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A MINLP Model for a Minimizing Fuel Consumption on Natural Gas Pipeline Networks

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Abstract: The problem of minimizing fuel consumption on natural gas pipeline networks is addressed. A mixed-integer nonlinear programming model for a special case of this problem will be presented and discussed. In addition, our computational experience on evaluating an outer approximation with equality relaxation and augmented penalty method is shown. The results, using different networks topologies over different type of compressor units, show how this model can be solved effectively.

Key words: Mixed-integer nonlinear programming, natural gas, pipeline networks

1. Introduction

Natural gas is transported by pressure throughout a pipeline system. This transmission produces energy loss caused by the existing friction between the gas and the pipeline's inner wall, and for the heat transfer between the gas and the environment. Compressor stations installed in the network compensate for this energy loss by increasing the pressure to keep the gas moving. Typically, the compressor stations consume in fuel about 3 to 5 % of the total gas flown through the network (Wu, 1998). This becomes significant as about thousand of millions of cubit feet of gas are transported every day. Hence the importance of finding a better way to operate these compressor stations through a pipeline system.

There are several variations of this problem depending on the modeling assumptions and the decisions to be made. One of the modeling assumptions made in most of the previous works is that the number of compressor units to be working within each compressor station is fixed. In our work, we consider this as a decision variable hence the model becomes a mixed-integer nonlinear problem (MINLP).

The problem is typically modeled as a non-linear network flow problem where decision variables are mass flow rate at each arc and pressure drop at each node. Examples of this representation are shown in Figures 3, 4, and 5, where the arcs represent either compressor stations or pipelines and the nodes represent supply, transshipment or demand points.

In this work we present a MINLP model for the problem of minimizing the fuel consumption in a pipeline network. Our decision variables are the pressure at each node of our network, the mass flow trough the pipeline, and the number of compressor units that have to be on within each station. We present a computational experience by evaluating an outer approximation with equality relaxation and augmented penalty method, which solves two kinds of problem: one called the master problem for solving the non-linear constrains and the sub-problem, which considers the mixed-integer part. This methodology can handle the fact that the objective function or the feasible domain can be non-convex. See Floudas (1995).

In our preliminary findings, we have seen that it is possible to solve small problems for certain kind of compressor units optimally, specially when applying a pre-processing phase (scaling the equations), but it is quite complicated finding a feasible solution for the others.

2. Problem Description

These are the modeling assumptions.

- We assume that the problem is in steady state. This is, our model will provide solution for systems that have been operating for a relative large amount of time. Transient analysis would require increasing the number of variables and the complexity of this problem.
- The network is balanced. This means that the sum of all the net flows in each node of the network is equal to zero. In other words, the total supply flow is driven completely to the total demand flow, without loss. We know that compressor stations are feed with some of the fuel driven trough the pipelines, and for sustaining this assumption we consider the cost of this consumption as an extra cost in our model named opportunity cost that represents the amount we should spend if we bought the fuel from third parties.
- Each arc in the network has a pre-specified direction.
- There are a pre-specified number of identical centrifugal compressors connected in parallel in each compressor stations.

2.1 Model

In this work, parameters and data are represented with upper case letters, while variables are represented in lower case.

Parameters:

Vs:	Set of supply nodes
Vd:	Set of demand nodes
V:	Set of all nodes in the network
Ap:	Set of pipelines arcs
Ac:	Set of compressor station arcs
A:	Set of all arcs in the network; $A = Ap \cup Ac$
U_{ij}:	Arc capacity of pipeline (i,j); $(i,j) \in Ap$
R_{ij}:	Resistance of pipeline (i,j); $(i,j) \in Ap$
N_{ij}:	Upper bound on the number of compressor units station (i,j); $(i,j) \in Ac$
P_i^L, P_i^U :	Pressure limits at each node; L = lower bound, U= upper bound; $i \in V$
b_i :	Net mass flow rate at each node; $b_i > 0$ if $i \in Vs$, $b_i < 0$ if $i \in Vd$, $b_i = 0$ otherwise

Variables:

x_{ij}:	Mass flow rate in arc (i,j); $(i,j) \in A$
p_i:	Pressure at node i; $i \in V$
n_{ij}:	Number of compressor units working at station (i,j); $(i,j) \in Ac$

Formulation:

Objective function $$\min \sum_{(i,j)\in Ac} g_{(i,j)}(x_{ij}, p_i, p_j) \quad (1)$$

Balance flow equation in each node, where $\sum_{i\in V} b_i = 0$ $$\sum_{\{j|(i,j)\in A\}} x_{ij} - \sum_{\{j|(i,j)\in A\}} x_{ji} = b_i$$

Pipeline capacity $$x_{ij} \le U_{ij} \qquad (i,j)\in Ap$$

Gas flow dynamics in each pipeline (steady state) $$p_i^2 - p_j^2 = R_{ij}x_{ij}^2 \qquad (i,j)\in Ap$$

Pressure range $$P_i^L \le p_i \le P_i^U \qquad i\in V$$

Operational limits at each compressor station $$\left(\frac{x_{ij}}{n_{ij}}, p_i, p_j\right)\in D_{ij} \qquad (i,j)\in Ac \quad (2)$$

$$x_{ij}, p_i \ge 0, n_{ij} \in \{0,1,2,\ldots,N_{ij}\} \quad (3)$$

It is important to mention that a compressor station is composed of several identical centrifugal compressors, connected in parallel that might be turned on or turned off, see Figure 1.

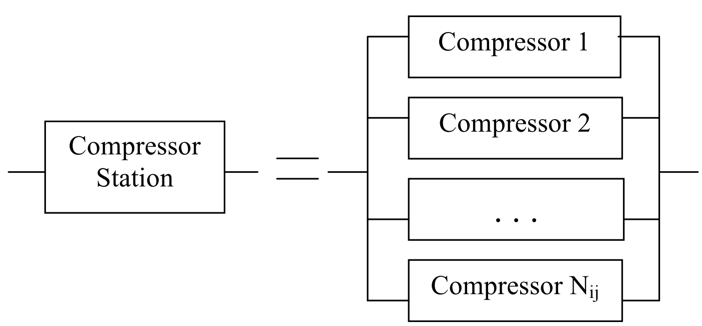

Figure 1. Representation of a compressors station

For a single centrifugal compressor unit (i,j), its domain is determined by the variables x_{ij} (flow through the arc ij), p_i (inlet pressure) and p_j (outlet pressure).

Now, when considering N_{ij} units within the station, the flow x_{ij} through the station can be equally split into the number of compressor stations working. The flow trough each unit becomes x_{ij}/n_{ij} so

$\left(\frac{x_{ij}}{n_{ij}}, p_i, p_j\right)$ must satisfy D_{ij} from equation (2). A more detailed description can be found in Wu (1998).

So it has been found (Wu et. al, 2000) that the domain D_{ij} of a centrifugal compressor (i,j) is defined by:

$$\frac{h_{ij}}{s_{ij}^{\ 2}} = A_H + B_H\left(\frac{q_{ij}}{s_{ij}}\right) + C_H\left(\frac{q_{ij}}{s_{ij}}\right)^2 + D_H\left(\frac{q_{ij}}{s_{ij}}\right)^3$$

From previous work (Wu et al., 2000) constraint (2) can be expressed as:

$$h_{ij} = \frac{ZRT_s}{m}\left[\left(\frac{p_i}{p_j}\right)^m - 1\right]$$

$$q_{ij} = ZRT_s \frac{x_{ij}}{p_i}$$

where the followings parameters are assumed known with certainty:

A_H, B_H, C_H, D_H	Constants, which depend on the type of compressor (typically estimated by least square method).
T_s	Gas temperature
Z	Gas compressibility factor
R	Gas constant
m	= (k-1)/k, where k is the specific ratio
R^L	Surge (lower limit of q_{ij}/s_{ij})
R^U	Stonewall (uper limit of q_{ij}/s_{ij})

and the following auxiliary variables are introduced:

q_{ij}	Inlet volumetric flow rate in compressor (i,j); (i,j) ϵ Ac
h_{ij}	Adiabatic head of compressor (i,j); (i,j) ϵ Ac
s_{ij}	Compressor speed that should between $S_{\min} \le S \le S_{\max}$, where speed S_{min} = minumun speed and S_{max} = maximum speed are known.

Variables h_{ij}, q_{ij} and s_{ij} are directly known to the operator; however, given the mapping from (h_{ij}, q_{ij}, s_{ij}) to (x_{ij}, p_i, p_j), it is preferable to work on the latter space from the network optimization perspective. Figure 2 illustrates this domain in the (x_{ij}, p_i, p_j) space for x_{ij} fixed.

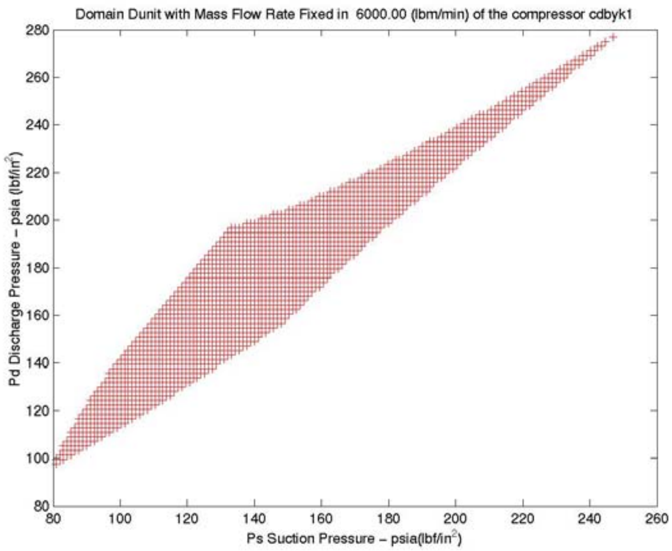

Figure 2. Domain of a compressor unit, with x_{ij} fixed in 6000 lbm/min

As we can appreciate of the domain for a centrifugal compressor is non-convexity.

It is know from previous work (Villalobos-Morales and Ríos-Mercado, 2002) that a good approximation to the real cost function is given by:

$$g_6(x_{ij}, p_i, p_j) = x_{ij}\left[A_6\left(\frac{x_{ij}}{p_i}\right)^2 + B_6\left(\frac{p_j}{p_i}\right)^2 + C_6\left(\frac{x_{ij}}{p_i}\right)\left(\frac{p_j}{p_i}\right) + D_6\left(\frac{x_{ij}}{p_i}\right) + E_6\left(\frac{p_j}{p_i}\right) + F_6\right],$$

where A_6, … , F_6 are known constants.

It is well know that the behavior of each compressor is non-linear. Furthermore, the feasible domain in (2) is a non-convex set. In addition, the objective function is also non-convex. These features make this problem particularly nasty.

Now, some MINLP solvers will allow binary variables only. In that case, the model would have to be modified in the following way. A binary variable n_{ijk}, which is equal to one if the k-th compressor of station compressor (i,j) is working, and 0 otherwise. Then we add the equation $\sum_k x_{ijk} = n_{ij} \quad \forall (i,j) \in Ac$; and allow n_{ij} to become a real variable.

3. Previous Work

3.1 Fixed Number of Compressor Units

We now highlight the most relevant contributions addressing the special case where the number of units is fixed and therefore not a variable in the model. From the optimization perspective, most of the approaches have been based on dynamic programming techniques.

The main advantages of DP are that a global optimum is guaranteed and that no linearity can be easily handled. Disadvantages of DP are that its application is practically limited to networks with simple structures, such as linear or tree-like topologies (see Figures 3 and 4), and that computation increases exponentially in the dimension of the problem, usually refered as the curse of dimensionality. In topologies with no cycles, it has been showed that the flow variables can be uniquely determined and thus eliminated from the problem. DP then focuses on finding an optimal set of pressures. Among the most relevant work we can cite Wong and Larson (1968), Lall and Percell (1990), and Carter (1998), who worked on a nonsequential DP algorithm to handle cyclic networks when the mass flow rate variables are fixed. For a more detailed description of DP applied to gas networks, the reader is refered to Ríos-Mercado (2002).

Gradient search techniques, such as the generalized reduced gradient method are also a choice. Advantages of the GRG method are that it avoids the dimensionality issue and that it could be applied to networks with cycles. However, since the GRG method is based on a gradient search method, there is no guarantee to find a global optimum, especially in the presence of discrete decision variables, so it may stall at local minima. The most significant work in this respect is due to Percell and Ryan (1987).

Other related work include Osiadacz (1987), who worked on numerical simulations of gas pipeline networks with no optimization involved; Osiadacz and Swierczewski (1994) and Osiadacz (1995), who used hierarchical optimization techniques; Wu, Boyd and Scott (1996), who used a mathematical model for the fuel cost minimization over a single unit compressor station; Kim, Ríos-Mercado, and Boyd (2000), who proposed an approximation algorithm that iteratively adjusts the flow variables in a heuristic way and then finds an optimal set of pressures; and Ríos-Mercado et al. (2002), who develop a technique to reduce the size of the network at pre-processing.

3.2 Number of Units Not Fixed

To the best of our knowledge, the only work dealing with the number of units as a variable is that of Wu et al. (2000). However their model is not quite a MINLP. They first determinate, at first level, the amount of flow through the compressor station, and then, at a second level, figure out the optimal number of units for that particular flow. That approach of course limits the search for a global optimum. Since our idea is treat all variables, at the same level, this is what motivates the choice of handling this problem as a MINLP, which becomes the main purpose of this work.

4. Proposed Solution Procedure

As we have seen in the previous section, some researchers have considered as decision variables the pressure drop at each node of the network and the mass flow transported in the pipeline. The variation we are trying to handle is to consider simultaneously that in each compressor station there is a number of compressors connected in parallel and in dependence of the flow, we will decide how many compressor to turn on for transporting the fuel. This means adding another decision variable of the integer type. We will try to solve this kind of problem considering simultaneously both variables types (continuous and integer), which makes this problem a MINLP.

Among the most popular methodologies for solving MINLP models we find:

1. Generalized Benders Decomposition (GBD)
2. Branch and Bound (BB)
3. Outer Approximation (OA)
4. Feasibility Approach (FA)
5. Outer Approximation with Equality Relaxation (OA/ER)
6. Outer Approximation with Equality Relaxation and Augmented Penalty (OA/ER/AP)

These approaches are better described in Floudas (1995).

We have chosen the outer approximation with equality relaxation and penalty augmented method because this can handle the non-convexity in the objective function, the domain or both. We know however, given the non-convexity of our model, global optimums are not necessarily guaranteed (Floudas, 1995).

The OA/ER/AP method, due to Grossmann et. al (2001) at the Engineering Design Research Center (EDRC) at Carnegie Mellon University, is implemented in a software called DICOPT. DICOPT (Grossman et al., 2001) is a solver available in GAMS (Brooke, Kendrick, and Meeraus, 1992) for solving MINLPs. The algorithm solves iteratively a series of NLP and MIP sub-problems. In the full version of the paper we will include a detailed discussion of the algorithm, and highlight the algorithmic parameters that were evaluated.

5. Computational Work

The purpose of this work is twofold. First, we would like to be able to solve a large number of instances of this problem and to show better solutions can be reached than those obtained by approaches that consider a fixed number of units within each compressor. Then, we evaluate the performance of algorithmic parameters to asses the effectiveness of the method on this type of problems. This will include finding the best parameters that yield high quality solutions.
In order to do that, we have implemented the model in GAMS. First, we consider a simple topology (see Figure 3), which consists of 6 nodes (one demand, one supply), 5 arcs (2 compressors and 3 pipelines). For this topology, 9 different types of compressors, with data taken from real-world units, were tested. The model was run on a Sun Ultra 10 under Solaris 7 OS.

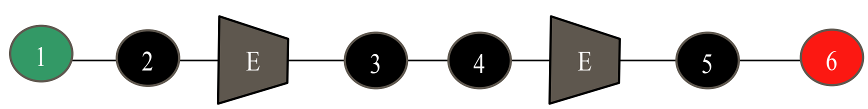

Figure 3. Linear topology.

We first ran the problem setting net flow values of 400 MMCFD (1 MMCFD = 10^6 cubic feet per day) and found numerical difficulties. Only two of nine compressors were solved. In other instances, we found that some Jacobean elements were too large, so that the algorithm was unable to find a solution.

So we increased the flow to 950 MMCFD and applied a pre-processing phase, which consisted of scaling some of the constraints. The results are shown in Table 1. As can be seen the algorithm found optimal or feasible solutions in 5 of the 9 instances. This illustrates the importance of an appropriate scaling in the preprocessing phase, but it also shows further work is necessary at pre-processing to derive models with no numerical difficulties. For the problems solved, we can also observed that most of the time was spent on solving the MINLP sub-problem.

The compressor's name is allocated in the first column in Table 1. The model status column indicates the stopping criteria used by DICOPT, where "Intermediate non integer" means that the solver failed in the NLP sub-problem, "Integer solution" means that the solver was able to find feasible solution, and "Locally optimal" means that a local optimal solution was found. The third column shows the numerical value of the objective function that represents the consumption cost. The fifth column shows how long the solver takes to find the solution, and the lasts two columns show the total time and percentage taken by each sub-problem.

Compressor type	Model status	Objective function	Number of iteration	Duration	CPLEX (time, %)	CONOPT (time, %)
Cdbnk1	Intermediate non integer	4255115.4	1007	2.616	2.26 86.24	0.36 13.76
Cdbnk3	Integer solution	9817343.8	2547	1.37	1.32 96.35	0.05 3.65
Cdbyk2	Intermediate non integer	5353535.7	1117	3.741	3.48 93.05	0.26 6.95
Cdryk1	Intermediate non integer	3781068.3	49	0.079	-	-
Cdsnk1	Locally optimal	4277220.1	56	0.102	0.10 100	0 0
Cdbnk2	Intermediate non integer	4255115.4	1007	2.501	2.18 87.21	0.32 12.79
Cdbyk1	Integer solution	4265334.1	1334	14.676	14.63 99.66	0.05 0.34
Cdhrk1	Integer Solution	4157115.7	441	3.443	3.41 99.13	0.03 0.87
Cdryk2	Integer solution	4429228.1	275	0.805	0.77 96.27	0.03 3.73

Table 1. Results of experimentation.

In those instances where the algorithm failed to find a solution, it has been observed that the maximum iteration number is reached, and the solution is infeasible.

That happens when an NLP sub-problem cannot be solved to optimality. Some NLP solvers terminate with a status other than optimal if not all of the termination criteria are met. For instance, the change in the objective function is negligible (indicating convergence) but the reduced gradients are not within the required tolerance. Such a solution may or may not be close to the (local) optimum. Another explanation is that the NLP sub-problem fails resulting in a non-optimal but feasible solutions. Sometimes an NLP solver cannot make further progress towards meeting all optimality conditions, although the current solution is feasible. Further work is under way now to attempt to exploit the current problem structure so we can deal with these difficulties successfully.

This is an ongoing research. We are still working on pre-processing to address the numerical difficulties obtained when applying the algorithm. It is expected that the full version at the paper will contain results for all instances. In addition, the full version of the paper will contain optimal results (not shown here) for other type of topologies (illustrated in Figures 4 and 5) and a comparison to the approach, which uses a fixed number of compressors.

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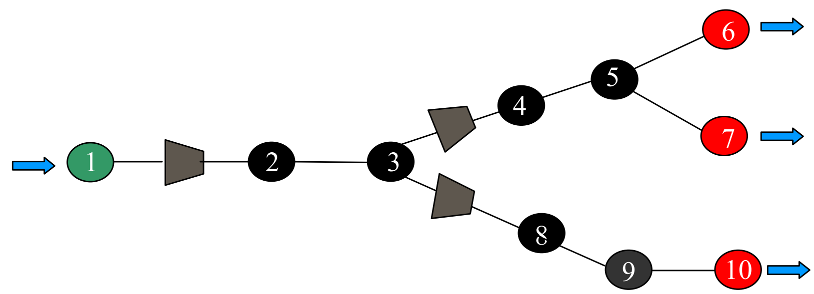

Figure 4. Example of a tree topology.

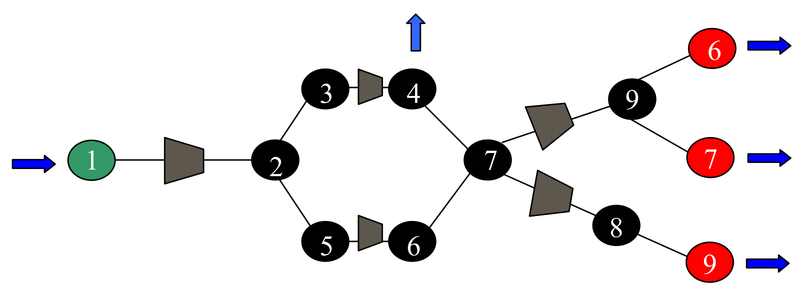

Figure 5. Example of a topology with cycles.

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