A Heuristic for Minimum Cost Steady-State Gas Transmission Networks

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Abstract

In this paper we consider the problem of minimizing fuel cost on steady-state gas transmission problems on looped networks. We present a mathematical formulation, and propose a heuristic based on a two-stage iterative procedure. At a first stage, gas flow variables are fixed and optimal pressure variables are found via dynamic programming. At the second stage, the pressure variables are fixed and an attempt is made to find a set of flow variables that improve the objective function by exploiting the underlying network structure.

Keywords : natural gas, transmission networks, fuel minimization, heuristics

1 Introduction

A gas transmission network for delivering natural gas involves a broad variety of physical components such as pipes, regulators, and compressor stations to name a few. As the gas travels through the pipe, gas pressure is lost due to friction with the pipe wall. Some of this pressure is added back at compressor stations, which raise the pressure of the gas passing through them. In a gas transmission network, the overall operating cost of the system is highly dependent upon the operating cost of the compressor stations in a network. A compressor station's operating cost, however, is generally measured by the fuel consumed at the compressor station. Hence, the goal is to minimize the total fuel consumption used by the stations while satisfying specified delivery requirements throughout the system.

Gas transmission network problems differ from traditional network flow problem in some fundamental aspects. First, in gas networks, a pressure variable is defined at every node in addition to the flow variables representing mass flow rates through each pipe. Second, in addition to the network flow conservation constraint set, there exist two other type of constraints: (1) a nonlinear equality constraint on each pipe, which represent the relationships between the pressure drop and the flow; and (2) a nonlinear non-convex set for each compressor station, which represents the feasible operating limits for pressure and flow within the station.

In this paper we present a mathematical model for this problem (Section 2) and propose a heuristic solution procedure, which is described in Section 3.

2 Problem Statement and Mathematical Formulation

Let $G = (N, L, M)$ be a directed network defined by a set N of n nodes, a set L of l pipes, and a set M of m compressor stations. The mass flow rate on a pipe $(i, j) \in L$ is represented by u_{ij}, and the mass flow rate through a compressor station $(i,j) \in M$ is represented by v_{ij}. Note that each compressor station is represented by a special pipe which connects a pair of nodes $(i,j) \in M$, where i and j are the corresponding suction and discharge nodes, respectively. Let u, v be the vectors of u_{ij}'s and v_{ij}'s, i.e., $u = \{u_{ij}, (i,j) \in L\}, v = \{v_{ij}, (i,j) \in M\}$, and let w be the vector defined by $w = (u,v)^T$. Let $p = (p_1, ..., p_n)^T$ be the pressure vector with p_i the pressure at node i. Let $s = (s_1, ..., s_n)^T$ be the source vector with s_i the source at node i. If s_i is positive (negative), this corresponds to the gas supply limit (demand requirement) at node i. For the steady-state model, the sum of the sources is assumed to be zero, i.e., $\sum_{i=1}^{n} s_i = 0$.

The flow balance equation at a node has the following meaning: the sum of flows coming out of the node is equal to the sum of the flow entering the node. It can be represented as

$$\sum_{j:(i,j)\in L\cup M} w_{ij} - \sum_{j:(j,i)\in L\cup M} w_{ji} = s_i, \quad \forall\ i \in N, \tag{1}$$

where w_{ij} represents either u_{ij} if $(i,j) \in L$ or v_{ij} if $(i,j) \in M$.

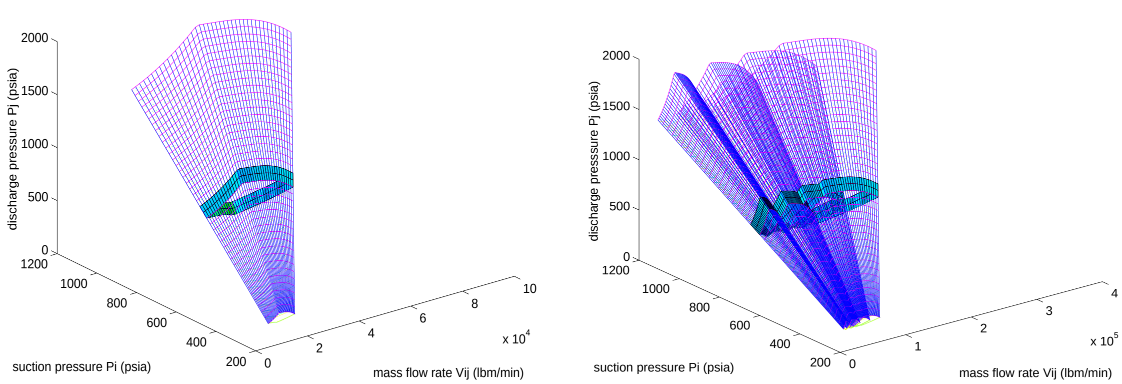
The physical law that relates the flow in the pipe to the difference of pressure at its two ends for high-pressure networks is given, as discussed in Osiadacz (Osiadacz, 1987), by the Weymouth's formula:

$$p_i^2 - p_j^2 = k_{ij} u_{ij}^2, \qquad \forall\ (i,j) \in L, \tag{2}$$

where k_{ij} is a constant whose value depends on the pipe physical properties.

The physical operational limits at each compressor station is another set of constraints, which includes the maximum/minimum compressor speed ratio, the maximum/minimum allowable volumetric flow rate. A compressor station is typically of many compressor units (which in turn can be of many types) arranged in different configurations settings. Let us assume that each compressor station (i,j) has k centrifugal compressor units hooked up in parallel.

Let D_{ij}^k denote the feasible compressor domain for variables (v_{ij}, p_i, p_j), and let $g_{ij}^k(v_{ij}, p_i, p_j)$ denote its corresponding fuel cost function. Recent work by Wu et al. (Wu et al., 1999) contains a detailed explanation about the structure of the domain D_{ij}^k, and the behavior of the fuel consumption function g_{ij}^k. Figure 1 from (Wu et al., 1999) shows an example of domain D_{ij}^{unit} ($k = 1$ centrifugal compressor unit) and a compound domain D_{ij}^4.



(a) D_{ij}^{unit} for a compressor station with a single compressor unit

(b) D_{ij}^k for compressor station (i,j) with $k = 4$ compressor units in parallel

Figure 1: Feasible domain of the compressor station of Wu et al.

The fuel cost function, g_{ij}^{unit}, in a single compressor unit is computed by

$$g_{ij}^{unit}(v_{ij}, p_i, p_j) = a_{ij} v_{ij} \left\{ \left(\frac{p_j}{p_i} \right)^m - 1 \right\}, \qquad \forall\, (v_{ij}, p_i, p_j) \in D_{ij}^{unit}, \tag{3}$$

where a_{ij} and m are constants which are determined by the specific type of compressors involved.

In our work, we use function g'_{ij}, extended version of g_{ij}^{unit}, which is given by $g'_{ij} = a_{ij} v_{ij} \{ (\frac{p_j}{p_i})^m - 1 \}$, $(v_{ij}, p_i, p_j) \in D_{ij}^k$. The mathematical formulation of the problem is given by

$$\begin{aligned}
&\text{minimize} && \sum_{(i,j)\in M} g'_{ij}(v_{ij}, p_i, p_j), && (4a)\\
&\text{subject to} && \sum_{j:(i,j)\in L\cup M} w_{ij} - \sum_{j:(j,i)\in L\cup M} w_{ji} = si, \quad \forall\, i \in N, && (4b)\\
& && p_i^2 - p_j^2 = k_{ij} u_{ij}^2, \quad \forall\, (i,j) \in L, && (4c)\\
& && (v_{ij}, p_i, p_j) \in D_{ij}^k \subset R^3,\ (i,j) \in M. && (4d)
\end{aligned}$$

The difficulty in solving this type of problems arises from the presence of non-convexity in both the set of feasible solutions and the objective function. In addition, the type of underlying network topology becomes a crucial issue. That is, for a non-looped network topology, dynamic programming approaches have been applied with relative success. See Ríos-Mercado (Ríos-Mercado, 1999) and Carter (Carter, 1998) for details of the dynamic programming algorithms.

These procedures rely heavily on theoretical results establishing that, for this type of systems, the involved flow variables can be determined in advance, and thus, eliminated from the problem. For network topologies with loops, the problem becomes more difficult because the flow variables can not be uniquely determined, so they indeed have to be explicitly treated in the model. This type of looped networks become the main focus of this work.

3 A Heuristic Solution Procedure

Let $x^0 = (v^0, p^0)$ be an initial feasible solution to problem (4). For a tree structured gas transmission network, flow variables v are uniquely determined. However, for looped networks, one may obtain better a objective function by modifying the current flow setting v^0. For this purpose, we introduce the residual network concept (Ahuja et al., 1993). The residual network was originally introduced to find the optimal flow (or to prove its optimality) in minimum cost network flow problems. We define the residual network with respect to the current flow vector v° as follows. We replace each arc (i,j) in the original network by two arcs, a forward arc (i,j) and a backward arc (j,i): the arc (i,j) has cost c_{ij} and the arc (j,i) has cost $-c_{ij}$.

In our heuristic flow modification step, the costs of the residual network are approximated by the derivatives of the objective function with respect to the flow on each compressor station, that is,

$$c_{ij} \approx a_{ij} \left\{ \left(\frac{p_j}{p_i} \right)^m - 1 \right\}, \tag{5}$$

where p_i, p_j are the current solution values delivered by dynamic programming with fixed flow variables. This cost c_{ij} is assigned at each forward edge of the residual network, while $-c_{ij}$ is assigned at each backward edge.

The cycle cost τ_C, total cost of the cycle C in a residual network, is defined by

$$\tau_C = \sum_{(i,j)\in M_C} \delta_{ij}(C) \cdot c_{ij}, \tag{6}$$

where $\delta_{ij}(C)$ equals 1 if (i,j) is contained in the cycle C and (i,j) is a forward arc of $G'(v^\circ)$, -1 if $(j,i) \in C$ and (j,i) is a backward arc of $G'(v^\circ)$, and 0 otherwise, and M_C is the set of compressor stations located in the cycle C. If τ_C is negative, then we call a negative cycle and denote it as C^-.

Modification of the flow is done by augmenting flow through a negative cycle C^-. That is, if there exists a negative cost cycle C^-, then we augment positive flow through C^-, and hence update the current flow setting. This flow modification step can be represented as

$$v^{new} = v^0 + \lambda \cdot \delta(C^-), \tag{7}$$

where $\lambda > 0$ is the positive amount of flow which will be added through the cycle, and $\delta(C^-)$ is the vector of $\delta_{ij}(C^-)$, a vector representing the negative cycle C^-. The flow modification step can be viewed as a nonlinear programming algorithm in which we try to find a direction (a vector of flow modification) such that by moving λ units in this direction, the objective function decreases. In our heuristic procedure, a negative cycle vector $\delta(C^-)$ corresponds to the search direction.

The value λ is bounded below by zero and above by $\overline{\lambda}$, which can be obtained by considering the complex inequality constraint set D_{ij}, $(i,j) \in C^-$. If $\overline{\lambda} = 0$, then the algorithm stops. Otherwise, we set $\lambda = \overline{\lambda} > 0$.

For the newly obtained flow setting v^{new}, we need to find pressure variables, which requires to rerun dynamic programming with fixed flow setting v^{new}. If dynamic programming with v^{new} has no feasible solution or no improvement has been achieved, we reduce the size of λ by setting $\lambda = \gamma\lambda$, where $0 < \gamma < 1$, and run dynamic programming until we get a desirable result. We now provide a step-by-step summary of the algorithm.

Step 1 Find an initial feasible solution $x^0 = (v^0, p^0)$.

Step 2 Construct the residual network G', and find a negative cycle C^- with negative cost τ_{C^-}.

Step 3 If $|\tau_{C^-}| < \varepsilon$, where ε is a small number, stop. Otherwise, go to Step 4.

Step 4 Set $\lambda = \overline{\lambda}$. If $\lambda = 0$, stop. Otherwise,

(a) Modify the current flow v^t by $v^{t+1} = v^t + \lambda \cdot \delta(C^-)$.

(b) Calculate pressure values using dynamic programming with modified flow v^{t+1}.
If dynamic programming yields infeasible solution, or $g^{t+1} - g^t > 0$, then set $\lambda = \gamma\lambda$, with $0 < \gamma < 1$, and go to (a). Otherwise, go to *Step 2*.

Our current ongoing research involves the computational implementation of the heuristic.

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