Compressor Driven Networks Flows

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Abstract: A typical pipeline network for delivering natural gas requires a tremendous amount of fuel per day to operate the compressor stations driving the gas. Efficient design and operation of these complex networks can substantially reduce airborne emissions, increase safety, and decrease the often multi-million dollar daily operating costs. The goal of this research is to propose and develop effective solution methods for optimization problems arising in gas pipeline networks. This paper addresses the minimization of fuel cost incurred by the compressor stations driving the gas. We present a mathematical model for the steady-state case and discuss a lower bounding scheme based on several model simplifications.

Introduction: We consider the problem of finding a minimum cost configuration when transporting natural gas over a pipeline network. What determines the cost is the fuel consumption of the compressor stations driving the gas. This is a non-convex nonlinear mixed-integer programming problem, which is highly challenging from the computational standpoint. Since finding optimal solutions for this problems is extremely difficult, we focus on deriving both upper and lower bounds of the optimal solution. These procedures are then used to assess the quality of the proposed solutions. In this paper we describe a mathematical model for the fuel cost minimization of steady-state gas pipeline networks and discuss a lower bounding scheme.

Problem Description: This problem has a network underlying structure [1] where the arcs are associated with either pipelines (set A_p) or compressor stations (set A_c), and the nodes (set V) represent the physical connection among arcs. The decision variables are the mass flow rate (or simply the flow) x_{ij}, for each arc $(i, j) \in A_p \cup A_c$, and the gas pressure p_i, for each node $i \in V$, and the number of compressor units r_{ij} to be turned on within each compressor station, for each $(i, j) \in A_c$. The conservation of flow is maintained at every node. The gas pressure, however, diminishes as it is transported. The compressor stations are thus used to increase the gas pressure and keep the gas going. In each compressor station, a cost $h_{ij}(x_{ij}, p_i, p_j, r_{ij})$ (that depends on the flow x_{ij}, the in-going pressure p_i, the out-going pressure p_j, and the number of compressor units r_{ij}) is incurred [2]. In addition, operational limits on the compressor define a feasible domain D_{ij} for the variables x_{ij}, p_i, p_j, r_{ij}. In the pipeline arcs, flows and pressures are related by the nonlinear equality: $p_i^2 - p_j^2 = k_{ij} x_{ij} |x_{ij}|$, where k_{ij} is a constant obtained from the pipeline characteristics [2].

The problem consists of minimizing the total cost derived from the fuel consumption at each compressor station

$$h(x, p, r) = \sum_{(i,j)\in A_c} h_{ij}(x_{ij}, p_i, p_j, r_{ij})$$

subject to: (1) conservation of flow at each node, (2) compressor station constraints (given by domains D_{ij}), (3) pipeline constraints (given by the above nonlinear equalities), (4) simple pressure bounds, and (5) variable restrictions (e.g. x_{ij} , p_i non-negative, and r_{ij} integer). It is important to note that the cost functions h_{ij} are general non-convex and nonlinear, and that the domains D_{ij} define non-convex feasible sets.

Solution Approaches: There are three elements in the model that make the problem hard: (a) the non-convexity of the objective function, (b) the non-convexity of the sets D_{ij} , and (c) the presence of the pipeline constraints (3). Therefore we sort to developing both lower and upper bounds that allow us to assess the quality of feasible solutions.

The first step towards a lower bounding procedure was to develop a convex piece-wise linear under-estimator of the objective function. Then, to deal with (b), a linearly-constrained convex superset of set D_{ij} was derived. The details of these developments and a lower bounding scheme (ignoring the pipeline constraints) are contained in [3].

The next step is to use this scheme within a broader lower bounding procedure aimed at incorporating the nonlinear pipeline constraints (3). One way to achieve this is by Lagrangian relaxation and subgradient optimization [4] and is now being pursued.

The other part of the research deals with deriving upper bounds for this problem, i.e., relatively quick approximation algorithms that deliver high quality feasible solutions. This is still an on-going work.

Future Work: The Lagrangian lower bounding approach consists of relaxing the hard (pipeline) constraints and placing them in the objective function with associated penalty costs (Lagrange multipliers). It can be shown that once the multipliers are fixed to some value, the corresponding subproblem is an indefinite quadratic program subject to linear constraints. Although this is also a hard problem, it can actually be solved by cGOP (global optimization software for problems with special structure described in [5]). This procedure is applied iteratively for a sequence of multipliers generated by subgradient optimization that guarantee a non-decreasing sequence of valid lower bounds for the original problem. Another of our short-term targets is the computational evaluation of the bounding procedures. This will allow us to empirically evaluate the effectiveness of the proposed approaches.

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