

The remainder of the paper is organized as follows. Section 2 first reviews the relevant literature on locations models in related problems. This is followed in Section 3 by the formal definition and mathematical formulation of the main problem and its extensions. This section is followed by Section 4, where the case study results are presented. Finally, the conclusion and future directions are discussed in Section 5.

2 Related literature review

The literature on facility location models and methods applied in health care management has been quite active over the past few years. Our problem is focused on the location of public health care services. A survey in the context of the public sector is presented by Marianov and Serra [36]. Important efforts have been made in its application to health care problems. Some important surveys are proposed by Ahmadi-Javid et al. [1], Güneş et al. [30], Rais and Viana [42], Li et al. [34], Daskin and Dean [16], Brotcorne et al. [10], and Rahman and Smith [41].

The MCLP was proposed in 1974 by Church and Revelle [13] and White and Case [47]. The MCLP is a classic problem in the literature of the facility location. This problem is designed for finite resources that are unable to cover all demand. The objective is to find the best subset of p locations that maximize the covered demand. A demand point is covered if the distance to a facility is equal to or lower than a critical value. Reviews of covering problems can be found in Farahani and Hekmatfar [26], Snyder [44], García and Marín [28]. Some recent surveys of the MCLP applications are presented by Berman et al. [8] and Farahani et al. [27].

In particular, the survey presented by Ahmadi-Javid et al. [1] reviews facility location works related to health care from the year 2000 to 2016, 54% of the problems were related to emergency facilities such as (ambulance stations, trauma centers, or emergency off-site public access devices), while the rest of them were related to non-emergency facilities such as primary health centers. Location problems based on the MCLP represent 35% of works with 48 papers, and partial coverage is a characteristic only used in 10% of the works, all of them in emergency applications [45, 2, 35, 37, 11].

The models addressed in this research are based on the MCLP with partial coverage. In this problem, the classical binary coverage is replaced by a continuous parameter between zero and one calculated by a particular decay function. The more distance between a demand point and the facility, the value approaches zero. The first idea of the gradual covering was described by Church and Roberts [14]. The general concept of using a decay function in the MCLP is introduced in Berman and Krass [6], employing a step-wise function in a network version of the problem, providing a formulation and an effective heuristic procedure. In Berman et al. [7], the decay function was named as the non-ascending general decay function with two pre-specified threshold distances. They show how this problem can be transformed into the uncapacitated facility location problem when the set of potential facilities is discrete. The two previous works are a generalized idea of the problem addressed by Drezner et al. [21] for a single facility MCLP. A decay function is used in a capacitated version of the MCLP by Pirkul and Schilling [40]. The demand points beyond the coverage radius are allocated to facilities with available capacity, but the decay function allocated them to a nearby facility. A method based on a Lagrangian relaxation is used to find efficient solutions for instances up to 625 demand nodes. An overview of the gradual cover models can be found in Berman et al. [8].

Another continuous related work with a linear function in a planar space is found in Drezner et al. [22] for a single facility location. The authors proposed a B&B algorithm that produced a very efficient performance for instances up to 10,000 demand points. In Karasakal and Karasakal [32], the term "partial coverage" is introduced for the MCLP taking the same considerations of previous works for multiple facility locations. A solution procedure for large instances (up to 1,000 nodes, 40 potential sites) is proposed using a Lagrangian relaxation.

Recently, some extensions of the gradual cover location problem have been proposed. Tavakoli and Lightner [45] proposed an MCLP-based model for allocating vehicles and the location of facilities for emergency medical services (EMS) minimizing the amount of population not covered. A goal programming problem to locate EMS stations and find the minimum number of vehicles satisfying the performance levels is proposed in Alsalloum and Rand [2]. The probability of covering a demand within the target time is minimized in the first objective, and the second objective ensures that any demand arising located within the target time will find at least one ambulance available. In Eiselt and Marianov [23], the gradual covering is applied to the set covering location model, including the quality of service as a decision criterion. Lim et al. [35] proposed an extension of the MCLP that includes a minimum level of covered demand in the system and a flexible number of locations to be opened for the ambulance location problem. In Naoum-Sawaya and Elhedhli [37], a two-stage stochastic optimization model for the ambulance redeployment problem is proposed minimizing the number of relocations over a planning horizon while an acceptable service level is maintained. Drezner and Drezner [18] proposed an alternative objective function of maximizing the minimum cover of every demand point, ensuring that every demand point is covered as much as possible and there are no demand points with low cover. An ascent algorithm and tabu search were evaluated for instances up to 900 demand points.Chan et al. [11] proposed a multi-responder and gradual cover problem for automated external defibrillators in a probabilistic extension of the MCLP. The main contribution lies in developing a mixed-integer linear formulation equivalents or tight and easily computable bounds. Bagherinejad et al. [4] included the joint partial coverage when a demand point is covered by multiple facilities, developing multiple heuristics for networks up to 900 demand points. Bagherinejad et al. [4] included the gradual covering concept and the cooperative coverage in a single problem. A simulated annealing and tabu search were used to solve instances up to 150 demand nodes. In Drezner et al. [20], the gradual cover competitive facility location problem is proposed, which captures the market share by new facilities in a continuous space. Other recent applications using a gradual function are presented by Küçükaydın and Aras [33] for the location of multi-type facilities that include customer preference, by Erkut et al. [24] for ambulance location problem that includes a survival function, by Dogan et al. [17] for a multi-objective location of preventing health care facilities, and by Yücel et al. [48] for the location of mobile medical locations.

The variation of the coverage radius in a gradual covering location problem has been proposed by Drezner et al. [19] for a single facility and by Bashiri et al. [5] for multiple facilities. Eydi and Mohebi [25] introduce the MCLP with gradual coverage and variable radius over multiple periods. The variable cost directly impacts the coverage radius, and the facility capacity was considered. A simulated annealing algorithm was proposed to solve the problem.

Table 1 summarizes the characteristics of the most important contributions related to the MCLP with partial coverage. The last three rows correspond to the models evaluated in this paper. Columns three to six are features considered in these models. In the last column the particular features that differentiate each work are shown. Model 1 is similar to the one proposed in Karasakal and Karasakal [32], but in this case, the problem was assessed in a case study with a larger data-set (up to 188,026 demand points, 1,835 potential sites, and 856 opened facilities) in a public health care system. Model 2 incorporates two types of facilities, considering that some facilities are already installed in the system. The coverage level is calculated considering the interaction of the existing facilities. Model 3 is similar to model 2 but incorporates a multi-institutional system where the coverage of demand points can be expanded with the collaboration between institutions. As far as we know, this last problem has not been addressed in previous works.

Table 2 shows the methods used to solve the most closely related problems, the software or solver used, and the instances sizes of the case studies. The instances used in our case study are the largest concerning the number of demand points and facilities to be opened, but they occupy second place in the number of candidate sites. The number of candidate facilities and opened facilities tested can be classified in the group of the large-scale instances of the MCLP. All instances of our formulations were optimally solved by the B&B algorithm of CPLEX. Many recent improvements have been made in the performance of exact methods to solve integer programming problems together with the optimization in the use of computational resources and the development of new technologies. These advances avoided the need to develop alternative heuristic methods for solving problems addressed and the size of the instances proposed.

As contributions of our paper, we propose novel alternative formulations of the maximal covering location problem with partial coverage applied to a real critical problem, as is the second-level health cares services in developing countries. The case study applied to the Mexican health care system allows us to explore the benefit of making decisions using the solutions of the proposed models. We also propose estimating the coverage rates with the interaction of existing facilities and with the case of a multi-institutional scheme.

Table 1: Characteristics of works related to MCLP with partial coverage.

Paper	Year	Variable radius	Existing facilities	Facility types	Single objective	Other features
[6]	2002	Yes	No	No	Yes	Multiple levels of coverage
[32]	2004	No	No	No	Yes	
[45]	2004	No	Yes	No	Yes	Multiple vehicles are located at each facility
[3]	2007	No	No	No	No	Demand covered twice, minimization of the total travel distance for uncovered demand
[39]	2015	Yes	No	No	Yes	Applied to the hub location problem
[11]	2016	Yes	Yes	No	Yes	The level of coverage is probabilistic
[46]	2016	No	Yes	Yes	Yes	Two types of partial coverage are maximized, multi-type vehicles, allocation of vehicles to demand points
[4]	2018	Yes	No	No	Yes	Cooperative covering and location-allocation features
[25]	2018	Decision variable	No	No	No	Minimize costs, multiple time periods and capacitated service
[9]	2019	Yes	No	No	Yes	Joint partial coverage and co-location of facilities.
[33]	2020	Yes	No	Yes	Yes	Customer preferences and maximization of the profit
[12]	2021	No	No	No	Yes	A minimum distance between adjacent facilities
Model 1		Yes	No	No	Yes	
Model 2		Yes	Yes	Yes	Yes	
Model 3		Yes	Yes	Yes	Yes	Multiple institutions

Table 2: Solution methods and case studies of works related to MCLP with partial coverage.

Paper	Method	Software/Solver	Demand nodes	Candidate sites	Opened facilities
[6]	B&B and Greedy heuristic / LP-Relaxation	Cplex	400	400	80
[32]	Heuristic: Lagrangian relaxation based solution procedure		1,000	40	24
[3]	Lexicographic optimization and different versions of the Fuzzy goal programming	Cplex 8.0	50	50	8
[39]	B&B	Cplex 12.4 / Gurobi 5.0.2	81	22	20
[11]	B&B	Cplex 12.1	11,701	5,000	200
[46]	B&B	GAMS/BARON	420	17	6
[4]	Cooperative covering and location-allocation features	Heuristic: Simulated anneling and Tabu search	150	150	20
[25]	Heuristic: Simulated anneling	Cplex	100	100	5
[9]	B&B and Heuristics: greedy heuristic, ascent heuristic, and Tabu search.		400	400	133
[33]	B&B / Lagrangian relaxation / LS	Cplex 12.8	1,000	250	Variable
[12]	B&B	OPL	104	104	15
Model 1	B&B	Cplex 20.1.0	188,026	1,835	856
Model 2	B&B	Cplex 20.1.0	179,716	1,081	250
Model 3	B&B	Cplex 20.1.0	512,666	2,710	250

3 Formulation

3.1 The coverage rate (a_{ij})

The service coverage is defined by a critical distance around each facility. However, the limited number of facilities that can be opened implies that not all demand points will be covered. A non-ascending function is used to enlarge the coverage of a facility to a second critical distance and avoid an abrupt ending of the coverage. This type of function was proposed by Berman et al. [7]. The level of coverage is gradually decreased in the gap between these two critical distances. This change lets to assign the coverage to a demand point, even if this coverage is gradual.

The partial coverage decreasing function is calculated for each candidate site. Its equation is given by:

$$a_{ij} = \begin{cases} 1 & \text{if} \quad d_{ij} \leq l_j \\ \frac{u_j - d_{ij}}{u_j - l_j} & \text{if} \quad l_j < d_{ij} < u_j \\ 0 & \text{if} \quad d_{ij} \geq u_j \end{cases} \tag{1}$$

where:
a_{ij} is the coverage rate of the candidate facility located at site j for the demand point i,
d_{ij} is the distance between the demand point i and the potential facility located at site j,
l_j^s is the primary coverage radius of the potential facility located at site j,
u_j^s is the secondary coverage radius of the potential facility located at site j.

In Equation (1), if the distance between a demand point i and a candidate facility site j is less than or equal to the primary coverage radius, this point is fully covered ($a_{ij} = 1$). If the distance is equal to or greater than the secondary coverage radius, the demand point is not covered ($a_{ij} = 0$). If the distance is between these two critical bounds, the demand point is partially covered ($0 < a_{ij} < 1$). In most problems, bounds l and u are fixed for all the facilities. In this paper, we used variable bounds according to the demographic characteristics where each candidate site is located.

3.2 The variable coverage radius (r_j)

For a large-scale problem, the characteristics of the territory and population vary widely from one region to another. The regions can be classified into urban and rural areas. In urban areas, the population density is very high, and moderate travel distances can mean a lot of travel time. On the other hand, rural areas have a very low population density and vast territory. It is evident that the coverage radius should not be the same in both types of regions. For this reason, the coverage radius is adjusted according to the area where the potential site is located. This practical consideration was initially proposed by Berman et al. [7].

The type of function to estimate r_j depends on the characteristics of the particular application of the problem. In the case of partial coverage, each coverage radius is divided into two bounds: l_j and u_j. We propose determining a coverage radius for each candidate site and then determining the two bounds based on this value. The primary coverage radius can be estimated as a proportion of the variable coverage radius according to Equation (2). The secondary coverage radius can be determined as a proportional increase of l_j according to Equation (3). However, there are many ways in which these bounds can be determined.

$$l_j = \Delta_1 r_j \tag{2}$$

$$u_j = (1 + \Delta_2) l_j \tag{3}$$

3.3 Model 1: The maximal covering location problem with partial coverage

This problem is introduced by Berman et al. [7] and Karasakal and Karasakal [32], and can be applied to new second-level services that can be installed in an existing hospital network. These kinds of services can include new technologies or emergency services such as the one required to face the Covid-19 pandemic. The objective of the problem is to maximize the demand covered through the location of facilities that will supply the service. We denote as an "active facility", a candidate location that is opened in the solution of the problem, and an "inactive facility" to the one that is not open. A demand point is fully covered if this is located at a distance lower or equal to the primary coverage radius of an active facility. A demand point is partially covered if the distance is between the primary and the secondary critical radius of the closest active facility. A demand point is not covered if it is located at a distance greater than the secondary critical radius of any active facility.

The indices, parameters, and variables are described below:

Indices and sets:

- M Set of demand points; $i \in M$.
- N Set of candidate sites where the service can be installed; $j \in N$.
- N_i Subset of N such that $a_{ij} > 0$ for a given demand point $i \in M$, $j \in N_i$.

Parameters:

- h_i Demand of service in demand point i; $i \in M$.
- a_{ij} Coverage rate of the potential facility located at site j for the demand point i according to Equation (1); $i \in M$ and $j \in N$.
- p Maximum number of candidate sites where service will be installed.

Variables:

- Y_j Binary variable equal to 1 if the service is installed in candidate site j; 0, otherwise.

X_{ij} Binary variable equal to 1 if candidate site j is an active facility and it has the highest coverage rate for the demand point i among all other active facilities; 0, otherwise.

The problem formulation is as follows:

$$\max \quad \sum_{i \in M} h_i \left(\max_{j \in N} \{a_{ij} Y_j\} \right) \tag{4}$$

$$\text{subject to} \quad \sum_{j \in N} Y_j \leq p \tag{5}$$

$$Y_j \in \{0, 1\} \qquad j \in N \tag{6}$$

The objective function (4) maximizes the sum of demand covered of each demand point multiplied by the highest coverage rate according to the active facilities. Constraints (5) define the maximum number of sites where the service will be installed according to p. The binary variables are defined in constraints (6). Note that the objective function (4) is a piece-wise linear function that can be easily linearized according to Berman et al. [7] as follows:

$$\text{maximize} \quad \sum_{i \in M} \sum_{j \in N_i} h_i a_{ij} X_{ij} \tag{7}$$

$$\text{subject to} \quad \sum_{j \in N} Y_j \leq p \tag{8}$$

$$\sum_{j \in N_i} X_{ij} \leq 1 \qquad i \in M \tag{9}$$

$$X_{ij} \leq Y_j \qquad i \in M, j \in N_i \tag{10}$$

$$Y_j \in \{0, 1\} \qquad j \in N \tag{11}$$

$$X_{ij} \in \{0, 1\} \qquad i \in M, j \in N_i \tag{12}$$

The auxiliary binary variable X_{ij} determines the highest coverage rate for the demand point i among all active facilities. In case of a tie, one active facility is randomly chosen. According to constraints (9), only one variable associated with each demand point i can be equal to one. The variable X_{ij} related to the highest coverage rate will be equal to one in the optimal solution because this benefits the objective function. If a demand point i is not covered in the solution, all its associated variables X_{ij} will be equal to zero. According to constraints (10), if a candidate site j is not opened in the solution, all the associated X_{ij} are equal to zero. The nature of the additional decision variables is described in constraints (12).

The size of the problem could be an important issue when we are attempting to solve large-scale instances. The set of demand points must include only demand points such that $\sum_{j \in N} a_{ij} > 0$ because any other demand point is not covered in any solution of the problem. Another important aspect is to consider only the variables X_{ij} such that $a_{ij} > 0$ because any other variable will be zero in the solution.

Furthermore, the final coverage of each demand point in a solution can be determined with the following equation:

$$Z_i = \sum_{j \in N_i} a_{ij} X_{ij} \qquad i \in M, \tag{13}$$

where Z_i is the highest coverage rate of the demand point i. The demand points can be classified as follows:

- Fully covered ($Z_i = 1$), if at least one active facility is located at a distance equal to or lower than l_j from the demand point i.
- Partially covered ($0 < Z_i < 1$), if the closest active facility is located at a distance greater than l_j but lower than u_j from the demand point i.
- Non-covered ($Z_i = 0$), if all active facilities are located at a distance greater than or equal to u_j from the demand point i.

3.4 Model 2: The MCLP with partial coverage with existing facilities

In a real scenario, when we are evaluating an existing service to include new locations, there exist facilities that will continue supplying the service, there are facilities that do not supply the service but could start it (we refer to this as a new service being installed in the facility), and there are candidate sites for installing new facilities if required. We consider these three cases in the MCLP with partial coverage in the following problem. This is motivated by the fact that hospitals could incorporate additional services if they need them, and the investment will be much lower than installing new hospitals. However, new facilities could also be required because there are not even installed hospitals in some areas. The set of candidate sites must be expanded to include existing facilities where the service is being supplied, candidate facilities where the service can be installed, and candidate sites to build new facilities. The notation is the following:

Indices and sets:

G_i Set of existing facilities that supply the service and they cover fully or partially demand point i; $j \in G_i$.
N_1 Candidate facilities where new service can be installed; $j \in N_1$.
N_2 Candidate sites for installing a new facility; $j \in N_2$.
N $= N_1 \cup N_2$; $j \in N$.
N_i Subset of N such that $a_{ij} > 0$ for demand point i; $j \in N_i$.

Parameters:

p_1 Maximum number of candidate existing facilities where service can be installed.

p_2 Maximum number of new facilities that can be installed.

The set of existing facilities where the service can be installed and the set of potential sites must be managed separately in the model because they imply different costs and the number of locations to be opened must be defined separately unless a budget constraint will be defined. We note that all existing facilities that provide services can be removed from the formulation to reduce the size of the problem. However, they affect the coverage rate of potential sites if demand points are inside their coverage radius. In that case, the coverage rates must be estimated by comparing the additional benefit in the coverage of each demand point. The equation to update the coverage rate is the following:

$$a'_{ij} = \max\{a_{ij} - \max_{l \in G_i}\{a_{il}\}, 0\} \qquad i \in M, j \in N \tag{14}$$

In this problem, we can consider only demand points such that $\sum_{j \in N} a'_{ij} > 0$. The formulation of the problem is the following:

$$\begin{aligned}
\text{maximize} \quad & \sum_{i \in M} \sum_{j \in N_i} h_i a'_{ij} X_{ij} && && (15)\\
\text{subject to} \quad & \sum_{j \in N_1} Y_j \leq p_1 && && (16)\\
& \sum_{j \in N_2} Y_j \leq p_2 && && (17)\\
& \sum_{j \in N_i} X_{ij} \leq 1 && i \in M && (18)\\
& X_{ij} \leq Y_j && i \in M, j \in N_i && (19)\\
& Y_j \in \{0,1\} && j \in N && (20)\\
& X_{ij} \in \{0,1\} && i \in M, j \in N_i && (21)
\end{aligned}$$

3.5 Model 3: The MCLP with partial coverage for a multi-institutional scheme with existing facilities

In this problem, we consider the case where there are multiple institutions in a system, and they can share an existing service. An example of this system is the Mexican health care system divided into multiple public institutions that serve different population segments, but there is a real motivation to create collaboration among them. In this problem, each demand point has a demand rate for each institution. The demand of an institution can be covered by facilities of other institutions, but this coverage has a lower impact on the objective function according to a parameter λ. The options where the service can be installed in existing facilities or candidate sites to open new facilities to supply the service are also considered. The notation is similar to the formulation of model 2 with the following definitions:

Indices and sets:

K Set of institutions in the system; $q, k \in K$.

G_i^k Set of existing facilities of institution k that supply the service such that $a_{ij} > 0$ for demand point i; $j \in G_i^k$, $j \in G_i^q$.

N_i^k Subset of N_i for institution k; $j \in N_i^k$, $j \in N_i^q$.

N_1^k Candidate existing facilities of institution k where the service can be installed, $j \in N_1^k$.

N_2^k Candidate sites where a new facility can be installed to supply the service for institution k, $j \in N_2^k$.

Parameters:

λ Real value between zero and one that represents the coverage level of other institutions regarding the coverage level of the titular institution.

Variables:

X_{kij} Binary variable equal to 1 if demand of institution k in demand point i is covered (partially or fully) by candidate location j; 0, otherwise; $i \in M$, $j \in N_i$.

Parameters h_{ki}, p_1^k, and p_2^k are also updated with the additional index k. If a demand point of an institution is already covered by one or more facilities of the same institution or any other institution, we must consider the additional benefit in the coverage (a'_{kij}) of the potential locations if one of these is installed. The parameter b_{ki} is used to calculate the current demand coverage of institution k in demand point i, including the coverage of facilities of the same institutions ($j \in G_i^k$) and facilities of other institutions ($j \in G_i^q | q \neq k$), but these last ones are multiplied by λ.

$$b_{ki} = \max\{\max_{j \in G_i^k}\{a_{ij}\}, \max_{j \in G_i^q | q \neq k}\{\lambda a_{ij}\}\} \qquad k \in K, i \in M \tag{22}$$

The benefit in the coverage rate is calculated by subtracting b_{ki} from a_{ij}. If this value is negative, the benefit is equal to zero. The equation to calculate a'_{kij} is the following:

$$a'_{kij} = \max\{a_{ij} - b_{ki}, 0\} \qquad i \in M, j \in N \tag{23}$$

In this problem, we can remove the demand points such that a'_{kij} is equal to zero to reduce the size of the problem.

The formulation of the problem is the following:

$$\text{maximize} \quad \sum_{k\in K}\sum_{i\in M} h_{ki}\left(\sum_{j\in N_i^k} a'_{kij}X_{kij} + \sum_{j\in N_i^q|q\neq k} \lambda a'_{kij}X_{kij}\right) \tag{24}$$

$$\text{subject to} \quad \sum_{j\in N_1^k} Y_j \leq p_1^k \qquad k\in K \tag{25}$$

$$\sum_{j\in N_2^k} Y_j \leq p_2^k \qquad k\in K \tag{26}$$

$$\sum_{j\in N_i} X_{kij} \leq 1 \qquad k\in K, i\in M \tag{27}$$

$$X_{kij} \leq Y_j \qquad k\in K, i\in M, j\in N_i \tag{28}$$

$$Y_j \in \{0,1\} \qquad j\in N \tag{29}$$

$$X_{kij} \in \{0,1\} \qquad k\in K, i\in M, j\in N_i \tag{30}$$

The objective function (24) maximizes the coverage of demand of all institutions. If the coverage is provided by a facility of another institution, the coverage rate is multiplied by λ. Constraints (25) determine the number of existing facilities where the service will be installed for each institution, and constraints (26) determine the number of new facilities that will supply the service for each institution. The demand of each institution at each demand point can be covered by one facility either of the same institution or by another according to constraints (27). The nature of variables is given by constraints (28)–(30).

4 Case Study

4.1 Experiment settings

The case study is based on five second-level services in the Mexican public health care system: gynecology (S1), pediatric care (S2), internal medicine (S3), trauma care (S4), and orthopedic care (S5). Mexico is formed by 32 federal states, which are grouped into eight regions, as illustrated in Figure 1. According to INEGI [31], the population of Mexico is distributed among 192,247 demand points. In Table 3, the number of demand points, the population in 2015, the land area in a square kilometer (km^2), and the population density in inhabitants by km^2 is shown by federal states (FS), by regions (RG), and globally (GB). Some demand points are impossible to cover since they are located in regions that are non-viable to install hospitals around. These demand points were removed from instances because they can not be covered by any candidate site.

The number of candidate locations for each service is shown in Table 4. The data is grouped by federal states, by regions, and globally. This network is composed of three types of locations: (i) the

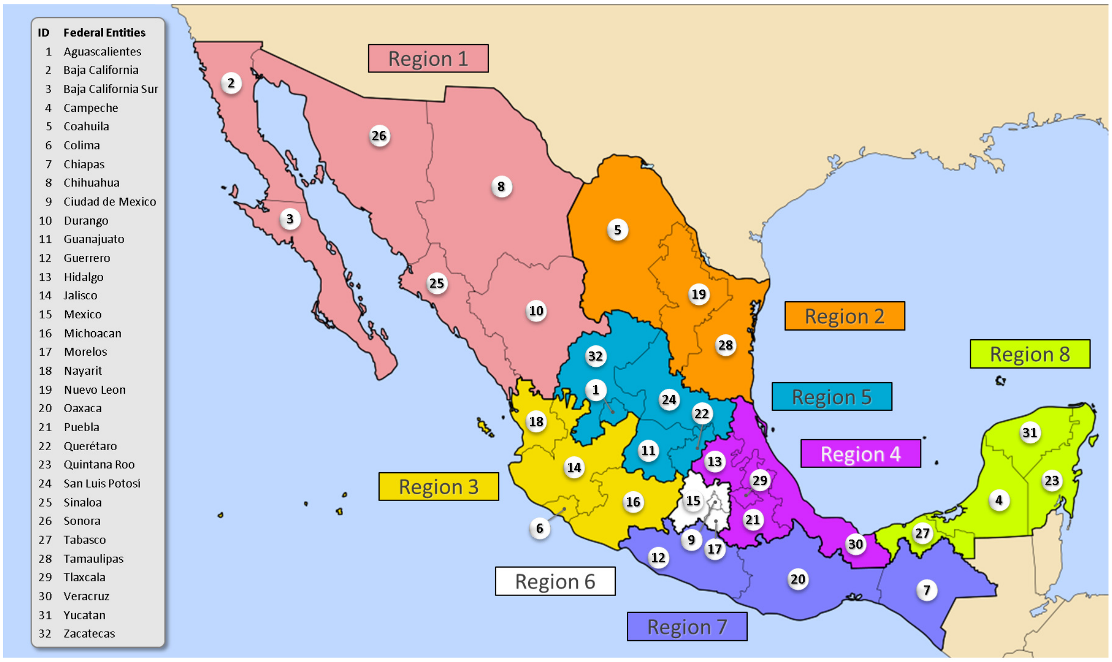

Figure 1: Map of Mexico divided into regions and federal states.

existing hospitals that supply the service (EH); (ii) a set of candidate hospitals where the service can be installed (CH); and (iii) a set of potential locations where new hospitals can be built (NH). This notation to describe the facility types will be taken up later. The hospital network was obtained from the General Directorate of Mexican Health Information (`http://www.dgis.salud.gob.mx`), and the set of potential sites was selected from demand points with demand levels higher than one thousand inhabitants. The coordinates of demand points and candidates' locations were determined with the Universal Transverse Mercator system to calculate the Euclidean distances between both sets of locations.

The proportion of the population at every demand point that requires a second-level service is assumed to be constant in the system for experimental purposes. Under this assumption, we used the number of inhabitants at each demand point as the demand rate for each service. Therefore, the same demand points and demand rates are used in all the evaluated services with different candidate sites. This assumption is made because there are currently no sufficient data to estimate the actual demand for each service by demand point.

4.1.1 The variable coverage radius for the Mexico case

The population density distribution of the 2,457 municipalities (counties) of Mexico in 2015, obtained from INEGI web site (`http://www.inegi.org.mx`), is shown in Figure 2. The demand points are grouped by municipalities in Mexico, and this is the lowest level with data on population density. The municipalities in the horizontal axis are sorted by population density, and the

Table 3: Characteristics of regions and federal states of Mexico.

RG	FS	Demand points	Population (inhabitants)	Area (km^2)	Density (inh./km^2)
1		38,561	15,153,881	756,528	20
	2	4,547	3,315,324	73,163	45
	3	2,850	711,755	74,610	10
	8	12,257	3,556,193	247,413	14
	10	5,794	1,754,557	123,364	14
	25	5,845	2,966,059	57,370	52
	26	7,268	2,849,993	180,608	16
2		16,434	11,515,490	296,000	39
	5	3,825	2,954,643	151,595	19
	19	5,265	5,119,385	64,156	80
	28	7,344	3,441,462	80,249	43
3		24,308	14,320,861	171,073	84
	6	1,235	711,133	5,783	123
	14	10,946	7,844,542	78,597	100
	16	9,427	4,584,277	58,599	78
	18	2,700	1,180,909	28,094	42
4		33,236	18,411,659	130,952	141
	13	4,714	2,858,291	20,821	137
	21	6,400	6,168,806	34,309	180
	29	1,294	1,272,782	3,997	318
	30	20,828	8,111,780	71,824	113
5		25,202	13,500,980	184,326	73
	1	1,989	1,312,485	5,616	234
	11	8,995	5,853,370	30,607	191
	22	2,717	2,038,304	11,691	174
	24	6,829	2,717,719	61,138	44
	32	4,672	1,579,102	75,275	21
6		6,897	27,009,984	28,725	940
	9	547	8,918,636	1,495	5,967
	15	4,846	16,187,575	22,351	724
	17	1,504	1,903,773	4,879	390
7		37,833	12,717,915	230,666	55
	7	20,047	5,216,982	73,311	71
	12	7,290	3,533,200	63,597	56
	20	10,496	3,967,733	93,758	42
8		9,776	6,854,240	166,836	41
	4	2,778	899,642	57,516	16
	23	1,993	1,462,399	44,825	33
	27	2,499	2,395,221	24,731	97
	31	2,506	2,096,978	39,764	53
GB		192,247	119,485,010	1,965,105	61

Table 4: Classification of candidate sites by service, region, and federal state.

		S1				S2				S3				S4				S5			
RG	FS	EH	CH	NH	Tot.	EH	CH	NH	Tot.	EH	CH	NH	Tot.	EH	CH	NH	Tot.	EH	CH	NH	Tot.
1		102	19	186	307	97	25	186	308	69	17	186	272	27	27	186	240	23	32	186	241
	2	16	1	21	38	19	2	21	42	11	1	21	33	7	3	21	31	7	3	21	31
	3	6	0	20	26	5	1	20	26	5	1	20	26	2	2	20	24	2	2	20	24
	8	20	5	53	78	16	8	53	77	9	6	53	68	0	6	53	59	2	4	53	59
	10	15	3	32	50	14	4	32	50	12	1	32	45	3	4	32	39	2	5	32	39
	25	23	4	15	42	22	5	15	42	19	3	15	37	9	2	15	26	1	10	15	26
	26	22	6	45	73	21	5	45	71	13	5	45	63	6	10	45	61	9	8	45	62
2		65	19	68	152	66	15	68	149	49	15	68	132	26	16	68	110	7	35	68	110
	5	23	5	19	47	23	4	19	46	22	3	19	44	10	4	19	33	3	11	19	33
	19	21	11	28	60	22	10	28	60	11	7	28	46	7	5	28	40	3	9	28	40
	28	21	3	21	45	21	1	21	43	16	5	21	42	9	7	21	37	1	15	21	37
3		84	23	122	229	71	33	122	226	55	16	122	193	39	14	122	175	14	39	122	175
	6	5	0	4	9	5	0	4	9	4	0	4	8	3	1	4	8	0	4	4	8
	14	37	9	69	115	25	22	69	116	16	6	69	91	15	6	69	90	8	13	69	90
	16	29	9	33	71	29	8	33	70	27	5	33	65	13	7	33	53	5	15	33	53
	18	13	5	16	34	12	3	16	31	8	5	16	29	8	0	16	24	1	7	16	24
4		140	20	127	287	124	29	127	280	95	39	127	261	33	41	127	201	26	48	127	201
	13	23	2	21	46	22	3	21	46	17	4	21	42	9	4	21	34	1	12	21	34
	21	48	9	36	93	40	14	36	90	30	15	36	81	4	19	36	59	19	4	36	59
	29	12	1	15	28	13	0	15	28	8	3	15	26	1	5	15	21	4	2	15	21
	30	57	8	55	120	49	12	55	116	40	17	55	112	19	13	55	87	2	30	55	87
5		86	17	65	168	91	10	65	166	51	19	65	135	29	10	65	104	11	27	65	103
	1	4	2	3	9	4	2	3	9	5	0	3	8	5	0	3	8	0	5	3	8
	11	35	9	10	54	41	1	10	52	17	13	10	40	14	3	10	27	3	13	10	26
	22	5	1	12	18	5	1	12	18	4	0	12	16	3	1	12	16	3	1	12	16
	24	22	3	22	47	20	4	22	46	10	5	22	37	3	5	22	30	5	3	22	30
	32	20	2	18	40	21	2	18	41	15	1	18	34	4	1	18	23	0	5	18	23
6		214	25	115	354	186	54	115	355	85	16	115	216	26	40	115	181	38	30	115	183
	9	49	12	12	73	34	28	12	74	26	0	12	38	2	17	12	31	16	4	12	32
	15	156	11	83	250	144	23	83	250	53	14	83	150	18	22	83	123	22	19	83	124
	17	9	2	20	31	8	3	20	31	6	2	20	28	6	1	20	27	0	7	20	27
7		110	31	62	203	86	47	62	195	61	31	62	154	27	17	62	106	7	36	62	105
	7	35	17	19	71	27	21	19	67	28	12	19	59	12	4	19	35	3	12	19	34
	12	29	8	13	50	26	9	13	48	13	9	13	35	7	5	13	25	2	10	13	25
	20	46	6	30	82	33	17	30	80	20	10	30	60	8	8	30	46	2	14	30	46
8		55	13	67	135	55	11	67	133	37	14	67	118	15	15	67	97	8	22	67	97
	4	10	3	14	27	10	2	14	26	10	0	14	24	5	0	14	19	2	3	14	19
	23	9	3	19	31	7	5	19	31	5	4	19	28	2	5	19	26	2	5	19	26
	27	20	5	11	36	23	1	11	35	16	7	11	34	7	6	11	24	1	12	11	24
	31	16	2	23	41	15	3	23	41	6	3	23	32	1	4	23	28	3	2	23	28
GB		856	167	812	1,835	776	224	812	1,81	502	167	812	1,481	222	180	812	1,21	134	269	812	1,215

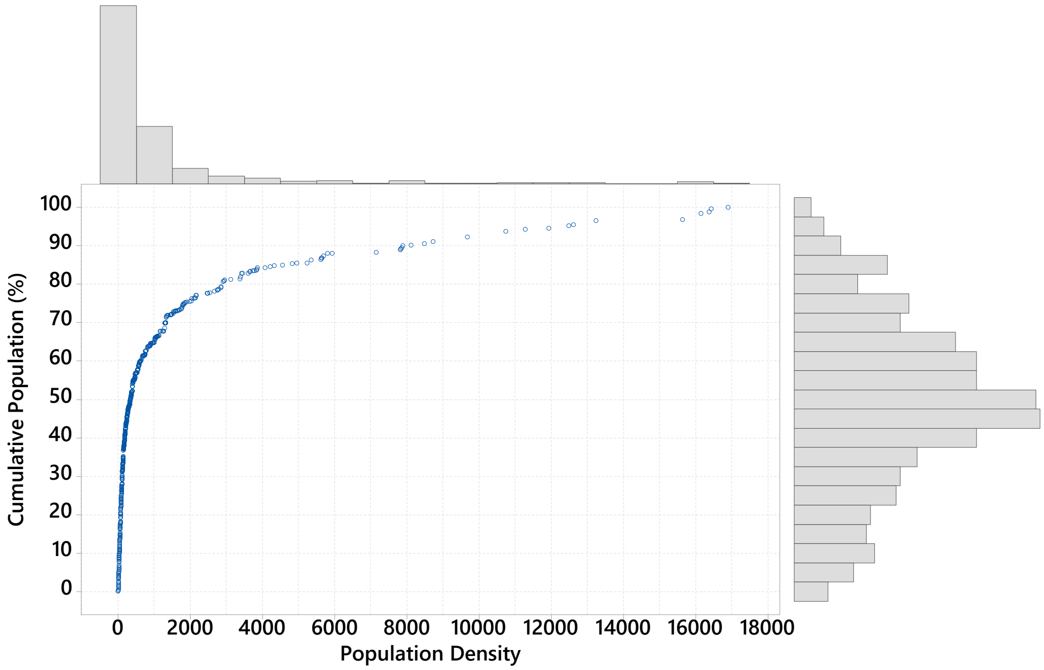

Figure 2: Classification of municipalities according to the population density.

cumulative population is shown in the vertical axis. We can note that half of the population lives in areas with a population density lower than 400 inhabitants per square kilometer (inh/km^2). Three-quarters of the population lives in a territory with a population density lower than 2,000 inh/km^2, and the remaining population (25%) lives in a territory between 2,000 to 17,000 inh/km^2. We propose a logarithmic function with high sensitivity to low population density rates, but that includes the entire threshold of values of the population density rates. This function calculates the coverage radius according to the population density of each municipality where a candidate site is located.

The function to estimate a variable coverage radius is presented in Equation (33). The graphical representation of the coverage radius function applied to the municipalities of Mexico is shown in Figure 3. The function is adjusted based on a minimum and a maximum coverage radius. These limits are adjusted in a range of population density rates ($\delta_{\min}$, $\delta_{\max}$). Two coefficients must be determined to adjust the function: α and β.

The notation in the equations is the following:

r_j Variable coverage radius of location j.
$r_{\max}$ Maximum coverage radius.
$r_{\min}$ Minimum coverage radius.
δ_j Population density of location j.
$\delta_{\max}$ Maximum population density.
$\delta_{\min}$ Minimum population density.
α Exponent value of the logarithm calculated by Equation (31).

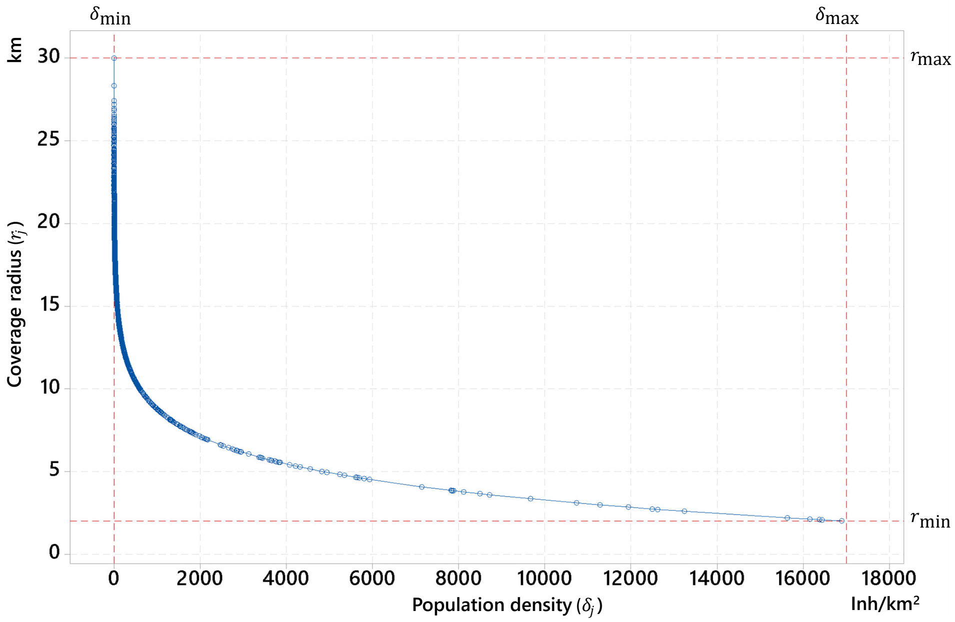

Figure 3: Graphical representation of the coverage radius.

β Adjustment coefficient calculated by Equation (32).

$$\alpha = \frac{r_{\max} - r_{\min}}{\log_{10}(\delta_{\max}) - \log_{10}(\delta_{\min})} \tag{31}$$

$$\beta = \log_{10}(\delta_{\max})^{\alpha} + r_{\min} \tag{32}$$

$$r_j = \beta - \log_{10}(\delta_j)^{\alpha} \tag{33}$$

For experimental purposes, the values of some parameters were fixed. The minimum and maximum population densities were based on the population density of Mexico (2015). The minimum coverage radius was taken from the average distance of hospitals in Mexico City because it is the most populated city with the largest number of hospitals at the same time. The maximum coverage radius was fixed to 30 km because this distance is reachable in rural areas of Mexico. The parameters α and β were calculated with Equations (31) and (32), respectively. The values of all these parameters are the following:

$$r_{\min} = 2\,\text{km} \qquad \delta_{\min} = 0.14\,\text{inh./km}^2$$
$$r_{\max} = 30\,\text{km} \qquad \delta_{\max} = 17,000\,\text{inh./km}^2$$
$$\alpha = 2.3917 \qquad \beta = 25.2976$$

In the case of partial coverage, the coverage radius must be extended to a secondary coverage radius. In this case, limits were set as follows:

$$l_j = r_j$$
$$u_j = 2l_j$$

The secondary coverage radius of a hospital for the most populated areas is near 4 km, and for the least populated areas is 60 km.

4.2 Solution method

The branch-and-bound algorithm from the CPLEX callable library, version 20.1.0, with a C++ API was used to find the optimal solution for each instance. The experiments were carried out in an Intel Core i7-5600U at 2.60GHz with 16GB of RAM under Windows 10 operating system.

In Table 5, the worst-case execution time is shown for each model. The number of demand points ($|M|$), the number of candidate sites ($|N|$), and the total number of locations that can be opened (p) are shown. We can see that in all the cases, CPLEX found the optimal solution of the corresponding integer linear programming model in less than one hour of CPU time, even though some instances can be classified as large-scale instances.

Table 5: Worst-case execution time of the B&B algorithm by model.

	Service	$\|M\|$	$\|N\|$	p	CPU time (s)
Model 1	S1–S5	192,247	1,835	856	1,337
Model 2	S5	179,716	1,081	150	271
Model 3	S2	512,666	2,710	150	1,141

4.3 Model 1 assessment

In the case of model 1, the problem was solved for the five second-level health care services in three different clustering levels: by federal states, by regions, and globally. In Section 4.3.1, we compared the advantages of solving the entire instance rather than solving it in segmented instances. In Section 4.3.2, the solution of global instances is compared with the actual distribution of the services. The objective is to compare the outcomes and benefits of using an optimization model to improve access to second-level health care services.

4.3.1 Solving the problem at different clustering levels

This experiment aims at identifying the advantages of solving the problem in a single large-scale instance instead of dividing it into multiple instances. The actual planning of infrastructure in Mexico is made locally, in most cases throwing away the potential of a unified resource planning. The number of opened hospitals was fixed to the existing hospitals that supply the service. The candidate sites include the existing hospitals where the service is operating (EH), additional candidate hospitals that do not supply the service (CH), and new potential sites where new hospitals can be installed (NH). The number of candidate sites for each service by federal state, by region, and globally are shown in Table 4. The problem was first solved by states-wide instances, all the locations obtained from the optimal solutions were unified as a single solution, and these were evaluated as a single global instance. This means that hospitals of other federal states can cover a demand point. Then, the regional instances were solved and combined similarly to evaluate a single solution. Finally, a global instance was solved to compare the results between the solutions obtained with the previous cases.

Table 6 summarizes the results for each instance type and each service. The first and second columns identify the service and instance type previously defined in Section 4.1. The number of instances that were solved of each type is shown in column three. The number of total facilities opened by each instance type is shown in column four. For example, there are 32 optimal solutions of FS (Federal State) instances, the optimal locations were unified in a single global solution with 856 opened facilities, and then, the global objective value is calculated and shown in column five. The objective value is also represented as a percentage of the total demand in the sixth column. For example, 93.7% of demand was covered with the solution of the first row. The number of locations opened of each type is shown in the following three columns. The percentage of demand and demand points covered are classified according to the coverage level (full, partial, and null) in the following columns. In all the cases, the best objective value was reached with the solution of the global instance. The same occurs with the percentage of covered demand and the number of covered demand points. The objective function as a percentage of the total demand shows that solutions of global instances compared with state-wide instances were higher in 2.6%, 3.2%, 2.8%, 7.0%, and 9.4% for services S1 to S5, respectively. Significant improvements in the objective function were found when the coverage level is lower in the system, as in the case of S4 and S5. This result encourages the integration of the services planning as an integral system to increase the current coverage.

According to the case, parameter p was fixed to the number of existing hospitals in each federal state, each region, and globally. In some state-wide instances, it was observed that the optimal solution required a lower number of facilities than p to cover the maximum possible demand. For example, the total number of open locations in S1 was equal to 802 for the federal states instances when the maximum number of locations that could be opened was 856. This means that in some federal states, more locations are operating than the ones required according to the objective function. However, in the global instance of S1, the entire possible locations were opened because locations not required in some federal states were required in other states to improve their coverage. This fact is another advantage of solving the problem as a single global system.

Table 6: Summary of results by service and type of instance.

S	Type	Inst.	p	Obj. value (x1000)	Obj. value (%)	Facility type EH	CH	NH	Total	Cov. dem. (%) Full	Partial	Null	Cov. dem. pts (%) Full	Partial	Null
S1	FS	32	856	111,916	93.7	456	79	267	802	85.0	12.9	2.2	47.7	37.6	14.7
	RG	8		112,872	94.5	453	79	267	799	86.1	12.3	1.6	50.1	38.7	11.2
	GB	1		114,978	96.2	434	82	340	856	87.9	11.6	0.5	56.4	39.0	4.6
S2	FS	32	776	110,243	92.3	413	94	243	750	83.6	13.2	3.1	44.0	36.9	19.2
	RG	8		111,526	93.3	395	108	269	772	84.4	13.2	2.4	46.8	38.1	15.1
	GB	1		114,085	95.5	375	115	286	776	86.8	12.3	1.0	53.5	39.4	7.1
S3	FS	32	502	103,963	87.0	249	82	162	493	75.8	17.6	6.6	34.5	35.3	30.1
	RG	8		105,514	88.3	240	89	173	502	76.7	17.9	5.4	36.7	37.9	25.5
	GB	1		107,309	89.8	239	93	170	502	77.4	18.7	3.9	38.7	39.9	21.4
S4	FS	32	222	81,693	68.4	91	51	80	222	54.6	24.1	18.4	17.7	25.0	51.0
	RG	8		88,528	74.1	78	56	88	222	59.5	23.5	17.1	18.6	26.5	55.0
	GB	1		90,035	75.4	82	55	85	222	60.2	24.4	15.4	18.7	27.8	53.6
S5	FS	32	134	68,022	56.9	48	28	58	134	48.6	14.0	32.9	8.5	11.2	75.4
	RG	8		76,781	64.3	32	35	67	134	53.5	17.5	28.9	10.3	15.4	74.2
	GB	1		79,286	66.4	32	38	64	134	54.0	20.2	25.8	11.0	18.5	70.5

4.3.2 Comparison between existing system and the optimal location found by solving the model

One of the main goals of our research is to provide a way to improve the existing system. To this end, we compare the existing locations of hospitals with an optimal solution yield by the proposed model, which purpose is to improve access to services.

In Table 7, we compare two cases: (A) the solution of the MCLP with partial coverage, fixing the existing locations, and (B) the optimal global solution based on a set of candidate locations composed of the sets EH, CH, and NH previously described. The structure is similar to Table 6. The additional demand covered in the case of B was 8%, 11%, 14%, 27%, and 23% for S1 to S5, respectively. In some cases, such as S4 and S5, the improvement in the objective function in case B was 56% and 52%, respectively. The impact in the objective function was greater when the coverage level in the system was lower, as occurred with S4 and S5. For example, the percentage of demand not covered by S4 in case A is 40.2%, being 64.4% of demand points, while the demand not covered in case B is reduced to 15.4% being 53.6% of demand points.

This experiment allows us to find the percentage of well-located hospitals based on the objective function of model 1. This percentage is estimated by the number of existing hospitals in the optimal solution. This percentage is 51%, 48%, 48%, 37%, and 25% for services S1 to S5, respectively. If the network of existing hospitals that do not supply the services is used to install the service, the government could improve the demand coverage with significant savings because this will be less expensive than installing new hospitals. In the optimal solutions, the percentage of this type of location selected was 10%, 15%, 19%, 25%, and 28% for services S1 to S5, respectively.

Table 7: Comparison of results between the existing system and the optimal location found by solving the model.

Service	Solution	Objective value (x1000)	(%)	Facility type EH	CH	NH	Total	Covered demand (%) Full	Partial	Null	Covered locations (%) Full	Partial	Null
S1	A: Actual	105,464	88.3	856	0	0	856	78.8	16.0	5.2	39.0	37.9	23.1
	B: Optimal	114,978	96.2	434	82	340	856	87.9	11.6	0.5	56.4	39.1	4.6
S2	A: Actual	100,697	84.3	776	0	0	776	74.7	16.9	8.4	35.7	36.5	27.8
	B: Optimal	114,084	95.5	376	114	286	776	86.8	12.3	1.0	53.5	39.4	7.1
S3	A: Actual	90,623	75.8	502	0	0	502	61.3	25.4	13.3	28.6	34.6	36.8
	B: Optimal	107,309	89.8	239	93	170	502	77.4	18.7	3.9	38.7	39.9	21.4
S4	A: Actual	57,566	48.2	222	0	0	222	36.7	23.0	40.2	13.7	21.9	64.4
	B: Optimal	90,035	75.4	82	55	85	222	60.2	24.4	15.4	18.7	27.8	53.6
S5	A: Actual	52,035	43.5	134	0	0	134	33.9	18.1	48.1	6.5	10.4	83.1
	B: Optimal	79,286	66.4	33	37	64	134	54.0	20.2	25.8	11.0	18.5	70.5

4.4 Model 2 assessment

As stated in Section 1, another of the critical problems faced in the actual health care system is the lack of coverage of the existing facilities because most of the second-level health care units are concentrated in the largest cities of the country. In this particular case, we are interested in locating new orthopedic care services because this is the service with the lowest coverage rate of the five evaluated services, and it is one of the essential second-level services that present a lack of coverage. To this end, the location of new orthopedic care services (S5) was solved using model 2 under the following specifics.

Different numbers of new locations (p) were assessed in the experiments: 10, 20, 30, 40, 50, 100, 150, 200, and 250. In Table 8, the main characteristics of the actual system are shown. The demand is 113.5 million inhabitants to be covered, divided into 163,231 demand points. The service is currently supplied by 134 hospitals. The percentages of demand and demand points covered by these facilities are shown in the table. The number of candidate sites to use in the problem is shown in the last two columns. These sites include 269 candidate hospitals that can supply the service and 812 candidate sites where new hospitals can be located.

Table 8: Characteristics of the existing system for orthopedic care service.

Demand	Demand points	Existing facilities	Covered demand (%) Full	Partial	Null	Covered demand points (%) Full	Partial	Null	Candidate Sites CF	NH
113,523,531	163,231	134	34	18	48	7	10	83	269	812

Three types of instances were solved for each value of p: (i) the case where the service is installed in existing hospitals that do not supply the service (CH), (ii) the case where only new hospitals can be opened without including existing facilities (NH), and (iii) the case where the service can be installed in existing or new hospitals (CH+NH). In case (i), constraints (17) do not exist; in case (ii), only constraints (17) exist, and in the last case, constraints (16) and (17) are replaced by:

$$\sum_{j \in N_1^k} Y_j + \sum_{j \in N_2^k} Y_j \leq p \tag{34}$$

Where p is the maximum number of locations to be opened. In Figure 4, the objective values of all instances are compared, and the detailed results are shown in Table 9. The objective value of model 2 represents the additional demand covered by the new locations. In the fourth column of the table, the global objective value considering the existing facilities is shown. We can observe that there is no significant difference between objective values of instances under cases (i) and (ii). However, when both types of candidate sites are available, as in case (iii), there is an average improvement of 22% in the objective value. In Table 9, we can see that more demand and demand points are fully and partially covered with the solutions in case (iii). The optimal value of the objective function and the number of new locations have no linear relationship. The growth on the objective value reduces as the number of new locations increases as shown in Figure 4.

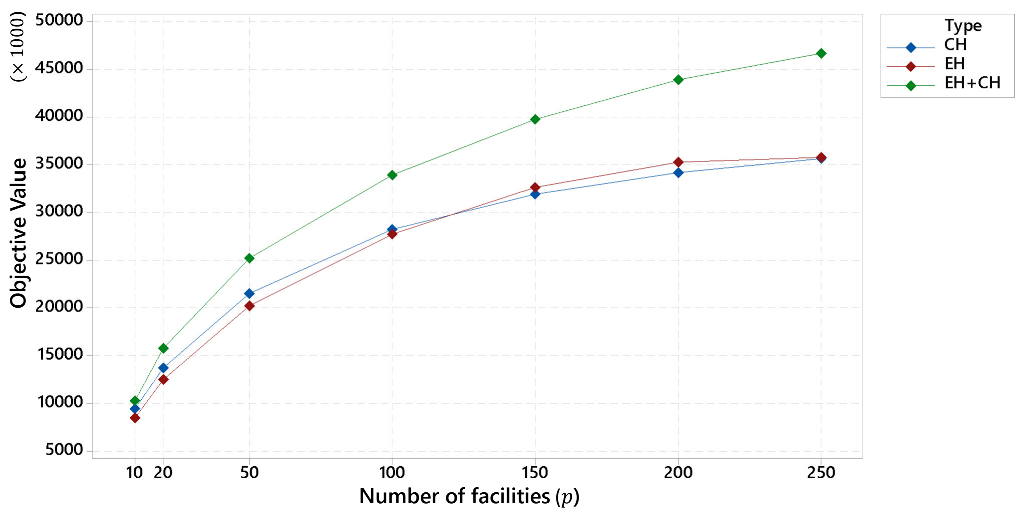

Figure 4: Objective value by type of candidate sites and number of new locations.

Table 9: Comparison of results between evaluated scenarios of model 2.

	p	Objective value ($\times 10^3$) Model 2	Global	Covered demand Full	Partial	Null	Covered demand points Full	Partial	Null
Existing facilities	0		52,035	18.1	48.1	33.8	6.5	10.4	83.1
Case (i): CH	10	8,483	60,519	40.0	19.8	40.3	7.2	11.9	80.8
	20	12,493	64,528	42.6	20.8	36.5	7.8	13.2	79.0
	30	15,738	67,773	44.3	22.4	33.3	8.5	14.4	77.1
	40	18,179	70,215	45.8	22.8	31.5	9.3	16.0	74.7
	50	20,190	72,225	47.1	23.6	29.3	10.3	17.4	72.3
	100	27,718	79,753	51.9	26.0	22.0	14.8	23.0	62.2
	150	32,611	84,646	56.0	26.0	18.0	18.7	27.6	53.7
	200	35,259	87,294	58.6	25.3	16.1	21.3	30.4	48.4
	250	35,746	87,782	59.3	25.0	15.7	22.7	30.4	46.9
Case (ii): NH	10	9,438	61,474	40.7	18.9	40.3	7.5	12.4	80.1
	20	13,727	65,762	43.4	19.6	37.0	8.2	13.7	78.1
	30	16,998	69,033	45.9	19.4	34.8	8.9	14.4	76.7
	40	19,504	71,539	47.7	19.5	32.9	10.0	15.4	74.6
	50	21,509	73,544	49.1	19.9	31.0	10.8	16.7	72.6
	100	28,216	80,251	54.2	20.5	25.3	15.0	21.6	63.4
	150	31,903	83,938	57.5	20.5	22.1	18.1	24.9	57.0
	200	34,185	86,220	59.4	20.8	19.9	20.9	27.5	51.7
	250	35,635	87,670	61.1	20.6	18.3	23.4	31.0	45.7
Case (iii): CH+NH	10	10,272	62,308	41.5	19.1	39.5	7.4	12.2	80.4
	20	15,746	67,781	45.4	19.5	35.0	8.2	13.9	77.9
	30	19,502	71,537	47.8	20.2	32.0	8.9	14.8	76.3
	40	22,681	74,716	50.1	20.2	29.7	9.4	15.6	75.0
	50	25,182	77,217	52.1	20.2	27.7	10.5	17.2	72.4
	100	33,899	85,934	58.1	22.4	19.5	15.7	23.9	60.4
	150	39,751	91,787	62.5	22.7	14.8	19.6	28.2	52.2
	200	43,890	95,926	66.1	22.2	11.7	23.3	31.1	45.6
	250	46,669	98,704	69.1	21.3	9.6	26.4	34.1	39.5

In Figure 5, the graphical representation of a solution of model 2 is shown for the location of ten additional orthopedic services under case (iii). Blue circles represent the coverage radius of hospitals that already supply the service, the radius of the new services installed in existing hospitals are represented by pink circles, and the new hospitals opened are represented by red circles. We can see that the new locations are opened in zones with high population density not covered by existing facilities. The population density is represented on the red scale, as shown in the population density map. Additional coverage of 9% of total demand was obtained with the ten new locations, reaching a total demand coverage of 55% if we considered all the previous installed services. There is still 39.5% of the demand out of the coverage radius of any hospital, and this demand is spread over 80.4% of demand points.

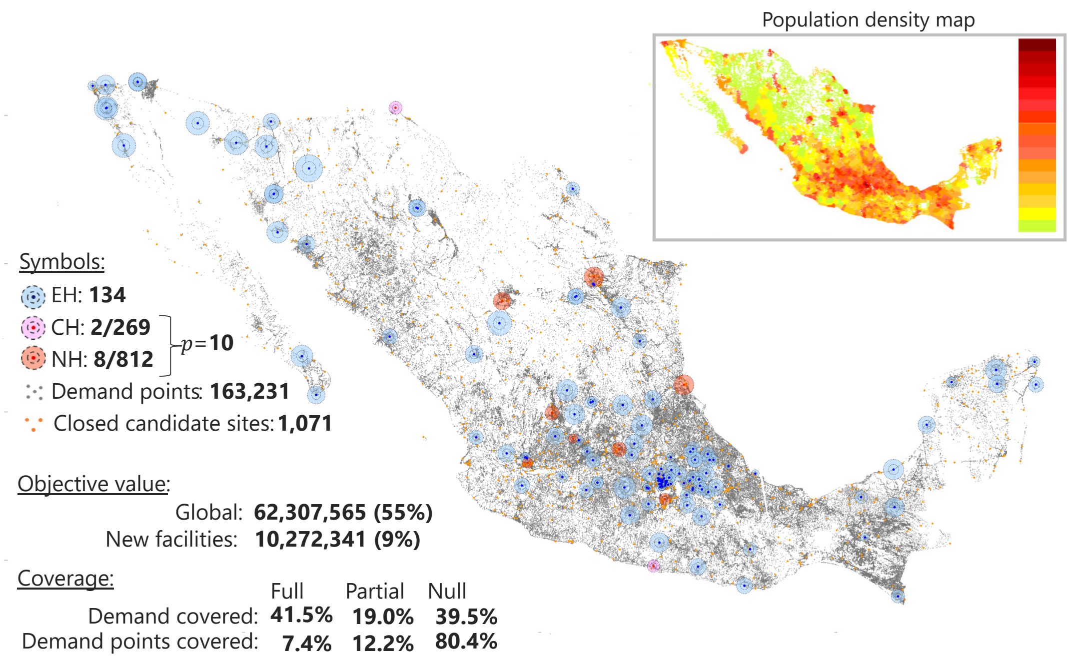

Figure 5: Location of orthopedic care service in the map of Mexico for $p = 10$ using model 2.

4.5 Model 3 assessment

The three main public institutions of Mexico were evaluated in this multi-institutional version of the MCLP. For this experiment, pediatric care (S2) was evaluated because there is one of the essential services in all the country, and there is enough information about its availability in published data. Three public institutions were simulated in the experiment based on actual institutions: I1 composed of institutions SSA, SME, and IMSS-Bienestar (for uninsured population); I2 representing the institution ISSSTE (for public sector workers); I3 representing the IMSS (for private-sector workers). In Table 10, the main characteristics of the instance are shown. The demand rates were randomly generated following uniform distribution based on the population level of each demand point with the distributions $\gamma = 1 - \alpha - \beta$, $\alpha = \text{Unif}(0, 0.2)$, and $\beta = \text{Unif}(0, 0.2)$ for institutions I1, I2, and I3, respectively. The existing hospitals that supply the services are shown in the fourth column. The percentage of demand and demand points covered by these hospitals are shown in the following columns. The candidate locations are classified in existing hospitals where the service can be installed and candidate locations where new hospitals can be built. Demand points that are fully covered by existing hospitals are not considered in the problem, and demand points that are partially covered are updated according to Equation (23). The level of participation in the demand coverage of other institutions is determined by λ. This value was ranged from 0 to 1 by increments of 0.1. The number of locations to be opened in the system (p) was assessed as follows: 10, 20, 50, 100, 150, and 200 facilities. One global instance for each combination of p and λ was assessed. The

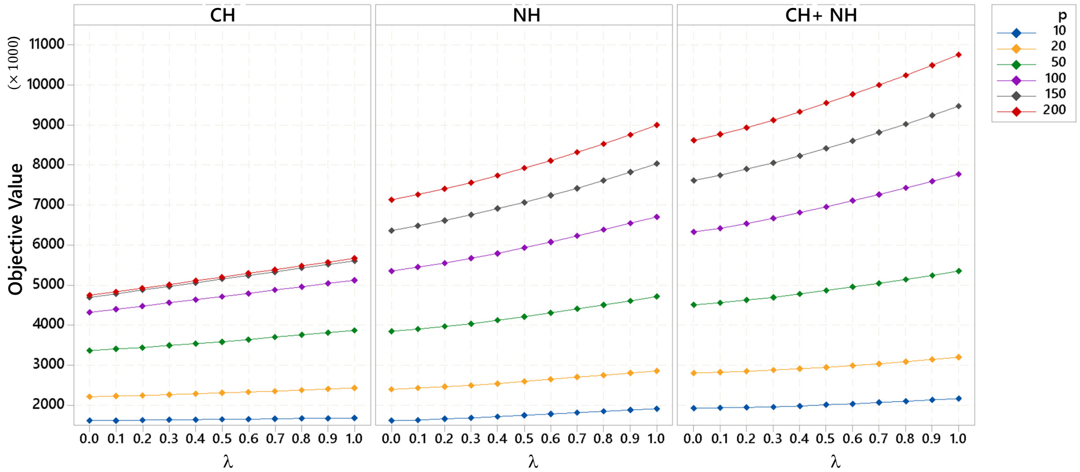

Figure 6: Objective values of model 3 with different values of λ and p for each case.

three types of cases defined for model 2 were also used in this experiment, but the proportion of facilities that can be opened for the three institutions was 0.8, 0.1, and 0.1 for institutions I1, I2, and I3, respectively.

Table 10: Current coverage in the multi-institutional scheme for service 4.

Institution	Demand	Demand points	Current facilities	Covered demand (%)			Covered demand points (%)			Candidate Sites	
				Full	Partial	Null	Full	Partial	Null	CH	NH
I1	96,060,917	150,947	772	71	14	15	21	27	52	221	812
I2	11,513,225	183,187	120	41	16	43	5	10	86	25	812
I3	11,900,297	178,532	193	54	13	33	7	12	80	28	812
Total	119,474,439	512,666	1,085							274	2436

Figure 6 shows the results for all combinations of p and λ. The objective values are compared among different values of λ and p for each one of the three cases. We can observe that the objective value is higher when λ tends to 1 with an average increase of 19.7% regarding solutions with $\lambda = 0$. The results of the three types of cases are also contrasting. The objective values are higher when new hospitals are opened because the number of candidate options is much higher than hospitals that currently do not supply the service. However, the most significant improvement in the coverage is obtained when both types of locations are available in the problem.

5 Conclusions

In this paper, we revisited the MCLP with partial coverage and proposed some extensions to solve some problems related to locating second-level health care services. These models are motivated by a need to improve access to these services in developing countries. For the partial coverage, a decay function is used based on two critical coverage bounds. A logarithmic function was proposed to determine the coverage radius of each candidate site based on the population density. Large-scale instances of the problem are considered due to the high number of demand points and facilities that form the health care system. Model 1 aims at locating new services that will be installed in a hospital network. Model 2 is proposed for the case when the service is already working, and it is provided in a hospital network; there are two types of new locations: existing facilities where the service can be installed and new facilities where the service can be supplied. In model 3, we proposed the multi-institutional version of the previous model, where the demand can be covered by facilities from other institutions.

Our case study, based on real-world data from the Mexican Health Care System, revealed very interesting results. A first important finding is that we identified some advantages from solving the global instances over region-wide or federal state-wide instances. For example, the improvement from state-wide to global instances ranged from 2.6% to 9.4% of additional demand covered. There is more impact in the objective function when the actual demand coverage is low such as in the case of S4 and S5 that covers 48.2% and 43.5% of the total demand, respectively. In another experiment, we observed an improvement in the percentage of demand covered from 8.0% to 27.2% when we compare the current locations of the service with the optimal location using model 1. For the actual location of the services, many other important aspects were considered. However, the benefit in the demand coverage is directly associated with the improvement in the service accessibility, which can be considered in the decision process of locating new facilities.

We note that the current disposition of the services is not focused on the improvement of access. There is a high concentration of facilities in the largest cities, leaving the rural areas without coverage. This is associated with the high level of demand found in urban areas. However, the infrastructure planning must include every type of region and population sector. As proposed in this research, this type of model could improve access to these services as a priority. The capacity of the facilities is another important issue that must be evaluated carefully. However, capacity planning could be analyzed in a follow-up study once the location of a hospital has been determined.

The branch-and-bound algorithm successfully solved all instances of the main problem and its extensions. The optimal solution of all instances was found in a reasonable CPU time, with the highest taking 1,337 CPU seconds for solving an instance of model 1. This model was the most difficult to solve because the high number of facilities to be opened.

Future research on this problem is oriented to evaluating additional features of the second-level health care services such as the type of service: normal or emergency services. For some services the capacity must be included as part of the location problem without ignoring the service coverage. The feature of joint coverage among facilities can extend the coverage of some services. For example, some facilities do not have operating rooms but can provide outpatient care. All these features must be evaluated for large-scale instances. In this sense, the development of alternative solution methods such as meta-heuristics algorithms could be a valuable asset to be pursued.

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