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A LOCATION-ALLOCATION HEURISTIC FOR A TERRITORY DESIGN PROBLEM IN A BEVERAGE DISTRIBUTION FIRM

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Abstract: In this paper a real-world territory design problem arising in a beverage distribution firm is addressed. The firm seeks to find a partition of a set of geographic units into a given number of territories in such a way that a dispersity measure of the formed territories is minimized. In addition it is required that the territories are balanced with respect to two different node activity measures. A heuristic methodology based on a location-allocation technique is proposed to solve this problem. This location-allocation scheme consists of a two-stage iterative process where territory centers are first located and then customers are allocated to centers. This methodology has been used with relative success for territory design problems involving single balancing constraints. In our work, we extend this approach and propose an adaptation of this technique to handle multiple balancing constraints and contiguity constraints simultaneously. In addition a local search technique is applied to improve solution quality. Several experiments were conducted and the results show a good performance of the heuristic.

1. INTRODUCTION

Territory design can be viewed as the problem of grouping small geographical areas into larger geographical clusters called territories. This kind of problems arises in different applications such as political districting, design of school territories, social facilities, emergency services, sales and service territory design. The problem studied in this article is motivated by a beverage company. The company needs to distribute geographically its customers in the city according to certain planning criteria. The company seeks balanced territories (similar in size) with respect to each of two different activity measures (number of customers and sales volume). Territory contiguity is also required, that is basic units can reach each other by traveling within the territory. In addition, compact territories are desired, that is, customers within a territory are close to each other; and finally it is desired a fixed number of territories.

In the literature a technique called location-allocation has been studied and applied with relative success to solve this kind of problems (Kalcsics, Nickel, and Schröder, 2005). This scheme consists of a two-stage iterative process where territory centers are first located and then customers are allocated to centers. However this technique has been designed to solve only problems involving single balancing constraints.

In this paper we extended this technique and proposed an adaptation to handle multiple balancing constraints and contiguity constraints simultaneously. In addition a local search technique developed for this kind of problem (Ríos-Mercado and Fernández, 2006) is applied to improve the solution quality. The algorithm was implemented and tested on several instances based on information provided by our industrial partner. The results show the efectiviness of the proposed approach, as it was able to obtain solutions of good quality (both in terms of its compactness measure and feasibility with respect to the balancing constraints). Also an experiment was conducted in order to determine the appropriate value for the heuristic's parameter.

2. PROBLEM DESCRIPTION

A graph $G=(V,E)$ is used to model the problem, where a customer or basic unit i is represented by a node, and an edge between nodes i and j exists if nodes i and j represent adjacent blocks. Every node $i \in V$ has the following parameters:

geographical coordinates (c_i^x , c_i^y) and two measurable activities. Let w_i^a be the measure of activity *a* in node *i*, and $a \subset A$, where A={1,2} is the set of activities in each node. A territory is represented by a subset of nodes $V_k \subset V$, and clearly each node must be assigned to one territory only. The number of territories is a given parameter *p*.

A solution to this problem must balance every territory with respect to each activity. Due to the unique assignment constraint and to the discrete structure of the problem is practically imposible to get perfectly balanced territories. To address this issue, we measure the degree of balance by computing the relative deviation of each territory with respect to its ideal weight given by $\mu^a = \sum_{\forall i} w_i^a / p$. Another important constraint is the contiguity of each territory. This means that each territory in the solution must induce a connected subgraph of *G* to guarantee the existence of a path entirely contained in the territory for every pair of nodes.

Finally it is desired to get compact territories so that customers are near to each other. There are several measures of compactness studied in the literature (Shirabe, 2005). It has been shown that the *p*-center and *p*-median distance functions yield reasonable compactness measures. In our case we use a dispersity function of the form $\sum_{i \in V} \sum_{j \in V} d_{ij} x_{ij}$ where x_{ij} takes the value of 1 if unit *j* is assigned to territory with center in *i* and 0 otherwise, $i, j \in V$; and d_{ij} represents the euclidean distance between the nodes *i* and *j*. The motivation for this choice is that this dispersity measure produces solutions that are easier to handle in the allocation phase of our heuristic. Moreover, this measure was compared to one based on a *p*-center distance function. We found that they have similar optimal solutions in instances under study. Nevertheless, the *p*-center function measure deteriorates the performance in the allocation phase.

Then the problem is stated as finding a *p*-partition of *V* , satisfying all the specific planning criteria, that minimizes the dispersity measure. A mixed integer model is proposed as follows.

(P_0) Minimize

$$f(x) = \sum_{i \in V} \sum_{j \in V} d_{ij} x_{ij} \tag{1}$$

Subject to

$$\sum_{i \in V} x_{ij} = 1 \qquad j \in V \tag{2}$$

$$\sum_{i \in V} x_{ii} = p \tag{3}$$

$$\sum_{j \in V} w_j^a x_{ij} \le (1+\tau^a)\mu^a x_{ii} \qquad i \in V, a \in A \tag{4}$$

$$\sum_{j \in V} w_j^a x_{ij} \ge (1-\tau^a)\mu^a x_{ii} \qquad i \in V, a \in A \tag{5}$$

$$\sum_{j \in \bigcup_{v \in S} N^v / S} x_{ij} - \sum_{j \in S} x_{ij} = 1 - |S| \qquad i \in V, S \subset V / (N^i \cup \{i\}) \tag{6}$$

$$x_{ij} \in \{0,1\} \qquad i, j \in V \tag{7}$$

Objective (1) is the measure for territory dispersity that represents the sum of the euclidean distances between each node and its center. Constraints (2) guarantee that each node *j* is assigned to one territory only. Constraint (3) forces the number of *p* territories. Constraints (4)-(5) represent the territory balance with respect to each activity measure establishing the territory size within a range (measured by a tolerance parameter τ^a) around the average size μ^a . The upper bound balancing constraints (4) also ensure that if no center is placed at *i*, no customer can be assigned to it. Constraints (6) guarantee the connectivity of the territories where N^i represents the set of nodes which are adjacent to node *i*. These constraints were proposed by Drexl and Haase (1999), and they are similar to the constraints used in routing problems to guarantee the connectivity of the routes. Note that, as usual, there is an exponential number of such constraints. This model was tried to be solved to optimality using GAMS modeling software without the connectivity constraints. Small instances of 100 nodes were solved to optimality but parameter tuning was required in order to improve processing time. Nevertheless, greater instances of 500 nodes were not solved in a reasonable time. Due to the fact that current instances are even greater than those of 500 nodes, the use of a heuristic procedure is proposed.

3. PROPOSED SOLUTION METHODOLOGY

The proposed heuristic is based on a location-allocation procedure. This consists of two iterative phases: first a location phase where the centers of the territories are chosen, and then an allocation phase where the basic areas are assigned to these centers. These two phases are iteratively performed until a stopping criterion is met. As a stopping criterion we consider a given number of iterations without solution improvement. The phases of the heuristic are described in the next sections.

3.1 Location Phase

The choice of territory centers has a considerable impact on the resulting territories in such a way that a “bad” selection of centers will seldom yield acceptable territories. In this phase a new configuration of territory centers is found from the last allocation phase. The best center for each current territory is easily recomputed as the node that has the minimum of the maximum distances to each node within the territory. This is, a 1-median problem is solved for each territory.

We would like to emphasize that at the beginning we do not have a configuration of territory centers. One way to obtain this initial configuration is to use an existing *p*-median heuristic in order to get relative good centers to use in the next phase. There are several papers that address this issue, such as the work of Resende and Werneck (2004) and Rosing and Revelle (1997). One way to get an initial configuration is solving the MILP until a feasible solution is found. However, this procedure is not appropriate for larger instances (in size). Therefore a GRASP heuristic proposed by Caballero-Hernández, Ríos-Mercado, and López (2007) was used to get the required initial configurations.

3.2 Allocation Phase

As input to this phase the configuration of the centers located in the previous phase is used. Let $V_c \subset V$ the set of *p* territory centers. We could reformulate problem P_0 with a smaller number of variables and constraints, but it has been showed in the literature that the resultant problem is still a hard problem (Kalcsics, Nickel, and Schröder, 2005). A new formulation is proposed to take advantage of the problem structure and also to get better balanced territories. The tolerance is reduced to 0 in one activity and the integer constraints on the assignment variables are relaxed. The resulting model is basically a transportation problem:

P_1^a Minimize
$$f(x) = \sum_{i \in V} \sum_{j \in V} d_{ij} x_{ij} \tag{8}$$

Subject to
$$\sum_{i \in V_c} x_{ij} = 1 \qquad j \in V \tag{9}$$

$$\sum_{j \in V} w_j x_{ij} = \mu \qquad i \in V_c \tag{10}$$

$$x_{ij} \geq 0 \qquad i \in V_c, j \in V \tag{11}$$

This problem can be solved efficiently using a solver that implements the simplex method. It is necesary to emphasize that there are two problems, one for each activity. In the optimal solution of each model P_1^a the territories are perfectly balanced. However, it is possible that the unique assignment constraint is not satisfied due to the continuous variables. A basic area j for which more than one variable x_{ij}, $i \in V_c$ has positive values is called *split area* or just *split.* Note that this means that this basic area has been assigned to two or more territories. For a basic (optimal) solution of this model it is easy to prove that there are at most p-1 splits due to the fact that there are at most p-1 no basic variables when the model is solved by the simplex method.

Now it is necessary to round off the fractional variables of every split to one (one variable) or zero (the other variables). So the problem is how to round these variables in order to keep the perfect balance as much as possible. This problem is called the *split resolution problem* and the methodology designed to solve it is explained in the next section.

3.2.1 Split Resolution Problem

For each split it is necessary to decide to which territory or center it will be assigned. In order to repair a possible nonconnected territory we first insert the split node to a non connected territory where the addition of this node repairs the connectivity. After that we need to solve every split node that has not been yet assigned to a territory. The criteria that will guide these assignments are:

i) Connectivity: A split node that causes the territory to become not connected will be discarded.

ii) Compactness and balance impact: An impact measure is computed in order to choose the territory with the minimum impact. This measure is defined as follows:

$\psi(X) = \lambda F(X) + (1-\lambda)G(X)$, where:

$F(X) = \sum_{i \in V} \sum_{j \in V} d_{ij} x_{ij}$ is the dispersity function

$G(X) = \sum_{k=1}^{p} \sum_{a \in A} g^a(X_k)$ is the balance function,

with $g^a(X_k) = (1/\mu^a)\max\{\omega^a(X_k) - (1+\tau^a)\mu^a, (1-\tau^a)\mu^a - \omega^a(X_k), 0\}$

Every split is repaired according to these two criteria in order to avoid non-connected territories and also to not deteriorate the obtained balance and the compactness.

3.3 Local Search

After each iteration of the location-allocation procedure a local search proposed by Ríos-Mercado and Fernández (2006) and adapted to this problem is applied. In this local search a basic unit is tried to be moved from one territory to another. The merit function is defined as follows: $\psi(X) = \lambda F(X) + (1-\lambda)G(X)$ using the same terms that in the split resolution.

A neighborhood N(S) consists of all solutions reachable from S by moving a basic unit i from its current territory *t(i)* to a neighbor territory *t(j),* where j is the corresponding basic unit in territory *t(j)* adjacent to i, without creating a non-contiguous solution. Such a move is denoted by *move(i, j)* and is illustrated in the figure, where *move(i, j)* is represented by arc *(i, j)* (depicted in bold). Note that *move(i, j)* is allowed only if $V_{t(j)} \cup \{i\}$ is connected (which is always the case if arc *(i, j)* exists), and $V_{t(j)} / \{i\}$ remains connected. In practice an additional stopping criterion, such as *limit_moves,* is added to avoid performing the search for a relatively large amount of time. So the procedure stops as soon as a local optimal is found or the number of moves exceeds *limit_moves*.

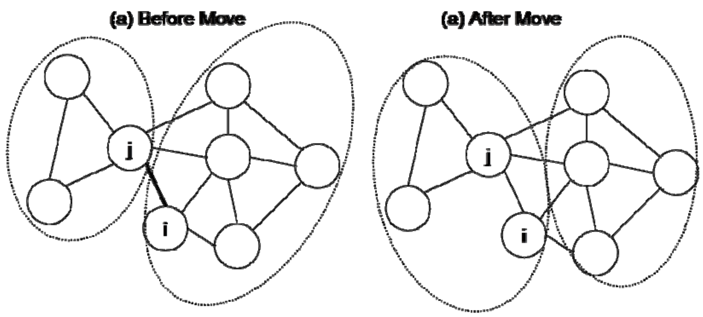

Figure 1. Movement from One Territory to Another

4. EMPIRICAL EVALUATION

This heuristic was implemented with the Sun C++ compiler workshop 8.0 under the Solaris 9 operating system and run on a SunFire V440. For the experimentation, we used a set of randomly generated instances based on data from our industrial partner. The topology of the instances was randomly and uniformly generated as a planar graph with coordinates in the range from 0 to 500 for each node. After that, for each node, both activities measures were generated from uniformly distributed random variables in a range provided by our industrial partner. The size of the test instances are 500 and 1000 basic units with a tolerance of 10% and 5% in balance, 20 instances were obtained for each combination of size-tolerance.

First, it is necessary to calibrate the parameter λ for the heuristic. All the instances were solved and their objective values were compared in order to find which value yields better results. This experiment is shown in Table 1. The values in columns 3, 4, 5 and 6 represent the number of times that the heuristic with the fixed value in the parameter λ produces the best solution compared to the other parameter values.

Table 1. Calibration of Parameter λ

Size	Balance Tolerance	λ =0.7	λ =0.8	λ =0.9	λ =1.0
500	10%	4	15	1	0
500	5%	5	13	2	0
1000	10%	2	16	2	0
1000	5%	3	15	2	0

As it is observed, the best values are obtained with the value 0.8 in the parameter λ. Consequently, in the following experiments a value of λ = 0.8 is used.

Table 2. Experimentation Results

Size	Balance Tolerance	% Improvement LS	Execution time average	Iterations average	% Connectivity Average
500	10%	46.45 %	192.76 seg.	47.43	97.5 %
500	5%	47.34 %	201.23 seg	48.02	94.4 %
1000	10%	51.23 %	289.48 seg	63.31	93.1 %
1000	5%	50. 67 %	291.53 seg	65.29	91.2 %

Table 2 display the following results. The first two columns indicate the characteristics of the set of instances. The third column shows the average improvement obtained when the local search is applied after each algorithm's iteration. The fourth column indicates the average execution time (in seconds) to solve an instance. The iterations needed to solve an instance (in average) are shown in the fifth column, and the last column represents the percentage obtained of connected solutions, that is the proportion of feasible solutions with respect to the contiguity constraints that are obtained throughout the execution of the algorithm.

As can be observed, the location-allocation phase of the heuristic was improved in a good enough percentage, meaning that the local search is essential to get better solutions. An important result was obtained regarding the execution time: even though the instances size was incremented, the execution time and the computational effort was not significantly deteriorated. This is because of the nature of the procedure; the most expensive part in the algorithm is the resolution of the splits, but the number of splits is determined by the number of desired territories *p*. In this case this number was the same for both sizes of our instances.

Another important result of the procedure was the percentage of connected solutions. Even though our method does not guarantee that all territories in each solution are connected, the procedure designed to solve the splits was very effective since over 90% of the times the procedure yield connected solutions.

5. CONCLUSIONS

In this paper we adressed a territory design problem from a specific application from a beverage distribution firm. A heuristic methodology based on a location-allocation technique was proposed in order to get good solutions for the problem. The general location-allocation technique was adapted to handle multiple balancing constraints and contiguity constraints simultaneously. To the best of our knowledge, this has not been tried in the literature. In addition, we proposed a way to get connected territories taking advantage of problem structure. We found that the performance of this technique is related to the number of requested territories. So, this technique has a good behavior and it is recommended when the number of desired territories is small compared to the number of basic areas, even in large instances. We also got a better insight of the structure of our problem and we noticed that the connectivity in the territories is easily obtained with this simple heuristic in the most of the algorithm iterations. The local search plays an important role in the solution approach, so it could be convenient to conduct more research in order to improve even more this phase. In addition, another contribution was the study for the parameter λ used in the heuristic.

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