Approximating the Fuel Consumption Function on Natural Gas Centrifugal Compressors

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Abstract

In the gas industry there are three important stages which are production, transportation and sales. In particular, in the transportation phase, the objective is to transport gas from some production or storage centers to different distribution centers at the least cost possible. To achieve this, compressor stations are placed at some points of the network to keep the gas moving. The function representing the incurred cost of fuel at each compressor unit (centrifugal type), which is installed in the network, is typically nonlinear, nonconvex, and difficult to evaluate computationally. Optimization algorithms for this problem have to evaluate this function many times. Due to this, a better approach may be using approximation functions, which are easier to evaluate than the real function. In this paper, we perform a computational evaluation with several approximation functions over a set of collected data from nine compressors units. The results confirm that one these proposed functions does a very good job at approximating the real function.

1. INTRODUCTION

Natural gas is transported throughout a pipeline network system. The gas flows throughout the network, and losses energy and pression due to both friction between the gas and the pipe inner wall, and heat transfer between the gas and its environment. To overcome this loss of energy and to keep the gas moving, compressor stations, which consume part of the transported gas, are installed in the network. The transportation cost is important because the amount of gas being transmited yearly in any system is huge. The decision making problem consists of figuring out how to operate the compressor stations, with the aim of transporting the gas from storage or production centers (where gas is inyected) to the different distribution centers (where gas is taken out), at least cost. The function representing the fuel consumption in a compressor is nonlinear and nonconvex. Its evaluation is complicated and, since a typical algorithm for solving nonlinear optimization problems (such as generalized reduced gradient and steepest descent [1], for instance), requires evaluating this objective function many times, CPU time turns out to be relatively high. Because of this, several approximation functions, whose evaluation is less expensive, have been proposed.

These functions were evaluated in [4] using data for one centrifugal compressor unit. It was observed that one of the tested functions outperformed the other ones. In this paper, we extend this evaluation to a wider collection of compressors (nine in total) with data taken from industry. This becomes the main contribution of this work. The results from this evaluation confirm that one function does a very good at approximating the objective function. The maximum relative error for this function is observed to be less than 3%. Therefore, we conclude that this function can represent faithfully the real objective function and can be used in future works in this area.

2. COMPRESSOR UNITS

There are two main types of compressor units which are centrifugal and reciprocating. In this work, we consider centrifugal units because they are more frequently found in industry. Their construction is simple, allowing for continuous operation during large periods of time.

The following equations describe the feasible operation domain for a centrifugal compressor unit in terms of the variables Q (volumetric flow rate), H (adiabatic head) and S (compressor speed).

$$\frac{H}{S^2} = A_H + B_H\left(\frac{Q}{S}\right) + C_H\left(\frac{Q}{S}\right)^2 + D_H\left(\frac{Q}{S}\right)^3 , \tag{1}$$

$$S^L < S < S^U ,$$

$$Q^L < Q < Q^U ,$$

where A_H, B_H, C_H and D_H are constants which depend on the compressor unit and are typically estimated by applying the least squares method to a set of collected data of Q, H and S. S^L and S^U represent minimum and maximum compressor speed, respectively. Q^L and Q^U are parameters that indicate the minimum and maximum volumetric flow rate limits, respectively.

Each compressor has certain performance associated with it. This performance is known as the compressor efficiency. This becomes a very important factor for any analysis since the higher the efficiency the lower the fuel consumption. The compressor efficency η is described as follows:

$$\eta = A_E + B_E\left(\frac{Q}{S}\right) + C_E\left(\frac{Q}{S}\right)^2 + D_E\left(\frac{Q}{S}\right)^3 \tag{2}$$

where A_E, B_E, C_E and D_E are also estimated in the same way as in (1). From the network modeling point of view, working in terms of mass flow rates and pressures is preferred because the mass flow rate is kept at each node of the network. This is not the case for the volumetric flow rates. Therefore a transformation from the original compressor domain (in H, Q, and S) to a domain including the variables (w,Ps,Pd), where w is the mass flow rate through the compressor, Ps is the suction pressure, and Pd is the discharge pressure, is made.

Since the main goal of a compressor is to increase the gas pressure to keep it flowing through the system, we have that Ps<Pd. The relationship between this (w,Ps,Pd) domain and the (H,Q,S) domain is established by the following:

$$H = ZRTs\left[\left(\frac{Ps}{Pd}\right)^m - 1\right], \tag{3}$$

$$Z = ZRTs\left(\frac{w}{Ps}\right), \tag{4}$$

where m = (k-1)/k, k is the specific heat ratio, Z is the gas compressibility factor, R is the gas constant, and Ts is the average temperature, assumed constant. Figure 1 shows the operation envelope in Q, S and H for a single centrifugal unit.

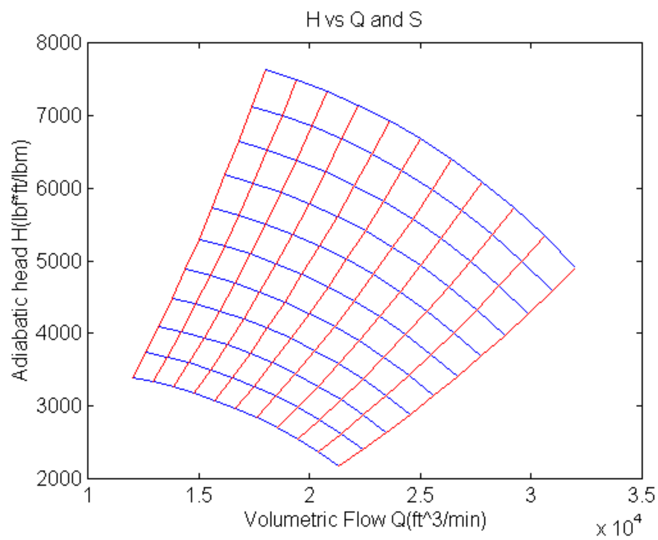

Figure 1: Operation envelope in Q, S and H (single centrifugal unit).

3. FUEL COST FUNCTION

The fuel cost g for a centrifugal compressor is given by the following function:

$$g(w, Ps, Pd) = \frac{\alpha \ wH}{\eta},$$

where α is a positive constant, which for simplicity is assumed to be equal to 1 throughout this work. Note that H is a function of Ps and Pd. This function tell us the work the compressor has to perform for transporting certain amount of mass flow rate (w) at some efficiency value. As it can be seen, the main computational cost for evaluating η as a function of (w,Ps,Pd) comes from evaluating the denominator. To evaluate this, it is necessary: (a) computing H and Q from (3) and (4), respectively, (b) obtaining S from (1), which implies finding the roots of a function, and (c) evaluating (2). A more detailed study of this cost function can be found in [4].

As it can be seen, doing this procedure every time we wish to evaluate a single point in the domain (w,Ps,Pd) requires a CPU time relatively high. Since typical algorithms for nonlinear optimization problems have to evaluate the objective function many times, it is not recommended to use this type of functions. One way to deal with this problem is to use approximation functions. In [5], six polynomial functions for approximating fuel cost function were proposed. The authors came to the conclusion that one the these functions was superior to the others. However, one limitation of that work was that the evaluation was carried out for only one compressor unit. Of course, in order to generalize this result is necessary to carry out a evaluation over a wider set of compressor units, which is the main part of this work. The approximation functions used for this evaluation are shown below.

$$g_1(w,Ps,Pd) = A_1 w + B_1 Ps + C_1 Pd + D_1$$

$$g_2(w,Ps,Pd) = A_2 w^2 + B_2 wPs + C_2 wPd + D_2 Ps^2 + E_2 PsPd + F_2 Pd^2 + G_2 w + H_2 Ps + I_2 Pd + J_2$$

$$g_3(w,Ps,Pd) = Ps\left(A_3 \frac{w}{Ps} + B_3 \frac{Pd}{Ps} + C_3 \right)$$

$$g_4(w,Ps,Pd) = Ps\left(A_4 \left(\frac{w}{Ps}\right)^2 + B_4 \frac{w}{Ps}\frac{Pd}{Ps} + C_4 \left(\frac{Pd}{Ps}\right)^2 + D_4 \frac{w}{Ps} + E_4 \frac{Pd}{Ps} + F_4 \right)$$

$$g_5(w,Ps,Pd) = w\left(A_5 \frac{w}{Ps} + B_5 \frac{Pd}{Ps} + C_5 \right)$$

$$g_6(w,Ps,Pd) = w\left(A_6 \left(\frac{w}{Ps}\right)^2 + B_6 \left(\frac{Pd}{Ps}\right)^2 + C_6 \frac{w}{Ps}\frac{Pd}{Ps} + D_6 \frac{w}{Ps} + E_6 \frac{Pd}{Ps} + F_6 \right)$$

4. COMPUTATIONAL EVALUATION

To carry out this experiment, we used Matlab 4.2c.1[3] in a Pentium1 PC with 16Mb of RAM. First, the coefficients of each approximation function were estimated using the least squares method using a sample of 1000 points in the (w,Ps,Pd) domain. Then, we did the function evaluation. To do this, we generated a 10x10x10 mesh in the (w,Ps,Pd) domain. In each grid point, we evaluated each function, tallying the relative error of each aproximation function with respect to the original fuel consumption function. This was done in each one of the nine compressor units. The relative error was calculated as | g_real( ) - g_approximate( ) | /g_real( ). The parameter values used were: isentropic exponent k =1.287, compressibility factor Z=0.95, and R=85.2 (lbf-ft/lbm-°R).The data for the compressor units were taken from [2].

Table 1 shows the results of the evaluation. In each cell the maximum relative error for each compressor unit (row) and function (column) is shown. Functions g_2 and g_4 are not shown since their errors were very large. We observed that function g_6 had a better approximation than the other functions. In eight out of nine compressor units the error of g_6 is less than 3%, and even though an error of 9.8% is observed in the other compressor, this is certainly better than the other functions. These results verify that, indeed, g_6 consistently outperfoms the other ones over each of the compressor units tested.

	Functions			
Name of compressor	**g1**	**g3**	**g5**	**g6**
CPID SNARLIN-K1	25.63	25.63	8.76	0.86
CPID RAKEEY-K1	25.99	55.99	9.02	0.55
CPID RAKEEY-K2	27.91	27.91	8.56	2.71
CPID HAMPER -K1	32.66	32.66	21.04	9.82
CPID BELLVAN-K1	30.15	30.15	9.69	1.05
CPID BELLVAN -K2	30.15	30.15	9.69	1.05
CPID BELLVAN-K3	61.27	61.27	15.23	2.13
CPID BETHANY-K1	54.08	54.08	15.11	2.53
CPID BETHANY-K2	20.40	20.40	8.28	0.68

Table 1: Maximum relative error (%) of the approximation functions.

5. CONCLUSIONS

In this work several approximation functions for the fuel consumption function in a centrifugal compressor were evaluated. The experiment was done over nine different compressors with data taken from industry. It was observed that one of these functions, g_6, approximated the real function very well. Therefore, this function can be recomended to be used in future works as it is easier to evaluate than the real function. Future work on this project includes the development of a model based on an algebraic modeling system, and then a through evaluation of different nonlinear programming algorithms on this problem.

Acknowledgments: This research has been supported by the Mexican National Council for Science and Techonology (Grant No. J33187-A) and by Universidad Autónoma de Nuevo León through its Scientific and Technological Research Support Program (Grant No. CA555-01).

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