An Integrated Framework for Control and Management of Forest Fires Using Potential Fire Risk Indices[1]

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Abstract

The objective of this research is to develop an integrated methodology for forest fire control, mainly addressing the problem of how to locate and deploy firefighting brigades in order to maximize the protected area considering its different classes; that is, to maximize the weighted sum of the number of available resources for the region, respecting the time limit of the arrival of the resources to extinguish the fire. The distributed resources should cover the most considerable amount of the geographic area previously classified for its importance level. This methodology integrates (i) a geographical information system (GIS), (ii) a module that computes risk maps of the study zone based on important indices from literature such as the fire potential index, the ignition index, the dynamic behavior index, and the energy behavior sub-index, (iii) a mixed-integer linear programming (MILP) module, and (iv) a post-processing module. The MILP module applies a branch-and-bound algorithm to determine the optimal location of the resources. The overall solution methodology is illustrated and discussed in a case study from the state of Chiapas, Mexico. x

Keywords: Wildfire management; fire potential index; resource location; resource deployment; mixed-integer linear programming.

1 Introduction

Like other disturbances, fire is a factor that is naturally present in many ecosystems and landscapes throughout the world [3]. However, its appearance and use within forest ecosystems are viewed controversially in management or conservation [12]. Every fire season, large forest masses are lost throughout the planet. An annual affectation of these ecosystems has been calculated at an average of over 1043.6 million hectares [10]. Consequently, there has been increased attention to forest fires throughout the world. In Mexico's case, forest fires are a phenomenon widely distributed in many forest ecosystems, mainly in protected natural areas, such as parks, biosphere reserves, among others [1]. According to Mexican statistics, the number of fires is increasing, only until September 2020, about 333,000 ha have been affected, recording more than 5,580 fires distributed across the country [5].

The control of forest fires has been a challenge for all nations, even the most developed, which have the technology and economic resources for acquiring specialized means of transportation for fire control [13]. However, on occasions, they have been unable to prevent or reduce them. This fact has led the institutions and academic and scientific community to reformulate several matters with respect to what should be the best institutional policy for resolving the problem.

The initial implementation of an attack for the extinction of fires in the days before an occurrence is an essential part of the planning of forest fires [11]. So, when fires occur, the resources for extinguishing them are deployed to prevent their propagation [35]. It has been recognized that a rapid initial attack on a forest fire within an established period prevents it from causing significant damage [17]. At the same time, most fire administrators find themselves limited with respect to the initial attack [21]. The implementation of optimization models has been demonstrated to serve as a support for decision making. Therefore these models could be used to efficiently distribute the resources to ensure timely response to call for service [14, 25].

Those in charge of forest fire management should decide the amount and type of transport (land vehicles, helicopters, planes) to acquire, the number of firefighters to hire, and which fire extinguishing equipment to buy, to minimize the cost of the resources and at the same time to rapidly satisfy the demands that vary throughout a fire season. This decision-making process becomes complicated because the occurrence of fires and the need for means of fire extinction is very variable, both in time and space. Most of the fire management organizations should deploy their resources to satisfy their daily demands. Therefore, it is not surprising that many operations research specialists have studied forest fire management problems throughout the world, given that this is a highly relevant problem of decision-making [18].

Forest fire management has been involved in developing various methodologies of analysis based on operations investigation techniques, which serve as tools for creating policies. This development is reflected in the growing number of applications, some of which are discussed by Martell et al.

[20].

This paper's main contribution is to present a methodology or tool that integrates both a pre-processing step that computes potential fire risk indices and a decision-making step that places fire brigades. The second step is to build and solve an optimal resource location problem modeled as an integer programming model. In our work, we base our study on a model proposed by Dimopoulou and Giannikos [7] that maximizes the weighted sum of demand points that are covered in a protected area given its different risk classes, respecting the number of available resources and the arrival time limit to extinguish the fire (it is variable for each zone). The formulation used is an integer linear programming model that determines the deployment of resources so that any fire can be attacked within a specified time limit. The distributed resources should cover the largest amount of the geographic area previously classified by its level of importance. The developed tool allows the calculation of different important indices in fire behavior, through dynamic or historical data, for different periods of the year and different hours of study. An important contribution is the map processor based on different indices of fire behavior. A generator of instances has been created, taking as input the fire potential index maps. A solution has been given to the mathematical model for the distribution of brigades to a case study of Chiapas's state.

The rest of the paper is organized as follows. The following section describes and defines the problem in greater detail. In Section 2, we review the most important works using operations research tools for optimal locations of resources for wildfire management. Section 3 describes the resource location problem, including its relevant assumptions and its integer programming model. This is followed by Section 4, where the main integrated solution methodology is described. This methodology includes two main components. Section 4.2 presents the mathematical expressions for calculating the potential fire indices for the classification of the area. Section 4.3 describes the classification of the study area. Our empirical work is presented in Section 5. Different test scenarios are used varying the meteorological conditions, and a case study is shown for the state of Chiapas, Mexico. Finally, closing remarks are drawn in Section 6.

2 Decision Making Models in Wildfire Management

There has been extensive research on wildfire propagation and containment [35]. There are many issues at stake that are interrelated such as fire evolution, the ecological impact of fires, socio-economic, and associated fire management problems. Some recent review papers on this are due to Miller and Ager [23] who present a review on advances in wildfire management by risk analysis, and Minas et al. [24], who present a review of operations research methods applicable to wildfire management, Thompson et al. [31] present a review paper focusing on decisions related to the rare larger and longer-duration fire events, where the scope and scale of decision-making can be far broader than initial response efforts, and where determining and demonstrating efficiency of strategies and actions can be particularly troublesome. They organize their review around key decision factors such as context, complexity, alternatives, consequences, and uncertainty, and for illustration contrast fire management in Andalusia, Spain, and Montana, USA. Two of the largest knowledge gaps relate to quantifying fire impacts to ecosystem services, and modeling relationships between fire management activities and avoided damages. The relative magnitude of these and other concerns vary with the complexity of the socioecological context in which fire management decisions are made. They conclude their review by discussing topics for future research, including expanded use of the economics toolkit to better characterize the productivity and effectiveness of suppression actions, integration of ecosystem modeling with economic principles, and stronger adoption of risk and decision analysis within fire management decision-making.

Dunn et al. [9] present a nice review, describing limitations in existing operations research models from the perspective of large fire management decisions. They identify a broader set of objectives, decisions, and constraints to be integrated into *the next generation operations research models*. They claim that the inclusion of these changes will support evaluating a suite of response options and the efficient resource packages necessary to achieve response objectives, aiding decision maker's ability to minimize responder exposure while reducing the social, ecological, and economic impacts of wildfires.

2.1 Deterministic Optimization Approaches

In terms of decision-making problems in a wildfire, the location of resources and their selection for the initial attack (that is, the actions carried out by the first resources to arrive at a wildfire to protect lives and property, and prevent further extension of the fire) has been studied in the literature throughout operations research tools. In particular, integer programming models have been developed for these types of problems. Martell [19] presents an excellent review that focuses on using of operations research and management science (OR/MS) methods to address the suppression of potentially destructive wildfires, mainly involving works before 2014. More recent work includes Rodríguez-Veiga et al. [28] who consider aerial resource deployment. They present two MIP models to address the problem of how to allocate the aerial resources to flight routes and refueling points.

Rashidi et al. [27] study the vulnerability of landscapes to wildfires based on the worst-case scenario ignition locations' impact. Using this scenario, they model wildfires that cause the most considerable damage to a landscape over a given time horizon. The landscape is modeled as a grid network, and the spread of a wildfire is modeled using the minimum travel time model. To assess the impact of a wildfire in the worst-case scenario, they develop an integer programming model to optimally locate the ignition points so that the resulting wildfire results in the maximum damage. Their results indicate that the worst-case wildfires, on average, have more than twice the impact on landscapes than wildfires with randomly located ignition points.

Wei et al. [33] present a containment strategy for large fires based on the concept of potential wildland fire operation delineations (PODs). They show how multiple PODs can be clustered together to form a“box” referred to as the *response POD* (or rPOD). Fire lines are built along the boundary of an rPOD to contain a large fire. They indicate how assets such as communities and infrastructure within an rPOD can be protected through *point zone protection*. The authors introduce a mixed-integer program model to optimally aggregate PODs into an rPOD to coordinate containment and point protection to maximize net value change under different fire weather scenarios and resource availability constraints. They test their approach in the Lolo National Forest in western Montana, USA.

Matsypura et al. [22] propose a multi-period mixed-integer programming framework to determine the optimal spatial allocation of prescribed burning activities over a finite planning horizon. In contrast to the existing fuel management optimization literature, they model fuel accumulation with Olsonâ€™s equation. To capture potential fire spread along with irregular landscape connectivity considerations, they use a graph-theoretical approach that allows them to exploit graph connectivity measures (e.g., the number of connected components) as optimization objectives. They solve the resulting model by a general purpose MILP solver and present a heuristic for handling larger instances.

Heyns et al. [15] study the problem of selecting multiple tower sites from a large number of potential site locations for optimal locating specialized tower-mounted cameras that would aid early forest fire detection. They develop metaheuristics for this hard combinatorial location problem and tested it in a case study in the Nelspruit region in South Africa currently monitored by the Forest-Watch detection system. They found that visibility cover superior to that of the existing system in the region is achieved and obtained in several days, whereas traditional approaches typically require months of speculation and planning.

2.2 Stochastic Approaches

In related work, involving optimization and simulation tools simultaneously, Wei et al. [32] tested a simulation and optimization procedure to transfer crews end engines between dispatch zones in Colorado (central United States) and into Colorado from out-of-state. This study aimed to show how sharing fire engines and crews between fire suppression dispatch zones may help improve the utilization of fire suppression resources. The results showed that improving the accuracy in predicting daily resource demands decreases the engine and crew transport costs by up to 40%.

Pereira Pacheco and Claro [26] present a stochastic programming model for addressing interannual fire management as a problem of multistage capacity investment in a portfolio of management resources, enabling fuel treatments and fire preparedness. They consider wildfires as the demand, with uncertainty in the severity of the fire season and in the occurrence, time, place, and severity of specific fires. In their empirical testing using a hypothetical test landscape, they found, among their findings, that the value of postponement increases significantly for scenarios of increased uncertainty (higher volatility) and higher escape costs, as also does the optimal budget (although not proportionally to the changes in the escape costs).

Belval et al. [2] present a stochastic programming model addressing dynamic decisions taken over time as fire behavior evolves. Such considerations include spatial restrictions for fire crew travel and operations. Crew safety is also considered. Fireline quality issues are accounted for by comparing control line capacity with fireline intensity to determine when a fireline will hold. The model assumes crews may work at varying production rates throughout their shifts, providing flexibility to fit work assignments with the predicted fire behavior. Nonanticipativity is enforced to ensure solutions are feasible for all modeled weather scenarios. Test cases demonstrate the model's utility and capability in a raster landscape.

2.3 Integrated Frameworks

There have also been some efforts to develop integrated forest fire control systems For instance, Dimopoulou and Giannikos [8] develop a framework using GIS, mathematical programming, and simulation modules. Other frameworks focus on risk assessment. Thompson et al. [30] present a framework incorporating risk analysis, stochastic simulation, and multi-criteria optimization modules. Kalhor [16] proposes an integrated framework for wildfire risk mitigation decision making at wildland-urban interfaces. To the best of our knowledge, our approach is the first to use and incorporate the computation of potential fire risk indices into the pre-processing stage.

3 Problem Statement

The main objective is to find the optimum deployment of the resources for extinguishing forest fires. The demand points and possible locations are known. The set of demand points is that of the potential deployment sites. Each demand point has an assigned weight depending on the importance of the zone. The optimal location of the resources is obtained in such a way so as to maximize the weighted sum of demand points given the importance of the zone, under the following requirements:

- A demand point may be covered by more than one resource.
- There are a limited number of resources.

The main goal is to extinguish the forest fire as quickly as possible and thus minimize the potential risk through the development of strategies of early alert for disasters, elaborate and execute development plans to offer resistance to such disasters and to help in the reduction of the fires, given that uncontrolled forest fires have harmed the local landscape and economy.

It is necessary to classify the study area according to its risk to generate key demand points. In order to classify the zones of highest risk or importance, we need to know the risk potential index that makes it possible to determine risk as a function of the existing vegetation and as a function of the topographical relief and the influence exerted by meteorology. The detailed description of the calculation of this index is found in Section 4.2.

3.1 Integer Programming Model

For paper completeness sake, we present the model as introduced by Dimopoulou and Giannikos [7]. The following notation is used for defining parameters and variables.

Sets and indices

I**:** Set of candidate zones for the location of resources

J**:** Set of demand points

K**:** Set of resources

Parameters

$N(j,k)$**:** Set of candidate zones that can cover the demand of point j for resources of type k; $j \in J$, $k \in K$

w_j**:** Weight according to the necessity of protection of point j; $j \in J$

S_k**:** Number of available resources of type k; $k \in K$

$N(j,k)$ is the set of candidate sites that can cover demand point j for type k resources. More specifically, let $T^{\max}(j)$ be the time limit for covering demand point j, based on the convention adopted according to the classification of the zone. The values of $T^{\max}(j)$ for each class are found in Table 3. If we denote by $t(i,j,k)$ the time it takes a type k resource to travel from location i to demand point j, then $N(j,k)$ is given by $N(j,k) = \{i \,|\, t(i,j,k) \leq T^{\max}(j)\}$.

Decision variables

$$x_{ik} = \begin{cases} 1 & \text{resource of type } k \text{ is placed in zone } i;\ i \in I, k \in K \\ 0 & \text{otherwise} \end{cases}$$

$$y_j = \begin{cases} 1 & \text{if the demand of point } j \text{ is covered by at least one resource; } j \in J \\ 0 & \text{otherwise} \end{cases}$$

Model

$$\text{maximize} \quad f(x,y) = \sum_{j \in J} w_j y_j \tag{1}$$

$$\text{subject to:} \quad y_j - \sum_{i \in N(j,k)} \sum_{k \in K} x_{ik} \leq 0 \qquad j \in J \tag{2}$$

$$\sum_{i \in I} x_{ik} = S_k \qquad k \in K \tag{3}$$

$$x_{ik}, y_j \in \{0,1\} \qquad i \in I, j \in J, k \in K \tag{4}$$

The objective function (1) represents the weighted maximization of the number of demand points covered. The different values of w_j express the different needs of protection. Given that some of the demand points cannot be covered due to the limited number of resources, the demand points that require more protection will have greater weight. Constraints (2) specify that a demand point j is covered only if at least one resource is placed at a point that can cover j within the time limit. Constraints (3) guarantee the location of all of the available resources. Finally, Constraints (4) indicate the binary nature of the decision variables.

This linear integer programming model is known to be NP-hard [4]; however, the linear relaxation of the problem is relatively tight. Thus, instances of as many as 11,000 demand points and 22,000 binary variables have been resolved by commercial branch-and-bound methods in quite reasonable times (less than 5 seconds) using a relative optimality gap of 1×10^{-4} as stopping criterion. This gives confidence that the problem is relatively tractable despite its inherent computational complexity.

4 Methodology

4.1 An Integrated Framework

As mentioned before, the solution methodology integrates three important aspects to be discussed in this section. First, the potential risk of fires is taken into account by computing appropriate indices based on geographic and soil properties of the area of interest [29]. This computation is discussed in detail in Section 4.2. Based on these indices, the following step is to classify the areas according to their risks and importance. This classification is needed as input to the optimization model in the following step. The area classification is discussed in Section 4.3. Finally, the third step consists of solving an integer programming model where we must decide where to locate each type's resource to maximize the weighted coverage of the demand points. For this step, we use the model introduced by Dimopoulou and Giannikos [7], and it is described in Section 3.1.

This is a description of the solution methodology.

Step 1 – Image capturing: In this step, the area of interest must be captured or defined for the ensuing steps. This can also be taken from corresponding GIS databases. In the case of Mexico, SEMARNAT (Mexican Secretary for NAtural Resources) has a dedicated web site at `http://infoteca.semarnat.gob.mx/website/geointegrador/mviewer/viewer.htm`.

Step 2 – Map generation: In this step, the goal is to generate the related maps of the study zone. From the same web site at SEMARNAT, one can obtain fuel maps, topograpahic maps, and road maps, with all informtion needed for the study.

Step 3 – Index computation: With the previous maps, we proceed to compute all related indices as described below, and determine the class type for each area in terms of its defense priority index.

Step 4 – Instance generation: Once we have determined the area class type for the entire region, we procede to generate the demand points of interest and the potentical location points (sets I and J in the model, see Section 3.1) along with all required parameters to complete the problem instance to be solved.

Step 5 – Problem optimization: The instance generated in the previous step is fed as input to the optimization solver. In this case the integer programming problem is solved by branch and bound. The output is the values of variables x (where to locate the resources) and y (what demand points are covered), along with the objective function value.

Step 6 –Postprocessing: A graphical display of the solution just found.

Details on the map generation step can be found in [6]. An illustrative example is given in Section 4.4. Next, we describe in detail the key elements of the solution methodolgy.

4.2 Potential Risk of Fires

The potential risk of fires is unknown beforehand but can be calculated based on meteorological parameters and data of the surface of the terrain [34]. This index, known as the fire potential index (FPI), requires for its use a set of inputs, including those relative to the spatial identification of the territory (digital model of the terrain), those corresponding to the identification of the fuel models, which should be charted and digitalized, and finally, information on the variability of the meteorological parameters for the time where the degree of evolution of the risk is of interest.

The FPI is the result of the sum of three components. The information supplied by these components represent the three phases that characterize the organization and consolidation of the combustion in the presence of the forest fire.

The ignition index (I_{ig}) is used to determine the facility presented by the accumulations of fine dead plant material to enter combustion through the application of a heat source, which indicates the higher or lower predisposition presented by the fuels for accepting caloric energy and commencing the reactions of oxidation that determine combustion.

The dynamic behavior index (I_{cd}) is used to evaluate the greater or lesser facility of the fuels affected by the ignition to give continuity to the reactions of oxidation as a function of their own combustibility, of the influence they receive from the slope of the terrain and the wind velocity. It indicates of the materialization of the spatial evolution presented by the active front in the initial stages of the propagation. In this index, the model is supplied with the fuels' intrinsic properties concerning the structure and spatial organization facing the faculties possessed for extending, via energetic transmission, the combustion. With this index's design, the dynamic effect after ignition is incorporated into the model.

The last component of the model, denominated energy behavior sub-index (I_{ce}), incorporates the valuation of the consolidated phase of the combustion. This sub-index incorporates the complete consolidated combustion phase once the fire started and the oxidation phase is completed. In the first version of the model, this sub-index includes in the outcome, the characteristics of the propagation, and the results of the energy release were obtained combined the velocity of propagation, length of the flame, linear intensity of the advance front, and the heat per area unit [34]. In the more advanced model, the energy release component has been modified. The equation of this sub-index is determined by a mathematical harmonization of two fire behavior components, the heat by unit area, and the length of the flame:

$$I_{\text{ce}} = \sum_i \frac{2FL_i HUA_i}{(FL_i + HUA_i)} \times \frac{A_i}{A_{\text{t}}} \tag{5}$$

where FL_i is the assigned weight from the flame length (Table 1), and HUA_i is the assigned weight from the heat per unit area (Table 1), A_i is the area of each fuel model distribution and A_{t} is the size of total study area managed within each cell or pixel. All I_{ce} variables were calculated with the BEHAVE system [29].

For the application of the expressions that provide the quantified values of each of the components, the following criteria are considered in selecting the meteorological values. In wind velocity, the most frequent is taken. For this purpose, a database is required over a long time period. For the thermo-hygrometric values, the hours of highest solar radiation are considered.

The expressions that comprise the FPI I_{fpi} are shown below.

Table 1: Values assigned for determining the dynamic and energy behavior sub-indices. (Source: Rodríguez y Silva et al. [29]).

Rate of spread (m / min)	Flame length (m)	Heat per unit area (kcal / m^2)	Value (assigned weight)
0–10	0–0.5	0–380	1
11–20	0.51–1.0	381–1265	2
21–30	1.10–1.5	1266–1415	3
31–40	1.51–2.0	1416–1610	4
41–50	2.10–2.5	1611–1905	5
51–60	2.51–3.0	1906–2190	6
61–70	3.10–3.5	2191–4500	7
71–80	3.51–4.0	4501–6630	8
81–90	4.10–4.5	6631–8000	8
> 90	> 4.5	> 8001	10

$$I_{\text{fpi}} = I_{\text{ig}} + I_{\text{cd}} + I_{\text{ce}} \tag{6}$$

$$I_{\text{ig}} = \frac{PI_{\text{mi}} CI_{\text{mi}} S_{\text{mi}}}{S_{\text{tc}}} \tag{7}$$

$$I_{\text{cd}} = \frac{CD_{\text{mi}} S_{\text{mi}}}{S_{\text{tc}}} \tag{8}$$

$$I_{\text{ce}} = V_{\text{p}} + A + I_{\text{l}} \tag{9}$$

Equation (7) represents the ignition index (I_{ig}). Equation (8) represents the propagation's dynamic behavior after ignition at the most frequent wind velocity (I_{cd}). Equation (9) represents the energy behavior index (I_{ce}). It is obtained from the sum of the weights assigned to each interval in which the values have been divided after resolving fire behavior equations. V_{p} represents the velocity of the flame front propagation (m/min.) A is the flame length (m). I_{l} is the linear intensity of the advance flame front (Kcal/m/s). These indices depend on the following parameters:

PI_{mi} = The probability of ignition of fuel model mi, at the hour of the day of highest solar radiation.

S_{mi} = Surface occupied by model mi in the area of analysis.

S_{tc} = Total surface of the area of analysis.

CI_{mi} = Coefficient of ignition characteristic of model mi.

CD_{mi} = Assigned weight according to the table for propagation velocity, with most frequent wind velocity and maximum slope of model mi.

Again, all these parameters were computed with the BEHAVE system [29].

Supression Difficulty Index

To correctly plan preventive action in each zone, it is necessary to complement the previous indices by means of an index that evaluate the difficulty of fire supression. This fire supression difficulty index I_{dex} is computed as follows:

$$I_{\text{dex}} = \frac{I_{\text{ce}}}{I_{\text{mi}} + I_{\text{ald}}} \tag{10}$$

where I_{mi} represents the mobility index and I_{ald} denotes the defense opening index. This index is determined by the following equation:

$$I_{\text{ald}} = T_{\text{rh}} C_{\text{p}} \tag{11}$$

with T_{rh} representing the weight assigned to the defense opening line rate according to the fuel models by teams of supression specialists and C_{p} denoting the adjustment coefficient according to the existing slope in each of the areas used by the fuel models.

Again, all these parameters were computed with the BEHAVE system [29].

Defense Priority Index

Finally, once the potential fire risk and supression difficulty indices are computed, we are in a position to compute the defense priority index I_{pd} which represents the importance of each zone. This index is computed as a combination of the previous indices as follows:

$$I_{\text{pd}} = (0.45) I_{\text{fpi}} + (0.55) I_{\text{dex}} \tag{12}$$

To identify the danger as a function of this index, Table 2 is used. The first column shows the values of index I_{pd} and the second column indicates the danger class classification.

Table 2: Classification of the defense priority index.

I_{pd}	Danger class
0.0 − 0.7	1 (Low risk)
0.8 − 1.5	2 (Medium risk)
1.6 − ∞	3 (High risk)

This classification table is used to give each zone a class type and feed the optimization model. The computation of this inde for each zone is required.

4.3 Area Classification

To clearly define sets I and J from the MILP model (1)-(4), it is necessary to know to know the study area's classification. The risk potential index (defined in the previous section) allows us to define the zone's importance. There are three classes: low, medium, and high risk. For each class, there is a distance threshold (or maximum distance) between every pair of demand points must satisfy. Naturally, for class 1 (high risk) the distance between the demand points is shorter compared to class 2 and 3. Also, each class has a time limit for resource arrival.

Table 3 shows the time limit in which the vehicle of the brigades should arrive to extinguish the fire depending on each class. For example, $T^{\max}(j) = 25$ for any point j contained in Class 1.

Table 3: Time limit of classes $T^{\max}(j)$.

Class	Time limit (min)
1	25
2	50
3	80

Apart from the time limit, the distance depends on the quality of the road and the average speed of the vehicle. Consequently for area 1 which is the most important and of the shortest time, the distance between its demand points is shorter than in class 2 and 3. Table 4 shows the distance between the demand points of each class [34].

Table 4: Maximum distance of the demand points.

Class	Distance (km)
1	15
2	30
3	45

The importance of the zone is classified by indices, which depend on the characteristics of the study area. To make the classification of the area, certain calculations are required. The risk indices are in function of the existing vegetation and in function of the relief and of the influence that meteorology exerts on the zone.

4.4 Example of Application of Methodology

We now provide an example on how the methodology is applied to a region in the state of Chiapas, Mexico. First, all geographic information from the site of interest is captured in Steps 1 and 2 of the integrated framework from Section 4.1. Then, the computation in Step 3 is carried out as follows. First, potential fire risk indices are computed (according to Table 2, Section 4.2), obtaining a risk map as shown in Figure 1. In this map, the priority index 1 (low risk), 2 (medium risk), and 3 (high risk), are indicated by color blue, yellow, and red, respectively. Then, the area is classified according to its importance as explained in Section 4.3. A grid is built obtaining a map shown in Figure 2.

The information from Step 3 is used in Step 4, to generate the demand points accordingly. We

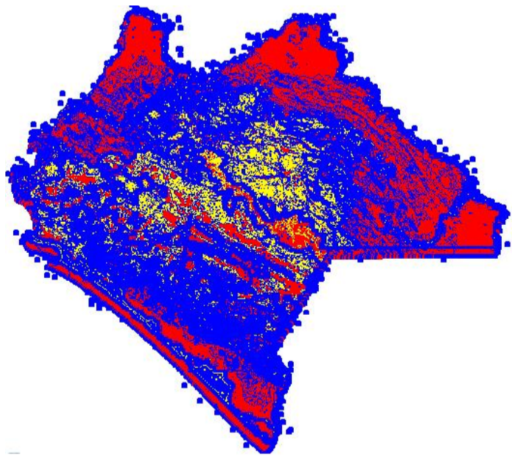

Figure 1: Fire risk map.

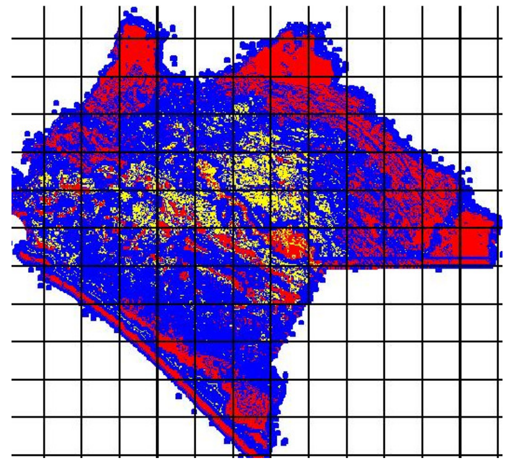

Figure 2: Map grid.

obtain the map shown in Figure 3. In this figure, the blue points represent demand points, that is, set J from model (1)–(4).

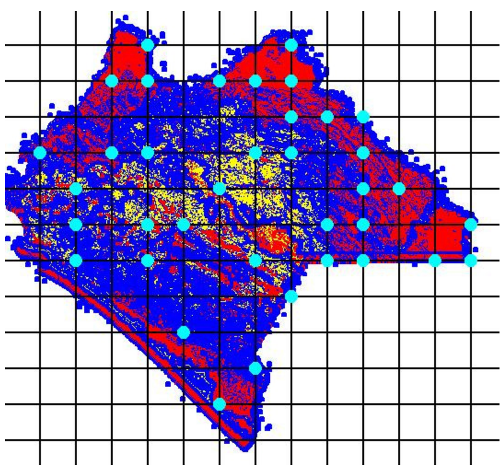

Figure 3: Location of demand points.

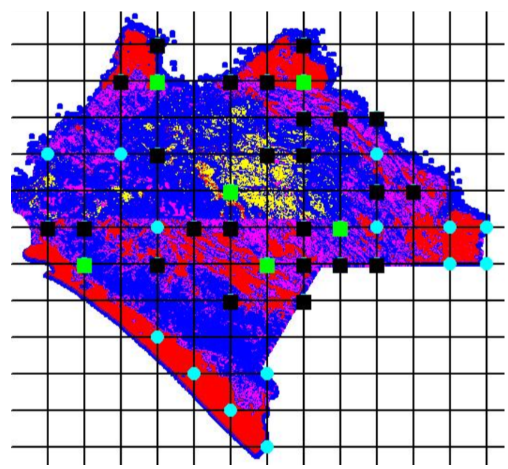

Figure 4: Optimal solution.

Once the parameters are set and the demand points are generated, we proceed to optimize model (1)–(4) in Step 5. A solution looks like the one in Figure 4.

The green square points indicate the optimal location of the resources. The black square points indicate the demand points that are covered under the optimal decision. The blue points indicate the demand points that are not covered under this decision.

5 Case Study

The presented methodology was applied to a case study in the state of Chiapas, Mexico. The state of Chiapas is located in the southeast of the Mexican Republic, occupying a surface of 73,211 km^2.

Geographically it is located between 17°59' and 14°32' latitude north, and between 90°22' and 94°14 longitude west.

The set of tests that are described below have the principal objective of evaluating the solutions obtained with the optimization model for the different test scenarios. In addition, they are designed to evaluate the performance of the branch and bound method in the resolution of some instances of the model.

The solution method used for the resolution of this problem is branch and bound (BAB), for each of the test scenarios. Table 5 shows the results obtained. In the first columns (1-4) the number of scenario is indicated along with the different parameters of the meteorological conditions. The different variations are: temperature (T), humidity (H), wind velocity (V) and where b is low and a is high. For temperature (T), we consider low if it is 0-5°C and high from 35-40°C. Humidity is low in 0-10% and high in 80-100%. Wind velocity is low when it is 0-4 km/h and high if it is 20-24 km/h.

Table 5: Percentage of covered demand points, 7 brigades.

Instance	T	H	V	Demand points	Covered points	% covered	O.F. Value
1	b	b	b	28	18	64.29	23
2	b	b	a	41	30	73.17	41
3	b	a	b	28	22	78.57	27
4	b	a	a	33	31	93.94	35
5	a	b	b	28	22	78.57	28
6	a	b	a	41	30	73.17	39
7	a	a	b	26	22	84.62	29
8	a	a	a	34	27	79.41	34

The fifth column shows the number of demand points generated in the pre-process. As is described in the pre-processing, scenarios 2 and 6 generate a greater number of demand points due to the fact that conditions are unfavorable, that is, there are zones of higher risk. The last three columns show the results obtained after resolving the optimization model. The sixth and seventh columns show the number of demand points covered and the associated percentage of each one of the resolved instances, respectively. As can be observed, the highest percentage of points covered is for scenario 4. It can be observed that a higher number of demand points does not necessarily imply a higher percentage of points covered. The eighth column shows the value of the objective function of the associated problem of maximization.

Analyzing the values of the objective function, we make the following observations: the highest value is that of instance 2, however, it does not coincide with that of highest percentage of points covered. This is due to the fact that demand points have been covered that required greater protection. Although the number of points covered is not the highest, it is highest in the value of the objective function.

Another study has been made to see the effect the increment of the number of brigades has on the number of demand points covered and value of the objective function. The instances are generated based on scenario 3, where the meteorological conditions are: high temperature, high humidity, low wind velocity. A set of demand points of size 28 was created and tested for 5, 7, 9 and 11 brigades.

Table 6: Performance of instance 3 with different number of brigades.

Instance	Brigade No.	Demand points	Covered points	% Covered	O.F. Value
3A	5	28	19	67.86	25
3B	7	28	24	85.71	30
3C	9	28	26	92.86	32
3D	11	28	28	100	34

Table 6 shows the results of this study where in the first column we have the name of the instance and in the second the number of brigades. The third column shows the number of demand points which is the same for all, given that it is the same scenario, the next column presents the points covered. The fifth and sixth columns indicate the number of demand points covered and the value of the objective function, respectively. As can be observed, the greater the number of brigades available, the higher the number in the objective function and the higher percentage of points covered, naturally. For example, if the number of brigades is increased from 5 to 9, the percentage of points covered increases by 25 % and the value of the objective function is increased to 7. It can be observed that the increment from 5 to 7 brigades increases the objective function in 5 units, but from 7 to 9 the increment is only of 2 units. That is, the increase of 5 to 7 is more critical than of 7 to 9 brigades. Figure 5(a) shows the value of the objective function, depending on the number of brigades deployed for scenario 3.

Table 7: Performance of instance 2 with a different number of brigades.

Instance	Brigade No.	Demand points	Covered points	% Covered	O.F. Value
2A	5	41	28	68.29	35
2B	7	41	32	78.04	39
2C	9	41	34	82.92	41
2D	11	41	36	87.80	43
2E	13	41	38	92.62	45
2F	15	41	40	97.56	47
2G	16	41	41	100.00	48

The same study is carried out for scenario 2, where the meteorological conditions are: low temperature, low humidity, high wind velocity, months of May-June, at 13:00 hrs. Table 7 shows the results of the instances generated from scenario 2. For example, if the number of brigades is increased from 5 to 9, the percentage of covered points is incremented by 14.34 % and the value of the objective function is increased by 6. Figure 5(b) specifies the value of the objective function as a function of the number of brigades deployed for scenario 2. An increment of 10 % can be seen of points covered with the increase of 5 to 7 brigades and an increase of the objective function of 4 units. For the case of the increase of 7 to 9 brigades, the growth both in the objective function and in the percentage of points covered is lower. In Figure 5(c) we observe the percentage of non-covered points for scenario 2, depending on the number of brigades that are deployed in the state of Chiapas. For the most extreme conditions 16 brigades are required to cover the entire state, and guarantee an effective initial attack.

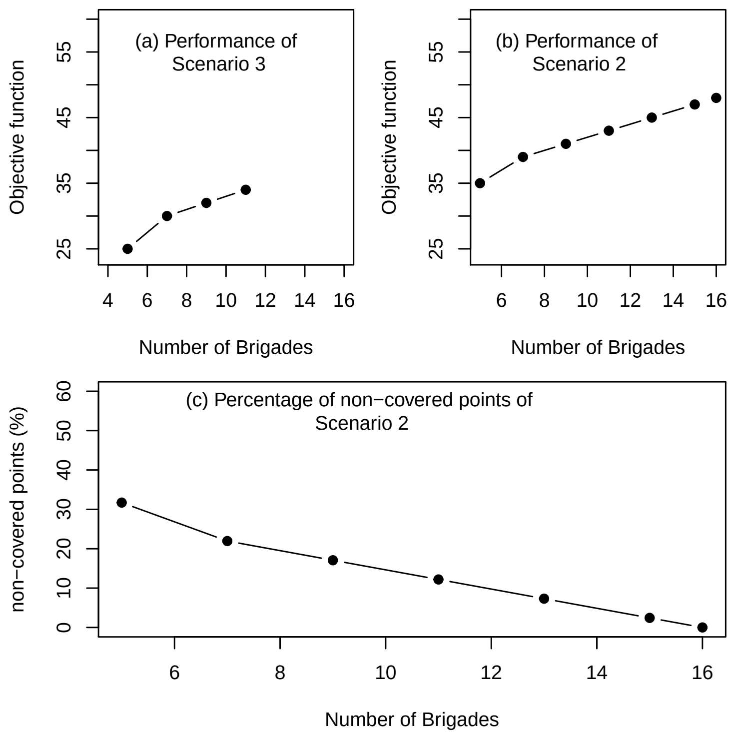

Figure 5: (a) Variation of the objective function for scenario 3; (b) Variation of the objective function for scenario 2; (c) Comparison of non-covered points and number of brigades deployed in scenario 2 .

In the instances that were resolved, the computational time for the solution of the model was from 0.2 to 3 seconds. Instances of as many as 11,000 demand points were resolved, with 21,500 binary variables and the computational time was no more than 3 seconds. The time guarantees us that large instances of this problem can be resolved and solutions can be obtained in a very reasonable time.

6 Conclusions

The present study focuses on a real-world problem arising from forest fires in Mexico, whose principal problem is determining where to place fire fighting brigades. The realization of this research includes the detailed study of forest fires, the factors that affect these fires and their behavior, and creating a tool that aids in this decision-making process. The problem of controlling forest fires is of interest throughout the world. In particular, to the best of our knowledge, Mexico does not have a fire control tool like the one proposed in this work. Also, no previous work concerning optimization models for fire management in Mexican forests exists.

The optimization problem was modeled as a mixed-integer linear programming model based on a model from the literature. It should be mentioned that suitable adjustments were made to make use of the model. The MILP model was solved by a state-of-the-art commercial branch-and-bound (BAB) solver. The computational study of the problem was divided basically into two parts. First, calculations were made of the risk indices for the creation of maps of forest fire risk. Second, the solutions to the instances for different test scenarios were analyzed.

The computational study of the problem was divided basically into two parts. First, calculations were made of the risk indices for the creation of maps of forest fire risk. Second, the solutions to the instances for different test scenarios were analyzed.

This research's principal contribution is the development and computational implementation of a tool that integrates the calculation of different important indices (more details of this calculation can be found in Díaz Romero [6]) for the fire behavior and the solution to a problem of deploying brigades. As far as we know, this is the first work that presents an integrated calculation of these indices and the optimization that determines the best possible deployment of resources for the study of forest fires in Mexico. The tool provides a base for making decisions on the deployment of brigades. It should be mentioned that the tool not only provides the solution to the decision-making problem but also calculates and provides forest fire risk maps as a function of different meteorological conditions. These maps can be useful for the creation of prevention programs in high-risk zones.

The methodology has been applied to a case study in the state of Chiapas. The area was classified according to the fuel models of Rothermel. A calculation was made of the different indices of fire behavior. Risk maps were generated for different meteorological conditions. A study was made of the objective function's behavior under a different number of brigades and the percentage of demand points covered.

For the cases tested in the paper, the BAB method could find optimal solutions in all cases. The approximate number of binary variables fluctuated between 80-–200. Besides, a larger case with over 22,000 binary variables was generated and resolved. However, because this is an NP-hard problem, there is no guarantee that an optimal solution can be found for larger instances. Even by incrementing the number of resources, the combinatorial aspect of the problem makes it more challenging. Therefore, an extension of the present work would be the development of heuristics for finding feasible solutions in a reasonable time for handling larger instances.

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