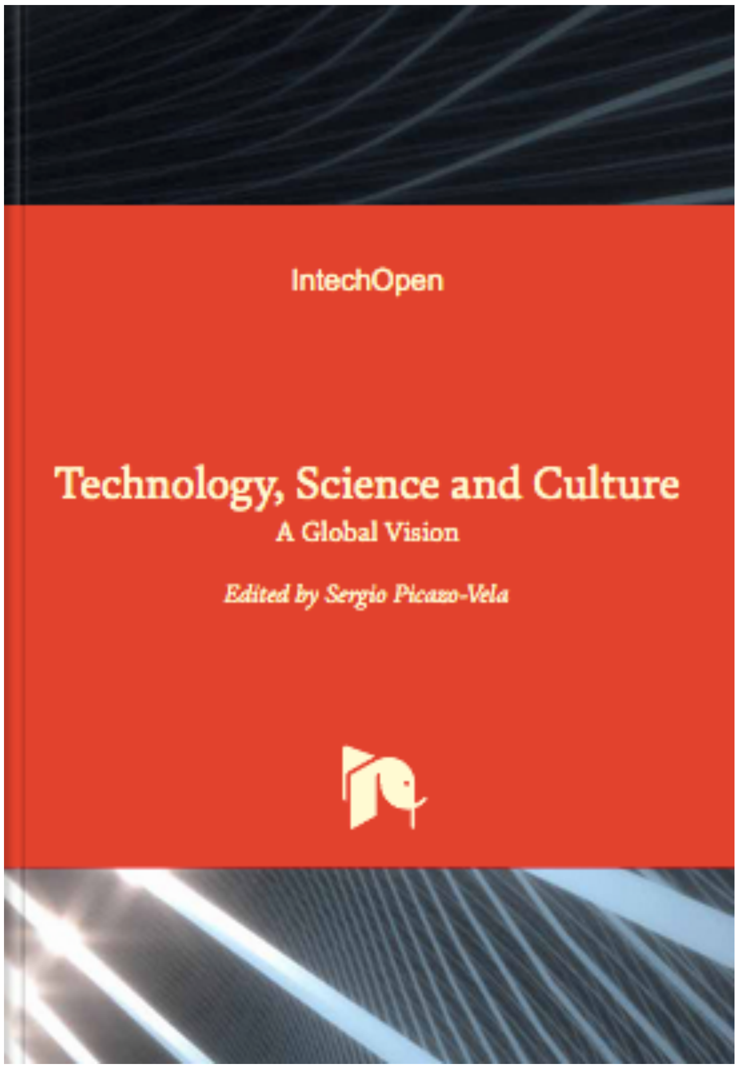
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# Contents

Chapter

# Comparison of Dispersion Measures for a Territory Design Problem

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## Abstract

Territory design consists of dividing a geographic area into territories according to certain planning criteria. In most applications, it is desired that the resulting territories are balanced, connected, and compact. We analyze different ways of measuring dispersion, each cast in a particular mixed-integer linear programming model. One is based on p-centers and the other on p-medians. The experimental work includes a comparison between these two models in terms of robustness.

**Keywords:** territory design, p-center, p-median, dispersion measures, integer programming

## 1. Introduction

Territory design deals with the discrete assignment of basic units (BUs) (such as zip-code areas, blocks, etc.) into clusters with restrictions defined by planning criteria. The need for a territory design plan is present in several social planning contexts such as political districting and commercial territory design. The motivation to divide a geographical area is related to the fact that smaller areas are easier to manage. In practice, a territory design requires a lot of time and effort and, in most cases, needs to be performed recurrently as the environment evolves and new needs develop. Thus, it is desired to have an automatic or computational method to support this decision-making process.

Mathematical formulations of territory design problems have been developed since 1965. As noted in Kalcsics et al. [1], most of the research related to territory design problems is tied to specific applications and thus have specific planning criteria accordingly. However, three types of requirements can be identified in most territory design applications: balance, connectivity, and compactness. Balance refers to having territories of the same size, and a size is defined upon an activity measure related to the problem application. For example, the size of the territory may be defined as the number of inhabitants in an area or the amount of workload involved. Connectivity constraints require all the BUs of a territory to be connected. Compactness is another spatial characteristic that is rather vaguely defined as having the BUs in a territory as close together as possible.

Compactness is crucial in most applications of territory design because this leads to shorter (more inexpensive) routes when distributing product or visiting the BUs later. Having territories that are as independent as possible is the aim of most applications of territory design. Despite the importance of the compactness constraint, in the literature, there is no consensus regarding the best practice for measuring it.

Compactness measures used in territory design models can be categorized into two main classes: one that considers the geometrical shapes of territories and another one which is concerned with the distances between BUs within a territory. The first type of measures compares spatial characteristics of the shape of territories with regular geometric shapes such as circles or convex polyhedra. On the other hand, distance-based measures use diverse dispersion measures to describe how close together BUs within a territory are. Bear in mind that maximizing compactness is equivalent to minimize dispersion. These types of compactness measures are often used as objective functions in integer programming minimization problems.

1. Geometric measures of compactness

The compactness measure used in Kalcsics et al. [1] and Butsch [2] is based on the geometrical shape of districts. Both divisional methods take into consideration the convex hulls of districts to decide the placement of the bisecting division in each iteration. Thus, the solutions obtained by their methodology are inherently compact.

2. Distance-based measures of compactness

This type of measures describes how close together the BUs within a territory are. Diverse measures of dispersion are used for this purpose since maximizing compactness is equivalent to minimizing dispersion. For instance, dispersion could be measured by the maximum distance between two nodes in a district as in Ríos-Mercado and Salazar-Acosta [3] and in Gliesch et al. [4]. If district centers are defined in a model, they are useful in dispersion metrics that use the aggregation of the distances from BUs to their assigned territory center. A first method to aggregate distances may be done by a simple sum [5–7] or a weighted sum [1] with weights corresponding to activity measures. A second method of aggregation is done by a metric called moment of inertia which is basically the sum of the squared values of distances [8].

In this short chapter, we present a comparison between two distance-based measures of dispersion that involve the definition of a territory center. These metrics are the p-center measure and the p-median measure. In the following section, we describe the models and the methodology used for this comparison. Next, we present some empirical results and finally the conclusions.

## 2. Mathematical models

The aim of this chapter is to compare the performance of the p-center and the p-median metrics for dispersion when used as the objective function of models for a territory design problem. The models used for this analysis were introduced by Ríos-Mercado and Fernández [9] and by Salazar-Aguilar et al. [5]. We consider a version of these models without connectivity constraints.

Below we show the mathematical formulation of the p-center-based model (PC). Let $V$ be the set of BUs that represent the geographical area of study. In addition, let $A$ be the set of activity measures that are considered for the balance constraints of the practical problem.

For all the BUs in $V$, the following parameters are defined:

- $d_{ij}$: Euclidean distance between the BUs $i$ and $j$.
- $w_i^{(a)}$: value of activity measure $a$ in $A$ that corresponds to BU $i$.

In addition, let $x_{ij}$ be the binary decision variable that indicates if BU $i$ belongs to the territory centered at BU $j$ $\left(x||_{ij} = 1\right)$ or not $\left(x||_{ij} = 0\right)$. Clearly $x_{ii} = 1$ indicates that BU $i$ is a territory center.

Objective function (3) minimizes the p-center dispersion measure. This measure is defined as the maximum distance between any basic unit in a territory and its center. Equation (4) represents the constraint of having only a predefined number of territory centers $p$. Constraints (5) ensure the unique assignment of each BU to exactly one territory. The balance constraints are defined by (6) and (7) in which the lower bounds ($LB(a)$) and the upper bounds ($UB(a)$) are established to the resulting sizes of district according to each activity measure $a$ as follows:

$$LB(a) = (1 - \tau) \sum_{i \in V} w_i^{(a)} \tag{1}$$

$$UB(a) = (1 + \tau) \sum_{i \in V} w_i^{(a)} \tag{2}$$

where $\tau$ is a tolerance parameter that in this case has a value of 0.05.

$$\min \text{z} = \max_{i,j \in V} \left\{ d_{ij} x_{ij} \right\} \tag{3}$$

Subject to:

$$\sum_{i \in V} x_{ii} = p \tag{4}$$

$$\sum_{j \in V} x_{ij} = 1 \; \forall i \in V \tag{5}$$

$$\sum_{i \in V} w_i^{(a)} x_{ij} \geq (1 - \tau(a))\mu(a) x_{jj} \; \forall j \in V \tag{6}$$

$$\sum_{i \in V} w_i^{(a)} x_{ij} \leq (1 + \tau(a))\mu(a) x_{jj} \; \forall j \in V \tag{7}$$

$$x_{ij} \in \{0, 1\} \; \forall i, j \in V \tag{8}$$

The only difference between this model and the p-median-based model (PM) is the objective function which is defined in (9). The p-median-based objective is to minimize the total distance between each BU in a territory and its center.

$$\min \text{z} = \sum_{i,j \in V} d_{ij} x_{ij} \tag{9}$$

## 3. Experimental work

This section describes the methodology implemented to compare both dispersion methods. We tested 100 artificially generated instances based on real-world data from a commercial territory design problem of different sizes. For each instance, we obtained the optimal solutions for both the PC and the PM models using the CPLEX solver. For each instance, the optimal solution (the values of the decision variables $x_{ij}$) of the p-center problem ZC* was evaluated with the objective function of the p-median problem. The resulting value PM(ZC*) was compared with the optimal value of the p-median problem PM(ZM*) as follows:

$$RD_{PM} = \frac{PM(ZC) - PM(ZM)}{PM(ZM)} \tag{10}$$

In contrast, for each instance, the optimal solution of the p-median problem *ZM* was evaluated with the objective function of the p-center problem resulting in PC(ZM*). Accordingly, this value was compared with the optimal value of the p-center problem PC(ZC*) as follows:

$$RD_{PC} = \frac{PC(ZM) - PC(ZC)}{PC(ZC)} \tag{11}$$

We name these measures as "relative differences" for each dispersion measure ($RD_{PM}$ for the p-median and $RD_{PC}$ for the p-center measures). Relative differences describe how far the solution of one model is from the other under each dispersion measure.

The instances tested had two activity measures and five districts to be formed and ranged in sizes of 60, 80, 100, 120, and 150 BUs. We tested 20 instances of each size. The results are shown in the following figure (**Figure 1**). The horizontal axis shows the test instance numbers. Test instances are numbered with respect to their sizes. That is, instance numbers 1–20 correspond to the test instances with 60 BUs, instance numbers 21–40 correspond to the test instances with 80 BUs, and so on. In green, we show $RD_{PM}$, and in blue, we show $RD_{PC}$.

As can be seen from the figure, relative differences $RD_{PM}$ (green) are generally better (closer to zero) than $RD_{PC}$ (blue). (Only on five instances, the $RD_{PC}$ (blue) values were very close to zero.) What this means is that *ZC*, the optimal solution from the PC model, is generally closer to the optimal solution to the PM model.

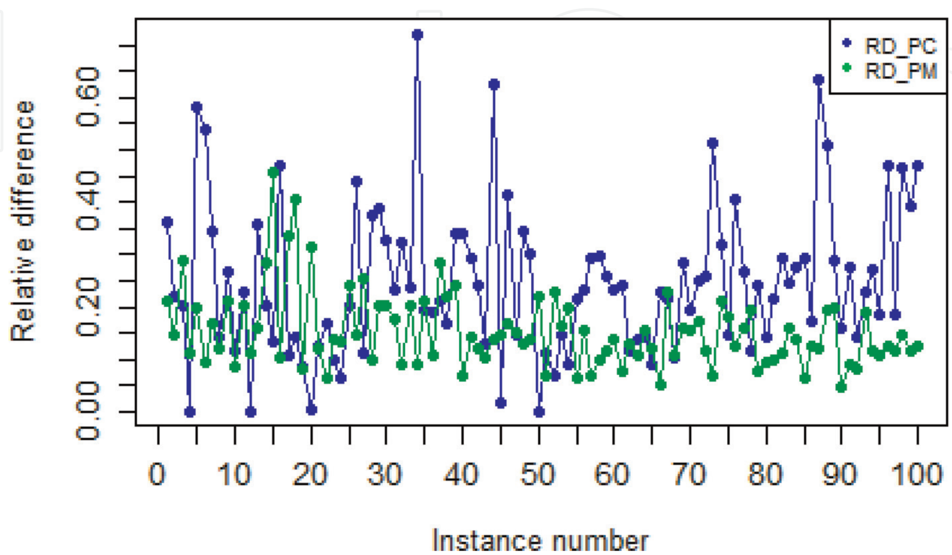


**Figure 1.**
*Relative difference in dispersion metrics.*

Therefore, by using ZP*, we are better off under both models than using ZM*. We conclude then that model PC is more robust than model PM.

## 4. Conclusion

In this chapter, we have shown a comparison of two models for a territory design problem with different dispersion measures for dispersion: the p-center and the p-median. Both models were tested with 100 artificially generated instances, and the optimal solutions obtained were evaluated with the corresponding dispersion measure to compare both models. The optimal solution of the PM model ZM* was evaluated with the objective function of the PC model. Its value was compared with the optimal solution of the PC model with the defined measure $RD_{PC}$. The same was done for the optimal solution of the PC model ZP*, and the results were compared with $RD_{PM}$. The relative differences $RD_{PC}$ were lower than $RD_{PM}$ for most instances. A relative difference closer to zero means that the assignment of BUs to territory centers obtained from the optimal solution of one model gave a dispersion value that is very close to that of the optimal solution of the other model. These results show that the PC model is more robust compared to the PM model. Future work will consider the comparison of models with other dispersion measures which are not center-based since in practice the definition of a territory center has no practical meaning.

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