An Extended Unit Restriction Model with Environmental Considerations for Forest Harvesting[1]

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Abstract

This paper addressed a forest harvesting problem with adjacency constraints, including additional environmental constraints to protect wildlife habitat and minimize infrastructure deployment costs. To this end, we propose an integer programming model to include those considerations during the optimization of the harvest regime of a Mexican forest. The model considered was based on the Unit Restriction Model, a benchmark approach that merges the management units before the optimization process. The resulting model, namely the Green Unit Restriction Model (GURM), and the benchmark model (URM) from the literature were tested with the forest Las Bayas using information obtained from the SiPlaFor project from the Universidad Juárez. The proposed model was solvable in all tested instances. A sensitivity analysis study over a core data set of test instances was carried out on the different parameters of the GURM model to determine optimal configurations for the specific case study. Several environmental measures are assessed in our experimental work. Among the parameters assessed were the distance value between pairs of units harvested in the same period, the distance value between pairs of units considered natural reserve, timber volume to be harvested, green-up period, and minimum forest reserve area.

An interesting observation from the experiments was that the maximum area inversely affected the URM and URM-Green models, larger areas resulted in a reduced number of management units in the URM model, thus reducing the computational time to solve the instance of the problem, but in this case, at the expense of a reduced profit. One of the interesting findings was that, in all experiments under all different factors, harvesting every 5 or 6 years yields better profits than harvesting every 10 or 12 years. The current standard in the Mexican system is to harvest every five years.

Keywords: Forest management; Integer programming; Adjacency constraints; Unit restriction model; Green-up constraints; Clear-cut regime

1 Introduction

The collaboration of the operations research and forest science communities has been very successful over the past few decades. The use of operations research analytical tools has had a positive impact in many areas of forestry management problems such as strategic, tactical, and operational planning, wildfire management, conservation, and the use of OR to address environmental concerns [32].

One particular area that has received considerable attention is harvest scheduling problems, which in essence involve strategic and tactical decisions on how to harvest a forest, seen as a collection of stands, in such a way that maximum timber volume or profits are obtained. These decisions are subject to requirements that assure sustainable forest growth. Many of these planning requirements are set forth by government authorities and are designed to preserve wild habitats.

Among these problems, a particular class considers *adjacency constraints*, which are requirements that prevent the harvesting of adjacent stands or forest management units. The original motivation of these types of constraints was to prevent soil erosion. However, recently there have been other environmental benefits such as preserving the habitat of wildlife or creating a barrier around the recently harvested area until it recovers some of its biomass. These problems consider a regime of clear-cuts distributed along a planning horizon that is divided into periods. An important requirement limits the maximum adjacent area that is allowed to be cut at any given period.

In the literature, several approaches have been studied to address the adjacency issue. Two well-known approaches are the Unit Restriction Model (URM) and the Area Restriction Model (ARM) [27]. Both approaches differ on how the forest management units, or simply units, were defined out of single forest stands. The URM assumes that these management units are merged a priori, while the ARM forms the management sets out of the stands during the solution process.

This paper considers a harvest scheduling problem with adjacency requirements with additional environmental considerations to help preserve wildlife and the forest itself. Our model includes the preservation of old-growth forest reserve, green-up period requirements, tree biological maturity preservation, and distance-based requirements. These are further explained in Section 3. These requirements have been proposed independently for other forest management problems; however, to the best of our knowledge, ours is the first model to consider all these simultaneously under the URM approach. In addition, the model is studied under a case study from the state of Durango, Mexico. Despite being the eleventh country [17] with timber resources globally, Mexico has not systematically used these tools to manage its forests at the level of the nations mentioned above.

To assess our model, we used a case study from Las Bayas. Las Bayas is a piece of land owned by the Universidad Juárez, located in Durango, Mexico. Its geographic and forest measurements information is available through the SiPlaFor project [33]. We use real-world data estimates of timber pricing. A sensitivity analysis study over a core data set of test instances was carried out on the different parameters of the URM-Green model to determine optimal configurations for the specific case study.

The rest of the paper is organized as follows: A literature review is presented in Section 2. Section 3 describes the problem, its requirements, and modeling assumptions. It also includes the integer programming model. Section 4 describes the experimental work in detail, including a description of how the test instances are constructed and a full assessment of the case study. Finally, Section 5 summarizes the main findings and discusses future work.

2 Literature Review

The use of operations research tools and methodologies to address many decision-making problems in forestry management has been very active over the past 50 years [6]. Many problems involving operational, tactical, and planning decisions have been addressed from the optimization perspective. Some recent surveys can be found in Ezquerro et al. [16], Belavenutti et al. [3], Rönnqvist et al. [32]. In particular, this section is focused on a unique set of harvesting problems involving adjacency constraints. Adjacency-constrained models were first introduced in the seminal paper of Murray [27]. The author introduced two models: the Area Restriction Model (ARM) and the Unit Restriction Model (URM). The ARM approach is based on the idea of limiting, at a particular given period, the total area of adjacent harvesting units to a given upper bound. In contrast, the URM handles the adjacency constraints by establishing that two adjacent units cannot be simultaneously harvested in the same period. There is a third class of adjacency-constrained models that attempt to improve on the limitations of the ARM. These are the so-called hyper-unit models [36, 37, 38], where one hyper unit is generated a priori as a candidate cluster for aggregated forest areas from each forest unit by a predefined rule to meet lower and upper size requirements.

Although the ARM perspective increases the number of potential solutions to choose from, the addition of clustering processes within the computational optimization stage causes the problem to become more complex. Many solutions to ARM-based models have been studied in the past, including heuristic approaches [2, 10, 12, 30] and exact optimization schemes such as the path formulation [25], the clique-cluster packing formulation [19, 24], and the bucket formulation [14]. Since this paper is based on the URM, we focused the review on URM-based approaches mainly. Stochastic adjacency models are out of the scope of this paper, but the reader is referred to the work of Wei and Murray [34].

In the early years, significant effort was devoted to developing exact and heuristic algorithms [28, 29, 35]. Manning and McDill [23] presented a computational study examining optimal parameter settings to improve the efficiency of solving harvest scheduling models with adjacency constraints. They used ILOG's Cplex 11.2. A total of 160 randomly generated hypothetical forests were created with 50 or 100 stands and four age-class distributions. Mixed-integer programming problems were formulated with four different adjacency constraint types, two Unit Restriction Model (URM) adjacency constraints (Pairwise and Maximal Clique) and two Area Restriction Model (ARM) formulations (Path and Generalized Management Unit). A total of 640 problem sets –where a set is a typical forest size, age-class distribution, and adjacency constraint type– were tuned to determine the optimal parameter settings and then solved at both the default and optimal settings. In general, they observed that mean solution time was shorter for a given problem set using the optimal parameters compared to the default parameters. The discussed results provide a simple approach to decrease the solution time for solving mixed-integer forest planning problems with adjacency constraints.

Borges et al. [7] presented a simulated annealing metaheuristic to a harvest scheduling problem with URM adjacency constraints. They assessed the performance of their metaheuristic under three new methods aimed at introducing biased probabilities in the management unit (MU) selection and compared them to the conventional method that assumes uniform probabilities. The new methods were implemented as a search vector approach based on the number of treatment schedules describing sequences of silvicultural treatments over time and standard deviation of the net present value within MUs (Methods 2 and 3, respectively), and by combining the two approaches (Method 4). They also presented three hundred hypothetical forests (datasets) for three different landscapes characterized by different initial age class distributions (young, normal, and old). Each dataset encompassed 1600 management units. The methods evaluation was carried out through objective function values, the first feasible iteration, and time consumption. They found that introducing a bias in the MU selection improves the solutions compared to the conventional method (Method 1). However, an increase in computational time was observed in general for the new methods. By comparing results, method 4 was the best alternative as it produced the best average and maximum objective function values alongside lower time consumption than Method 2 and for most datasets. Although Method 4 performed very well, Methods 2 and 3 should not be neglected since maximum objective function values were obtained by these methods for a considerable number of datasets.

Dong et al. [15] presented another simulated annealing metaheuristic to a class of forest harvest scheduling problems under URM and ARM constraints. Four hypothetical grid datasets with different age class distributions (i.e., young, normal, older, and spatially organized) and one real dataset from northeastern China were used to illustrate how 2-OPT moves can intensify a search within high-quality areas of solution space and thus produce higher-valued solutions as compared to the sole use of 1-OPT moves. Finally, extreme value theory was employed to estimate the global optimum solution and evaluate the heuristic solutions' quality. They found out that the 2-OPT technique produced consistently better solutions than the 1-OPT technique in terms of the values of the mean and maximum solutions and significantly decreased the standard deviations associated with the sets of solutions. The maximum solution values could attain results above 98% of the estimated optimal values.

Kašpar et al. [22] assessed the time efficiency of solving URM models with different adjacency constraints using a commercial solver. Their results indicate that the type of adjacency constraints can have significantly affect the solving time, and therefore it could be a crucial factor for the time required for developing forest plans. They noted that pairwise adjacency constraints may be sufficient today for addressing problems with forest harvest scheduling constraints.

Gharbi et al. [18] presented a new mixed-integer linear programming model classified as both a unit restriction approach and an area restriction approach. They needed to generate a feasible cluster in the first place to formulate the model. However, contrasting with other approaches, the authors state that there is no need to generate specific model constraints representing computationally burdensome clusters for large cases. The authors described and analyzed their approach by comparing it with the most efficient works in the literature. Comparisons were made from the modeling and computational points of view. Results showed that the proposed model was competitive concerning modeling complexity and formulation size.

3 Methods

3.1 Discussion of Environmental Requirements and Model Assumptions

The URM models from the literature do not consider environmental considerations in their respective formulations [19, 24, 27]. This work proposes a new model with the following additional constraints:

Primary forest reserve: A traditional measure to protect wildlife species is the spatial assignment of natural reserves, known in the literature as an *old-growth* or *primary forest* reserve. Primary forest reserve constraints have become more relevant in recent years because these areas may shelter valuable ancient trees and endangered animal species. Carvajal et al. [11] discuss important considerations for integrating old-growth concerns into forest planning with adjacency constraints. They also discuss establishing a requirement that assures a minimum primary forest area, typically between 10 and 30% of the total area. These requirements are also considered in our model.

Green-up period: Green-up constraints induce all units adjacent to a recently harvested unit to remain standing for many consecutive periods [8]. This way, a protective barrier is assigned until the harvested unit restores its forest mass while we avoid deforesting large areas at any given time of the planning horizon. In addition, this measure reinforces a better harvesting distribution by promoting continuous areas of mature woodland at seeding age adjacent to growing stands. In Murray [26], green-up constraints are defined in the URM formulation by extending the number of periods in which adjacent clear-cuts to a recently harvested unit are proscribed. We follow this approach in our paper.

Biological maturity: The forest age distribution is one of the most important environmental aspects in harvesting models. A well-distributed age structure at the landscape level ensures sustained yield of forest products and provides a diverse habitat for wildlife [1]. A way to achieve this goal is to protect the trees of a unit before reaching their age of biological maturity. This parameter can be seen as a threshold from which it is feasible to harvest a basic unit. To the best of our knowledge, the previous optimization work on forest management uses the age threshold mainly at establishing optimal rotation age for carbon sequestration [31]) or as a part of heuristic methods for penalizing cuts in immature stands [5, 4, 9]. We incorporate such a perspective in this work.

Distance Requirements: The consideration of distance can be seen from two points of view: the environmental, that seeks to maintain a reasonable distance between natural reserves, to allow the wildlife to have a safe habitat; and a second consideration, from the economic perspective, as each harvesting period also implies the deployment of infrastructure such as new routes and collection areas, since the reduction of the dispersion of the harvested management sets also reduces the costs of operation. Distance requirements have been mainly used to establish old-growth patches within a maximum distance to help wildlife species as deers and sables to survive and reproduce. Maximum distance constraints assume that beyond a specific threshold, the functional habitat for such species starts decreasing [21]. In addition, we also consider a similar set of constraints to establish a maximum dispersion between units to be simultaneously harvested to reduce transportation costs [6].

The following model assumptions are considered:

- The problem is deterministic; that is, all parameters are known with certainty.
- The management policy must respect the maximum clearcut area, adopted by law or by voluntary guidelines.
- Clearcut per management unit is the harvesting method to be used.
- The wood volume's growth pace is linear and depends on the average annual increase of standing trees.
- The regeneration barrier length should allow the recovery of most of the trees of a unit after its harvesting.
- The harvesting profit in a specific period is a function of the wood potential, prices, and production costs per cubic meter of wood of the tree species in the unit.
- Selling prices per cubic meter of wood are static over the whole planning horizon.

- The age threshold for the trees of a unit to become candidates for an old-growth forest must be greater than the length of the planning horizon ($\epsilon_{\text{nat}} > \delta_{\text{PH}}$).
- For simplicity, we assume that all management units have a surface between 51 and 100% of the maximum clearcut area. Namely, the simultaneous harvest of any pair of adjacent units violates the maximum area.
- Age of (manually constructed) units corresponds to the age of the youngest trees into it, so that no clear-cut interferes in the maturing vegetation process. Age of trees in units is assmued homogeneous.
- Strong and weak adjacency types (see Section 1) are used to avoid harvests between neighboring units, such that simultaneous harvests have no point of contact to reinforce the continuity of forest.

3.2 Green Unit Restriction Model (GURM)

Indices and Sets

I : Set of harvesting units; $i \in I$.
T : Set of time periods in the planning horizon; $t \in T$.
N_i : Set of harvesting units adjacent to harvesting unit $i \in I$.

Parameters

v_{it} : Volume of timber collected in harvesting unit i in period t; $i \in I$, $t \in T$.
β_{it} : Profit obtained by harvesting unit i in period t; $i \in I$, $t \in T$.
L_t : Lower bound for the timber volume to be harvested in period $t \in T$.
U_t : Upper bound for the timber volume to be harvested in period $t \in T$.
r : Regeneration period in years.
m_i : Maturity age of trees in harvest unit $i \in I$.
ϵ_{it} : Age of trees in unit i in period t; $i \in I$, $t \in T$.
a_i : Area (in hectares) of unit $i \in I$.
$A_{\min}$: Minimum area of native forest reservation (in hectares).
δ_{PH} : Planning horizon length in years.
ϵ_{nat} : Age of trees at which the unit can be considered as a natural reserve.
ϕ_{ij} : Euclidean distance between units i and j;. $i, j \in I$.
$\Lambda_{\max}$: Maximum distance between each pair of units harvested in the same period.
$\Gamma_{\max}$: Maximum distance between each pair of units considered as natural reserve.
M : A large enough constant.

Decision Variables

x_{it} : Binary variable equal to 1 if unit i is harvested in period t, and 0 otherwise; $i \in I$, $t \in T$.

z_i : Binary variable equal to 1 if unit i is considered as a natural reserve, and 0 otherwise; $i \in I$.

Integer Programming Model

$$\text{Maximize} \quad z = \sum_{i \in I} \sum_{t \in T} \beta_{it} x_{it} \tag{1}$$

$$\text{subject to} \quad \sum_{t'=t-r}^{t+r} \left(x_{it'} + x_{jt'} \right) \leq 1 \qquad t \in T,\, i \in I,\, j \in N_i \tag{2}$$

$$\sum_{t \in T} x_{it} \leq 1 \qquad i \in I \tag{3}$$

$$\sum_{i \in I} v_{it} x_{it} \geq L_t \qquad t \in T \tag{4}$$

$$\sum_{i \in I} v_{it} x_{it} \leq U_t \qquad t \in T \tag{5}$$

$$m_i x_{it} \leq \epsilon_{it} \qquad t \in T,\, i \in I \tag{6}$$

$$z_i + \sum_{t \in T} x_{it} \leq 1 \qquad i \in I \tag{7}$$

$$\sum_{i \in \Omega_I} a_i z_i \geq A_{\min} \tag{8}$$

$$z_i = 0 \qquad i \in I \,:\, \epsilon_{i1} + \delta_{\text{PH}} < \epsilon_{\text{nat}} \tag{9}$$

$$\phi_{ij} x_{it} - M(1 - x_{jt}) \leq \Delta_{\max} \qquad i, j \in I,\, t \in T \tag{10}$$

$$\phi_{ij} z_i - M(1 - z_j) \leq \Gamma_{\max} \qquad i, j \in I \tag{11}$$

$$x_{it} \in \{0, 1\} \qquad i \in I,\, t \in T \tag{12}$$

$$z_j \in \{0, 1\} \qquad j \in I \tag{13}$$

The objective function (1) seeks to maximize the harvesting profit throughout the planning horizon. Constraints (2) prevent any neighboring unit pair from being harvested simultaneously before the green-up period r. Constraints (3) assure a unit to be harvested at most once during the planning horizon. Constraints (4) and (5) guarantee the lower and upper limits of the timber volume harvested in each period. Constraints (6) guarantees that only the units that have reached the maturity age in the current period are considered for harvesting. Constraints (7)-(9) help us model the primary forest reserve constraints. Constraints (7) assure that no unit can be harvested (at any time) and simultaneously be considered a primary forest. Constraints (8) guarantee a minimum area exclusively dedicated to the primary forest reserve. As young or immature trees do not satisfy primary forest requirements, minimum age constraints must be imposed to prevent impractical assignments. To this end, constraints (9) ensure that units with relatively immature trees are not assigned to the primary forest reserve. Constraints (10) establish the maximum distance allowed between any pair of units harvested in the same period to decrease the operational costs. Constraints (11) and set the distance limit between any pair of natural reserve management units to guarantee the habitat of the wildlife. Finally, (12) and (13) express the nature of the binary variables.

The URM, as proposed by Murray [27], considers minimizing (1) subject to (3)–(5) and constraints

$$x_{it} + x_{jt} \leq 1 \qquad t \in T,\, i \in I,\, j \in N_i$$

instead of (2). Therefore, our model can also be seen as an extension of the URM.

The URM is NP-hard [19, 20]. Despite its inherent computational complexity, the URM has been solved relatively well in instances with up to 80,000 stands over 10 time periods by branch and bound [18].

4 Experimental Work

4.1 Preprocessing

The implementation of the solution framework can be divided into two stages: (a) the construction of instances from the geographical, economic, and measurement information of the forest, and (b) the solution of the corresponding integer programming model.

The information about the individual stands, such as the projected volume of wood that a stand can yield at each period of the planning horizon, the profit that can be obtained from the timber mentioned above, and the average age of each stand, is calculated first using the available databases. In this particular case study in Mexico, the geographical data was obtained from the SiPlaFor project. SiPlaFor is an open access system designed to support the decision-making process in preparating and executing sustainable forest management planning on temperate forests of Mexico [33]. SiPlaFor also contains information of the species, age, samples of trees from each stand, among other factors, to calculate the potential volume of timber harvested at each period during the planning horizon, considering an even-aged whole-stand model. For this project, we obtained the profit from the previously calculated volume of wood per stand during each harvesting period and the proportion of species in the stand. The prices were taken from a CONAFOR technical report [13].

4.2 Case Study

In this paper, we present a case study of Las Bayas forest, a wooded terrain located in the state of Durango, in northwest Mexico, as depicted in Figure 1. Las Bayas is a forest comprised mainly of pines and oaks. Its selection was motivated by the following reasons:

- Las Bayas belongs to the Universidad Juárez del Estado de Durango (UJED), under the management of the Faculty of Forestry. Therefore, its database contains high-quality information from the stands, freely available to the public [33].
- Las Bayas information is available in the SiPlaFor project, also managed by the UJED. The project contains information about other forests in Mexico, which makes it easier for our approach to be applied to other forests, assuming that the information in the databases meets the minimum requirements for the model.

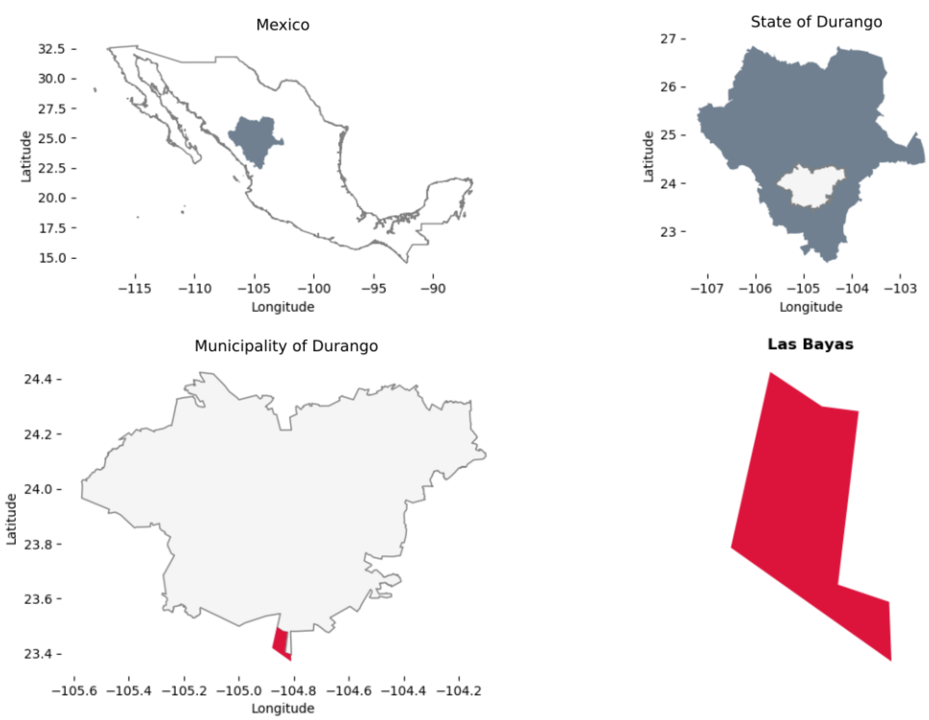

Figure 1: Geographic location of Las Bayas.

For this case study, we selected 118 stands, as shown in shaded areas in Figure 2. Those are some of the stands authorized to carry out timber extraction activities. Surrounding stands, instead, are considered natural reserves for agriculture of tourism.

4.3 Experimental Settings and Test Instance Generation

All models were implemented in the C++14 programming language, compiled in g++ version 7.4.0, and solved with the ILOG CPLEX 12.8 solver through its API. For test instance generation and data recollection, the programming language used was Python 3.7.4. The election of this language

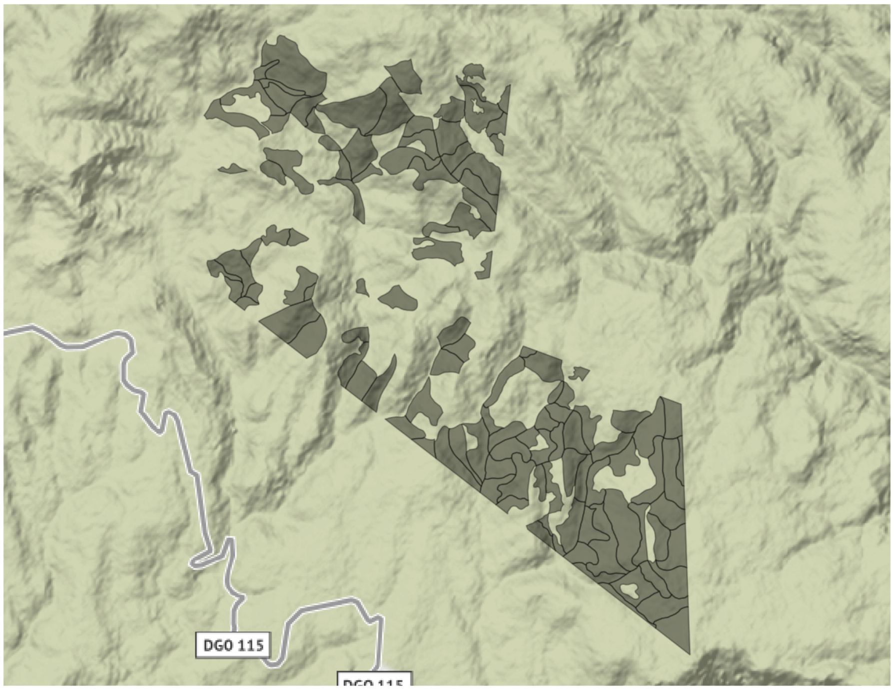

Figure 2: Subset of production stands from Las Bayas

was mainly because of the availability of powerful GIS (Geographic Information System) libraries, such as GeoPandas and pyproj. The experimentation was conducted in a server with an Intel Xeon E3-1240 v3 3.5 at GHz with 8 cores, with 16 GB of DDR3 RAM at 1600 MHz, in the Ubuntu 14.04.6 LTS operating system.

We formed forest management units by aggregating stands whose total area did not exceed a given maximum area. To this end, we studied three different values of maximum areas, namely 30, 40, and 50 ha. The length of the planning horizon was established at 60 years. For this length, different harvesting periods were evaluated: (i) 5 periods (that is, every 12 years), (ii) 6 periods (every 10 years), (iii) 10 periods (every 6 years), and (iv) 12 periods (every 5 years). The last corresponds to current practice. With these values, we formed the core instances, from which we built the instances for each experiment by varying the values described next and their default values.

The core instances are described in Table 1, where the first column is the name of the core instance, the second, third and fourth, columns show the maximum area of the management units of the instance, the number of periods, and the number of management units that it contains. Finally, the fifth and sixth columns show the number of decision variables and constraints of the resulting model for that instance.

The upper and lower bounds of the wood volume harvested at each period were calculated by considering the average potential yield of the forest at each period, divided by the number of periods, giving the average potential per period (*appp*). We also considered a volume variation percentage

Table 1: Description of the core instances

Instance	Max Area	Periods	MU	Vars	Cons
ins_urm_30_5_56_0_bayas_bigger	30	5	56	336	38579
ins_urm_30_6_56_0_bayas_bigger	30	6	56	392	44909
ins_urm_30_10_56_0_bayas_bigger	30	10	56	616	70637
ins_urm_30_12_56_0_bayas_bigger	30	12	56	728	83433
ins_urm_40_6_53_6_bayas_bigger	40	5	53	318	34586
ins_urm_40_7_53_7_bayas_bigger	40	6	53	371	40259
ins_urm_40_8_53_8_bayas_bigger	40	10	53	583	63323
ins_urm_40_9_53_9_bayas_bigger	40	12	53	689	74793
ins_urm_50_6_54_11_bayas_bigger	50	5	54	324	35885
ins_urm_50_7_54_12_bayas_bigger	50	6	54	378	41773
ins_urm_50_8_54_13_bayas_bigger	50	10	54	594	65703
ins_urm_50_9_54_14_bayas_bigger	50	12	54	702	77605

(vvp) of this value to be added or subtracted to the $appp$ value to obtain the upper and lower bound bounds ($upper = appp + vvp * appp$, $lower = appp - vvp * appp$). In this work, the default vvp value was established at 10% or 0.10.

The set of distances ($dist\_max\_manag$, $dist\_max\_natf$) that represents the maximum distances between any two pair of management units and native forest, respectively, at any period, was established with the default value $D1 = (8000m, 5500m)$. The minimum required fraction of timberland total area to be assigned as the native forest was set at 10% or 0.10. The regeneration years, that is, the number of years required before harvesting a management unit that is adjacent to a recently treated management unit, was set at $R3 = 20$ years.

The minimum percentage of the area considered for the native forest was established at $BN3 =$ 10% or 0.10. The time threshold was set at 3600 seconds. If this time is higher at the start of a new iteration, the optimization process is stopped. A second stopping criterion considered was a relative optimality gap of 0.01%.

4.4 Experiments and Results

This section describes the experiments conducted to determine the impact that different parameters of the instances, complementary to the core parameters, had on the objective value of the Green URM (GURM).

A different parameter was evaluated for each experiment, keeping the others with the default values mentioned in the previous section.

Core Parameters with Default Values

For this experiment, we evaluated the core instances, shown in Table 1, with the default values of the complementary parameters described earlier in Section 4.3. Such configuration resulted in a total of 12 test instances.

Table 2: Results for the core instances experiment.

Max Area	Periods	Obj. Value (mill. MXN)	Time sec.
30	5	\$90.82	22.58
	6	\$90.98	31.07
	10	\$92.47	1455.84
	12	\$92.49	2857.95
40	5	\$90.80	22.19
	6	\$90.82	39.01
	10	\$92.34	595.38
	12	\$92.48	129.92
50	5	\$90.86	31.71
	6	\$90.82	57.96
	10	\$92.42	607.46
	12	\$92.37	2446.49

The core parameters include three different maximum areas: $A1 = 30$ha, $A2 = 40$ha, and $A3 = 50$ha; and four different number of periods: $P1 = 5$, $P2 = 6$, $P3 = 10$, $P4 = 12$, which resulted in 12 test instances.

This experiment aims at having a baseline on which to compare the results obtained by varying the parameters individually. Table 2 shows the results obtained from the 12 instances evaluated in this experiment. The default configuration was feasible. The first and second columns show the maximum area and the number of periods, respectively. The third column shows the objective value in millions of Mexican pesos (Obj. Value (mill. MXN)), and the fourth column shows the time in seconds (Time sec.). All solutions were optimal, and the runtimes were reasonably short.

Distances

As mentioned before, distance is an essential factor to consider in forest management, both from an economic and the environmental point of view. From the economic perspective, our model addresses, in constraints (10), the cost of deploying the necessary infrastructure to carry out the harvest in a certain period, as the more dispersed the management areas to be harvested, the higher the cost of building roads, making trips between them or installing collection points.

From the environmental point of view, our model considers, in constraints (11), that native forest areas must maintain a certain maximum distance to fulfill their purpose of safeguarding the habitat of forest wildlife.

For this experiment, the pairs of distances of the 12 core instances were varied with four different values: $D2 = \{8500\text{m}, 5750\text{m}\}$, $D3 = \{9000\text{m}, 6000\text{m}\}$, $D4 = \{9500\text{m}, 6250\text{m}\}$, and $D4 = \{10000\text{m}, 6500\text{m}\}$, resulting in a total of 48 test instances.

The results of this experiment are shown in Table 3. All configurations were feasible. The first and second columns show the maximum area and the number of periods, respectively. The third, fourth, fifth, and sixth columns correspond to $D2$, $D3$, $D4$, and $D5$. Each one shows the objective value in millions of Mexican pesos (OV), the time in seconds (T). All solutions were feasible and optimal.

Table 3: Results for the distance experiment.

MA	P	D2		D3		D4		D5	
		OV	T	OV	T	OV	T	OV	T
	5	\$90.81	51.21	\$90.83	17.32	\$90.83	15.61	\$90.83	8.99
	6	\$90.97	43.70	\$90.97	18.89	\$90.97	14.96	\$90.97	11.66
30	10	\$92.46	1761.34	\$92.46	922.22	\$92.46	485.60	\$92.46	422.91
	12	\$92.48	2197.77	\$92.48	1656.72	\$92.48	875.89	\$92.48	729.87
	5	\$90.79	19.80	\$90.80	31.51	\$90.80	22.01	\$90.80	26.15
	6	\$90.81	31.18	\$90.82	44.12	\$90.82	38.41	\$90.82	51.28
40	10	\$92.33	806.95	\$92.33	1491.43	\$92.33	1206.59	\$92.33	884.48
	12	\$92.47	330.51	\$92.47	338.22	\$92.47	370.53	\$92.47	253.24
	5	\$90.85	34.02	\$90.87	22.47	\$90.87	27.94	\$90.87	10.38
	6	\$90.81	40.52	\$90.81	27.35	\$90.81	35.20	\$90.81	24.12
50	10	\$92.41	443.19	\$92.42	219.08	\$92.42	219.82	\$92.42	185.81
	12	\$92.37	2683.74	\$92.37	1798.33	\$92.37	843.27	\$92.37	1139.26

Figure 3 shows the results obtained. Each row corresponds to a group of instances with the same maximum area. The first four subplots correspond to the objective function value and the last four subplots to running time within each row. Each subplot displays the results for a given value of P (5, 6, 10, 12). Within each subplot, the set of distances defined in the experiment were evaluated.

As shown in Table 3 and Figure 3, the variation among the different distance values is negligible for a given fixed area and harvesting period. However, for a fixed distance value, it can be observed that harvesting every 5 or 6 years (P=12 or 10) yields better results than harvesting every 10 and 12 years (P=6 or 5).

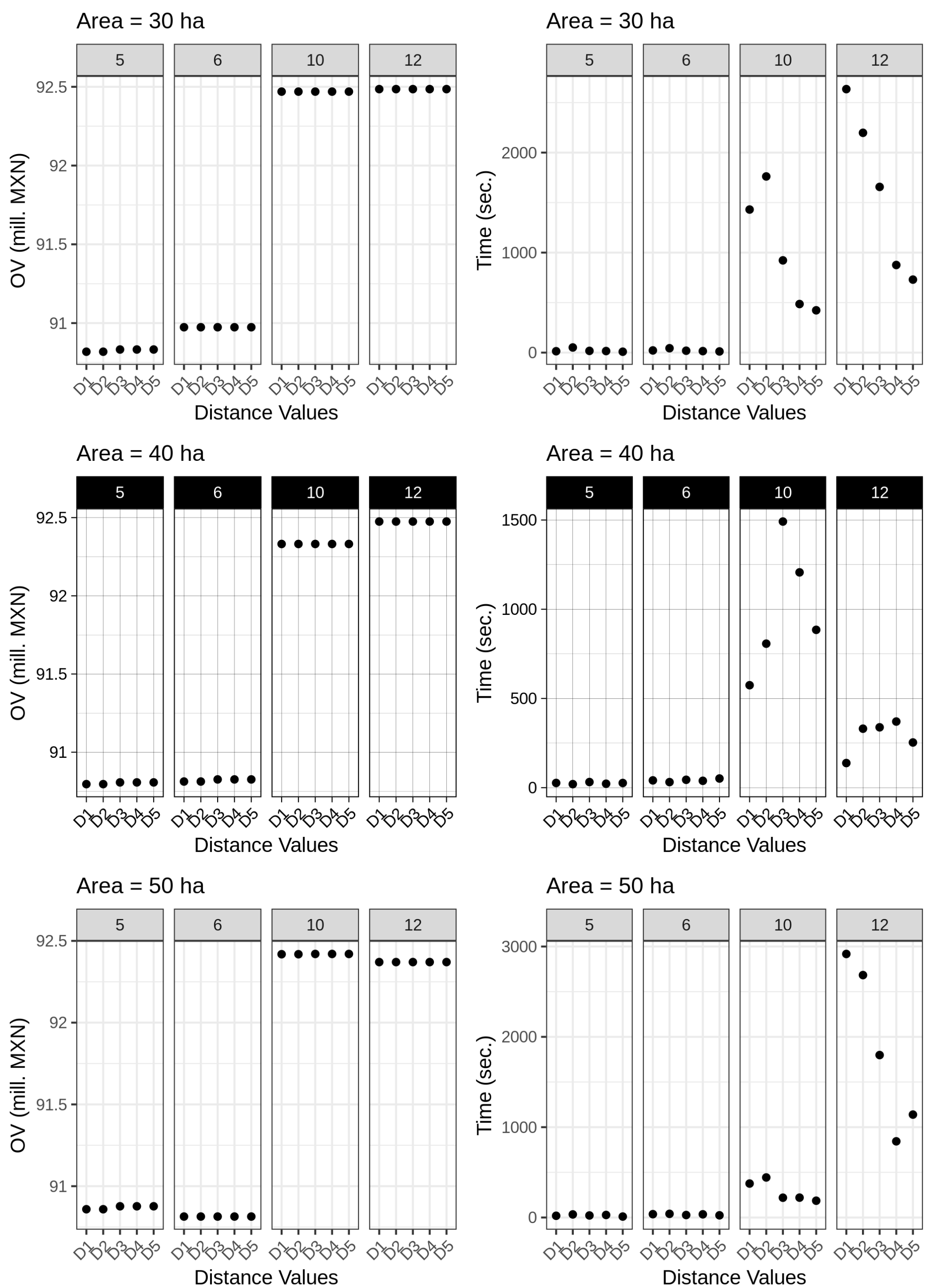

Figure 3: Results for the distances experiment.

Volume Percentage Threshold

Maintaining a constant harvest volume during each period is an important consideration so that the economic benefit also remains constant. However, it is not always possible for the timber volume to be the same every time.

Therefore, in constraints (4) and (5), our model includes lower and upper limits of timber volume per period, which, as they get close to each other, the harvest between each period will be more constant. However, this has the disadvantage that, if they were too strict, it could result in unfeasible solutions for that particular instance. On the other hand, the more separated they are, the easier it will be to find feasible solutions, but the volume will likely vary too much between periods.

In this experiment, the volume variation percentage of the 12 core instances were varied with four different values: $vvp2 = 12\%$, $vvp3 = 16\%$, $vvp4 = 20\%$, and $vvp5 = 24\%$, to determine how tight the lower and upper volume limits per period can be for Las Bayas. This configuration resulted in a total of 48 test instances.

Table 4: Results for the volume percentage threshold experiment.

MA	P	12		16		20		24	
		OV	T	OV	T	OV	T	OV	T
	5	\$91.06	43.80	\$91.55	19.42	\$91.96	24.77	\$92.34	9.88
	6	\$91.25	55.64	\$91.76	22.48	\$92.22	12.45	\$92.62	14.27
30	10	\$92.74	1134.61	\$93.24	448.33	\$93.70	328.53	\$94.11	190.61
	12	\$92.76	1102.56	\$93.31	466.54	\$93.74	548.90	\$94.21	226.04
	5	\$91.03	29.19	\$91.50	23.31	\$91.94	26.49	\$92.32	23.95
	6	\$91.12	67.57	\$91.65	27.93	\$92.15	11.62	\$92.52	48.00
40	10	\$92.63	894.32	\$93.21	52.06	\$93.61	91.80	\$94.07	95.93
	12	\$92.73	447.30	\$93.25	156.17	\$93.67	655.54	\$94.14	495.18
	5	\$91.11	39.00	\$91.57	32.60	\$92.01	13.39	\$92.37	19.21
	6	\$91.13	69.02	\$91.66	55.12	\$92.12	18.66	\$92.58	24.65
50	10	\$92.65	1501.14	\$93.16	960.72	\$93.68	103.25	\$94.04	615.73
	12	\$92.68	881.51	\$93.16	1718.59	\$93.68	609.68	\$94.07	1227.53

The results of this experiment are shown in Table 4. The first and second columns show the max area and the number of periods, respectively. The third, fourth, fifth, and sixth columns correspond to $vvp2$, $vvp3$, $vvp4$, and $vvp5$, respectively. For each one, the objective value is shown in millions of Mexican pesos (OV) and the time in seconds (T). All solutions were feasible and optimal.

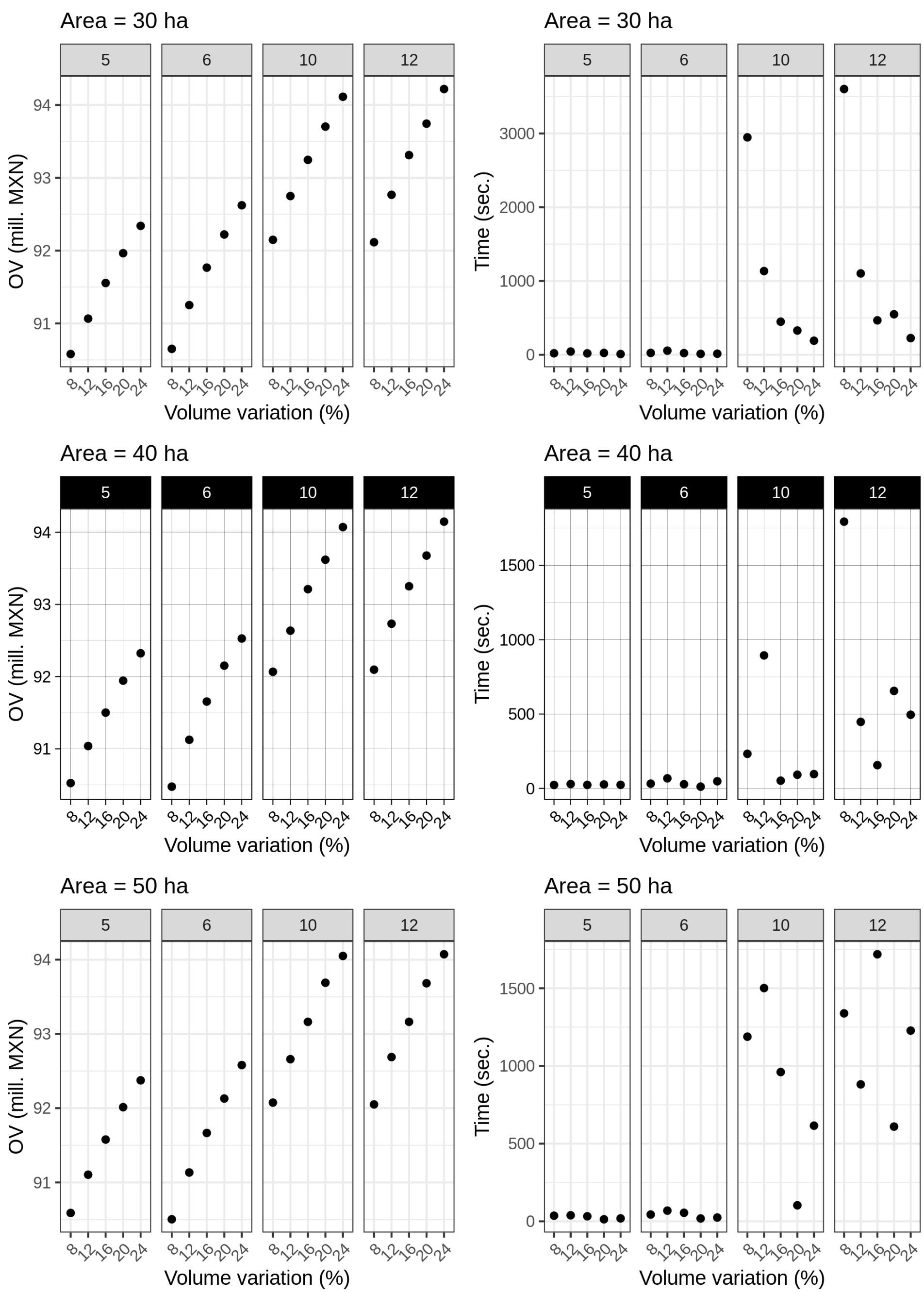

Figure 4: Results for the volume percentage threshold experiment.

Figure 4 shows the results obtained. Each row corresponds to a group of instances with the same maximum area. The first four subplots correspond to the objective function value and the last four subplots to running time within each row. Each subplot displays the results for a given value of P (5, 6, 10, 12). Within each subplot, the set of different values of *vpp* previously defined in the experiment were evaluated.

As we can see from Table 4 and Figure 4, for a given fixed area and harvesting period, the best results are observed for the largest value of $vpp = 24$. The decrease between this value and the smallest is less than 2%. About the variation among periods for a fixed value of volume, it can be observed that harvesting every 5 or 6 years (P=12 or 10) yields better profits than harvesting every 10 and 12 years (P=6 or 5). This result is consistent with the previous experiment.

Green-up Period

To avoid the formation of large deforested areas, even when adjacency restrictions are respected during the same period, the recovery period or green-up is included alongside the adjacency constraints of our model in (2).

The green-up period consists of avoiding, for several consecutive periods, the harvesting of management units adjacent to another one that has been recently treated in order to leave a protective barrier of standing trees that allows the unit to recover some of its forest mass.

The longer the green-up period, the larger the forest mass is recovered. However, it also limits much of the options for harvesting management units in future periods, resulting in unfeasible solutions.

For this experiment the green-up lengths of the 12 core instances were varied with four different values: $R1 = 10$, $R2 = 15$, $R4 = 25$, and $R5 = 30$. This resulted in a total of 48 test instances.

The results of this experiment are shown in Table 5. In this case, 42 configurations were feasible. The first and second columns show the maximum area and the number of periods, respectively. The third, fourth, fifth, and sixth columns correspond to $R1$, $R2$, $R4$, and $R5$. Each one shows the objective value in millions of Mexican pesos (OV) and the time in seconds (T). All the feasible solutions were optimal.

Figure 5 visually shows the results obtained. Each row corresponds to a group of instances with the same maximum area. The first column corresponds to the objective value obtained, and the second, the time it took for the method to find the optimal solution. The groups are divided into four subgroups that correspond to the periods considered for the core instances. For each period, the different green-up lengths R were evaluated.

As we can see from Table 5 and Figure 5, for a given fixed area and harvesting period, the best results are observed for the smallest value of $R = 10$. As R gets large, the objective function decreases by about less than 2%. About the variation among periods for a fixed value of the green-up period, it can be observed that harvesting every 5 or 6 years (P=12 or 10) yields better profits

Table 5: Results for the green-up experiment.

MA	P	10		15		25		30	
		OV	T	OV	T	OV	T	OV	T
	5	\$90.99	17.07	\$90.81	22.12	\$89.66	6.81	\$89.66	5.87
	6	\$91.62	21.12	\$91.62	24.44	\$90.97	22.39	\$88.26	8.45
30	10	\$93.06	1844.31	\$92.94	2423.81	\$91.24	46.89	-	-
	12	\$93.34	759.48	\$93.08	1755.67	\$90.29	830.10	-	-
	5	\$90.98	19.74	\$90.79	21.98	\$89.35	10.48	\$89.35	13.59
	6	\$91.60	24.36	\$91.60	21.44	\$90.81	35.99	\$87.44	9.97
40	10	\$93.02	889.15	\$92.85	958.92	\$91.22	52.44	-	-
	12	\$93.27	2889.52	\$92.92	3601.41	\$90.24	3228.85	-	-
	5	\$91.02	28.67	\$90.85	12.86	\$89.58	3.97	\$89.59	3.97
	6	\$91.65	25.72	\$91.65	20.68	\$90.81	12.99	\$88.11	5.64
50	10	\$93.05	2152.11	\$92.89	663.24	\$91.36	143.24	-	-
	12	\$93.32	959.59	\$93.00	1043.45	\$91.27	45.94	-	-

than harvesting every 10 and 12 years (P=6 or 5). This result again is consistent with the previous experiments.

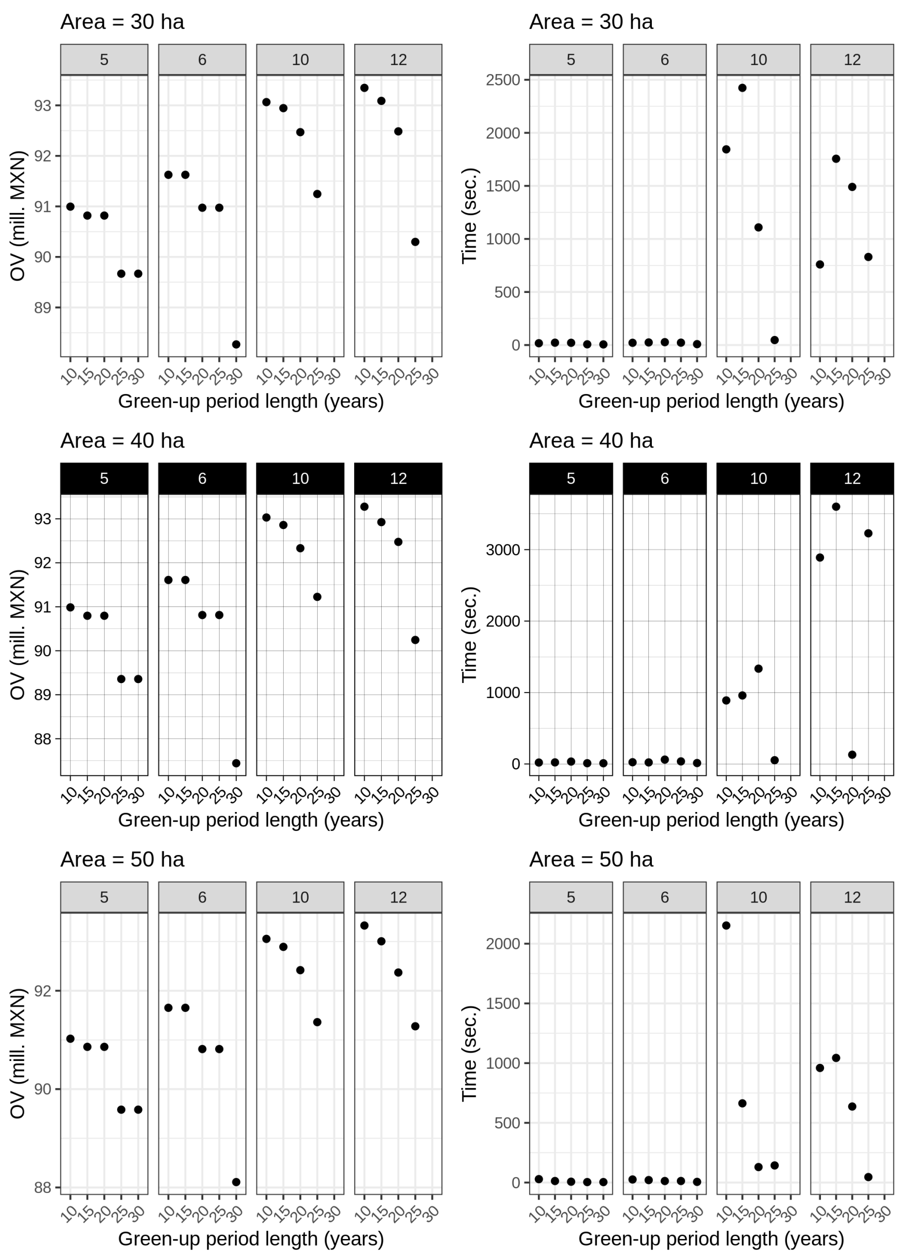

Figure 5: Results for the green-up experiment.

Minimum Forest Reserve Area

As already mentioned, maintaining a minimum forest reserve area percentage (set of constraints (7)–(9) of our model) is important as a measure to protect plant and animal species. In recent years, policies in which a minimum percentage of the forest must be preserved for this purpose have been established.

It is intuitive to deduce that the larger the minimum area of native forest, the lower the economic benefit such that the purpose of this experiment was to evaluate how relevant is the impact by testing different percentages of native forest.

To this end, the minimum forest reserve area of the 12 core instances was varied with four different values: $BN1 = 2\%$, $BN2 = 6\%$, $BN4 = 14\%$, and $BN5 = 16\%$, resulting in a total of 48 test instances.

Table 6: Results for the minimum forest reserve area experiment.

MA	P	2		6		14		16	
		OV	T	OV	T	OV	T	OV	T
	5	\$94.31	1.70	\$92.69	15.07	\$88.80	23.91	\$87.67	13.23
	6	\$94.37	2.02	\$92.84	9.57	\$88.95	31.34	\$87.80	12.59
30	10	\$96.00	70.62	\$94.36	402.06	\$90.40	1657.08	\$89.21	3602.28
	12	\$96.04	221.30	\$94.37	491.40	\$90.39	3600.78	\$89.22	3600.00
	5	\$94.28	3.21	\$92.66	23.48	\$88.74	28.83	\$87.62	44.07
	6	\$94.28	5.22	\$92.69	18.98	\$88.88	23.49	\$87.64	46.89
40	10	\$95.94	25.50	\$94.26	60.19	\$90.34	118.55	\$89.17	1700.11
	12	\$95.92	90.87	\$94.29	148.81	\$90.39	599.67	\$89.16	3600.00
	5	\$94.31	13.57	\$92.71	31.72	\$88.82	20.05	\$87.66	26.70
	6	\$94.27	13.82	\$92.71	17.87	\$88.79	20.58	\$87.64	40.25
50	10	\$95.94	212.98	\$94.32	228.60	\$90.29	893.42	\$89.13	3600
	12	\$95.90	1772.34	\$94.27	689.46	\$90.34	632.09	\$89.14	2842.95

The results of this experiment are shown in Table 6. The first and second columns show the maximum area and the number of periods, respectively. The third, fourth, fifth, and sixth columns correspond to $BN1$, $BN2$, $BN4$, and $BN5$. Each one shows the objective value in millions of Mexican pesos (OV) and the time in seconds (T). All the solutions were feasible and optimal.

Figure 6 shows the results obtained. Each row corresponds to a group of instances with the same maximum area. The first column corresponds to the objective value obtained, and the second, the time it took for the method to find the optimal solution. The groups are divided into four subgroups that correspond to the periods considered for the core instances. For each period, the set of four BN percentages previously defined in this experiment were evaluated.

As we can see from Table 6 and Figure 6, for a given fixed area and harvesting period, the best results are observed for the smallest value of $BN = 2$. As BN gets large, the objective function decreases by about 5-9%. About the variation among periods for a fixed value of minimum forest reserve area, it can be observed that harvesting every 5 or 6 years (P=12 or 10) yields better profits than harvesting every 10 and 12 years (P=6 or 5). This result again is consistent with the previous experiments.

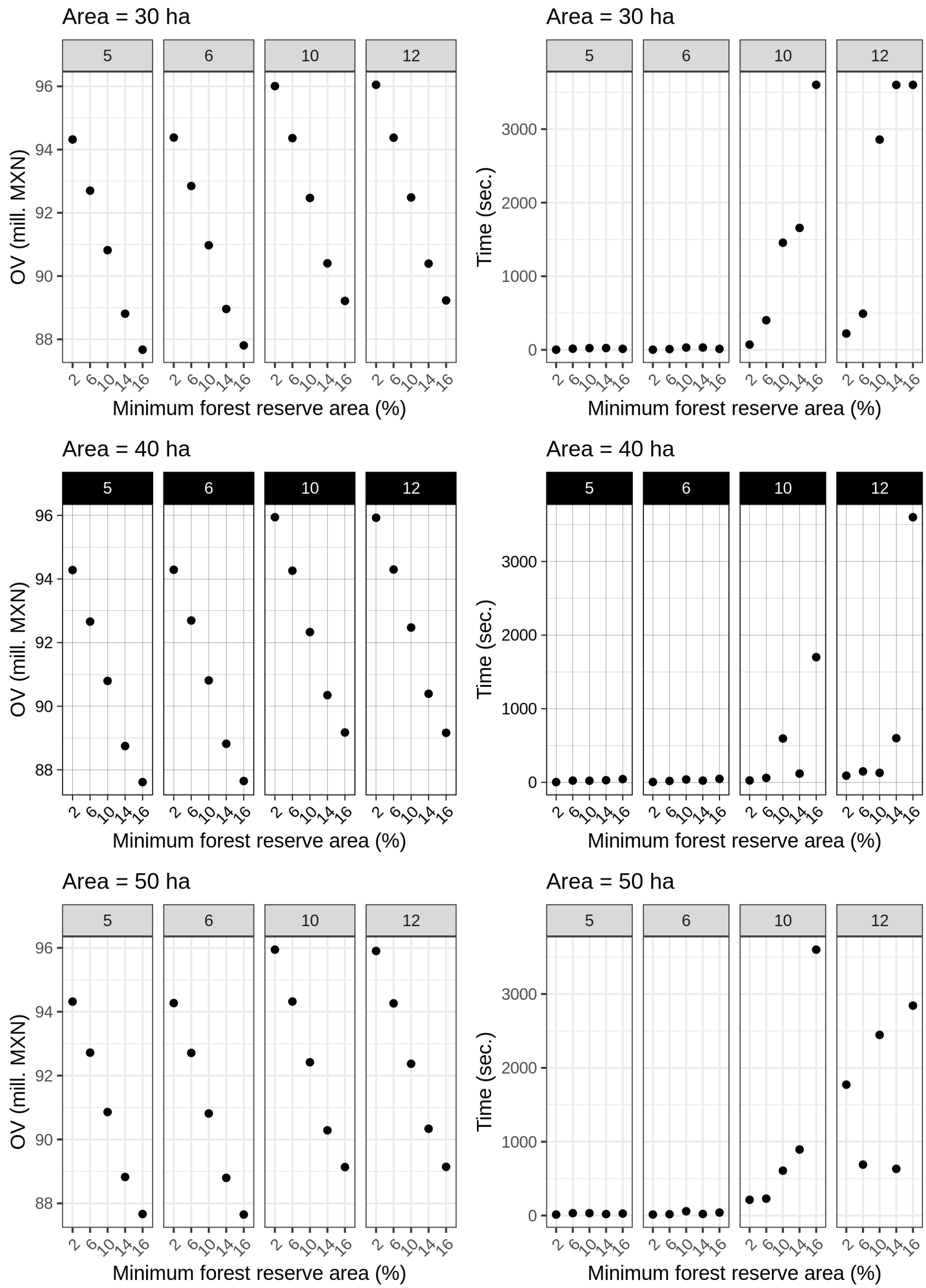

Figure 6: Results for the minimum forest reserve area experiment.

Example of a Harvesting Planning

Figure 7 shows an optimal plan obtained for instance `ins_urm_30_6_56_1_bayas_bigger`, with a $max\_area = 30$, $periods = 6$, management units $= 56$, and default complementary parameters of the instance required by the GURM, defined in Section 4.4, which correspond to the values of volume variation percentage, the maximum distance between native forest units, the maximum distance between management units, the minimum required area percentage of native forest, and years required before harvesting a management unit that is adjacent to a recently treated management unit.

We also solved the same instance but using the traditional URM model instead, that is, ignoring the additional constraints stated in (2) and (6)-(11, with default values. This solution is shown in Figure 8. Each color represents a period of harvesting, and in the case of the GURM, the white color represents a management unit turned into a natural reserve, and the gray color represents a sub-stand that was not considered in the harvesting.

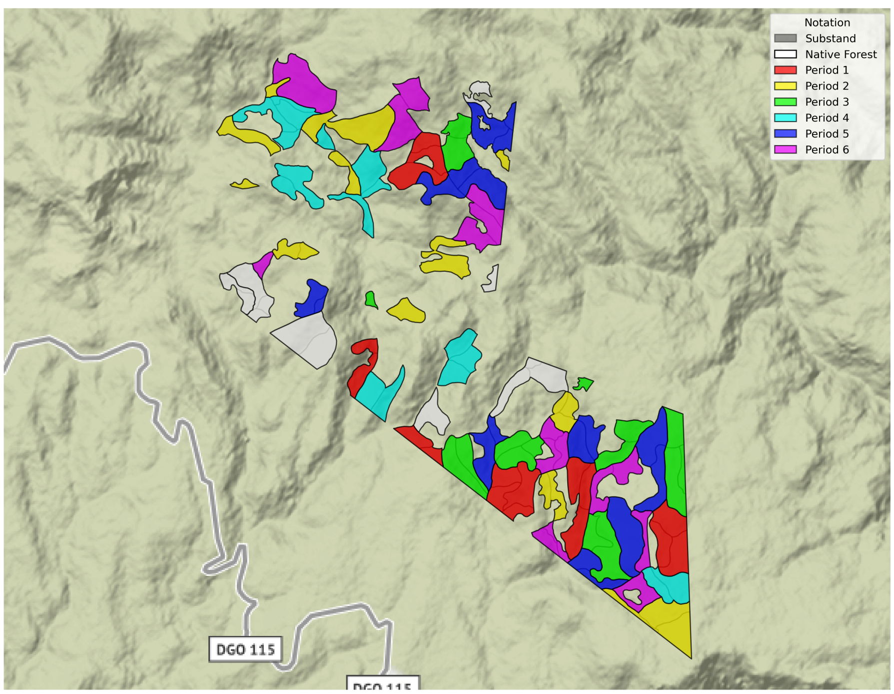

Figure 7: Optimal harvest plan obtained with the GURM.

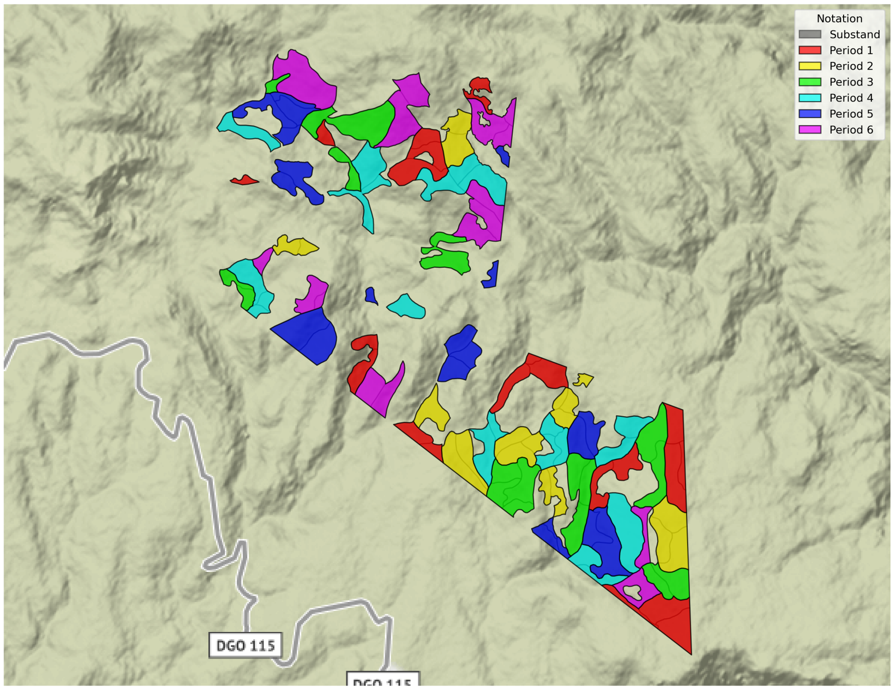

Figure 8: Harvest planning obtained with the traditional URM

Note that by contrasting the harvesting plans in both Figures, the GURM, unlike the traditional URM, enables a minimum area of native forest within a limited perimeter (see Figure 7, units in white color). The GURM also hinders the harvesting of too distant units in the same period, and thanks to the green-up barrier, it prevents the harvesting of neighboring units in consecutive periods. For instance, in Figure 8, we can find harvesting prescriptions for adjacent units in consecutive periods (1 and 2, 4 and 5, 5 and 6), while this is not observed in its counterpart in Figure 7, thanks to the green-up constraints incorporated in our model.

5 Conclusions

In this paper, we proposed the extension of a model from the literature with environmental constraints to solve the forest harvesting problem with adjacency to optimize the harvesting of a Mexican forest during a planning horizon. The model considered (GURM) was based on the Unit Restriction Model (URM), which considers the management of units beforehand. The resulting model GURM does not intend to compete with the ones from the literature in terms of maximizing harvesting profit but to maximize the profit while considering environmental constraints that protect the wildlife and the forest.

Another observation from the experiments was that the maximum area inversely affected the URM and GURM, larger areas resulted in a reduced number of management units in the URM model, thus reducing the computational time to solve the instance of the problem, but in this case, at the expense of a reduced profit.

As for the GURM, the change of values in the different constraints caused decreases in the OV of -3.23% at most for the default configuration. However, in most cases, the OV was hardly affected, and there were instances in which this value increased when some constraint thresholds were relaxed. Regarding the management and native forest distance constraints, the OV decreased by 2.17% for the benchmark when the corresponding thresholds varied. Regarding the volume threshold constraints by period, the OV decreased by 1.79% and increased by 0.28% compared to the benchmark instance, as the values were restricted and relaxed, respectively. Green-up constraints were among those that most negatively affected the objective value. In such a case, the economic benefit decreased from 1.93 to 3.11% when different configurations were compared versus the default setup. It was also observed that the native forest constraints significantly impacted the objective value. When suppressed, the OV increased by 2.02%. When strengthened, the OV decreased up to -3.23%.

Finally, in all experiments under all different factors, it was observed that harvesting every 5 or 6 years yields better profits than harvesting every 10 or 12 years. The current standard in the Mexican system is to harvest every five years.

Future work may include implementing and reformulating other models, specifically, the Area Restriction Method (ARM) that constructs a more extensive set of management units that could potentially increase the profit obtained during the harvesting planning.

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