RED-M 2007, Culiacán, Sinaloa, México, 5-8 de noviembre de 2007

A METAHEURISTIC ALGORITHM FOR A BI-OBJECTIVE SUPPLY CHAIN DESIGN PROBLEM [1]

Elias Olivares Benítez, José Luis González Velarde
Tecnológico de Monterrey, Campus Monterrey
Monterrey, Nuevo León, México
elias.olivares@itesm.mx, gonzalez.velarde@itesm.mx

Roger Z. Ríos Mercado
Universidad Autónoma de Nuevo León
San Nicolás de los Garza, Nuevo León, México
roger@mail.uanl.mx

Keywords: supply chain design, facility location, multiobjective optimization, transportation channel, metaheuristics

Abstract: *A supply chain design problem based on a two-echelon single-product system is addressed. The product is distributed from plants to distribution centers and then to customers. There are several transportation channels available for each pair of facilities between echelons. These transportation channels introduce a cost-time tradeoff in the problem to formulate a bi-objective mixed-integer programming model. The decisions to be taken are the location of the distribution centers, the selection of the transportation channels and the flow between facilities. Since this problem is NP-hard a metaheuristic algorithm was developed to solve it. The metaheuristic algorithm is composed of three methods. The constructive method generates solutions using a random strategy for opening distribution centers and a weighted greedy function to select the transportation channel. The improvement method uses local search with a dominance strategy. The combination method is used for post-processing and is based on a Path Relinking scheme. The proposed algorithm was compared with a previously developed ε-constraint based algorithm and over instances of different size. For the smallest instances, as expected, the reference algorithm was more efficient in terms of computing time and solution quality. However, for the largest instances with similar run times the metaheuristic algorithm achieved better results.*

1. Introduction

The problem of supply chain design is one of the components of Supply Chain Management (Chopra and Meindl, 2001). At the strategic level the managers must design the supply chain to operate with the minimum cost and to meet a level of customer service. The supply chain, also known as distribution network, is composed of facilities and transportation flows between facilities. These facilities perform different roles as suppliers, plants, warehouses, distribution centers and retailers. Some of the decisions implied in supply chain design are to determine the number, location and capacities of the facilities, to allocate products to facilities and to determine the flow of products between facilities (Simchi-Levi, Kaminsky and Simchi-Levi, 2000). Many models developed to design distribution systems are based on discrete location of facilities where a set of potential sites is known. The earliest models of this type were formulated by Baumol and Wolfe (1958), and Kuehn and Hamburger (1963). These and subsequent models have been formulated as mixed-integer programming problems that consider elements like the number of echelons, facility capacity, number of products, time periods, stochastic demand and side

[1] This research was supported by grant CAT-025 from Tecnologico de Monterrey.

constraints to include single or multiple sourcing and routing (Klose and Drexel, 2005). An element that has been considered scarcely in these models is the transportation channel used between facilities. The availability of different channels to transport the product between a pair of facilities is a feature of modern logistic services. These transportation channels can be seen as transportation modes (rail, truck, ship, airplane, etc.), shipping services (express, normal, overnight, etc.) or just as simple as the offer of transportation services from different companies. Transportation channels can be differentiated by parameters of time and cost. In general these parameters are negatively correlated with shorter times for the most expensive alternatives. This feature induces naturally to re-formulate the supply chain design problem as a bi-objective optimization model. Looking at the review by Current, Min and Schilling (1990) it is evident that the balance of these measures has not been studied extensively.

In this paper we present a metaheuristic algorithm to solve a supply chain design problem for a two-echelon distribution system. The problem has been introduced by Olivares-Benitez (2007) and was named "Capacitated Fixed Cost Facility Location Problem with Transportation Choices" (CFCLP-TC). Section 2 shows the problem description. The mathematical model and notation are explained in Section 3 with a remark about the computational complexity of the problem. Section 4 is dedicated to describe the algorithms developed to obtain the approximate set of efficient solutions for an instance. Section 5 shows some results of the computational experience with the algorithms described. The final conclusions are presented in Section 6.

2. Problem Description

The "Capacitated Fixed Cost Facility Location Problem with Transportation Choices" (CFCLP-TC) is based in a two-echelon system for the distribution of one product in a single time period. In the first echelon the manufacturing plants send product to distribution centers. The second echelon corresponds to the flow of product from the distribution centers to the customers. The number and location of plants and customers are known. There is a set of potential locations to open distribution centers. The number of open distribution centers is not defined a priori. Each candidate site has a fixed cost for opening a facility. The plants and potential locations for the distribution centers have a limited capacity. A supply constraint states that each customer is supplied at most by one distribution center. There are several alternatives to transport the product from one facility to the other in each echelon of the network. Each option represents a type of service with associated cost and time parameters. A scheme of the distribution network is shown in Figure 1.

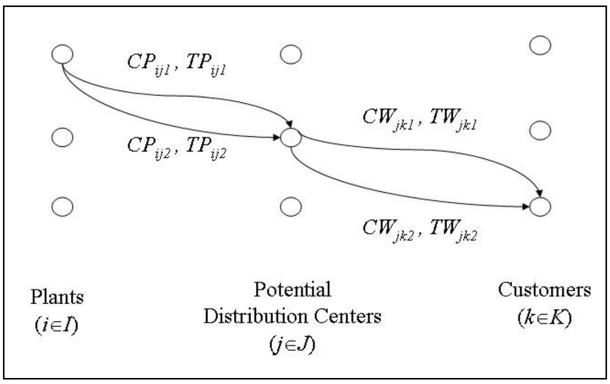

Figure 1 Capacitated Fixed Cost Facility Location Problem with Transportation Choices

The idea of this problem is to select the appropriate sites to open distribution centers and the transportation channels to be used, and the flow between facilities to minimize two objective functions simultaneously. The cost function combines the cost of transportation and the cost of facility opening. The time function considers the maximum transportation time along any path from the plants to the customers.

3. Model and Notation

The CFCLP-TC problem described previously is represented as a bi-objective mixed-integer programming model. Let I be the set of plants, J the set of potential distribution centers, K the set of customers, LP_{ij} the set of arcs between nodes i and j, and LW_{jk} the set of arcs between nodes j and k, for i in I, j in J, k in K. The parameters of the model are CP_{ijl} the cost of transporting one unit of product from plant i to distribution center j using arc ijl, CW_{jkl} the cost of sending one unit of product from distribution center j to customer k using arc jkl, TP_{ijl} the time for transporting any quantity of product from plant i to distribution center j using arc ijl, TW_{jkl} the time for transporting any quantity of product from distribution center j to customer k using arc jkl, MP_i the capacity of plant i, MW_j the capacity of distribution center j, D_k the demand of customer k, and F_j the fixed cost for opening distribution center j.

The decision variables of the model are X_{ijl} the quantity transported from plant i to distribution center j using arc ijl, Y_{jkl} the quantity transported from distribution center j to customer k using arc jkl, Z_j a binary variable equal to 1 if distribution center j is open and equal to 0 otherwise, A_{ijl} a binary variable equal to 1 if arc ijl is used to transport product from plant i to distribution center j and equal to 0 otherwise, and B_{jkl} a binary variable equal to 1 if arc jkl is used to transport product from distribution center j to customer k and equal to 0 otherwise.

The mixed-integer programming model has two objective functions:

$$\min(f_1, f_2)$$

$$f_1 = \sum_{i\in I}\sum_{j\in J}\sum_{l\in LP_{ij}} CP_{ijl} X_{ijl} + \sum_{j\in J}\sum_{k\in K}\sum_{l\in LW_{jk}} CW_{jkl} Y_{jkl} + \sum_{j\in J} F_j Z_j \tag{1}$$

$$f_2 = T \tag{2}$$

In this formulation, objective function (1) minimizes the sum of the transportation cost and the cost for opening distribution centers. Objective function (2) minimizes the sum of the maximum transportation time from the plants to the customers through each distribution center. This function was reformulated from equation (3) adding some constraints to eliminate the non-linearity:

$$f_2 = \min\left(\max_j \left(\max_{i,l}\left(TP_{ijl} A_{ijl}\right) + \max_{k,l}\left(TW_{jkl} B_{jkl}\right)\right)\right) \tag{3}$$

Due to space limitations the rest of the model is not included and the reader is invited to see the work of Olivares-Benitez (2007) for additional details. One of the sub-structures involved in the CFCLP-TC is the classical discrete facility location problem. Through several relaxations of Model 1 the structure of the uncapacitated facility location problem (UFLP) can be obtained. Cornuejols, Nemhauser and Wolsey (1990) have shown that the UFLP is NP-Hard. Therefore, the CFCLP-TC inherits that complexity to be considered NP-Hard too.

4. Solution Approach

Two algorithms were developed to solve the CFCLP-TC. The first algorithm is based on the ε-constraint method (Steuer, 1989; Ehrgott, 2005). The second is a population-based metaheuristic based on some principles of Scatter Search and GRASP. In addition, the proposed metaheuristic benefits from some ideas of the scheme proposed by González Velarde and Martí (2006). In both cases the idea is to generate the set of approximate efficient solutions for this problem.

The ε-constraint based algorithm uses objective f_1 as the main objective function and f_2 is transformed to a constraint with changing values ε^t for its right hand side. In each iteration t, a mixed-integer program (MIP) has to be solved. Because of the complexity of the problem related, a time limit of 3600 seconds was imposed to solve each MIP. The efficient frontiers obtained with this algorithm were used for comparison with the metaheuristic algorithm. This method, particularly intended for relatively small instances, is fully described by Olivares-Benitez (2007). In the present work, we propose a metaheuristic approach for handling larger instances.

4.1 Metaheuristic Algorithm

The metaheuristic algorithm is composed of three main methods. These are a constructive method, an improvement method, and a combination method. However, these methods use a basic procedure to construct a solution. This procedure is based on a decomposition of the problem. The solution is constructed hierarchically starting with the selection of the distribution centers to be opened. Each method uses a specific strategy to perform this selection. The next decision is the selection of the transportation channel between each pair of facilities. The selection of the transportation channel is done using a weighted greedy function. This greedy function has a component based on the transportation cost and other component based on the transportation time as shown in equations (4) and (5):

$$\phi\left(arc_{ijl}\right) = \lambda_c \frac{CP_{ijl}}{\max\limits_{i\in I, j\in J, l\in LP_{ij}}\left(CP_{ijl}\right)} + \lambda_t \frac{TP_{ijl}}{\max\limits_{i\in I, j\in J, l\in LP_{ij}}\left(TP_{ijl}\right)} \tag{4}$$

$$\phi\left(arc_{jkl}\right) = \lambda_c \frac{CW_{jkl}}{\max\limits_{j\in J, k\in K, l\in LW_{jk}}\left(CW_{jkl}\right)} + \lambda_t \frac{TW_{ijl}}{\max\limits_{j\in J, k\in K, l\in LW_{jk}}\left(TW_{jkl}\right)} \tag{5}$$

The weights are generated systematically in the constructive method and inherited through the rest of the algorithm. The transportation channel with the best value is selected. At this point the CFCLP-TC can be decomposed by echelon. First, the flow of product from distribution centers to the customers can be obtained solving a generalized assignment problem (GAP). Second, the flow of product from the plants to the distribution centers is obtained solving a transportation problem (TP). This basic procedure is called to construct a solution in each method.

The constructive method generates a number of solutions. The selection of the distribution centers to be opened is done randomly. The weights for the greedy functions λ_c and λ_t are generated systematically in a linear combination considering the number of solutions to be generated. These weights are used to select the transportation channel and the values are inherited through the rest of the algorithm. The improvement method uses local search and explores three types of neighborhoods. These correspond to movements of opening, closing and exchange of distribution centers. To accept one movement the dominance of the new solution is considered. If an infeasible or dominated solution is created by the movement, it is rejected. After a number of iterations applying the constructive and improvement methods, the combination method is used as a post-processing stage. It is based on Path Relinking to obtain a set of solutions for each pair of solutions from a reference set. The combination makes movements in the vector of values of the distribution centers. Again here the dominance of the new solutions is used to accept these movements.

A strategy of elitism is used to avoid losing solutions after each method and therefore converging toward the true efficient set. In the constructive and improvement methods, the solutions generated are used to update the approximate efficient set *NDS* using the dominance relation of the new solutions to those already existing in this set. From each method a reference set *RS* is constructed combining the solutions in the updated set *NDS* and the “diverse” solutions obtained from the method. In the post-processing stage the last set *RS* is used in the combination method. The solutions obtained in this method are used to update the approximate efficient set *NDS*. A scheme of these steps is shown in Figure 2.

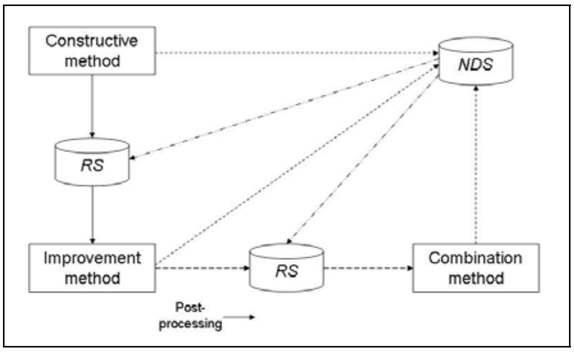

Figure 2 Scheme of the Metaheuristic Algorithm

5. Computational Experiments

Several instances were generated considering four main parameters: the number of plants, the number of potential distribution centers, the number of customers, and the number of transportation channels available between facilities. A combination of these parameters is considered an instance size because of the number of binary variables related. Five instances of each size were generated.

The ε-constraint based algorithm and the metaheuristic algorithm were coded in C. CPLEX 9.1 was used to solve the MIPs involved within the ε-constraint based algorithm, and the GAP and TP sub-problems involved within the metaheuristic. The algorithms were run in a 3.0 GHz, 1.0 Gb RAM, Intel Pentium 4 PC.

To make comparisons of the efficient frontiers obtained with the algorithms some metrics were used. The computing time and the number of non-dominated points $|S_i|$ are reported. The ratio R_{POS} (S_i) (Altiparmak et al., 2006) is calculated also. Additionally a special metric was developed considering the discretization of objective f_2. This metric is based on the ratio between two points with the same value of objective f_2 evaluated in objective f_1. The ratio is calculated with the value of the point from the efficient frontier obtained with the metaheuristic algorithm over the value of the point from the efficient frontier obtained with the ε-constraint based algorithm. The minimum (D_{min}) and average ($\overline{D}$) ratios along the efficient frontier are calculated. In Table 1 the results are shown for one instance of each size. The size code means number of plants - number of potential distribution centers - number of customers - number of transportation channels – number of instance. The results of the ε-constraint based algorithm are identified with the code [*ReC*] and the results of the metaheuristic algorithm are identified with the code [*MH*].

Table 1 Comparison of results from the metaheuristic algorithm [*MH*] and the ε-constraint based algorithm [*ReC*]

Size code	Total time (sec) [*MH*]	Total time (sec) [*ReC*]	$\|S_{ReC}\|$	$R_{POS}(ReC)$	$\|S_{MH}\|$	$R_{POS}(MH)$	$\overline{D}$	D_{min}
50-50-100-2-1	53715	24022	31	0.032	38	0.974	0.831	0.646
50-50-50-2-1	32901	24604	39	0.051	37	0.973	0.903	0.813
20-20-50-5-1	17756	24603	37	0.054	39	0.949	0.912	0.800
20-20-20-5-1	5029	24270	41	0.049	41	0.951	0.927	0.842
20-20-20-2-1	4680	22937	40	0.125	38	0.921	0.967	0.900
5-20-20-2-1	3615	22257	38	0.289	37	0.703	0.973	0.900
5-5-20-2-1	29	1982	31	1.000	20	0.050	1.042	1.000
5-5-5-5-1	92	141	38	1.000	32	0.125	1.020	1.000
5-5-5-2-1	75	6	32	1.000	22	0.364	1.028	1.000

The comparison of results for each metric must be made as follows. A greater value for $|S_i|$ and $R_{POS}(S_i)$ is better. These values indicate the size and quality of the efficient frontier. A lower value for metrics D_{min} and $\overline{D}$ indicate that the metaheuristic algorithm achieves lower cost (f_1) compared to the ε-constraint based algorithm, for the same time (f_2).

The comparison in Table 1 shows that the metaheuristic algorithm becomes competitive with the ε-constraint based algorithm for sizes over 20-20-20-5 in terms of computing time, size and quality of the efficient frontier obtained. A visual comparison of the efficient frontiers is shown in Figures 3 and 4 for a small instance and a very large instance respectively.

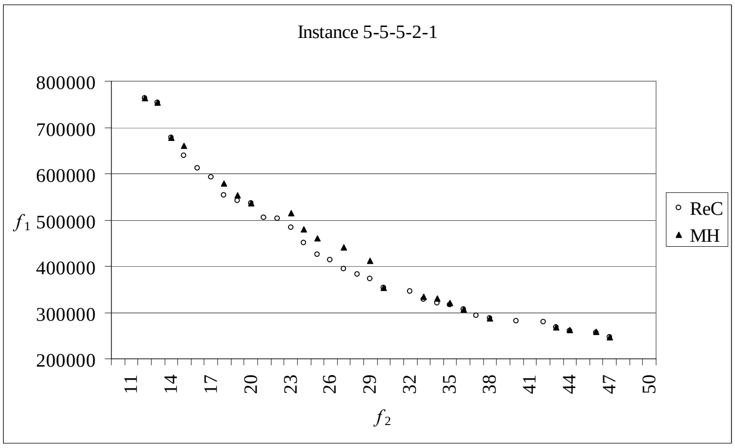

Figure 3 Comparison of the approximate efficient frontiers for instance 5-5-5-2-1

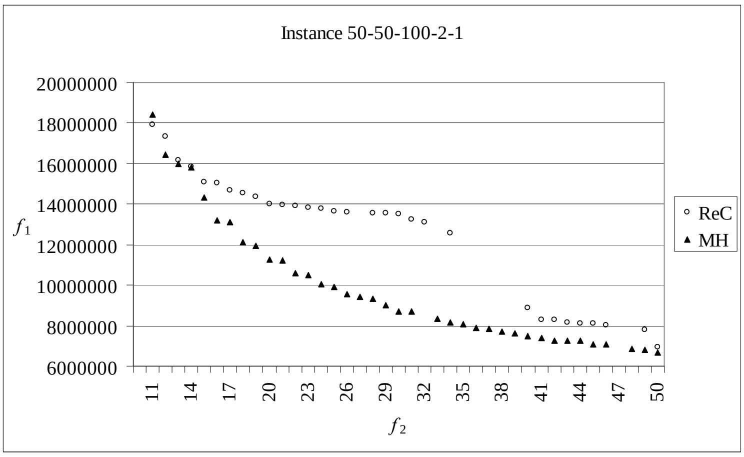

Figure 4 Comparison of the approximate efficient frontiers for instance 50-50-100-2-1

6. Conclusions

In the process of supply chain design many decisions have to be made and several aspects must be taken into account. However, an area of opportunity was identified in introducing the selection of transportation channel in the distribution network design. This decision produces a bi-objective problem where the total cost and transportation time must be optimized simultaneously. This problem was introduced by Olivares-Benitez (2007) and was named "Capacitated Fixed Cost Facility Location Problem with Transportation Choices" (CFCLP-TC). The mathematical model was formulated as a bi-objective mixed-integer program. The criteria to minimize are the total cost and the maximum time from the plants to the customers. The total cost is a combination of transportation cost and fixed opening cost. Because of the computational complexity of the problem a metaheuristic algorithm was developed to solve it. A reference algorithm was developed based on the ε-constraint method. The metaheuristic algorithm obtains solutions of better quality for large instances of the problem with competitive run times.

References

F. Altiparmak, M. Gen, L. Lin, T. Paksoy (2006). A Genetic Algorithm Approach for Multi-objective Optimization of Supply Chain Networks. *Computers & Industrial Engineering*, 51(1):197-216.

W. J. Baumol, P. Wolfe (1958). A Warehouse-Location Problem. *Operations Research*, 6(2):252-263.

S. Chopra, P. Meindl (2004). *Supply Chain Management: Strategy, Planning, and Operation*. Prentice Hall, Upper Saddle River, USA.

G. Cornuejols, G. L. Nemhauser, L. A. Wolsey (1990). The Uncapacitated Facility Location Problem. In P. B. Mirchandani and R. L. Francis (eds.), *Discrete Location Theory*, Chapter 3, pp. 119-171. Wiley, New York, USA.

J. Current, H. Min, D. Schilling (1990). Multiobjective Analysis of Facility Location Decisions. *European Journal of Operational Research*, 49(3):295-307.

M. Ehrgott (2005). *Multicriteria Optimization*. Springer-Verlag, Berlin, Germany.

J. L. González Velarde, R. Martí (2006). Adaptive Memory Programming for the Robust Capacitated International Sourcing Problem. *Computers & Operations Research*, forthcoming.

A. Klose, A. Drexl (2005). Facility Location Models for Distribution System Design. *European Journal of Operational Research*, 162(1):4-29.

A. A. Kuehn, M. J. Hamburger (1963). A Heuristic Program for Locating Warehouses. *Management Science*, 9(4):643-666.

E. Olivares-Benitez (2007). *Capacitated Fixed Cost Facility Location Problem with Transportation Choices*. PhD Dissertation, Tecnologico de Monterrey, Monterrey, Mexico, May.

D. Simchi-Levi, P. Kaminski, E. Simchi-Levi (2000). *Designing and Managing the Supply Chain: Concepts, Strategies and Case Studies*. McGraw Hill, Boston, USA.

R. E. Steuer (1989). *Multiple Criteria Optimization: Theory, Computation and Application*. Krieger Publishing Company, Malabar, USA.