GRASP Strategies for a Bi-objective Commercial Territory Design Problem

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Abstract In this work a problem motivated by a real-world case from a beverage distribution firm in Mexico is addressed. Different planning criteria are taken into account in order to create acceptable territory designs. Namely, each territory needs to be compact, connected and balanced according to two attributes (number of costumers and sales volume). We propose a bi-objective programming model and two solution procedures (B-GRASP and T-GRASP), this problem has not been addressed before to the best of our knowledge. B-GRASP and T-GRASP are based in a heuristic procedure best known as GRASP. The main difference between B-GRASP and T-GRASP is the way they consider the planning criteria during the construction phase. In B-GRASP, the construction attempts to find high quality solutions based on the optimization of two criteria: compactness and balancing according to the number of customers, demand is treated as a constraint. The construction phase in T-GRASP considers three objectives to be optimized: the compactness and the balancing with respect to the two attributes (number of customers and sales volume). Both procedures are evaluated on a variety of problem instances.

Key words: Bi-objective optimization, multiobjective optimization, territory design, GRASP

1 Introduction

The problem addressed in this paper arises from a beverage distribution firm in Mexico. Related work on this problem can be found in Ríos-Mercado and Fernández [5] and Segura-Ramiro et al. [6]. In those works the development of new models and solution procedures for a single objective version of this problem are included. In general, commercial territory design belongs to the family of districting problems that have a broad range of applications like political districting [1], school districting, and sales and service territory design [3, 8]. In most of these applications a single objective problem is considered, however in the real world is very common to pursue more than one criterion. In fact, looking at the literature on territory design (TD), there are a few works addressing these problems as multiobjective problems [7, 4]. Territory design (TD) is a hard task and it is very common in every enterprise dedicated to product sales and product distribution, specifically when the firm needs to divide the market into smallest regions to delegate responsibilities to facilitate the sales and distribution of goods. These decisions need to be constantly evaluated due to the frequent market changes such as introduction of new products or changes in the workload, which are factors that affect the territory design. Additionally, the multiple planning requirements that the firm wishes to satisfy and the large amount of customers that need to be grouped makes this difficult task even more critical. An efficient tool with capacity to provide good solutions to large problems is needed. The specific characteristics present in this concrete problem make it very unique, and not addressed before to the best of our knowledge. We introduced a bi-objective optimization model to represent the real situation. Two different GRASP strategies (B-GRASP and T-GRASP) are proposed and implemented aiming at finding a good approximation of the Pareto frontier. Each of these strategies consists of two main phases: construction and post-processing. In the construction phase a simultaneous territory creation is conduced and in the post-processing phase the neighborhood is explored in a similar way to that of the MOAMP procedure applied by Molina et al. [2]. We tested the proposed procedures on a set of instances and the test indicate that B-GRASP has better performance than T-GRASP. The paper is organized as follows. In Section 2 the problem is briefly described, it includes mathematical notation and a bi-objective optimization model. Section 3 shows details about the proposed solution procedures and Section 4 includes the experimental work. Finally we wrap in Section 5 with our conclusions.

2 Multiobjective Commercial Territory Design

2.1 Problem Description

In particular, the problem consists of finding a partition of the entire set of city blocks or basic units (BUs) into a fixed number (p) of territories, considering several planning territory requirements such as compactness, balance and connectivity. Compactness means customers within a territory should be relatively close to each other. Balance implies territories with similar size with respect to two attributes (number of customers and sales volume). Connectivity means BUs in the same territory can reach each other without leaving the territory. In addition, exclusive assignment from BUs to territories is needed. The problem is modeled by an undirected graph $G = (V, E)$, where V is the set of nodes (BUs) and E is the set of edges representing adjacency between blocks (BUs). That is, a block or BU j is associated with a node, and an edge connecting nodes i and j exists if i and j are adjacent. For each node $j \in V$ has associated some parameters such as geographical coordinates (c_x, c_y), and two measurable attributes (number of customers and sales volume) are defined. The number of territories is given by parameter p. It is required that each node is assigned to only one territory (exclusive assignment). The company wants balanced territories with respect to each of the attribute measures. Let us define the size of territory V_k with respect to attribute a as: $w^{(a)}(V_k) = \sum_{i \in V_k}(w_i^{(a)})$, where $a \in \{1, 2\}$ and $w_i^{(a)}$ is the value associated to attribute a in node $i \in V$. Another characteristic is that all of the BUs assigned to each territory are connected by a path contained totally within the territory. In addition, the BUs in each territory must be relatively close to each other (compactness). One way to achieve this requirement is to minimize a dispersion measure. We use a dispersion measure based in the objective of the p-median problem (p-MP). All parameters are assumed to be known with certainty. We propose a bi-objective optimization model for this commercial territory design problem. In this model the compactness and the maximum deviation with respect to the number of customers are considered as objectives and the remaining requirements are treated as constraints. Let $N^i = \{j \in V : (i, j) \in E \vee (j, i) \in E\}$ be the set of adjacent nodes to node $i; i \in V$. The Euclidean distance between j and i is denoted by d_{ji}, $i, j \in V$. The average (target) value of attribute a can be computed as $\mu^{(a)} = w^{(a)}(V)/p$, $a \in A$.

Due to the discrete structure of the problem and to the unique assignment constraint, it is practically impossible to have perfectly balanced territories with respect to each attribute. Let $\tau^{(2)}$ be the specific tolerance allowed by the company to measure the relative deviation from average territory size with respect to sales volume.

2.2 Bi-objective Optimization Model

Decision variables

$$x_{ji} = \begin{cases} 1 & \text{if a basic unit } j \text{ is assigned to territory with center in } i;\ i, j \in V \\ 0 & \text{otherwise} \end{cases}$$

In that sense $x_{ii} = 1$ implies i is a territory center.

$$\text{Min } f_1 = \sum_{j \in V} \sum_{i \in V} d_{ji} x_{ji} \tag{1}$$

$$\text{Min } f_2 = \max_{i \in V} \left\{ \frac{1}{\mu^{(1)}} \left| \sum_{j \in V} (w_j^{(1)} x_{ji}) - \mu^{(1)} x_{ii} \right| \right\} \tag{2}$$

$$\tag{3}$$

Subject to:

$$\sum_{i \in V} x_{ii} = p \tag{4}$$

$$\sum_{i \in V} x_{ji} = 1 \qquad \forall j \in V \tag{5}$$

$$\sum_{j \in V} w_j^{(2)} x_{ji} \geq (1 - \tau^{(2)}) \mu^{(2)} x_{ii} \qquad i \in V \tag{6}$$

$$\sum_{j \in V} w_j^{(2)} x_{ji} \leq (1 + \tau^{(2)}) \mu^{(2)} x_{ii} \qquad i \in V \tag{7}$$

$$\sum_{j \in \cup_{v \in S}(N^i \setminus S)} x_{ji} - \sum_{j \in S} x_{ji} \geq 1 - \mid S \mid \qquad i \in V; S \subset [V \setminus (N^i \cup \{i\})] \tag{8}$$

$$x_{j,i} \in \{0, 1\} \qquad i, j \in V \tag{9}$$

Objective (1) represents a dispersion measure based on a p-MP objective. In this sense, minimizing dispersion is equivalent to maximizing compactness. The second objective (2) represents the maximum deviation with respect to the target size related to the number of customers. So, balanced territories should have small deviation with respect to the average number of customers. Constraint (4) guarantee the creation of exactly p territories. Constraints (5) guarantee that each node j is assigned to only one territory. Constraints (6)-(7) represent the territory balance with respect to the sales volume as it establishes that the size of each territory must lie within a range (measured by tolerance parameter $\tau^{(a)}$) around its average size. Constraints (8) guarantee the connectivity of the territories. Note that, as usual, there is an exponential number of such constraints.

3 Proposed GRASP Procedures

In general, GRASP is a metaheuristic that contains good features of both pure greedy algorithms and random construction procedures. It has been widely used for successfully solving many combinatorial optimization problems. A GRASP is an iterative process in which each major iteration consists typically of two phases: construction and post-processing. The construction phase attempts to build a feasible solution S and the post-processing phase attempts to improve it. The motivation for GRASP in this application is because due to the fact that during the construction phase it is always possible to maintain the hard connectivity constraints (8) and the multiple objectives can be easily evaluated in a merit function. In addition GRASP is very attractive for generating diverse solutions.

Algorithm 1 General scheme for B-GRASP and T-GRASP $(\alpha, iter_{max}, f, max_{moves})$

INPUT $(\alpha, iter_{max}, f, max_{moves})$
α: GRASP RCL quality parameter
$iter_{max}$: GRASP iterations limit
f: Minimum node degree required to create a subgraph which is used to select initial seeds in the ConstructSolution method
max_{moves}: Maximum number of movements permitted in the post-processing phase.
OUTPUT D^{eff} set of efficients solutions
Λ: set of weights for greedy function selected in the range $[0, 1]$
$\Lambda \leftarrow$ generate (r); $\Lambda = \{\lambda_1, \lambda_2, ..., l\lambda_r\}$ $D^{eff} \leftarrow \emptyset$
$D^{pot}(S) \leftarrow \emptyset$: set of potential efficients solutions
FOR$(l = 1, ..., iter_{max})$
FOR EACH$(\lambda \in \Lambda)$

  $S \leftarrow$ ConstructSolution(α, f, λ)
  $D^{pot}(S) \leftarrow$PostProcessing(S, max_{moves})
  UpdateEfficientSolutions$(D^{eff}, D^{pot}(S))$

END FOR
END FOR
RETURN D^{eff}

Algorithm 1 shows the general scheme for the proposed GRASP. An instance of the commercial territory design problem, the maximum number of iterations $(iter_{max})$, the quality parameter (α), the minimum node degree (f) so that a node $i \in V$ can be selected as initial seed and the maximum number of permitted movements (max_{moves}) are the input. In order to find multiple initial solutions and to explore the solution space in a best way, a set of weights Λ is selected in such way that $\lambda \in \Lambda : \lambda \in [0, 1]$. The two phases are applied for each $\lambda \in \Lambda$. The following sections contain detailed information about the B-GRASP and T-GRASP strategies. For each iteration and each weight $\lambda \in \Lambda$ a construction phase and a local search phase is applied. The former returns a solution and uses two different strategies, namely B-GRASP and T-GRASP. The merit function in B-GRASP uses a weighted combination of the two original objectives. In contrast, in T-GRASP the balancing constraint (6)-(7) are relaxed and added to the merit function.

After the construction phase finishes, the obtained solution may be infeasible with respect to the sales volume. Then, in order to obtain feasible solutions, during the post-processing phase infeasibility is treated as objective to be minimized. This phase consists of systematically applying the local search sequentially to each of three objectives individually. That is, first local search is applied using f_1 as merit function in a single objective manner. After a local optima is found, the local search is continued with f_2 as merit function, and then f_3. Finally, f_1 is used in the final part of the cycle. During the search, the set of non-dominated solutions is updated at every solution. It is also clear that the order of this single objective local search strategy implies different search trajectories that is, optimizing in the order (f_1, f_2, f_3) generates a trajectory different from (f_2, f_3, f_1) for instance.

3.1 B-GRASP Description

This strategy follows the generic scheme of GRASP (Figure 1). A greedy function (10) during construction phase is a convex combination of two components weighted by λ which are related with the original objectives: dispersion measure (1) and maximum deviation (2). Post-processing phase consists of linking local searches. These main B-GRASP components are detailed as follows.

B-GRASP Construction Phase: In general, the construction phase consists of the assignment of BUs to territories keeping balanced territories with respect to the demand and looking for good objective values. Before the assignment process takes place p initial points are selected to open p territories, these points are the base for the assignment process. Previous work showed us this method is very sensitive to the initial seed selection. For instance, when some seeds are relatively close to each other the growth of the territory stops way before reaching balancing. This implies some territories end up relatively small territories. So a better spread of the seeds is needed. In order to obtain best initial seeds we select p disperse initial points that have high connectivity degree. Then, the construction phase starts by creating a subgraph $G' = (V', E(V'))$ where $i \in V'$ if and only if the degree of i, $d(i) \geq f$, where f is a user-given parameter. The seeds selection is made by solving a p-dispersion problem on G'. The p nodes are used as seeds to open p territories. Suppose we obtained $\{i_1, i_2, ...i_p\}$ disperse nodes. From these seeds we obtain a partial solution $S = (V_1, V_2, ..., V_p)$ such that $V_t = \{i_t\} : \forall t \in T, T = \{1, 2., ...p\}$.

Then, at a given B-GRASP construction iteration we consider p partial territories and attempt to allocate an unassigned node keeping balanced territories with respect to the demand. To do that, this method attempts to make assignments to the smallest territory (considering the demand). Let V_{t^*} be the territory with smallest demand, $c(t^*)$ is center of V_{t^*} and $N(V_{t^*})$ is the set of unassigned nodes adjacent to V_{t^*}. If $N(V_{t^*})$ is empty we take the next smallest territory and proceed iteratively. The cost of assigning a node j to territory V_{t^*} is given by the greedy function (10), this function weights the change produced in the objective values.

$$\phi(j,t^*) = \lambda f_{disp}(j,t^*) + (1-\lambda) f_{dev}(j,t^*) \tag{10}$$

Where

$$f_{disp}(j,t^*) = \frac{1}{d_{max}} \left(\sum_{i \in V_{t^*} \bigcup \{j\}} d_{ic(t^*)} \right) \tag{11}$$

$$f_{dev}(j,t^*) = \frac{1}{\mu^{(1)}} \max \left\{ w^{(1)} \left(V_{t^*} \bigcup \{j\} \right) - \mu^{(1)}, \mu^{(1)} - w^{(1)} \left(V_{t^*} \bigcup \{j\} \right) \right\} \tag{12}$$

and the normalization parameter

$$d_{max} = \frac{(N-p)}{p} \max_{i,j \in V} d_{ij} \tag{13}$$

Following the GRASP mechanism we build a Restricted Candidate List (RCL) with the most attractive assignments which are determined by a quality parameter $\alpha \in [0,1]$ (specified by user). The RCL is computed as follows:

$$\phi_{\min} = \min_{j \in N(t^*)} \phi(j, c(t^*)) \tag{14}$$

$$\phi_{\max} = \max_{j \in N(t^*)} \phi(j, c(t^*)) \tag{15}$$

$$RCL = \{ j \in N(t^*) : \phi(j, c(t^*)) \in [\phi_{min}, \phi_{min} + \alpha(\phi_{max} - \phi_{min})] \} \tag{16}$$

Then, a node i is randomly chosen from RCL. We update the territory $V_{t^*} = V_{t^*} \bigcup \{i\}$ and the center $c(t^*)$ is recomputed. This is the adaptive part of the GRASP. We proceed iteratively until all nodes are assigned. At the end of the process we obtain a p-partition $S = (V_1, V_2, ..., Vp)$ that could be infeasible with respect to the balance of sales volume. In a few words, the proposed construction procedure tries to build territories similar in size with respect to the demand attribute. The next component of B-GRASP is the post-processing or improvement phase.

B-GRASP Post-processing Phase: It consists of linking local searches which are applied to optimize one by one each one of the objectives (it is explained at the begging of the section 3). This process starts with the final solution obtained when the construction phase finishes. Then, we start with a S solution (p-partition of V) such that $S = \{V_1, ..., V_p\}$. Additionally, $\forall V_t \in S$ a center $c(t) \in V_t$ is associated and $\forall i \in V_t$ a territory index $q(i) = t$ is known. S may be infeasible with respect to the balancing constraints (6) and (7), so in this phase B-GRASP attempts to obtain feasible solutions and simultaneously it searches for solutions that represent the best compromise between the objective functions. In order to obtain feasible solutions during this phase, balancing constraints (6) and (7) are dropped and are considered as an additional objective function instead. There are three objectives that should be minimized: (i) dispersion measure,(ii) maximum deviation with respect to the number of customers, and (iii) infeasibility related with balancing of sales volume.

$$z_1(S) = \sum_{j \in V_t, t \in T} d_{jc(t)} \tag{17}$$

$$z_2(S) = \frac{1}{\mu^{(1)}} \max_{t \in T} \left\{ \max\{w^{(1)}(V_t) - \mu^{(1)}, \mu^{(1)} - w^{(1)}(V_t)\right\}\} \tag{18}$$

$$\eta(S) = \frac{1}{\mu^{(2)}} \sum_{t \in T} \max \left\{ w^{(2)}(V_t) - (1+\tau^{(2)})\mu^{(2)}, (1-\tau^{(2)})\mu^{(2)} - w^{(2)}(V_t), 0 \right\} \tag{19}$$

The Post-processing phase attempts to find potential efficient solutions in the neighborhood of S. For doing that, we define a neighborhood $N(S)$ which is the solutions set obtained by all possible moves such that a basic unit $i \in V_{q(i)}$ is reassigned to any adjacent territory $V_{q(j)}, q(j) \neq q(i)$ into the p-partition defined by S. A movement is permitted just when the assignment keeps connected territories. Each possible movement $move(i,j)$ deletes i from territory $q(i)$ and inserts it into territory $q(j)$, $(i,j) \in E, q(i) \neq q(j)$. For example, suppose we have a partition S with the structure $S = (..., V_{q(i)}..., V_{q(j)}, ...)$, if we select the $move(i,j)$, the neighbor solution $\overline{S}$ is given by $\overline{S} = (..., V_{q(i)} \setminus \{i\}, ..., V_{q(j)} \bigcup \{i\}, ...)$. The $move(i,j)$ is accepted only if this improves the value of the objective function that is been optimized in that moment.

The neighborhood exploration consists of linking local searches in a similar way to MOAMP applied by Molina et al. [2]. The linking of local searches is made considering different arrangement of objective functions. Suppose we select the optimization order as $(z_1(S), z_2(S), \eta(S))$, then the local searches are as follows: The first local search starts with S a final solution after construction phase and attempts to find the optimal solutions to the problem with the single objective $z_1(S)$ (17). Let S^1 be the best point visited at the end of this search. Then a local search is applied again to find the best solution to the problem with the single objective $z_2(S)$ (18) using S^1 as initial solution. After that, a local search is applied to find the best solution to the problem considering the single objective $\eta(S)$ (19) and the initial solution S^2 obtained in the before optimization. At this point, we solve again the problem with the first objective $z_1(S)$ starting from S^3. This phase yields at least 3 points that approximate the best solutions to the single objective problems that result from ignoring all except one objective function. During this phase only feasible solutions are kept. Additionally, efficient solutions may be found because all feasible points are checked for inclusion in the efficient set E. This efficient set E is updated according to Pareto efficiency, this check in is made over the original objectives: dispersion (17) and maximum deviation with respect to the number of customers (19).

Pareto efficiency. A solution $x^* \in X$ is efficient if there is no other solution $x \in X$ such that $f(x)$ is preferred to $f(x^*)$ according to Pareto order. That is, $x^* \in X$ is efficient if there is no solution $x \in X$ such that $f_i(x) \leq f_i(x^*) \forall i = 1, ..., g$ and at least one $j \in \{1, ..., g\}$ such that $f_j(x) < f_j(x^*)$. So in our case $g = 2$.

We can repeat the linking local searches using other arrangement of objectives. Each local search stops when the limit of iterations is reached or when the set of possible moves is empty.

3.2 T-GRASP Description

This procedure is very similar to the B-GRASP the main difference is in the construction phase. During this phase the greedy function (20) is a convex combination (22) of three components: dispersion measure (11), maximum deviation (12) and maximum infeasibility with respect to the upper bound of sales volume balancing (21). The procedure starts with p disperse points (obtained as in B-GRASP construction phase) and the cost of assigning a node i to territory t with center $c(t)$ is measured by a greedy function (20).

$$\gamma(j,t) = \lambda_1 f_{disp}(j,t) + \lambda_2 f_{dev}(j,t) + \lambda_3 f_{infeas}(j,t) \tag{20}$$

Where

$$f_{infeas}(j,t) = \frac{1}{\mu^{(2)}} \max\left\{(1+\tau^{(2)})\mu^{(2)} - w^{(2)}\left(V_t \bigcup \{j\}\right), 0\right\} \tag{21}$$

$$\lambda_1 + \lambda_2 + \lambda_3 = 1 \tag{22}$$

Note that (21) penalize only those assignments that make infeasible the balancing constraint given by (7). The post-processing phase of T-GRASP procedure is the same as in B-GRASP.

4 Experimental Results

In our experimental work we compare the efficient solutions obtained through both procedures B-GRASP and T-GRASP in order to determine the more robust procedure. We tested 10 instances which were randomly generated based on real-world data provided by the industrial partner. These have 1000 BUs, 50 territories and $\tau^{(2)} = 0.05$. The input parameters were $f = 2, \alpha \in \{0.0, 0.6, 1.0\}, \Lambda = \{0, 0.1, 0.2, ...1.0\}$ and 1500 was the maximum number of movements during the post-processing phase.

	$\alpha = 0.0$		$\alpha = 0.6$		$\alpha = 1.0$	
Instance	$\lvert E_{bg} \bigcap E_{all}\rvert$	$\lvert E_{tg} \bigcap E_{all}\rvert$	$\lvert E_{bg} \bigcap E_{all}\rvert$	$\lvert E_{tg} \bigcap E_{all}\rvert$	$\lvert E_{bg} \bigcap E_{all}\rvert$	$\lvert E_{tg} \bigcap E_{all}\rvert$
01	1	4	1	4	0	5
02	4	2	6	0	2	5
03	1	5	7	1	7	1
04	4	3	2	3	3	6
05	3	0	3	0	3	3
06	6	0	6	1	6	0
07	6	0	8	0	6	0
08	4	0	4	0	3	1
09	8	0	1	2	3	4
10	5	0	4	1	2	6

Table 1: Experimental results associated to the metric of quality for both strategies B-GRASP and T-GRASP

Our goal on this experimental work is to determine what is the best strategy (B-GRASP or T-GRASP) over all instance sets under previous specifications. Let E_{bg} be the set of the efficient solutions obtained by B-GRASP procedure and E_{tg} the set of efficient solutions obtained by T-GRASP. Suppose we put both efficient sets in a set E^* such that $E^* = E_{bg} \bigcup E_{tg}$. We apply the definition of Pareto efficiency in E^* and we obtain E_{all}. The last is the set of efficient solutions obtained from E^*, then $E_{all} \subseteq E^*$. Figure 1 shows an example of these sets. According to this, in Table 1 for each α value we have two columns which contain the metric of quality value associated to each strategy. For example, with $\alpha = 0.0$ in the instances set $\{01, 03\}$ T-GRASP is better than B-GRASP, however in the rest of the instances B-GRASP is definitively best. When $\alpha = 0.6$ T-GRASP is better than B-GRASP just in the instances $\{01, 04, 09\}$ and B-GRASP dominates to T-GRASP in the rest of the instances. Finally, when $\alpha = 1.0$ B-GRASP dominates to T-GRASP only in the instances set $\{03, 06, 07\}$. It means, B-GRASP dominates T-GRASP in most of the cases, according to this results.

5 Conclusions

Experimental work reveals us B-GRASP strategy is the best alternative to generate good approximations to the efficient front for the bi-objective commercial territory design problem addressed in this work. Most of the time B-GRASP reported better solutions than those reported by the procedure T-GRASP over all tested instances under different parameter configurations.

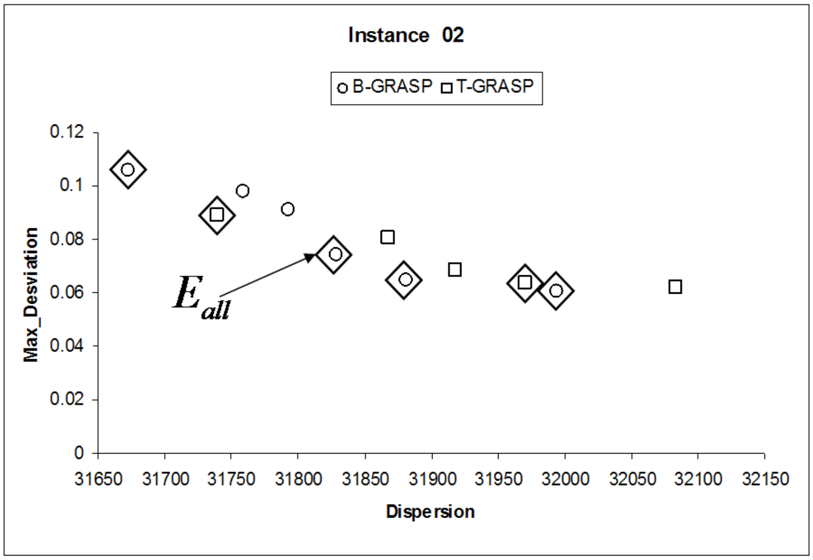

Fig. 1. Efficient fronts for instance 02 and $\alpha = 0.0$

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