A TABU SEARCH APPROACH FOR MINIMIZING FUEL CONSUMPTION ON CYCLIC NATURAL GAS PIPELINE SYSTEMS

Conrado Borraz-Sánchez **Roger Z. Ríos-Mercado***

Graduate Program in Systems Engineering
Universidad Autónoma de Nuevo León
San Nicolás de los Garza, NL 66450, México
e-mail: { *conrado, roger* }*@yalma.fime.uanl.mx*

Abstract

In this paper we propose a tabu search (TS) heuristic for the problem of minimizing fuel usage on cyclic natural gas transmission networks in steady-state. The problem is modeled as a nonconvex nonlinear program. Although effective approaches, mainly based on dynamic programming, exist for handling non-cyclic topologies, the cyclic case has not been address satisfactorily. In this work, we present a successful short-term memory strategy that overcomes local optimality, and provide empirical evidence, based on a number of instances using real-world data, of the superiority of the proposed TS over existing approaches.

Key-words: *Natural gas, pipeline systems, cyclic networks, nonconvex nonlinear programming, tabu search*

1. Introduction

In this paper, the problem of minimizing the fuel consumption incurred by compressor stations in the course of natural gas transmission is addressed. During this process, energy and pressure are lost, this owed to both friction between the gas and the pipes' inner wall, and heat transfer between the gas and the environment. As compressor stations once in a while increase its pressure, moving the gas through the network system, these normally consume about 3 to 5% of the transported gas (Schroeder, 1996). This transportation cost is significant because the amount of gas being transported at large-scale systems is enormous. Hence, the problem of finding out how to optimally operate the compressors driving the gas in a pipeline network becomes significantly important.

This problem is modeled as a nonlinear program (NLP), where the cost function is typically nonconvex, and the set of constraints is typically nonconvex as well. In general, a problem with these features is very difficult to solve (NP-complete).

We would like to emphasize that, during the past thirty years the research and implementation of methodologies have been focused on non-cyclic networks, having little success on cyclic topologies. Thus, cyclic topologies instances are regarded among the harder to solve. This work is reviewed in Ríos-Mercado (2002). As far as handling cyclic topologies, no effective methods have been developed to date, to the best of our knowledge.

* Corresponding author, tel. +52 (81) 1052-3328, fax +52 (81) 1052-3321

In this work, we propose an improved heuristic procedure based on tabu search (TS) for achieving high quality local optimal solutions on cyclic networks. TS is a technique that has proven successful for many combinatorial optimization problems. The procedure, in turn, makes use of a non-sequential dynamic programming technique, proposed by Carter (1998), to find a set of optimal pressures (for the pre-specified optimal flow). The empirical evidence shows that our procedure outperforms previous approaches, such as the GRG method (Flores-Villarreal and Ríos-Mercado, 2003). To the best of our knowledge, this is the first application of a tabu search heuristic to this type of problem.

2. Model Description

Assumptions: In the present paper, we make the following modeling assumptions. We assume that the problem is in steady state. This is, our model will provide solution for systems that have been operating for a relative large amount of time. The network is balanced. This means that the sum of all the net mass flow rates in each node of the network is equal to zero. Each arc in the network has a pre-specified direction. Each parameter is known (deterministic model). Identical centrifugal compressor units hooked-up in parallel within each compressor stations is assumed.

THE NLP MODEL

Parameters:

V: Set of all nodes in the network

V_s: Set of supply nodes ($V_s \subseteq V$)

V_d: Set of demand nodes ($V_d \subseteq V$)

A_p: Set of pipeline arcs

A_c: Set of compressor station arcs

A: Set of all arcs; $A = A_p \cup A_c$

U_{ij}: Arc capacity of pipeline (i,j); $(i,j) \in A_p$

R_{ij}: Resistance of pipeline (i,j); $(i,j) \in A_p$

P_i^L, P_i^U: Pressure range for node i; $i \in V$

B_i: Net mass flow rate at node i; $i \in N$.

$B_i > 0$ (<0) if $i \in V_s$ (V_d), $B_i = 0$ otherwise

Variables: x_{ij}: Mass flow rate in arc $(i,j) \in A$

p_i: Pressure at node i; $i \in V$

Formulation:

Minimize

$$\sum_{(i,j)\in Ac} g_{(i,j)}(x_{ij}, p_i, p_j) \qquad (1)$$

subject to

$$\sum_{\{j \mid (i,j) \in A\}} x_{ij} - \sum_{\{j \mid (j,i) \in A\}} x_{ji} = B_i \quad i \in V \qquad (2)$$

$$x_{ij} \le U_{ij} \quad (i,j) \in Ap \qquad (3)$$

$$p_i^2 - p_j^2 = R_{ij} x_{ij}^2 \quad (i,j) \in Ap \qquad (4)$$

$$P_i^L \le p_i \le P_i^U \quad i \in V \qquad (5)$$

$$(x_{ij}, p_i, p_j) \in D_{ij} \quad (i,j) \in Ac \qquad (6)$$

$$x_{ij}, p_i \ge 0 \qquad (7)$$

The objective function (1) is the sum of the fuel consumption at each compressor station in the network. Constraints (2)-(3) are the typical network flow constraints representing node mass balance and arc capacity, respectively, where $\sum_{i \in V} B_i = 0$ is assumed. Constraint (4) represents the gas flow dynamics in each pipeline of the network assuming steady state. Constraints (5) denote the pressure limits in each node. Constraint (6) represents the nonconvex feasible operating domain for compressor station (i,j). More details on this model, and on the nature of $g_{(i,j)}$ and D_{ij}, can be found in Wu et al. (2000).

3. Short-Term Memory Tabu Search

The basic concept of Tabu Search (TS), as described by Glover and Laguna (2001), is a meta-heuristic superimposed on another heuristic. TS explores the solution space by moving at each iteration from a solution x_o to the best solution in a subset of its neighborhood $V(x_o)$. TS is an iterative local search procedure that can start from any initial feasible solution and it is different from other local search techniques because the it allows moving out from the current solution to a solution that makes the objective function worse in the hope that it eventually will achieve a better solution. Thus, to avoid cycling, solutions possessing some attributes of recently explored solutions are temporarily declared tabu or forbidden. The duration that an attribute remains tabu is called its tabu tenure. This is one of the most important features of this algorithm. Here, components of the proposed NONDP_TS procedure are briefly discussed.

Trial Solution Generation: To generate a trial initial solution, we use an algorithm based on a non-sequential dynamic programming technique. This is a two-phase procedure where in the first phase a feasible set of flow variables is found. Then, for that fixed set of flows, an optimal set of pressures is determined by non-sequential dynamic programming (Borraz-Sánchez and Ríos-Mercado, 2004).

Neighbohood V(x): Then a list of neighbors of x, V(x), is generated. Let us define the notion of neighborhood V(x) for each solution $x \in \Re$. By definition V(x) is a set of solutions in $\Re$ reachable from x via a slight modification of Δp^{Vx} units.

$$V(x) = \left\{ x' \in \Re \;\middle|\; x' = x \oplus \Delta p^{Vx},\ \Delta p^{Vx} \in \Psi \right\}$$

where Ψ contains all possible modifications and $x' = x \oplus \Delta p^{Vx}$ means that x' is obtained by applying modification of Δp^{Vx} units to x. Note that the neighbor definition depends only on x, because for a given x, an optimal set of pressures can be found very effectively by DP.

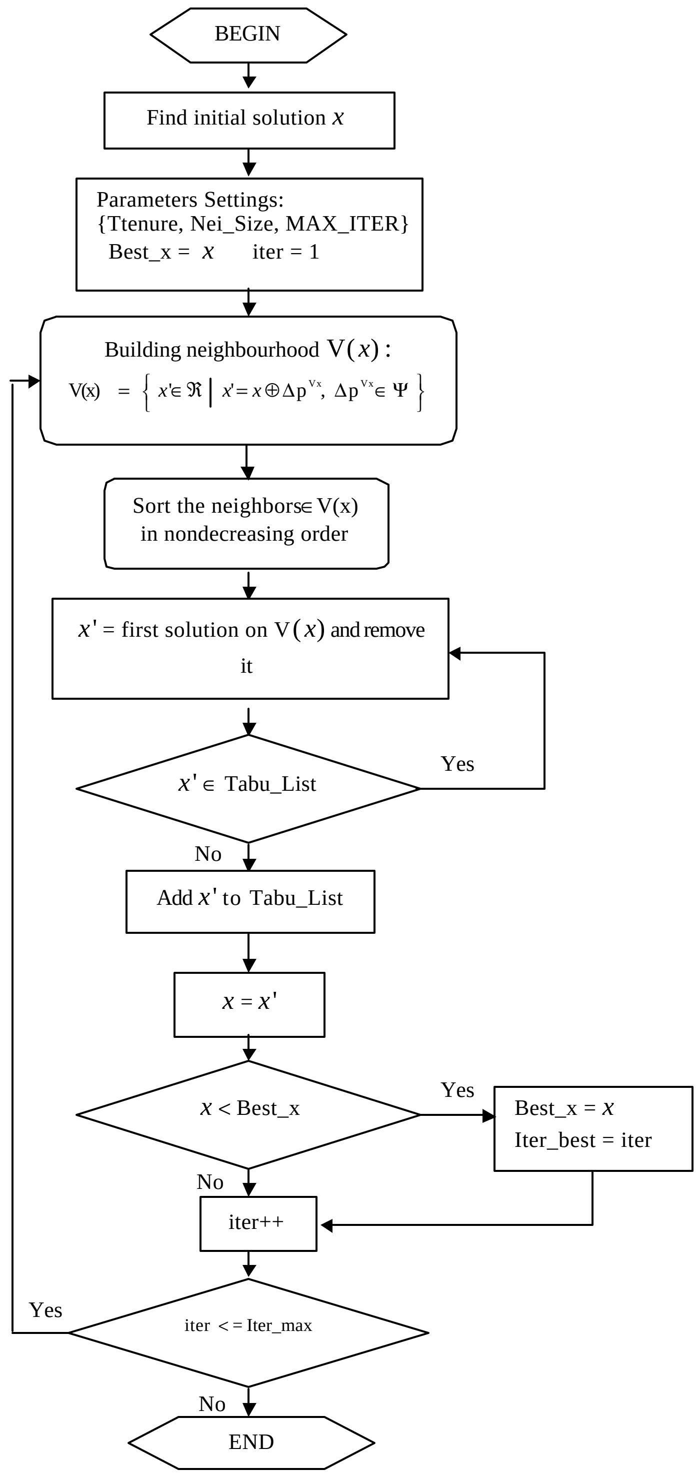

Figure 1: Flowchart of NONDP_TS heuristic algorithm.

Move Definition: At each step of the procedure, a subset $V^* \in \Re$ is generated and the local optimization problem

$$\min \left\{ g(x') \middle| x' \in V^* \subseteq \Re \right\} \text{ is solved.}$$

In order to escape from local minima, the idea is to move to the best neighbor x' in V^* even if $g(x') > g(x_o)$. Following a steepest descent approach, a move may result in a best possible improvement of the objective function value. Without additional control, however, such a process can cause a locally optimal solution to be re-visited immediately after moving to a neighbor, or in a future stage of the search process. To prevent the search from cycling between the same solutions, a tabu list is introduced.

Tabu List Restriction: The tabu list (TL), whose dimension strictly depends on the neighborhood selected, it is used to keep attributes that created the best solution in the past iterations for iterations so that they can not be used to create new solution candidates. As the iterations proceeds, a new attribute enters into a TL and the oldest one is released. Particularly, the size of TL is the control parameter of TS. The size of TL that provided good solutions usually grows with the size of $V(x)$.

The framework of the overall NONDP_TS procedure, depicted in Figure 1, is described in detail in the full version of this paper.

4. Empirical Evaluation

The proposed TS was developed in C++ and run on a Sun Ultra 10 workstation. In this computational evaluation the tabu length of 8 and stopping criteria of 100 iterations are used. All of the compressor-related data, described in Villalobos-Morales et al. (2003), was provided by a consulting firm in the pipeline industry.

Instance	GRG	TS	Gap (%)
net-c-6c2-C1	2,312,548	2,288,252	1.05
net-c-6c2-C4	1,393,061	1,393,001	0.04
net-c-6c2-C7	1,988,998	1,140,097	42.67
net-c-10c3-C2	Not found	4,969,352	N/A
net-c-10c3-C4	5,610,932	2,237,507	60.12
net-c-15c5-C2	6,313,810	4,991,453	20.94
net-c-15c5-C4	3,555,353	3,371,985	5.15
net-c-15c5-C5	Not found	7,962,687	N/A
net-c-17c6-C1	Not found	8,659,890	N/A
net-c-19c7-C4	Not found	8,693,003	N/A
net-c-19c7-C8	Not found	7,030,280	N/A

Table 1. Comparison of GRG and NONDP_TS.

Table 1 shows the excellent behavior of the NONDP_TS algorithm against the GRG method on cyclic networks (Flores-Villarreal and Ríos-Mercado, 2003). The instances tested are shown in the first column; immediately, the best objective values found by each method into the analysis are presented. The last column shows the percentage of NONDP_TS relative improvement over the solutions delivered by GRG. First, the NONDP_TS delivered solutions to all instances tested, whereas GRG failed for five of these. The results indicate that NONDP_TS procedure outperforms GRG in terms of solution quality.

We now present a comparative analysis showing the improvement achieved by the NONDP_TS approach when compared with the simple DP approach. In Table 2, the first column shows the instances tested, the second column shows the solution delivered by DP, the third column shows the best value found NONDP_TS, and finally, in its last column presents the relative improvement percentage from the DP solution. As can be seen, the improvement of NONDP_TS over the DP, is larger than 10% on 6 of 11 tested instances, and larger than 2% in 8 of the 11 instances. In only one of them the improvement is lower than 1%.

Instance	DP	DP_TS	Gap (%)
net-c-6c2-C1	2,317,794	2,288,252	1.27
net-c-6c2-C4	1,394,001	1,393,001	0.07
net-c-6c2-C7	1,198,415	1,140,097	4.86
net-c-10c3-C2	6,000,240	4,969,352	17.18
net-c-10c3-C4	2,533,470	2,237,507	11.68
net-c-15c5-C2	6,006,930	4,991,453	16.90
net-c-15c5-C4	3,669,976	3,371,985	8.11
net-c-15c5-C5	8,060,452	7,962,687	1.21
net-c-17c6-C1	9,774,345	8,659,890	11.40
net-c-19c7-C4	12,019,962	8,693,003	27.67
net-c-19c7-C8	8,693,003	7,030,280	19.12

Table 2. Comparison of simple DP and DP_TS.

In the next experiment, we attempt to assess the quality of the solution delivered by NONDP_TS. For this purpose, we derive a lower bound on the objective function as follows. First, we relax constraints (4), so the problem is reduced to a problem where we can optimize each compressor station individually. Still, the objective function is nonconvex, however we attempt to exploit the fact that it is now a function of three variables only. Furthermore, in many cases, some flow can be determined in advance, so the problem reduces to optimizing a two-variable function. For the purposes of comparison, we performed this by an exhaustive evaluation over a finite grid.

Table 3 shows these results. The first column displays the instances tested, the second and third columns show the lower bound and the best value found by the heuristic, respectively, and the last column shows the relative optimality gap obtained by NONDP_TS.

Instance	Lower Bound	$f^{TS}_{\max}$	Gap (%)
net-c-6c2-C1	2,287,470.58	2,288,252.53	0.03
net-c-6c2-C4	1,392,354.29	1,393,001.99	0.05
net-c-6c2-C7	949,909.48	1,140,097.39	16.68
net-c-10c3-C2	4,303,483.50	4,969,352.82	13.40
net-c-10c3-C4	2,015,665.98	2,237,507.93	9.91
net-c-15c5-C2	4,955,752.90	4,991,453.59	0.72
net-c-15c5-C4	3,103,697.48	3,371,985.41	7.96
net-c-15c5-C5	6,792,248.08	7,962,687.43	14.69
net-c-17c6-C1	8,129,730.11	8,659,890.72	6.12
net-c-19c7-C4	7,991,897.18	8,693,003.78	8.06
net-c-19c7-C8	5,897,768.92	7,030,280.45	16.1

Table 3. Solution quality.

As can be seen from the table, all of the tested instances have a relative optimality gap of less than 17%, 7 out of 11 instances had a relative gap of less than 10%, and 3 of these observed a gap of less than 1%. This shows the effectiveness of the proposed approach.

Finally, although our TS algorithm finds better solutions than the GRG method, it is more computationally expensive. However, any additional computation time leading to even marginal improvements can be easily justified since the costs involved in natural gas transportation are typically huge.

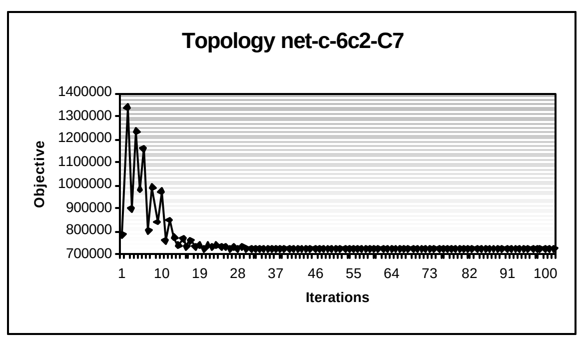

Figure 2. Convergence of the TS algorithm.

Figure 2 shows the convergence of the NONDP_TS algorithm on instance *net-c-6c2-C7*. Here it can be seen how the solution deteriorates but then it improves to a better solution, which illustrates how getting stuck at local optimality is overcome, which is the main advantageous feature of TS.

5. Closing Remarks

In this work we have presented a successful implementation of a short-term tabu search heuristic for solving the steady-state fuel cost minimization problem on cyclic topologies. The main contribution of this work is precisely on providing a method for handling cyclic topologies, outperforming existing approaches such as GRG, and simple non-sequential DP. To the best of our knowledge, this is the first application of TS to this type of problem.

There are still, though, many areas for further research. The proposed procedure is a basic short-term memory tabu search. Intensification and diversification components within the tabu search framework have not been explored. In addition, one of the great challenges is on addressing time-dependent systems.

Acknowledgments: Financial support for this research was provided by the Mexican National Council for Science and Technology (CONACYT grant J33187-A), and Universidad Autónoma de Nuevo León under its Scientific and Technological Research Support Program (PAICYT grants CA555-01, CA763-02, and CA820-04).

References

Borraz-Sánchez, C., and R.Z. Ríos-Mercado (2004). A non-sequential dynamic programming approach for natural gas network optimization. *WSEAS Transactions on Systems*, 3(4):1384-1389.

Carter, R.G., (1998). Pipeline optimization: Dynamic programming after 30 years. In *Proceedings of the PSIG Meeting*, Denver.

Flores-Villarreal, H.J., and R.Z. Ríos-Mercado (2003). Computational experience with a GRG method for minimizing fuel consumption on cyclic natural gas networks. In *Computational Methods in Circuits and Systems Applications*, pages 90-94. WSEAS Press, Athens, Greece.

Glover, F., and M. Laguna (2001). *Tabu Search*. Kluwer, Boston, USA.

Ríos-Mercado, R.Z., (2002). Natural gas pipeline optimization. In P. M. Pardalos and M. G. C. Resende (editors), *Handbook of Applied Optimization*, chapter 18.8.3, pages 813-825. Oxford University Press, New York.

Ríos-Mercado, R.Z., S. Wu, L.R. Scott, and E.A. Boyd. A reduction technique for natural gas transmission network optimization problems. *Annals of Operations Research*, 117(1-4):217-234, 2002.

Schroeder, D.W., (1996). Hydraulic analysis in the natural gas industry. In J.J.-W. Chen and A. Mital (editors), *Advances in Industrial Engineering Applications and Practice I*, pages 960-965, Cincinnati.

Villalobos-Morales, Y., D. Cobos-Zaleta, H.J. Flores-Villarreal, C. Borraz-Sánchez, and R.Z. Ríos-Mercado (2003). On NLP and MINLP formulations and preprocessing for fuel cost minimization of natural gas transmission networks. In *Proceedings of the 2003 NSF Research Conference*. Birmingham, Alabama.

Wu, W., R.Z. Ríos-Mercado, E.A. Boyd, and L.R. Scott (2000). Model relaxations for the fuel cost minimization of steady-state gas pipeline networks. *Mathematical and Computer Modeling*, 31(2-3):197-220.