

A Network Reduction Technique for Natural Gas Pipeline Networks

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Abstract

We address the problem of minimizing the fuel consumption incurred by compressor stations in steady-state natural gas transmission networks. In the real world, these type of instances are very large both in terms of the number of decision variables and the number of constraints, and very complex due to the presence of non-linearity and non-convexity in both the set of feasible solutions and the objective function. In this paper we present a study of the properties of gas pipeline networks, and exploit them to develop a technique that can be used to significantly reduce the size of the instances, without disrupting problem structure, making it more attractive for solution methodologies from the optimization stand point.

Keywords: natural gas, pipelines, transmission networks, preprocessing

Extended Abstract

In this paper we consider the problem of minimizing the fuel cost consumption incurred by compressor stations through natural gas transmission networks.

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This problem is represented by a network, where arcs correspond to pipelines and compressor stations, and nodes correspond to their physical interconnection points. The decision variables are the mass flow rates through every arc, and the gas pressure level at every node. At each compressor station, there is a cost function that depends on the inlet (suction) pressure, the outlet (discharge) pressure and the mass flow rate through the compressor. This cost function is typically non-convex and nonlinear. In addition the set of feasible solutions is typically non-convex as well.

In general, a problem with these characteristics is very difficult to solve. This can be clearly seen in many of the approaches that have been taken in past to deal with this problem such as those of Wong and Larson [9, 10], Tsal et al. [8], Percell and Ryan [6], Lall and Percell [4], Mallinson et al. [5], to name a few. The main contribution of our work is to provide a way to significantly reduce the size of the problem instances at pre-processing without disrupting problem structure. In fact, our approach has been successfully incorporated in recent work such as Wu et al. [12], Kim [2], and Kim, Ríos-Mercado, and Boyd [3].

For a more complete review on algorithms for pipeline optimization the reader is referred to the work of Carter [1] and Ríos-Mercado [7].

We now present a description of the problem and the mathematical formulation.

The objective function of the problem is the sum of the fuel costs over all the compressor stations in the network. This problem involves the following constraints:

- (i) mass flow balance equation at each node;
- (ii) gas flow equation through each pipe;
- (iii) pressure limit constraints at each node;
- (iv) operation limits in each compressor station.

The first two are also called steady-state network flow equations. We emphasize that while the mass flow balance equations (i) are linear, the pipe flow equations (ii) are nonlinear; this has been well documented in [11, 12]. For medium and high pressure flows, when taking into account the fact that a change of the flow direction of the gas stream may take place in the network, the pipe flow equation takes the following form:

$$p_i^2 - p_j^2 = c_{ij}u|u|^\alpha, \quad (1)$$

where p_i and p_j are pressures at the end nodes of pipe (i, j) , u is the mass flow rate through the pipe, α is a constant ($\alpha \approx 1$), and the pipe resistance c_{ij} is a positive quantity depending on the pipe physical attributes.

The steady-state network flow equations can be stated in a very concise form by using incidence matrices. Let us consider a network with n nodes, l pipes, and m compressor stations. Each pipe is

assigned a direction which may or may not coincide with the direction of the gas flow through the pipe. Let A_l be the $n \times l$ matrix whose elements are given by

$$a_{ij}^l = \begin{cases} 1, & \text{if pipe } j \text{ comes out of node } i; \\ -1, & \text{if pipe } j \text{ goes into node } i; \\ 0, & \text{otherwise.} \end{cases}$$

A_l is called the node-pipe incidence matrix. Similarly, let A_m be the $n \times m$ matrix whose elements are given by

$$a_{ik}^m = \begin{cases} 1, & \text{if node } i \text{ is the discharge node of station } k; \\ -1, & \text{if node } i \text{ is the suction node of station } k; \\ 0, & \text{otherwise.} \end{cases}$$

A_m is called the node-station incidence matrix. The matrix formed by appending A_m to the right hand side of A_l will be denoted as A , i.e., $A = (A_l \ A_m)$, which is an $n \times (l + m)$ matrix.

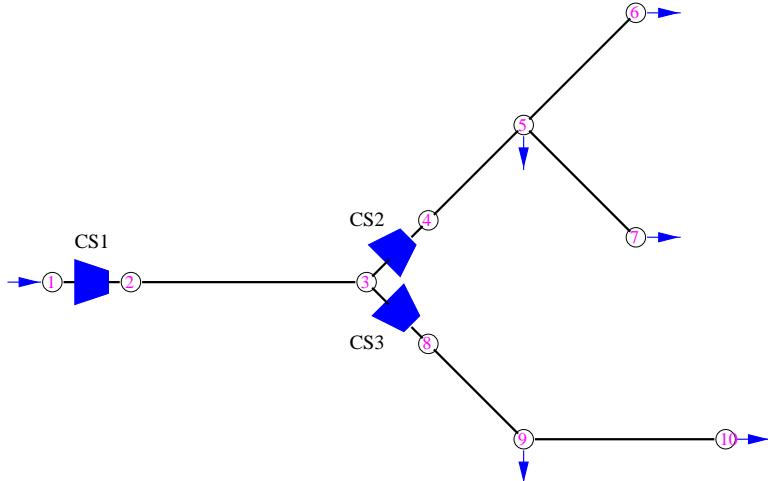


Figure 1: An example of a simple network

Figure 1 shows a simple network example with $n = 10$ nodes, $l = 6$ pipes, and $m = 3$ stations. Directions assigned to the pipes have been indicated. Note that all nodes, pipes, and stations have been

labeled separately. The matrices A_l and A_m for this network are given by

$$A_l = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 & -1 \end{pmatrix} \quad A_m = \begin{pmatrix} 1 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

where the i -th row in each matrix corresponds to node i , the columns in A_l correspond to pipes (2,3), (4,5), (5,6), (6,7), (8,9), and (9,10), respectively, and the k -th column in A_m to compressor station k (CS k) in the network. Note that in each matrix every column contains exactly two nonzero elements, 1 and -1 , which correspond to the two end nodes of the pipe or compressor.

Let $\mathbf{u} = (u_1, \dots, u_l)^T$ and $\mathbf{v} = (v_1, \dots, v_m)^T$ be the mass flow rate through the pipes and stations, respectively. Let $\mathbf{w} = (\mathbf{u}^T, \mathbf{v}^T)^T$. A component u_j or v_k is positive if the flow direction coincides with the assigned pipe or station direction, negative, otherwise. Let p_i be the pressure at node i , $\mathbf{p} = (p_1, \dots, p_n)^T$, and $\mathbf{s} = (s_1, \dots, s_n)^T$ be the source vector, where the source s_i at node i is positive (negative) if the node is a supply (delivery) node. A node that is neither a supply or delivery node is called a transition node and has s_i equal to zero. We assume, without loss of generality, the sum of the sources to be zero:

$$\sum_{i=1}^n s_i = 0. \quad (2)$$

The network flow equations can now be stated as the following:

$$\begin{cases} A\mathbf{w} = \mathbf{s} \\ A_l^T \mathbf{p}^2 = \phi(\mathbf{u}) \end{cases}$$

where $\mathbf{p}^2 = (p_1^2, \dots, p_n^2)^T$, $\phi(\mathbf{u}) = (\phi_1(u_1), \dots, \phi_l(u_l))^T$, with $\phi_j(u_j) = c_j u_j |u_j|^\alpha$ being the pipe flow equation at pipe j .

Now suppose the source vector \mathbf{s} is given satisfying the zero sum condition (2), and the bounds \mathbf{p}^L , \mathbf{p}^U of pressures at every node have been specified. The problem is to determine the pressure vector \mathbf{p} and the flow vector \mathbf{w} so that the total fuel consumption is minimized, that is,

$$\text{Minimize} \quad F(\mathbf{w}, \mathbf{p}) = \sum_{k=1}^m g_k(v_k, p_{in(k)}, p_{out(k)}) \quad (3)$$

$$\text{subject to} \quad A\mathbf{w} = \mathbf{s} \quad (4)$$

$$A_l^T \mathbf{p}^2 = \phi(\mathbf{u}) \quad (5)$$

$$\mathbf{p} \in [\mathbf{p}^L, \mathbf{p}^U] \quad (6)$$

$$(v_k, p_{in(k)}, p_{out(k)}) \in D_k \quad k = 1, 2, \dots, m \quad (7)$$

where v_k , $p_{in(k)}$, and $p_{out(k)}$ are the mass flow rate, suction pressure, and discharge pressure at station k -th, g_k is its corresponding cost function, and D_k is the feasible domain in which the triple variables $(v_k, p_{in(k)}, p_{out(k)})$ may vary, See [11, 12] for an in-depth study of the structure and properties of D_k and g_k . Note that

1. The feasible domains D_k are typically non-convex.
2. The fuel minimization functions g_k are nonlinear, non-convex and sometimes discontinuous.
3. The pipe flow equations (5) define a non-convex set.

In general, a problem with these characteristics is very difficult to solve. What we do in this paper is to propose a technique that significantly reduces the size of any instance to this problem, making it more tractable. This technique uses concepts from graph theory applied to natural gas pipeline networks.

In the full version of this paper we present two entire sections devoted to the most relevant results on graph theory and pipeline network flow equations related to our work, and a section where we develop the main theoretical results about uniqueness and existence of solutions.

Based on these developments, we end the full paper with a detailed description of the proposed network reduction method and show how to apply it in the two basic cases of network topologies, which are the most representative of real-world instances, and that represent the main contribution of our work.

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