

# Location of Primary Health Care Centers Covering a Set of Basic Services<sup>a</sup>

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## Abstract

We address the problem of locating primary health care centers, incorporating a set of basic services, and considering capacitated outpatient service. Complimentary services such as nutrition consultant, dental care service, psychological service, clinical analysis, and imaging services are modeled with covering constraints. The objective is to maximize the covered demand by the complimentary services while the total travel distance for the outpatient service allocation is constrained. The cost of opening new facilities and updating the existing ones in the network is restricted to a budget. Two auxiliary bi-objective integer linear programming models that help identify the trade-off between the total travel distance and the budget limit are proposed. A case study based on the Mexican public health care system is presented. Optimal solutions were found for a set of instances composed of 1,086 demand nodes and 411 candidate locations. The auxiliary models are solved by an augmented  $\varepsilon$ -constraint method. The empirical work shows the usefulness of the proposed models.

*Keywords:* Health care planning; Facility location; Integer programming; Bi-objective integer programming.

# 1 Introduction

The public health care system in Mexico is segmented into multiple institutions. The planning of the resources of each institution is based on the needs of its insured members. This circumstance brings many problems related to access equity and the quality of the institutions [4]. The lack of standardization in the design of health care facilities is one of the significant problems for establishing single joint planning of infrastructure among institutions. The Institute of Health for Well-being (INSABI, from *Instituto de Salud para el Bienestar*) is an institution created in 2020 to supply all health care services to the uninsured population. Formerly, each state was in charge of planning resources and the budget to invest in them. INSABI aims to centralize the significant decisions to reach equity through transparent processes.

According to a report [17] issued in 2015 by the Mexican Institute of Statistics, Geography, and Informatics (INEGI), the three leading causes of death in Mexico were cardiovascular diseases (25.5%), endocrine, nutritional, and metabolic diseases (17.5%), and malignant tumors (13%). The importance of the Primary Health Care Center (PHC) is owing to they are the first-level of action to prevent these diseases or provide a timely diagnosis. In a more general definition, the PHCs are the first contact points for the population to prevent and promote their health, and when it is required to provide outpatient care. These centers are the primary way of accessing specialized health care services.

Aligned with INSABI goals, this work's motivation is to create an adequate model for the planning of all primary health care facilities by using Operations Research tools. The objective is to maximize the access and quality of the first level of health care in public institutions. In the literature, most of the facility location problems related to primary health care focus on evaluating an only service. However, in developing countries such as Mexico, there is an urgent need for integrating more than an only service in the decision-making process. A group of services forms the basic set of primary health care services integrated by the outpatient consultation, nutrition care, dental care, psychological care, clinical analysis, and imaging. Even in some PHCs, some specialized services, such as gynecology or pediatrics, are provided.

The PHCs in the public sector of Mexico are classified in diverse levels of service. The most basic unit is formed by medical staff, including a general physician, one or two nurses, and a technical staff member. This group is referred to as the *basic kernel*. They provide outpatient service, and they are in charge of vaccination campaigns, promotion of health care, family planning, and the detection and control of chronic diseases. The number of basic kernels increases with the size of the PHC, including the complementary services that require additional staff and resources [34]. In Mexico, basic PHCs are located in rural areas, while the PHCs with the highest capacity and services are located in urban areas to maximize access to their services.

In this paper, it is addressed a facility location problem with a set of basic services, referred

to as FLPBS. In this problem, the outpatient service is considered as a capacitated service, each demand point is allocated to a single PHC, and the total travel distance (TTD) by the patients is handled as a constraint. The population assignment for the outpatient service is required to control the clinical record of each inhabitant and make possible the planning of the resources needed in each PHC. The complementary services are addressed as uncapacitated services with a critical coverage distance. The objective is to maximize the covered demand of each service by at least one PHC. The use of a maximal coverage distance is meant to ensure a minimum level of service to the largest population who require the service. The annual budget destined for infrastructure planning in health care to guarantee the total access to primary health care services is insufficient in Mexico. In that sense, a budget limit for the investment in new facilities or updating the existing ones is typically imposed. For this problem, a mixed-integer linear programming model (MILP) is introduced. As part of the solution methodology, two auxiliary bi-objective integer programming models are proposed for identifying feasible ranges for both the TTD bound and the budget. The first auxiliary model is used to identify both bounds' minimum value combination to get feasible solutions to the problem. In contrast, the second model is used to identify the maximum value combination to avoid the budget's sub-utilization. The Pareto front is obtained by applying the augmented  $\varepsilon$ -constraint method, AUGMECON2 [22].

Numerical experiments were carried out to assess the usefulness of the model and its auxiliary models. These experiments were based on real-world data from a region composed of 17 municipalities in the northern zone of the State of Mexico, evaluating 1,086 demand points. The information used to create the case study is based on the available data by the INEGI and the Mexican Ministry of Health (SS, for *Secretaría de Salud*).

The remainder of this paper is organized as follows. In Section 2, a literature review of facility location problems with applications in primary health care centers is presented. In Section 3, the integer programming model is presented, including the assumptions, the auxiliary models, and their interpretations. This is followed by Section 4 where the solution methodology is described. Section 5 presents an empirical assessment of the proposed model based on real-world data. Finally, some concluding remarks and discussion on future work are outlined in Section 6.

## 2 Literature Review

The Facility Location Problem (FLP) is one of the most studied problems in location theory. Different versions of the problem have been used and adapted to solve a large number of problems in the industry and the public sector. An extensive compilation of models is presented by Farahani and Hekmatfar [9]. Our research is applied to the health care area, which also has a large number of contributions. A recent survey on health care facility location is presented in Ahmadi-Javid et al. [1], but other important surveys have been done throughout the last decades by Papageorgiou [29],

Smith-Daniels et al. [39], Daskin and Dean [8], Rahman and Smith [31], and Rais and Viana [32]. Multi-objective models and methods on facility location are surveyed by Farahani et al. [10].

The FLPBS draws concepts and ideas from the Capacitated Facility Location Problem (CFLP) and the Maximal Covering Location Problem (MCLP). To the best of our knowledge, the FLPBS has not been addressed before. The Capacitated Maximal Location Problem (CMLP) is a related problem initially addressed by Chung et al. [5], and later studied by Current and Storbeck [7] and Pirkul and Schilling [30]. In this problem, the objective is to maximize the population assigned to a facility within a coverage distance, while the facilities are limited by their capacity. Yin and Mu [41] extends this model by incorporating multiple capacity levels in a similar way as the FLPBS. A recent application of this problem about locating drone launching sites for distributing resources is made by Chauhan et al. [3]. However, the main difference between CMLP and FLPBS is the number of services integrated into the problem. While CMLP evaluates capacity and coverage limits for the same service, FLPBS incorporates the main service as a capacitated service and the complementary service with coverage radius.

The Fixed Charge Facility Location Problem (FLP) with coverage constraints is another related problem proposed by Nozick [27]. This problem minimizes the total cost of serving a set of demand locations using the covering constraints to set a minimum level of uncovered demand. Although the objective function is not the same as the FLPBS, both problems have similar characteristics. However, the problem is also evaluated for a single service.

In this paper, two auxiliary models are proposed to find an efficient bound for the total travel distance with budget limit for the FLPBS. The first model, based on the capacitated  $p$ -median problem with a budget limit, must decide where to install new facilities or how to upgrade existing facilities. The second model is related to the FLP with the set of coverage constraints for the complementary services.

In Table 1, related works for the health care facility location problem are summarized, particularly the problems related to primary health care centers. We compared the proposed model for the FLPBS, with the other related works. The FLPBS takes classic characteristics of the facility location problem as it can be seen in the table, with the addition of the new characteristics previously mentioned. All the reviewed papers also considered the demand as a parameter. The travel distance/time is one of the most used parameters in the problems. The fixed and variable costs are evaluated by more than half of them. The facility capacity [14, 15, 24, 35, 36], as well as the incorporation of multiple services [13, 14, 18, 20, 25, 38, 40] are parameters used to a lesser extent. The use of both parameters was only presented in Griffin et al. [14], Shishebori and Yousefi Babadi [36], and the proposed FLPBS.

The facilities' location is the mandatory decision in this type of problem, while the allocation of demand is also evaluated in most of the problems. The demand coverage is evaluated in the FLPBS for the complementary services, and the demand coverage was only assessed by Smith et al. [38]

and Taymaz et al. [40].

The maximum travel distance is one of the most common constraints in this type of problem. In the FLPBS, the total travel distance is bounded. The maximum number of facilities is also a common constraint that we do not consider because the number of facilities is limited by the available budget.

Table 1: Survey of related facility location problem for primary health care facilities.

Paper	Model assumptions								Variables				Constraints							Modeling approach	Solution method	
	Demand	Travel time /distance	Facility capacity	Fixed and/or variable costs	Multiple servers	Multiple services	Elastic demand	Hierarchical system	Uncertainty	Location of facilities	Allocation of demand points	Number of resources required	Coverage of demand	Full coverage	Partial coverage	Maximum number of facilities	Maximum travel distance	Maximum available resources	Service capacity (min or max level)			Budget
This paper	✓	✓	✓	✓		✓				✓	✓	✓			✓		✓			✓	a	G
Marianov et al. [19]	✓	✓		✓						✓	✓					✓	✓			✓	a	GH
Marianov et al. [20]	✓	✓		✓		✓			✓	✓	✓					✓	✓			✓	a	G
Mitropoulos et al. [24]	✓	✓	✓							✓	✓			✓		✓	✓	✓		✓	ad	G
Ndiaye and Alfares [26]	✓			✓						✓	✓			✓		✓		✓		✓	a	G
Griffin et al. [14]	✓		✓	✓		✓				✓	✓					✓		✓	✓	✓	a	
Ratick et al. [33]	✓	✓		✓				✓		✓	✓					✓	✓			✓	a	G
Smith et al. [37]	✓	✓						✓		✓	✓					✓	✓			✓	a	G
Shariff et al. [35]	✓	✓	✓							✓	✓			✓		✓		✓		✓	a	GM
Güneş et al. [15]	✓	✓	✓		✓		✓			✓	✓	✓		✓		✓		✓		✓	a	
Cocking et al. [6]	✓	✓		✓						✓	✓			✓					✓	✓	a	G
Smith et al. [38]	✓	✓				✓		✓		✓	✓		✓	✓		✓	✓			✓	acd	G
Mitropoulos et al. [25]	✓	✓				✓		✓	✓	✓	✓			✓		✓		✓		✓	ade	G
Kim and Kim [18]	✓	✓		✓		✓				✓	✓			✓		✓		✓	✓	✓	a	GL
Ghaderi et al. [12]	✓	✓		✓						✓	✓			✓					✓	✓	b	GHM
Beheshtifar et al. [2]	✓	✓		✓						✓	✓			✓		✓		✓		✓	ad	M
Graber-Naidich et al. [13]	✓	✓		✓		✓				✓	✓			✓		✓		✓		✓	a	G
Shishebori et al. [36]	✓	✓	✓	✓				✓		✓	✓					✓		✓	✓	✓	af	G
Núñez Ares et al. [28]	✓	✓								✓						✓					a	O
Taymaz et al. [40]	✓	✓				✓		✓		✓		✓		✓	✓	✓					e	G
Modeling approach										Solution method												
a = Mixed-integer linear programming										G = General purpose branch-and-bound solver												
b = MINLP										L = Lagrangian relaxation												
c = Goal programming										H = Heuristics												
d = Multi-criteria decision making										M = Metaheuristics												
e = Stochastic programming										O = Other												
f = Robust optimization																						

As we can see from Table 1, most modeling approaches are mixed-integer linear programming

(MILP) models. There are a few of them that consider multi-objective or multi-criteria models [2, 24, 25, 38]. A few stochastic models have been studied as well as [36, 40].

In Smith et al. [38], a set of hierarchical models is proposed to locate public services. They proposed a MCLP that integrates the location of facilities, allocation, and coverage of demand decisions in a single problem as the same as the FLPBS, also considering multiple services and multiple facility levels. They limit the number of locations according to a number, while the FLPBS considers a budget limit. The main difference is the hierarchical structure in their formulation and the inclusion of a capacitated service in the FLPBS.

In summary, a novel model that integrates features from CFLP and MCLP for the location of primary health care centers in the public sector is proposed in this paper. Multiple facility types that have different costs and services are included. The main service is modeled similarly to CFLP since an allocation scheme is required to meet demand. Given the complementary services are limited, the MCLP is a good approximation to cover these services. The problem takes into account the current infrastructure because the objective is the continuous improvement of the system with a periodic capital investment to open new facilities or to improve the existing ones. This problem can be applied to the infrastructure planning of other developing countries with related health care systems.

### 3 Problem Description

There are two types of services: (1) the Main Service (MS) related to outpatient service provided by general physicians and (2) Complementary Services (CS) such as nutrition consultant, dental care service, psychological service, clinical medicine analysis, and imaging services. The outpatient service has a limited capacity based on the number of persons that can be affiliated with a facility. This number is determined by the facility type. Each demand point (locality) must be allocated to a single facility, and all demand must be covered. For complementary services, the coverage is based on a critical radius of the distance between facilities and localities.

Different types of facilities are evaluated in the problem. Each one has a limited capacity for the outpatient service and provides a set of complementary services. There is a setup cost for installing new facilities and an upgrading cost to expand the current facilities' capacity. A set of candidate locations is defined to install new facilities. Some factors, such as level of population, type of locality (urban or rural), and connectivity with other localities, are important to determine which type of facility is feasible in each candidate location.

For the outpatient service, the objective is to allocate all demand points limited by the facility's capacity while the Total Travel Distance (TTD) of the demand is constrained. For the complementary services, the objective is the maximization of the sum of the covered demand. The budget restricts the investment cost of installing and updating facilities. This cost will be named as Total

Cost (TC) in the remaining of the paper. In the following sections, when the TTD is mentioned, we refer to the total travel distance from demand points to the facilities to receive the outpatient service.

### 3.1 Formulation

The sets, parameters, variables, and the mathematical formulation of the FLPBS described as follows:

*Sets and indices:*

- $M$  Set of demand points (localities) ( $i \in M$ ).
- $N$  Set of candidate locations for new installation or upgrading of facilities ( $j \in N$ ).  $N$  is partitioned into two subsets,  $N_A$  and  $N_B$ .
- $N_A$  Subset of locations such that a new facility can be installed.
- $N_B$  Subset of locations such that a facility is already installed.
- $K$  Set of candidate facility types ( $k \in K$ )
- $K(j)$  Subset of candidate facility types to install or update at location  $j \in N$ ,  $K(j) \subseteq K$ .
- $S$  Set of services ( $s \in S$ ).
- $S(i)$  Subset of services that are required in demand point  $i \in M$ ,  $S(i) \subseteq S$ . Some demand point could be covered by service  $s$  by an existing facility that is not integrated in the problem.

*Parameters:*

- $\lambda^s$  The associated weight of service  $s \in S$  in the objective function.
- $F^k$  Fixed cost of installing facility type  $k \in K$ .
- $U_j^k$  Fixed cost of upgrading facility located at  $j \in N$  to facility type  $k \in K$ .
- $\mathcal{B}$  Available budget for installing or upgrading facilities.
- $D_{ij}$  Distance from demand point  $i \in M$  to the facility located at  $j \in N$ .
- $P_i$  Demand (number of people) of the main service at point  $i \in M$ .
- $C^k$  Capacity (number of people) of facility type  $k \in K$  for providing the main service.
- TTD Upper bound on the total distance traveled by population.



$R^s$  Critical distance of coverage for service  $s \in S$ .

$A_{ij}^{ks}$  The coverage parameter that is equal to 1 if a facility of type  $k$  in candidate location  $j$  covers the demand point  $i$  for service  $s$ . The coverage occurs when the facility of type  $k$  can provide service  $s$ , and when the distance between  $D_{ij} \leq R_s$  for service  $s$ .

*Decision variables:*

$Y_j^k$  Binary variable equal to 1 if a facility of type  $k$  is located at site  $j$ ; 0, otherwise.

$X_{ij}$  Binary variable equal to 1 if demand point  $i$  is allocated to facility located at  $j$  for the main service; 0, otherwise.  $X_{jj}$  represents a facility located at demand point located at  $j$ .

$V_i^s$  Binary variable equal to 1 if service  $s$  of demand point  $i$  is covered; 0, otherwise.

*Mathematical formulation:*

$$\text{(FLPBS) Maximize } \sum_{i \in M} \sum_{s \in S(i)} \lambda^s P_i V_i^s \quad (1)$$

$$\text{subject to } \sum_{j \in N} X_{ij} = 1 \quad i \in M \quad (2)$$

$$\sum_{i \in M} P_i X_{ij} \leq \sum_{k \in K} C^k Y_j^k \quad j \in N \quad (3)$$

$$X_{jj} \geq \sum_{k \in K} Y_j^k \quad j \in N \quad (4)$$

$$\sum_{k \in K(j)} Y_j^k \leq 1 \quad j \in N_A \quad (5)$$

$$\sum_{k \in K(j)} Y_j^k = 1 \quad j \in N_B \quad (6)$$

$$\sum_{j \in N_A} \sum_{k \in K} F^k Y_j^k + \sum_{j \in N_B} \sum_{k \in K} U_j^k Y_j^k \leq \mathcal{B} \quad (7)$$

$$\sum_{i \in M} \sum_{j \in N} P_i D_{ij} X_{ij} \leq \text{TTD} \quad (8)$$

$$V_i^s \leq \sum_{j \in N} \sum_{k \in K} A_{ij}^{ks} Y_j^k \quad i \in M, s \in S(i) \quad (9)$$

$$X_{ij} \in \{0, 1\} \quad i \in M, j \in N \quad (10)$$

$$Y_j^k \in \{0, 1\} \quad j \in N, k \in K \quad (11)$$

$$V_i^s \in \{0, 1\} \quad i \in M, s \in S \quad (12)$$

The objective function (1) maximizes the sum of demand covered by the complementary services. Each complementary service has a defined weight ( $\lambda^s$ ) in the objective function. Constraints (2) allocate each demand point to only one facility. The allocation of demand points is limited by the

capacity of each facility in constraints (3). In constraints (4), it is defined that demand points located in the same locality of a facility are allocated to this facility. Constraints (5) ensure that only one facility can be installed at most in a candidate location, while constraints (6) ensure that existing facilities remain the same or they can be updated. The number of additional and updated facilities is limited to a budget, according to the constraint (7). Constraint (8) defines an upper bound for the total distance traveled by the population in the allocation of demand points for the main service. In constraints (9), the variables  $V_i^s$  will take a value equal to 1 if the demand point  $i$  is covered for service  $s$  by at least one facility. The nature of decision variables is defined by constraints (10)–(12).

*Computational complexity:* To show that the FLPBS is  $\mathcal{NP}$ -hard, it is used a reduction from the MCLP as follows. Consider a particular instance of the FLPBS where there is only one facility type ( $K = \{1\}$ ) and one complementary service ( $S = \{1\}$ ) in the problem. Let the capacity of the outpatient service be greater than or equal to the total demand in the system ( $C \geq \sum_{i \in M} P_i$ ). Let the TTD bound be larger than or equal to the worst-case solution in the problem (e.g.  $TTD = \sum_{i \in M} P_i D_{ij^*}$  such that  $j^* = \arg \max_j \{D_{ij} | j \in N\} \forall i \in M$ ). It is assumed there are no current facilities in the system ( $N = N_A$ ). Let the cost in constraint (7) be fixed to one, and the budget represents the number of facilities to be opened ( $\sum_{j \in N} Y_j \leq \mathcal{B}$ ). Under this special instance, any possible value of the  $X_{ij}$  variables does not affect the objective function since constraints (3) and (8) will be inactive in the optimal solution. Therefore, the related constraints can be removed from the problem, and the remaining problem is just an instance of the MCLP. That is, the MCLP is polynomially reducible to the FLPBS. Clearly, the feasibility of the FLPBS can be checked in polynomial time. Since the MCLP is known to be  $\mathcal{NP}$ -hard [23] it follows that the FLPBS is also  $\mathcal{NP}$ -hard.

### 3.2 Auxiliary Formulations

Constraints (7)–(8) can be modeled as additional objective functions to be minimized in a multi-objective optimization problem. Instead of working with a set of solutions for Pareto front, the decision of which TTD bound and the amount of budget available for the improvement of the health care system are predefined by the design-makers. However, two auxiliary bi-objective integer linear programming models are proposed to find reasonable bounds for TTD when there is a budget limit. This method provides a broader perspective to understand the solution behavior.

#### Auxiliary Model 1 (AM1)

The first objective (13) minimizes the total distance traveled by the population from each demand point to the allocated facility. The second objective (14) minimizes the total cost of opening or updating the facilities of the system. Constraints (2)–(6), (10), and (11) remain in the model.

$$\text{Minimize } Z_1 = \sum_{i \in M} \sum_{j \in N} P_i D_{ij} X_{ij} \quad (13)$$

$$\text{Minimize } Z_2 = \sum_{j \in N_A} \sum_{k \in K} F^k Y_j^k + \sum_{j \in N_B} \sum_{k \in K} U_j^k Y_j^k \quad (14)$$

$$\text{subject to Constraints } (2) - (6), (10), (11)$$

### Auxiliary Model 2 (AM2)

This model is the same as the AM1 with the addition of constraints (17) which ensures that each demand point is covered at least by one facility for each complementary service

$$\text{Minimize } Z_1 = \sum_{i \in M} \sum_{j \in N} P_i D_{ij} X_{ij} \quad (15)$$

$$\text{Minimize } Z_2 = \sum_{j \in N_A} \sum_{k \in K(j)} F^k Y_j^k + \sum_{j \in N_B} \sum_{k \in K(j)} U_j^k Y_j^k \quad (16)$$

$$\text{subject to } \sum_{j \in N_A} \sum_{k \in K(j)} A_{ij}^{ks} Y_j^k \geq 1 \quad i \in M, s \in S(i) \quad (17)$$

$$\text{Constraints } (2) - (6), (10), (11)$$

### Interpretation of AM1 and AM2

In Table 2, a summary of the auxiliary models' features is presented. The main difference between the AM1 and the AM2 is the additional constraints that guarantee the coverage of each demand point in the AM2. Since the TTD bound and the budget are defined by the decision-maker, the Pareto front of the AM1 allows to identify the minimum TC to get feasible solutions in the FLPBS for a given TTD bound. On the other hand, the Pareto front of the AM2 allows identifying the maximum required budget to cover all demand points for the complementary services for a given TTD bound.

Table 2: Model features.

Model	Minimize TTD for the MS	Maximize coverage of CS	Minimize total cost
AM1	Objective function	Not evaluated	Objective function
AM2	Objective function	Constraint	Objective function
FLPBS	Constraint	Objective function	Constraint

To illustrate how both auxiliary models can be used to get useful bounds for the FLPBS, Figure 1 shows an illustrative example of the relationships among the models for a given instance. In this figure, the TTD vs. the TC of the solutions of the three models is plotted. The AM1 Pareto front is represented by the blue points, while the AM2 Pareto front is represented by the red points. In

both cases, the problem is integer, and the set of possible solutions is discrete. Each model has its own optimal solutions range, but the behavior is similar. The best TTD is found with the highest TC, and the lowest TC has the highest TTD. The solutions of the AM2 will have a higher cost than the solutions of the AM1 for a given TTD value because more facilities are opened or updated to guarantee the total coverage of demand for the complementary services. Therefore, the TTD bounds and the budget of the FLPBS must be inside the area between the two sets of solutions to be efficient. The area below the blue points will produce unfeasible solutions, and the area above the red points will produce solutions with not efficient use of the budget.

For example, the solution of three different points with the same TTD bound are represented in Figure 1 by  $P_1$ ,  $P_2$ , and  $P_3$ . The solution at  $P_1$  represents the minimum budget to get a feasible solution with  $TTD_1$  as the bound, the solution at  $P_3$  represents the maximum budget to cover all demand points with the same TTD bound, and the solution at  $P_2$  represents a solution between these two points. The range of coverage increases from 80% at  $P_1$  to 100% at  $P_3$ . The cost increment to obtain a solution from 80% to 100% is nonlinear because to cover the last remaining part of the demand becomes more expensive. Therefore, a partial increment in the budget starting from the minimum one required could get an important improvement in the coverage of demand as it is observed in the solution at  $P_2$ , which has a coverage of 95%. This is shown in Section 5.

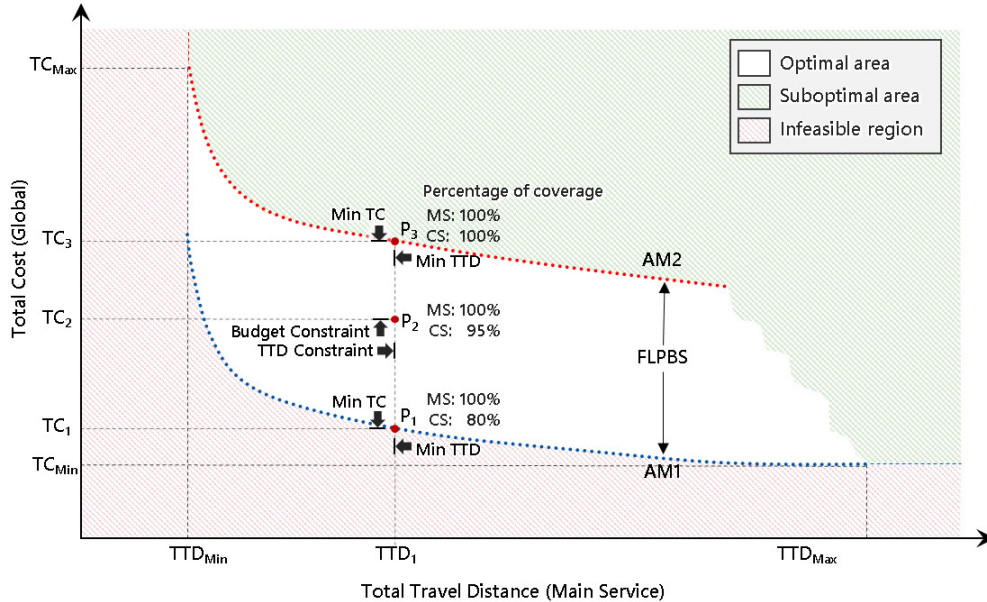


Figure 1: Graphical representation of AM1 and AM2 Pareto fronts.

The FLPBS is needed because the required budget to cover all demand could be very difficult to achieve in a real-world instance. However, AM2 can be used if there is no budget limit in the planning of the resources. FLPBS finds an efficient solution with a limited budget and with a predefined maximum TTD for the allocation of demand for the outpatient service. Since the TTD

metric can be challenging to interpret, an alternative equivalent metric is the Mean Travel Distance (MTD), which is calculated by dividing the TTD by the total demand. This value represents the mean distance that the population travels between demand points to facilities.

## 4 Solution Methodology

The augmented  $\varepsilon$ -constraint method (AUGMECON) was proposed by Mavrotas [21] for solving multi-objective programming models. An improved version of the algorithm (AUGMECON2) was later developed by Mavrotas and Florios [22]. This method is used to find the exact Pareto set of a multi-objective integer programming problem. The method avoids the production of weakly Pareto optimal solutions and accelerates the whole process by avoiding redundant iterations.

### 4.1 Applying AUGMECON2 for Solving the Auxiliary Models

The following implementation of the AUGMECON2 is proposed by keeping  $Z_1$  as the main objective function and  $Z_2$  as a constraint. However, the role of each objective function can be swapped. The notation used in this subsection is the following:

*Parameters and variables:*

- $q$     The total number of grids in the objective function range of  $Z_2$ .
- $r_1$     The range of possible bounds in the TTD constraint.
- $r_2$     The range of possible bounds in the budget constraint.
- $e_p$     The right-hand side coefficient in the budget constraint for the grid point  $p$ .
- $\epsilon$     A very small value, given by  $1 \times 10^{-9}$ .
- $b$     The bypass coefficient.
- $\mathcal{S}_1$     Slack variable for the TTD constraint.
- $\mathcal{S}_2$     Slack variable for the budget constraint.

The first step is to construct the payoff table that provides the extreme points of the optimal Pareto set. The problem is firstly solved by minimizing objective function  $Z_1$ . Then, the problem is solved again, minimizing  $Z_2$ , but including the previously found objective value of  $Z_1$  as a constraint bound. The values of  $Z_1$  and  $Z_2$  in the solution of the second model corresponds to an extreme solution of the optimal Pareto set. These steps are repeated, swapping the objective functions to found the second extreme point.

The payoff table provides the range of each objective function that is going to be used as constraints. The range  $r_2$  that corresponds to the objective  $Z_2$  is divided in  $q - 1$  intermediate equidistant grid points ( $e_p$ ). The density of the efficient set is controlled by the parameter  $q$ . There is a trade-off between the density of the efficient set and the computation time. The original problem is modified as follows:

$$\text{Minimize} \quad Z_1 - \epsilon \left( \frac{\mathcal{S}_2}{r_2} \right) \quad (18)$$

$$\text{subject to} \quad Z_2 + \mathcal{S}_2 = e_p \quad (19)$$

$$\begin{aligned} \text{constraints} \quad & (2) - (6), (10), (11), (17)^* \\ & \mathcal{S}_2 \geq 0 \end{aligned} \quad (20)$$

Note: (\*) Constraints (17) are only added for AM2.

For each grid point ( $e_p$ ), the model is solved to find an optimal Pareto set point. AUGMECON2 implements a slight modification in the objective function when more than two objectives are evaluated, which is not the case of these models. A second improvement is estimating the bypass coefficient to omit redundant iterations that find the same Pareto optimal solution. This coefficient is calculated as follows:

$$b = \left\lfloor \frac{q\mathcal{S}_2}{r_2} \right\rfloor \quad (21)$$

When the surplus variable  $\mathcal{S}_2$  is larger than  $r_2/q$ , the iteration can be omitted because no new Pareto optimal solution is generated. The coefficient  $b$  indicates how many consecutive iterations can be omitted.

This procedure is the same for both auxiliary models. The only difference is the addition of constraints (17) in AM2.

## 4.2 Solving the FLPBS

The Branch-and-Bound algorithm provided by conventional optimization software such as CPLEX is used in this work. Some slight modifications are proposed to the original FLPBS to find efficient solutions. The maximization of the complementary services coverage and the minimization of the TTD are independent objectives. The improvement of one objective does not mean the improvement of the other one. The TTD is constrained by a boundary value in the model, but when there are multiple optimal solutions, the TTD value found in the solution may not be the one with the lowest value. To ensure that the solution with the lowest TTD is found in the solution, the model is modified as follows using the previous notation of the AUGMENCON2 procedure.

$$\text{Maximize} \quad \sum_{i \in M} \sum_{s \in S(i)} \lambda^s P_i V_i^s + \epsilon \left( \frac{\mathcal{S}_1}{r} \right) \quad (22)$$

$$\text{subject to} \quad \sum_{i \in M} \sum_{j \in N} P_i D_{ij} X_{ij} + \mathcal{S}_1 = \text{TTD} \quad (23)$$

$$\begin{aligned} &\text{constraints} \quad (2) - (7), (9) - (12) \\ &\mathcal{S}_1 \geq 0 \end{aligned} \quad (24)$$

The range  $r$  is obtained from the payoff table as the difference between the extreme values of  $Z_1$  in both auxiliary models. This modification allows finding the best TTD when multiple optimal solutions are found in the problem because the algorithm maximizes the slack variable  $\mathcal{S}_1$ .

## MIP Start Strategy

When dealing with large-scale problems, some feasible solutions to the FLPBS could be challenging to achieve by the B&B algorithm. This was observed in the preliminary experimental work for scenarios close to the AM1 optimal Pareto front. A large amount of memory and time was spent trying to find a feasible solution. In this case, the B&B algorithm can start using an initial solution to avoid this problem. The solution is not required to be feasible, and it can be obtained from a related problem.

An alternate model (named Reduced\_FLPBS) is suggested. This model is formed by constraints (2)-(8) and (10)-(11), maximizing only the slack variable  $\mathcal{S}_1$ . This problem can be used to find an initial solution to the FLPBS. The number of constraints and variables is lower because constraints (9) are not considered. The feasible region is the same as that of the FLPBS, but with another objective function. This modification is observed to be extremely helpful, making the B&B algorithm converge a lot faster.

## 5 Empirical Work

### 5.1 Description of Case Study

The model is applied to a case study composed of 17 municipalities in the northern zone of the State of Mexico with a total of 1,086 demand points with an estimated population of 1.3 million inhabitants in 2019 and with a land area of 5,287 km<sup>2</sup> (Figure 2). This group of municipalities was chosen since they have similar characteristics of the population, and their primary health care centers belong to a group of three sanitary jurisdictions. Most of the information is obtained from publicly available data sources, while some information was generated based on real-world cases. The details of the sources are shown in Table 9 (Appendix A).



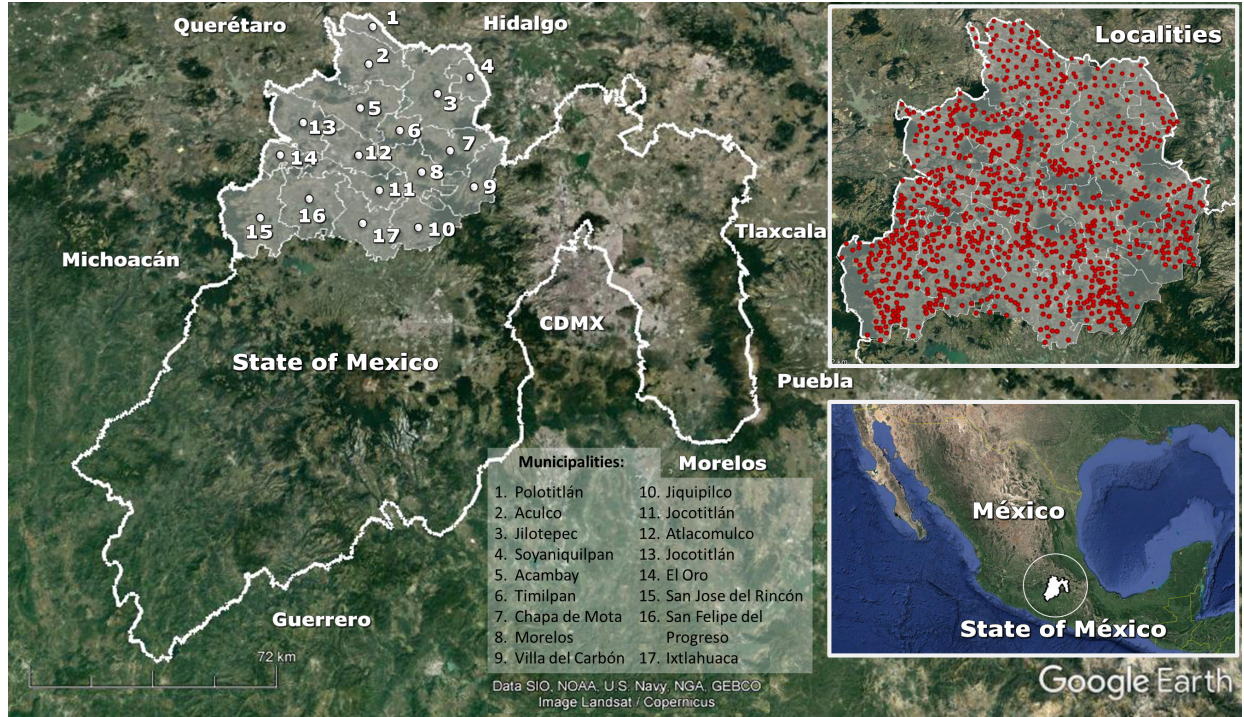


Figure 2: A map of the State of Mexico showing the 17 municipalities.

To give an idea of the population distribution around the area of study, Table 3 displays the localities classified in ranges according to the number of inhabitants. The ranges of the population are shown in the first column, the number of localities and population average are shown in the second and third columns, respectively. In the third- and second-to-last rows, the demand is classified by the type of locality (urban and rural). We can observe that most of the localities are rural (93%), with 63% of the population of the region. For these localities, 79% of them have less than 1,000 inhabitants on average. The urban localities represent 7% of localities, with 37% of the inhabitants of the region. Some facility types can only be installed in urban localities, as explained later. The last row shows the total number of localities and the total number of inhabitants in the region.

The types of health care facilities and their characteristics are shown in Table 4. For rural localities (R), only the first three facility types can be installed because these types of localities present in general low population density and they may be located in not very accessible places. All the facility types are available for urban localities (U). The outpatient service capacity is shown in column three; this capacity represents the number of people (#p) that can be permanently allocated to each facility type. The capacity of facility type 1 was set to 2,400 inhabitants, and the capacity of the following facility types is a multiple of this value. According to Secretaría de Salud [34], the basic capacity must be 3,000 inhabitants, but this was reduced to 2,400 inhabitants for experimental purposes. This change allows us to evaluate the location of new facilities to cover the outpatient



Table 3: Characteristics of the localities.

Population range	Number of localities	Population average
$\leq 100$	159	33
101 - 500	385	278
501 - 1,000	250	717
1,001 - 2,000	172	1,431
2,001 - 3,000	57	2,859
3,001 - 4,000	21	4,323
4,001 - 5,000	20	4,824
$> 5,000$	22	11,050
Urban localities	76	711
Rural localities	1,010	5,438
Total	1,086	1,131,193

service; otherwise, the problem would be limited to the allocation problem. The number of actual facilities and the total capacity are shown in the last two columns for each facility type. In the last row, the total number of facilities (294) and their capacity for the outpatient service is shown. If we compare the total capacity and the population, there is a capacity shortage of 425,593. The number of candidate locations to install new facilities is 117, integrating the network up to 411 potential facility locations.

Table 4: Types of health care facilities and the services provided.

Facility type	Category	Outp. service capacity	Setup cost	Compl. service coverage					Existing capacity	
				CS1	CS2	CS3	CS4	CS5	Facilities	Outp. service
1	U, R	2,400	4.0	0	0	0	0	0	179	429,600
2	U, R	4,800	6.7	0	0	0	0	0	83	199,200
3	U, R	7,200	14.5	1	1	1	0	0	27	64,800
4	U	9,600	16.8	1	1	1	0	0	2	4,800
5	U	12,000	22.9	1	1	1	1	0	3	7,200
6	U	14,400	31.7	1	1	1	1	0	0	0
7	U	16,800	33.0	1	1	1	1	1	0	0
8	U	19,200	35.7	1	1	1	1	1	0	0
9	U	21,600	36.9	1	1	1	1	1	0	0
10	U	24,000	38.4	1	1	1	1	1	0	0
11	U	26,400	41.7	1	1	1	1	1	0	0
12	U	28,800	42.6	1	1	1	1	1	0	0
Critical coverage radius (km)				6	12	12	18	24		
Total									294	705,600

The setup cost in million of pesos (M) is shown in the fourth column in Table 4. The complementary services are nutrition service (CS1), dental care (CS2), mental health services (CS3), clinical analysis (CS4), and radio-diagnosis and imaging (CS5). The weight in the objective function will be the same for all the services. The services available for each facility type are shown in the following five columns as binary values (1 if the service is provided, otherwise 0). In the penultimate row,

the critical coverage radius in kilometers for each complementary service is shown. These values are proposed for this experimental analysis. The costs of updating the facilities to other facility types are shown in Table 10 (Appendix A).

## 5.2 Computational Results

The Branch-and-Bound algorithm from the CPLEX callable library, version 12.8, with a C++ API was used to find the optimal solution to FLPSB and for solving the subproblems in the AUGMECON2 method. The experiments were carried out in an Intel Core i7-5600U at 2.60GHz with 16GB of RAM, under Windows 10 operating system. A relative gap tolerance of  $1 \times 10^{-6}$  was set as a stopping criterion without a time limit. Table 5 shows the payoff table of the AM1 and AM2 objective functions for the AUGMECON2 implementation. A set of 101 equidistant grid point was used for the budget constraint, while the TTD was minimized. Some statistics about the B&B CPU time for solving AM1 and AM2 are shown in Table 6. The number of instances (N), the mean, the standard deviation, the minimum and maximum values, the first and third quartiles, and the median of the CPU time are shown for each model. The last quarter of the AM1 instances close to the lowest budget were the most difficult to solve. From there on, all the instances were solved in reasonable running times. The smallest TTD objective value was the same in both auxiliary models. Still, the highest value was different because AM2 requires more facilities to ensure the coverage of complementary services, which helps reduce the TTD at the same time.

Table 5: Payoff table of AM1 and AM2.

Model	Objective	$Z_1$ (m)	$Z_2$ (\$ MX)
AM1	Min $Z_1$	426,420,121	1,082,556,795
	Min $Z_2$	1,745,757,649	371,484,933
AM2	Min $Z_1$	426,420,121	1,571,530,037
	Min $Z_2$	1,278,662,321	906,097,190

Table 6: Summary of CPU time (in seconds) statistics summary for the B&B.

Model	N	Average	Standard Deviation	Minimum	Q1	Median	Q3	Maximum
AM1	101	2,804	13,408	27	35	51	404	111,997
AM2	101	195	435	31	41	65	176	3,773

Figure 3 shows the solutions of the Pareto front of each auxiliary model. The values of the corresponding extreme points of each model are shown in the plot. A set of 18 samples for the FLPBS were chosen in the area between both Pareto fronts. The instances were grouped into two types to identify the effect of changing the TTD and budget bounds. In the first ones (FLPBS\_H), the budget limit was fixed, and the TTD bound was varied. In the second group of instances (FLPBS\_V), the TTD bound was fixed, and the budget was varied. Figure 3 shows the results of

all the FLPBS samples grouped by type. For instance, samples 1 to 3 (subgroup  $H_1$ ) have the same budget with different TTD bounds, and samples 10 to 12 (subgroup  $V_1$ ) have the same TTD bound with different budgets. The sample number and the coverage percentage of each sample are shown in parentheses near each point. The samples near AM1 Pareto front have the lowest coverage rates, while the ones near to the AM2 Pareto front have the highest coverage rates. The intermediate samples (2, 5, 8, 14, and 17) obtained percentages of coverage higher than 99%. These results show that a small increase in the budget, starting from the AM1 Pareto front, can significantly improve the percentage of the complementary services coverage. For the AM1 solutions, it was observed that the TTD could be reduced by 76% from the worst to the best value if the budget is increased 2.9 times. For AM2, the budget only requires an increase of 1.73 times, reducing the TTD by 67%.

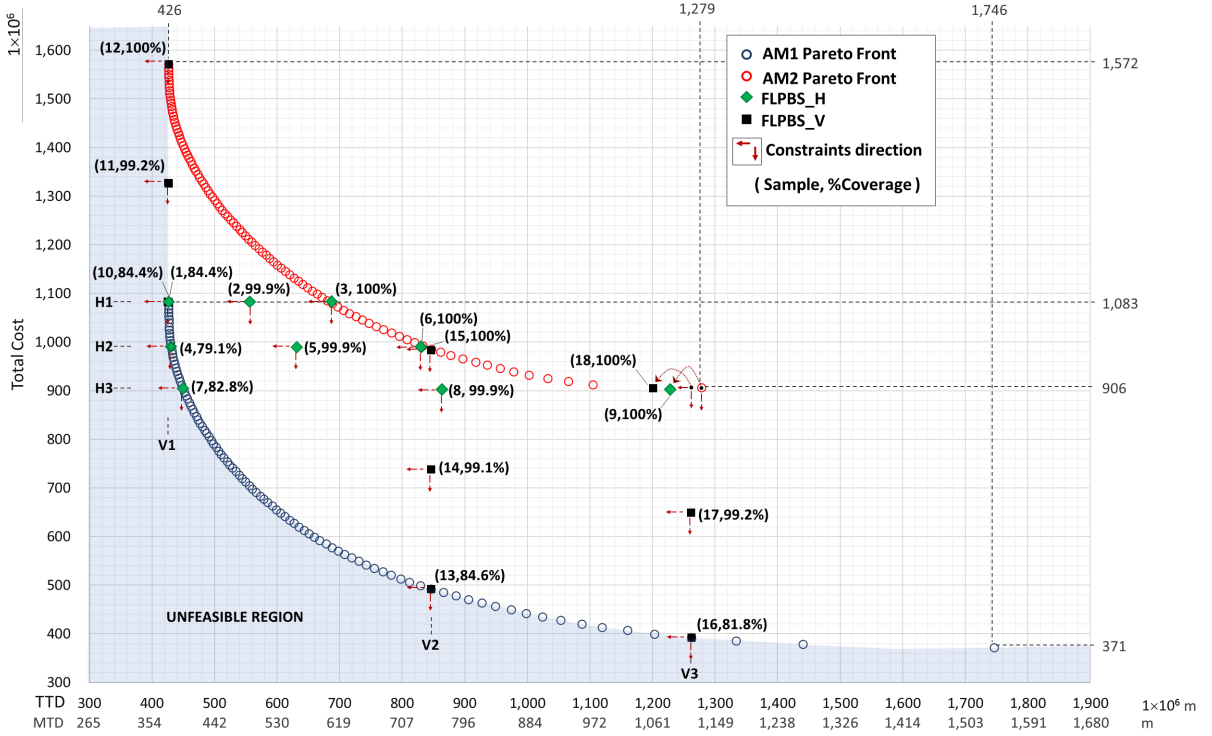


Figure 3: Graphical representation of the solutions to the AM1, the AM2, and the FLPBS.

A detailed summary of the results for each FLPBS sample is shown in Table 7. Each column's definition is described in Table 11 (Appendix A) to support the information's understanding. The number of existing facilities was 294, some of them were updated, and new facilities were opened in the solutions. The results show that new facilities are required in more proportion than updated facilities if the main objective is to reduce the TTD (samples 1, 4, 7, 10-12). In contrast, more updated facilities help to increase the demand covered by the complementary services (samples 3, 6, 9, 16-18).

When the available budget is not enough to get the desired coverage goal for the complementary

Table 7: Summary of results for the FLPBS instances.

Sample	Type	Constraints		Results						Main Service						Complementary Services					
				Facilities		Cost		TTD	MTD	% of Demand			Utilization		Mean Cov.	Percentage of coverage					
		Budget	TTD	New	Upd.	Tot.	New			Upd.	<1km	<5km	<10km	Mean		SD	(%)	CS1	CS2	CS3	CS4
1	H1	1,083	426	117	91	1,083	577	506	426	377	83	17	100	64	20	84.4	79	98	98	78	70
2	H1	1,083	557	62	85	1,082	386	696	557	493	78	22	100	72	22	99.9	99	100	100	100	100
3	H1	1,083	688	31	103	1,082	235	847	688	608	73	27	100	73	23	100.0	100	100	100	100	100
4	H2	990	430	116	80	990	554	436	430	381	83	17	100	67	21	79.1	78	97	97	76	48
5	H2	990	632	43	84	990	307	683	631	558	75	24	100	74	22	99.9	99	100	100	100	100
6	H2	990	833	14	104	990	143	847	831	734	69	30	99	75	23	100.0	100	100	100	100	100
7	H3	905	449	109	68	905	507	398	449	397	83	17	100	70	21	82.8	78	97	97	76	66
8	H3	905	864	13	91	902	157	745	863	763	69	30	99	76	23	100.0	100	100	100	100	100
9	H3	905	1,279	3	98	903	62	841	1,228	1,086	66	30	96	78	22	100.0	100	100	100	100	100
10	V1	1,083	426	117	91	1,083	577	506	426	377	83	17	100	64	20	84.4	79	98	98	78	70
11	V1	1,327	426	117	98	1,327	666	661	426	377	83	17	100	63	21	99.2	96	100	100	100	100
12	V1	1,572	426	117	123	1,572	664	908	426	377	83	17	100	61	22	100.0	100	100	100	100	100
13	V2	492	846	16	64	492	101	391	846	748	70	29	99	82	19	84.6	78	97	97	76	75
14	V2	739	846	14	74	739	140	598	846	748	70	30	99	78	21	99.1	96	100	100	100	100
15	V2	986	846	13	103	984	139	845	846	747	69	30	99	76	23	100.0	100	100	100	100	100
16	V3	393	1,262	1	60	393	4	389	1,262	1,116	66	28	95	87	17	81.8	78	97	97	76	61
17	V3	649	1,262	1	75	649	15	635	1,261	1,115	65	29	95	82	19	99.2	96	100	100	99	100
18	V3	906	1,262	3	99	906	62	844	1,201	1,061	66	30	96	78	23	100.0	100	100	100	100	100

services, the TTD bound of the main service can be enlarged to increase the demand coverage, waiving some quality in this objective. This effect is observed in samples type H1, H2, and H3. For instance, when the TTD bound was changed from  $426 \times 10^6$  m in sample 1 to  $557 \times 10^6$  m in sample 2, the coverage percentage increases from 84% to 99% with the same budget.

Samples type V1, V2, and V3 help to identify the impact in the coverage of complementary services when the budget is modified, but the TTD bound is kept as a fixed bound. For instance, the budget increase between samples 10 and 11 was about to \$244 M. This increased the covered demand from 84% to 99%.

The TTD value is challenging to interpret by itself. This parameter does not provide information about the distance variability between demand points and facilities. In Table 7, the demand percentages who travel a distance equal to or lower than 1 km, 5 km, and 10 km are shown. This indicator could help the decision-maker to identify the distribution of the distance allocation among the demand. For instance, in the worst-case solution, 96% of the demand travels less than 10 km.

Another important indicator is the utilization rate, which evaluates the demand allocated to each facility compared to its maximum available capacity for the outpatient service. A higher value of the mean utilization rates indicates a better use of the resources, while a higher standard deviation (SD) indicates an unequal distribution of demand among the facilities. The utilization rate of samples V1, V2, and V3 showed a decrease when the coverage of complementary services increased because more facilities or facilities with additional capacity were required to meet this objective. A lower number of facilities increase the utilization rate but with an increase in the TTD.

The results about the coverage of complementary services in a solution must be analyzed one by one. If some services have more priority in resource planning, they must have a higher weight in the

FLPBS objective function. In this experimental work, all services were equal-weighted. However, the service S1 has the lowest amount of covered demand in the solutions because its critical coverage radius (6 km) is the lowest. More facilities are needed for this service to extend its, increasing the total cost. Therefore, the critical coverage radius must be defined very carefully for each service because the percentage of coverage in the solution is affected by this parameter. Another observation in the results is that CS2 and CS3 have the same coverage radius and, they are available in the same facility types. This coincidence causes the coverage percentage is the same for both of them. Both services could be integrated as a single one in the model with greater weight in the objective function to reduce the complexity of the problems.

The incorporation of the slack variable  $S_1$  in the FLPBS objective function helps the algorithm to select the solution with the lowest TTD value when there are multiple optimal solutions in a given scenery. This was observed in samples 9 and 18 in Figure 3, which their TTD values were significantly lower than their TTD bounds.

### 5.3 Assessment of MIP Start Strategy

A set of 21 instances was tested to evaluate the implementation of the MIP start strategy with a budget range between the Pareto front of AM1 and AM2 and a TTD equal to or lower than 845,783 km. The stopping criterion was set to one hour of CPU time. The test instances were solved with the three schemes: the Reduced\_FLPBS, the FLPBS, and the FLPBS with the MIP start. A comparison among these schemes is shown in Table 8. As we can see from the table, all instances were optimally solved under the Reduced\_FLPBS model in less than one hour of CPU time, while only eight instances of the FLPBS were optimally solved. The solutions of the other eight samples were found with an average relative optimality gap of 0.05%. Five instances of the FLPBS were not solved since the B&B algorithm did not found an integer solution in one hour of CPU time. When the MIP start strategy was applied to the FLPBS, all samples were solved, eleven of them were optimally solved, and the remaining were solved with an average relative optimality gap of 0.04%. Finally, it can be observed in Figure 4 that instances of the FLPBS with the lowest budget value were not solved. Thus, it is evident that the problem becomes more difficult to solve when the constraints (7) or (8) become tighter, and the use of MIP start strategy can be useful to improve the performance of the B&B algorithm.

Figure 5 shows the solutions of the FLPBS with the MIP start. The TTD bound is fixed for these instances, and the budget is varied according to the auxiliary models' results. \$492 M is the minimum investment for the problem to be feasible, and \$985 M is the maximum investment to ensure the complete coverage of demand for all complementary services. Thus, the budget has a range of about \$466 M from the minimum required. In the plot, we can observe that only 5% of the budget range is needed to go from 85% to 91% of the covered demand (the greatest improvement),

Table 8: Comparison of solution schemes.

Termination criteria	Reduced_FLPBS			FLPBS			FLPBS + MIP start		
	Instances	Average		Instances	Average		Instances	Average	
		Relative gap (%)	CPU time (s)		Relative gap (%)	CPU time (s)		Relative gap (%)	CPU time (s)
Optimal solution found	21	0.00	165	8	0.00	1,155	11	0.00	1,288
Time limit:									
-Integer solution found	-	-	-	8	0.05	3,600	10	0.04	3,600
-No integer solution found	-	-	-	5	-	-	-	-	-

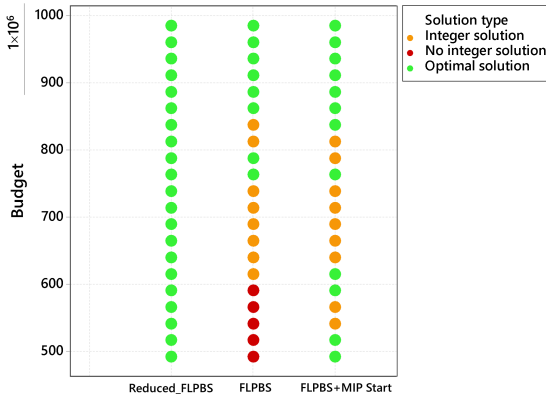


Figure 4: Classification of solutions for each model

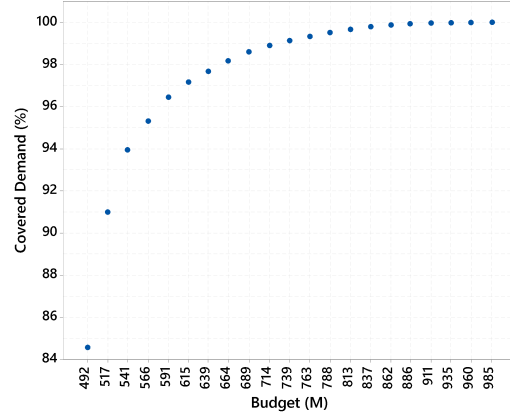


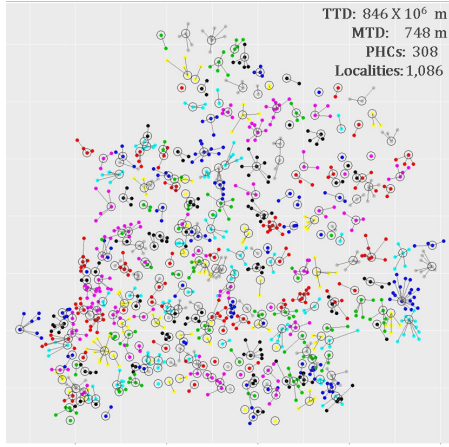
Figure 5: Comparison of solutions for the FLPBS with MIP start

and 80% of the budget range only improves the coverage from 96% to 100%. This logarithmic behavior is observed in the plot. We conclude that maximizing the coverage instead of guaranteeing the complete coverage of demand can produce significant savings in the investment of new facilities. A small increase in the minimum required budget can significantly improve the level of coverage.

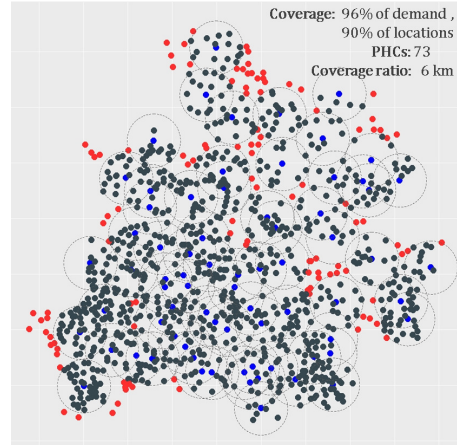
## 5.4 Graphical Results

Figure 6 shows the graphical representation of a solution. The result corresponds to instance number 14, according to Table 7. The allocation of the demand points for the main service is shown in plot 6(a). The coverage of the complementary services is displayed in plots 6(b)–6(f). The red points represent the demand points not covered by any facility. For the CS1, 10% of the localities are not covered, but these represent only 4% of the total demand. We can observe that the same solution was found for the CS2 and CS3 since they have the same coverage radius and availability in the facility types. For the CS4, 11 demand points representing 1.0% of localities, and 0.4% of demand were not covered. Finally, all demand was covered by the CS5.

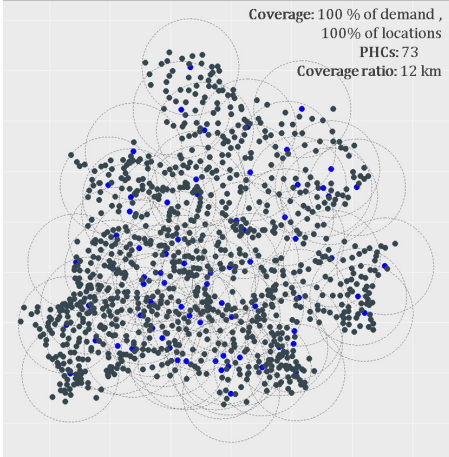




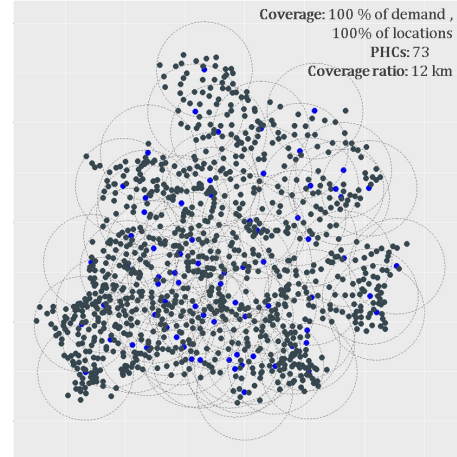
(a) Allocation of demand points for the MS.



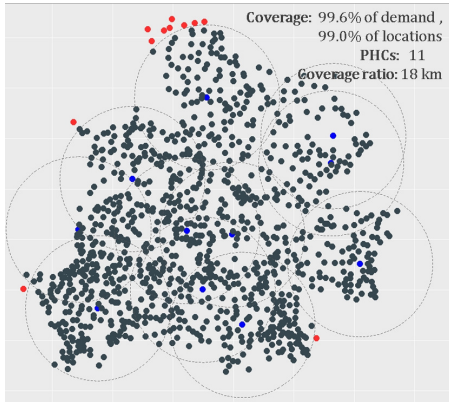
(b) Coverage of CS1.



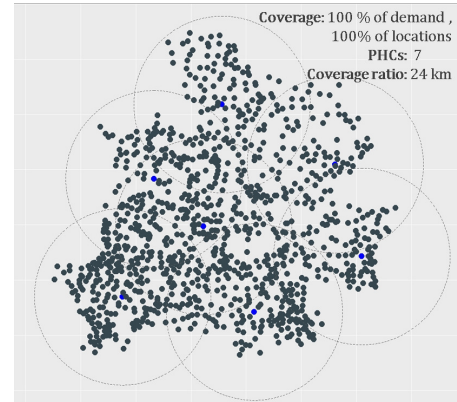
(c) Coverage of CS2.



(d) Coverage of CS3.



(e) Coverage of CS4.



(f) Coverage of CS5.

Figure 6: Graphical representation of instance 14 solution.

## 6 Conclusions

In the paper, we have studied the problem of locating primary health care centers with multiple services under different characteristics. The outpatient service is considered a capacitated service with demand allocation, and the demand coverage of complementary services is maximized. The problem is motivated by a real-world application in the Mexican Health Care System, and the results can be extended to developing countries with similar systems. This problem-solution allows the planning of primary health care infrastructure with an integral and standardized scheme. Since limited budgets always play a vital role, the objective is to find a solution under this tight requirement. The proposed model can be easily adjusted to handle other objectives.

A case study is presented using real-world data from the State of Mexico to assess the model. The case study results suggest a negative relationship between the total travel distance for the allocation of the outpatient service and the total costs of the solutions. This negative relationship is because more facilities are required to reduce the distance between demand points and facilities. The coverage of complementary services also increases the total cost when the coverage radius is relatively small or when the cost associated with the facility type that provides the service is relatively high. Besides, we introduced two auxiliary models, namely AM1 and AM2. The use of the auxiliary bi-objective programming models helps the decision-maker select efficient bounds for the main problem. The Pareto front of AM1 helps to identify the minimum budget required to get feasible solutions to the problem. In contrast, the Pareto front of AM2 determines the maximum required budget to cover all demand by the complementary services, both of them for a set of TTD bounds. Both auxiliary models were formulated as bi-objective integer programs and efficiently solved by the augmented  $\varepsilon$ -constraint method AUGMECON2.

Optimal solutions were always found for a network of 1,086 demand points and 411 candidate potential facility location points. It was challenging for samples with tight bounds to find an initial feasible solution by the algorithm, spending considerable time in this task. To this end, we implemented a start strategy using a reduced version of the FLPBS model for providing a feasible initial solution to the corresponding subproblem of the FLPBS. The experimental work shows the effectiveness of this strategy, causing a significant reduction in CPU time.

The solutions show that an additional partial increase of the budget starting from the AM1. Pareto front for a given TTD bound could significantly improve the coverage level of the complementary services. In that sense, the FLPBS could be used as a decision-making tool when the resources are finite for planning primary healthcare units. The solutions ensure the capacity feasibility for the outpatient service and provide a maximum total travel distance for the demand point to facilities. The demand covered for the complementary services is maximized to benefit as many users as possible under the limited budget. The coverage radius of each complementary service directly affects the quality of the solution. It requires a previous analysis based on the characteristics of the



real case to be solved.

It is clear that if the aim is to solve the model over a considerably more extensive region, using significantly more demand points and potential facility sites, the model may become intractable. The development of heuristics or decomposition techniques could be an important area of opportunity for further research in this area. Along this line, the model and technique presented in this paper can be valuable.

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## A Complementary Information for the Case Study

Table 9 shows each data source used to generate the instances of Section 5. The information obtained from each source is described, and the reference is provided in the last column.

Table 9: The data sources for the empirical assessment

Data	Description	Source	Ref.
Localities and their demand	Geographic location and population at 2010.	INEGI	[16]
	Projection of population at 2020.	CONEVAL	(a)
	Proportion of demand of each institution.	INEGI	[17]
Primary health care centers	The existing facilities, their location, their type and capacity.	MHM	(b)
Distance matrix	The geographic locations were converted to Universal Transverse Mercator system to found the euclidian distance from each point to the others.	INEGI	[16]
Facility types, their capacity, and their fixed and operative costs	All data related to the facility types.	MHM	[34]
(a) Web site: <a href="https://www.coneval.org.mx/">https://www.coneval.org.mx/</a>			
(b) Web site: <a href="http://www.dgis.salud.gob.mx">http://www.dgis.salud.gob.mx</a>			

Table 10 shows the updating cost of each facility type used in the Case Study in Section 5. The costs are presented in thousands of Mexican pesos. The cells with a “-” mark represent an infeasible combination.

Table 10: Cost of updating a facility

From:	To:										
	2	3	4	5	6	7	8	9	10	11	12
1	2,796	11,715	14,089	20,426	30,424	31,811	34,597	35,843	37,388	40,888	41,880
2	-	8,919	11,292	17,630	27,628	29,015	31,800	33,046	34,592	38,092	39,084
3	-	-	2,374	8,711	18,709	20,096	22,882	24,128	25,673	29,173	30,165
4	-	-	-	6,337	16,335	17,722	20,508	21,754	23,300	26,800	27,792
5	-	-	-	-	9,998	11,385	14,171	15,416	16,962	20,462	21,454
6	-	-	-	-	-	1,387	4,173	5,418	6,964	10,464	11,456
7	-	-	-	-	-	-	2,786	4,031	5,577	9,077	10,069
8	-	-	-	-	-	-	-	1,246	2,791	6,291	7,283
9	-	-	-	-	-	-	-	-	1,546	5,046	6,038
10	-	-	-	-	-	-	-	-	-	3,500	4,492
11	-	-	-	-	-	-	-	-	-	-	1,035

Table 11 contains the description of each parameter presented in Table 5 of Section 5.2. The second-to-last column indicates the units of each parameter. The last column shows the complete description of each parameter.

Table 11: Description of parameters of Table 5

Header		Units	Description	
Sample		The number of sample displayed in the Figure 3		
Type		The type of instance. V1, V2, V3 corresponds to the group FLPBS_V and H1, H2, H3 to the group FLPBS_H		
Constraints		Budget	millions of MX pesos	
		TTD	thousands of km	
			The value used in the RHS of constraint (7)	
			The value used in the RHS of constraint (8)	
Results	Facilities	New Upd.	number of units	
			The total number of new facilities	
			The total number of updated facilities	
	Cost	Tot. New Upd.	millions of pesos	
			The total cost of installing or updating facilities	
			The cost associated to install new facilities	
			The cost associated to update existing facilities	
Main service	TTD		thousands of km	
	MTD		meters	
	% of Demand	<1km		The total travel distance of demand points to facilities for the main service
		<5km	% of population	The mean travel distance in the solution
		<10km		The percentage of demand that travels less than 1 km to their facility
	Utilization rate	Mean		The percentage of demand that travels less than 5 km to their facility
		SD	% of population	The percentage of demand that travels less than 10 km to their facility
			The mean utilization rate of the facilities	
			The standard deviation of the utilization rate of facilities	
Complementary services	Mean cov.		% of population	
			The mean percentage of demand covered by the complementary services	
		% of coverage (CS1–CS5)	% of population	
			The percentage of demand covered by each complementary service	