Minimizing Cash-Out Penalty Costs: A Bilevel Programming Model

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Abstract: Natural gas pipeline operations inevitably involves imbalances. Most pipelines allow for some small imbalance tolerances. However, if imbalances are not resolved by the end of the month, the pipeline will penalize the shipper by imposing cash out prices. End-of-the-month imbalances are resolved by cash transactions between the pipeline and the shippers based on such prices. Depending on the amount of the imbalance position, this penalty may be onerous. Nevertheless, as long as one can anticipate the amount of penalty associated with a given imbalance position on any given day, day to day operations can be managed accordingly. The problem, is how to optimally managing shippers daily imbalance positions so as to minimize the cash-out penalty impossed by the pipeline company. We introduce a mixed integer bilevel programming model for this problem, and propose a conceptual simulated annealing heuristic for its solution.

Introduction: The natural gas industry has been going through a deregulation process since the mid 1980s leading to significant market changes. What this means is that now the decision making process of gas buying, selling, storing, transporting, and so on, is inmerse in a very complex world where producers, pipelines (transporters), and brokers, play all a very important role in the chain. There are many problems of practical importance, however, it is still surprising to find only a few that have been tackled by the operations research community. Guldman and Wang [4] account for part of that work.

The problem addressed in this paper arises when a natural gas shipping company draws a contract with a pipeline company to deliver certain amount of gas between pre-specified locations. What is actually deliver may be more or less of what was originally contracted (this is called an imbalance). When this situation occurs, the pipeline penalizes the shipper by imposing a cash-out penalty policy. Since this penalty is a function of the operating daily imbalances, an important issue for the shipper is how to carry out their daily imbalances so as to minimze their incurred penalty.

In this paper we present a mathematical framework for the problem of minimizing the cash-out penalty from the shipper's perspective. The problem is modeled as a mixed-integer bilevel linear programming problem (mixed-integet BLP or MIBLP), where the shipper plays the role of the leader (first level decisions) and the pipeline represents the follower (second level decisions). BLP is NP-hard in general. Mixed-integer BLP posses even a higher degree of difficulty as the typical concepts for fathoming in traditional branch and bound algorithms for mixed-integer programming (MIP) can not be directly applied to mixed-integer BLP. In order to find good solutions for this problem, we proposed a heuristic based on simulated annealing and discuss a few implementation issues related to BLP. Extensive computational work will be presented in a follow up paper as this represents work in progress.

Problem Description: Assume that a shipper has entered into a contract to deliver a given amount of natural gas from a receipt to a delivery meter in a given time frame. (In the following gas and natural gas are treated as synonyms.) The shipper must stipulate title transfer agreements with the meter operators and a transportation agreement with the pipeline. Under such agreements, the shipper nominates a daily amount of gas to be injected by the receipt meter operator into the pipeline and to be withdrawn by the delivery meter operator from the pipeline. The pipeline transports the gas from the receipt meter to the delivery meter.

Due to the nature of the natural gas industry, what is actually transported is inevitably different from what is nominated. Such a difference constitutes an imbalance. There exist operational and transportation imbalances. The first type of imbalances refers to differences between nominated and actual flows, while the latter involves differences between net receipts (receipts minus fuel) and deliveries. While pipelines allow for small imbalances, they issue penalties for both operational and transportation imbalances to the other parties. In the following the cash out penalties associated with operational imbalances are analyzed.

On the shipper side, an operational imbalance can be either positive or negative. A positive imbalance (negative) arises when the shipper leaves (takes) gas in (from) the pipeline. Alternatively, a positive (negative) imbalance means that the actual flow is smaller (greater) than the nominated amount of gas. A positive (negative) end-of-the-month imbalance implies a cash transaction from the pipeline (shipper) to the shipper (pipeline). Cash out prices are set in a way that whenever a shipper sells (buys) gas to (from) the pipeline, he does so at a very low (high) price. The relation between cash out price and imbalance position depends non-linearly on the average, maximum and minimum gas spot price for the past month.

Shippers daily nominate gas flows taking into account the constraints deriving from their buy/sell activity, their contractual constraints, and future market opportunities. The gas price is one of the major factors affecting their decisions. In the absence of cash out provisions, historically shippers would take out high cost gas in the winter from the pipeline (causing negative imbalances), and pay the transporter back with low cost gas in the summer. This corresponds to a speculative behavior by the shippers, whereby imbalances are created and managed as pseudo-storage in order to take advantage of movements in the gas price. Cash out penalties were designed in order to avoid such pricing arbitrages. In the present framework, shippers are concerned with avoiding costly cash out penalties.

Cash-Out Penalty Rules: The cashout penalty assessment is different from one company to the others. Most of the time, a pipeline will come up with an undisclosed procedure to determine the cashout penalty for their shippers. For this study, we will assume that a pipeline company will use the following rules in its cash-out procedure.

(1) At the end of the month, the shipper imbalance positions must be cashed out.

(2) In the process of cashing out these imbalances, the pipeline can reallocate the imbalances for this shipper in such a way that the final money transaction between both parties is minimized.

(3) The positive imbalance at any zone can be used to offset the negative imbalance at any other zone.

(4) For forward movement, the shipper needs to pay the transportation cost based on the forward movement volume.

(5) However, if the positive imbalance is moved backward, the shipper will get a back haul credit based on the backward movement volume.

(6) The final size of imbalance position (either positive or negative) cannot be greater that of the initial one.

Mathematical Model: As stated in the previous section, the decision making process for the shipper (leader) is to determine hot to carry out their daily imbalances such as to minimize the penalty that will be imposed by the pipeline (follower). It is assumed that all of the problem data is known with certainty.

The following notation is used to describe the model.

Indices and Sets

- i, j, k zone pool indices; $i, j, k \in J$
- t time index; $t \in T$

Data

- x_{ti}^L, x_{ti}^U bounds on daily imbalances at (end of) day t in zone i; $t \in T$, $i \in J$
- x_t^L, x_t^U bounds on total daily imbalances at (end of) day t; $t \in T$
- s_{ti}^L, s_{ti}^U bounds on balance swings during day t in zone i; $t \in T$, $i \in J$
- e_{ij} percentage of fuel retained for moving one dekatherm (dt) of gas from zone i to j; $i, j \in J$
- f_{ij} transportation charge for moving one dt of gas from zone i to j; $i, j \in J$, $i < j$
- b_{ij} backward haul credit for moving one dt of gas from zone j to i; $i, j \in J$, $i < j$
- r_i gas selling price at zone i; $i \in J$
- x_{0j} initial imbalance (start of day 1) in zone j; $j \in J$

Decision Variables

- x_{ti} imbalance at (end of) day t in zone i; $t \in T$, $i \in J$
- s_{ti} imbalance swing during day t in zone i; $t \in T$, $i \in J$
- y_i final imbalance at zone i; $i \in J$
- u_{ij} forward haul volume moved from zone i to j; $i, j \in J$, $i < j$
- v_{ij} backward haul volume moved from zone j to i; $i, j \in J$, $i < j$
- z total cash-out revenue for shipper

Auxiliar Variables

- p_i binary variable equal to 1 (0) if $x_{|T|,i}$ is nonnegative (nonpositive) (used in (2e)); $i \in J$
- q binary variable equal to 1 (0) if final imbalances are all nonnegative (nonpositive) (used in eq. (2f))

Here we provide the set of constraints involved in both the first and second level of the problem.

First Level Model:

Objective: Shipper's revenue.

$$\max \quad h_1(x,s,y,u,v,z) = z \qquad (1a)$$

Constraints:

$$x^L_{ti} \leq x_{ti} \leq x^U_{ti} \qquad t \in T, \quad i \in J \quad (1b)$$

$$s^L_{ti} \leq s_{ti} \leq s^U_{ti} \qquad t \in T, \quad i \in J \quad (1c)$$

$$x^L_t \leq \sum_{i \in J} x_{ti} \leq x^U_t \qquad t \in T \quad (1d)$$

$$x_{ti} = x_{t-1,i} + s_{ti} \qquad t \in T, \quad i \in J \quad (1e)$$

$$x_{ti}, s_{ti} \ \text{free} \qquad t \in T, \quad i \in J \quad (1f)$$

Second Level Model:

Objective: The penalty is determined by minimizing the amount of cash transactions. In many cases, both shipper and pipeline agree in a policy that represents a compromise between the two, so rather than maximizing revenue for shipper, it is agreed to minimize deviations from zero. So the follower objective is given by

$$\min \quad h_2(x,s,y,u,v,z) = |z| \qquad (2a)$$

Balance constraints: This constraint identifies the relationship between the imbalance at day $|T|$, forward and backward haul volumes, retained fuel, and final imbalance at zone i.

$$y_j = x_{|T|,j} + \sum_{i:i<j}(1-e_{ij})u_{ij} + \sum_{k:k>j} v_{jk} - \sum_{k:k>j} u_{jk} - \sum_{i:i<j} v_{ij} \quad j \in J \quad (2b)$$

Gas conservation: This constraint ensures no gas loss occurs.

$$\sum_{i \in I} y_i + \sum_{(i,j):i<j} e_{ij}u_{ij} = \sum_{i \in I} x_{|T|,i} \qquad (2c)$$

Note that $\sum_{(i,j)} e_{ij}u_{ij} \geq 0$, so $\sum_i y_i \leq \sum_i x_{|T|,i}$.

Zone upper bounds: This constraints prevents cyclic movements of gas. It simply states that, at any given zone, we cannot move more than any initial positive imbalance.

$$\sum_{i:i<j} u_{ij} + \sum_{k:k>j} v_{jk} \leq \max\{0, x_{|T|,j}\} \qquad j \in J \qquad (2d)$$

Forward haul upper bounds: These bounds prevent positive-to-positive and negative forward movement of imbalances.

$$u_{ij} \leq \begin{cases} x_{|T|,i} & \text{if } x_{|T|,i} > 0 \\ & \text{and } x_{|T|,j} < 0 \\ 0 & \text{otherwise} \end{cases} \qquad i,j \in J; i<j \qquad (2e)$$

Sign on final imbalances: This is a business rule that states that final imbalances for all zones must have the same "sign" (i.e., all nonpositive or nonnegative). sign, i.e., an imbalance must not change sign.

$$-M(1-q) \leq y_i \leq Mq \quad i \in J \qquad (2f)$$

where M is a large number and q is a binary 0–1 variable.

Shipper's revenue: This equation represents the revenue from ths shipper's point of view.

$$z = \sum_{i \in J} r_i y_i + \sum_{(i,j):i<j} b_{ij} v_{ij} - \sum_{(i,j):i<j} f_{ij}(1-e_{ij}) \qquad (2g)$$

Variable types:

$$y_i, z \quad \text{free} \qquad (2h)$$

$$u_{ij}, v_{ij} \geq 0 \qquad (2i)$$

$$q \in \{0,1\} \qquad (2j)$$

Constraint (2d) is a disjunctive constraint that can be linearized by replacing it with

$$\sum_{i:i<j} u_{ij} + \sum_{k:k>j} v_{jk} \leq Mp_j \qquad j \in J \tag{3a}$$

$$\sum_{i:i<j} u_{ij} + \sum_{k:k>j} v_{jk} \leq x_{|T|,j} + M(1-p_j) \qquad j \in J \tag{3b}$$

Constraint (2e) can be modeled by

$$u_{ij} \leq Mp_i \qquad i,j \in J \tag{4a}$$

$$u_{ij} \leq M(1-p_j) \qquad i,j \in J \tag{4b}$$

The Bilevel Program:

$$\begin{array}{lll} \max & z & (5) \\ \text{subject to} & \text{(1b)–(1f)} & (6) \\ & \min |z| & (7) \\ & \text{subject to} \quad \text{(2b)–(2j)} & (8) \end{array}$$

In the first level, there are $2|T||J|$ continouos variables and $3|T||J| + |T|$ constraints. In the second level, there are $|J|(|J|+1) + |J|$ continuous variables, $|J|+1$ binary variables, and $|J|(|J|-1) + 7|J| + 4$ linear constraints. However, once the first level variables $x_{|T|,i}$ are known, all binary variables in constraints (2e) are not needed, so we are left with only one binary variable (constraint (2f)), the same number of continuous variables, and $|J|(|J|-1) + 4|J| + 4$ linear constraints (with $|J|(|J|+1) + 2|J|$ out of these being simple bound constraints). The problem is classified in BLP literature as a mixed-integer Linear BLP, which is known to be NP-hard [2].

Solving Mixed-Integer Linear BLPs: BLPs are NP-hard, even for the linear case. Algorithms designed to solve integer programs generally rely on some form of separation, relaxation, and fathoming to construct even tighter bounds on the solution. Separation is usually accomplished by placing contradictory constraints on a single variable. This approach is directly applicable to mixed integer BLP. The natural relaxation derives from the removal of the integrality requirements on the variables. Fathoming, however, presents several difficulties as some of the typical rules employed in MIPs do not hold for mixed-integer BLPs. Details of a branch and bound implementation for mixed-integer BLP can be found in [2].

Heuristics: Most of the recent developments on heuristics for solving bilevel programs address the linear case. In particular, approaches based on meta-heuristics such as tabu search, genetic algorithms, and simulated annealing, have been applied with certain degree of success to solve some applications of linear BLPs (e.g., see Gendrau et al. [3], Anandalingam et al. [1]). Another technique is the grid search heuristic proposed by Bard [2], which makes use of parametric programming.

In this work, we propose a simulated annealing implementation for mixed-integer BLPs based on the ideas of Anandalingam et al. [1] for the linear BLP.

Simulated Annealing: Simulated annealing (SA) was derived from statistical mechanics with the aim of finding (near) optimal solutions to large-scale problems. It generalizes hill climbing methods (in the case of maximization) and eliminates their main disdvantage: dependence of the solution on the starting point. Moreover, it statistically promises to deliver a globally optimal solution in the limit. This is achieved by introducing a probability ρ of acceptance (that is the replacement of the current point by a new point): $\rho = 1$ if the new point provides a better value of the objective function. In general, $\rho > 0$ depends on the values of the objective function evaluated at the current and new points, and an additional control parameter known as the *temperature*, denoted by T. The lower the temperature, the smaller the chances for acceptance of the new point. During the execution of the algorithm, T is lowered in steps. Termination occurs for some small value of T for which virtually no changes are accepted anymore. Most of the applications of SA have targeted combinatorial optimization problems, but the technique has been adapted to continouos optimization problems as well (e.g., see Michalewicz [6]). For a more extense treatment of simulated annealing the reader is referred to Kirpatrick et al. [5].

Simulated Annealing for the Cash-Out Problem: Our implementation makes use of the fact that for a given (x, s), the follower's rational reaction can be obtained by solving a mixed integer program (7)–(8) wih only one discrete variable, that is, almost a linear program. This implies that only components of the vector (x, s) need to be generated randomly. Furthermore, it can be seen from the formulation that the decision variables driving the leader's decision are $x_{|T|,i}$, that is, the state of the imbalances at the very last time period. Hence, we procede to generate $x_{|T|,i}$ first, and then attempt to find a feasible (x, s) vector.
Notation:

F	objective of the leader as defined by eq. (1a) for current solution
F^*	best value of leader's objective function at current iteration
z^*	array that stores the solution associated with F^*
k^*	maximum allowable iterations with no change in optimal solution
α	temperature reduction parameter
T^*	minimum temperature allowed for annealing solution
$U[a, b]$	uniform distribution between a and b

The Simulated Annealing Heuristic:

Step 0: (Initialization) Set $F^* = +\infty$, temperature $T = T_{max}$, and $k = 0$.

Step 1: Generate solution (x^k, s^k) in the neighborhood of (x^{k-1}, s^{k-1}).

$$x_i^k \sim U[\min x_i^k, \max x_i^k]$$

Step 2: Solve (7)–(8) for (x^k, s^k); obtain (y^k, u^k, v^k) and current value of F.

Step 3: Compute $\Delta F = F^* - F$.
(3.1) If $\Delta F > 0$, then $F^* \leftarrow F$, $z^* = (x^k, s^k, y^k, u^k, v^k)$
(3.2) If $\Delta F < 0$, let $z^* = (x^k, s^k, y^k, u^k, v^k)$ with probability $\exp(-\Delta F/T)$.
(3.3) If $\Delta = 0$, put $k \leftarrow k + 1$.
(3.4) If $k > k^*$, go to Step 4; else go to Step 1

Step 4: Set $k = 0$ and lower temperature, $T \leftarrow \alpha T$

Step 5: If $T < T^*$ or other termination criteria met, stop; else go to Step 1.

Current Work: Our work in progress includes the computational implementation of the heuristic presented in this paper. A follow up paper will include our computational work including a full assesment of the quality of the solutions delivered by the method in terms of both their quality and computational requirements. Of particular practical importance is the study of the nature of the solutions for different data (initial imbalances) scenarios and its impact on the penalty cash-out costs impossed by the pipeline companies. This is a key issue for the decision makers in shipping companies of natural gas nowadays. Although the method is being developed for a particular application of mixed integer BLP, we expect to make progress toward the development of a heuristic for general mixed integer BLPs, for which practically no effective technique has been developed up to date.

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References

[1] G. Anandalingam, R. Mathieu, L. Pittard, and R. Sinha. Artificial intelligence based approaches for hierarchical optimization. In R. Sharda et al., editor, *Impact of Recent Computer Advances in Operations Research*. North-Holland, New York, 1989.

[2] J. F. Bard. *Practical Bilevel Programming: Algorithms and Applications*. Kluwer, Dordrecht, 1998.

[3] M. Gendreau, P. Marcotte, and G. Savard. A hybrid tabu-ascent algorithm for the linear bilevel programming problems. *Journal of Global Optimization*, 8(3):217–233, 1996.

[4] J.-M. Guldmann and F. Wang. Optimizing the natural gas supply mix of local distribution utilities. *European Journal of Operational Research*, 112(3):598–612, 1999.

[5] S. Kirkpatrick, C. D. Gelatt, and M. P. Vecchi. Optimization by simulated annealing. *Science*, 220(4598):671–680, 1983.

[6] Z. Michalewicz. Evolutionary computation techniques for nonlinear programming problems. *International Transactions on Operational Research*, 1(2):223–240, 1994.