Maximal Covering Location with Partial Coverage for Second-Level Specialized Health Care Services

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Abstract

The lack of access to Second-level Health Care Services (SHCS) in developing countries is primarily due to the scarcity of facilities and the limited investment of resources in the public sector. Access to these services directly relates to the distance the population travels to these facilities. In that sense, a maximal covering location problem can be helpful to maximize the impact of decisions related to the location of new SHCS. In this paper, we propose a model to guide the location of additional sites where second-level services can be installed in a network of public hospitals. The partial coverage and variable radius are considered in the problem to assess a large territory with different characteristics and population densities. The system is composed of multiple institutions that supply differentiated varying levels of coverage concerning their own demand and external demand. The objective of the problem is to improve the demand coverage in the system by locating new sites, since there are already sites offering different services. A case study in the Mexican public health system is conducted to assess four specialized SHCS. The obtained results evidence for the benefit of using optimization tools in the resource planning of SHCS.

Keywords: Health care planning; facility location; maximal covering location; partial coverage; integer programming.

1 Introduction

Second-level Health Care Services (SHCS) such as gynecology and pediatrics are essential in society. A large part of the population will require these services at various moments of their lives, and the demand for these services grows yearly [17]. In rural areas, the main problem is access to facilities that offer these services. In contrast, the problem is more related to capacity issues in urban areas. However, distance and time are critical to survival in emergencies in both cases.

The lack of access to SHCS in public hospitals is critical in developing countries such as Mexico. The investment in the health care infrastructure is limited and insufficient to ensure the total coverage of demand. However, there it is priority objective by the government to reach for universal access to SHCS, while avoiding inequality and discrimination in the decision-making process. Hence, each decision to invest new resources in the public sector must be taken, maximizing its impact on society. Mexico has a segmented health care system [31]. This type of system avoids making global decisions, and efforts are made individually by federal states or institutions. Recently, a change has been promoted to take federal decisions to invest resources to improve SHCS in Mexico [52]. This change aims to develop tools for the infrastructure planning as a whole system.

SHCS are available in most public hospitals under two schemes: Outpatient (ambulatory) and inpatient care. The former is any service patients receive without being admitted to a hospital or for a stay shorter than 24 hours. The latter is for patients admitted to a hospital to receive medical care. A distant hospital discourages patients from timely getting to their appointment with a health care specialist. On the other hand, inpatient services are frequently related to medical emergencies originating elsewhere. For instance, a woman may need to consult a gynecologist due to sudden complications during pregnancy.

Along with the capacity of these services, access is the main factor in the decision to select new locations. However, the capacity level can be adjusted according to the demand characteristics, but the location is permanent. Therefore, the location of services can be analyzed as a strategic first-level decision and capacity planning can be done based on the specific characteristic of the covered region after the location decision is taken. However, this decision can be made in single-stage planning based on the needs and context of the situation. Therefore, we propose this problem as a decision making tool that provides initial insight about the location of SHCS sites taking into account the global impact in the country from the coverage and access perspective. As a second-stage, a capacity planning model that includes the neighborhood environment and more specific demand patterns can be done once the general location of SHCS is determined in this problem.

Like many other developing countries, the main problem in Mexico is the geographic distribution of specialists [34]. For instance, 54.2% of them are located in 3 out of the 32 federal states of the country. In Mexico City, there are 505.7 specialists per 100,000 inhabitants, while the federal state with the lowest rate has 35.9 specialists per 100,000 inhabitants. This difference is because most second and third-level hospitals are located in the country's biggest cities. However, gynecology and pediatrics services have become more widely needed because the demand is distributed throughout the territory at different levels. Therefore, these services should be available not only in urban areas, but also in rural areas, through their installation in community hospitals.

In the Operations Research (OR) field, the maximal covering location problem (MCLP) is typically used in the health care area to locate emergency services such as ambulance stations or emergency centers [29]. However, recent works have extended its use to many other applications, such as the location of primary health care centers or hospitals [1]. In this case, we address the location of SHCS. Karasakal and Karasakal [36] consider the MCLP in the presence of partial coverage. In our work, we extended the model proposed by Karasakal and Karasakal [36] to a health care system with multiple institutions, in which each institution may provide (partial) demand coverage not only to its own beneficiaries but also for the beneficiaries of other institutions. Partial coverage is supposed to avoid an abrupt ending of coverage. Each candidate site has a different coverage critical distance due to the extensively evaluated territory composed of rural and urban areas with varying population densities. Since the system is already operating with existing sites providing the service, the coverage rate of each candidate site must be adjusted considering the interaction with these facilities. In this sense, the model's objective is to maximize the demand covered by the existing facilities and to improve the coverage of demand partially covered if a new site enhances the coverage.

The case study to be presented is based on the Mexican Health Care System (MHCS) for four services: Gynecology, pediatric care, internal medicine, and trauma care/orthopedics. The coverage distance for each candidate site is based on the population density of the place where is located. We evaluated the effect on the demand coverage, according to the collaboration level of the institutions. The impact on the demand coverage is evaluated on two cases: (i) a set of candidate hospitals that do not provide service, and (ii) in a set of candidate locations where no hospitals are currently operating. Then, we evaluate the benefit of centralizing this planning decision to solve the model in a single global instance instead of multiple federal or regional instances. Finally, we intend to find out the location of a new service in the existing network of hospitals, evaluating the impact on the demand coverage according to the number of new sites opened. The results of these experiments encourage using these types of models as part of the decision-making process in the location of public SHCS to optimize the impact of limited resources on society.

As a first contribution of our paper, we extend a maximal covering location problem with partial coverage to handle SHCS with multiple institutions. This is motivated by a real-world case from the Mexican health care system. Another contribution is the proposal of a gradual coverage function for the multi-institutional system. We also present a detailed case study from the MHCS that allows us to assess the benefit of making decisions using the solutions of the proposed model. We also suggest estimating the coverage rates with the interaction of existing facilities in a multi-institutional scheme. To solve these models, we used CPLEX's branch-and-bound (B&B) solver. All instances were optimally solved, managing to solve large instances up to 47,549 demand points, 3,583 candidate sites, and 1500 selected sites.

The remainder of the paper is organized as follows. Section 2 reviews the relevant literature on location models in related problems. This is followed by Section 3 presenting the formal definition and mathematical formulation of the problem. Then, Section 4 presents our empirical work, including the case study. Finally, the conclusion and future directions are discussed in Section 5.

2 Related literature review

The literature on facility location models and methods applied in health care management has been quite active over the past few years. Our problem is focused on the location of public health care services. A survey in the context of the public sector is presented by Marianov and Serra [41]. Significant efforts have been made in the application to health care problems. Some important surveys are proposed by Güneş et al. [32], Ahmadi-Javid et al. [1], Rais and Viana [50], Li et al. [39], Daskin and Dean [18], Brotcorne et al. [11], and Rahman and Smith [49].

The maximal covering location problem (MCLP) was proposed by Church and ReVelle [14] and White and Case [57]. The MCLP is a classic problem in the literature on facility location. This problem is designed for finite resources that cannot cover all the demand. The objective is to find the best subset of p locations that maximizes the covered demand. A demand point is covered if the distance to a facility is equal to or lower than a critical value. One feature of MCLP is that it can be structured as a p-median problem [15]. Therefore, solution procedures for the p-median problem, even heuristics, can be applied to solve MCLP. There is a number of heuristic and exact method to solve this problem, specially, for large instances. For instance, a simulated annealing was proposed by Murray and Church [46]. An approach based on an heuristic concentration was proposed by Rosing and ReVelle [53]. A Lagrangean relaxation and a dual-based heuristics with branch and bound were proposed by Galvão and ReVelle [30] and Downs and Camm [20], respectively. A greedy randomized adaptative search procedure is proposed by Resende [51].Genetic algorithms are proposed by Arakaki and Lorena [3] and Tong et al. [55].

In particular, the survey presented by Ahmadi-Javid et al. [1] reviews facility location works related to health care from 2000 to 2016. Among these, 54% of the problems were related to emergency facilities such as ambulance stations, trauma centers, or emergency off-site public access devices, while the rest were related to non-emergency facilities such as primary health centers. Location problems based on the MCLP represent 35% of works with 48 papers, and partial coverage is a characteristic only used in 10% of the works, all of them in emergency applications [54, 2, 40, 47, 12].

The model addressed in this research is based on the MCLP with partial coverage. In this problem, the classical binary coverage is replaced by a continuous parameter between zero and one calculated by a particular decay function. The greater the distance between a demand point and the facility, the more the value of this parameter approaches zero. A partial covering model can be seen as a particular application of the MCLP [57]. The first work that introduced the concept of gradual covering was Church and Roberts [16]. Later, Drezner et al. [24] applied this concept for a single facility MCLP, and Pirkul and Schilling [48] used a decay function in a capacitated version of the MCLP. The general idea of using a decay function in the MCLP was introduced in Berman and Krass [7], employing a step-wise function in a network version of the problem, providing a formulation and an effective heuristic procedure. In Berman et al. [8], the decay function was named the non-ascending general decay function with two pre-specified threshold distances. They showed how this problem could be transformed into the uncapacitated facility location problem when the set of potential facilities was discrete. An overview of gradual covering location models can be found in Berman et al. [9].

Gradual coverage has also been used in continuous facility location. For instance, a related work with a linear function in a planar space is found in Drezner et al. [25] for a single facility location. The authors proposed a B&B algorithm that produced an efficient performance for instances up to 10,000 demand points. In Karasakal and Karasakal [36], the term "partial coverage" was introduced for the MCLP taking the same considerations of previous works for multiple facility locations. A solution procedure for large instances (up to 1,000 nodes, 40 potential sites) was proposed using lagrangian relaxation.

Recently, some extensions of the gradual covering location problem have been proposed. Tavakoli and Lightner [54] proposed an MCLP-based model for allocating vehicles and the location of facilities for emergency medical services (EMS), minimizing the amount of population not covered. A goal programming problem to locate EMS stations and find the minimum number of vehicles satisfying the performance levels was proposed in Alsalloum and Rand [2]. The probability of covering a demand within the target time was minimized in the first objective, and the second objective ensured that any demand arising within the target time would find at least one ambulance available. In Eiselt and Marianov [26], the gradual covering was applied to the set covering location model, including the quality of service as a decision criterion. Lim et al. [40] proposed an extension of the MCLP that includes a minimum level of covered demand on the system and a flexible number of locations to be opened for the ambulance location problem. In Naoum-Sawaya and Elhedhli [47], a two-stage stochastic optimization model for the ambulance redeployment problem was proposed to minimize the number of relocations over a planning horizon while maintaining an acceptable service level. Drezner and Drezner [21] proposed an alternative objective function of maximizing the minimum cover of every demand point, ensuring that every demand point was covered as much as possible and there were no demand points with low cover. An ascent algorithm and tabu search were evaluated for instances up to 900 demand points. Chan et al. [12] proposed a multi-responder and gradual covering problem for automated external defibrillators in a probabilistic extension of the MCLP. The main contribution lies in developing mixed-integer linear formulation equivalents or tight and easily computable bounds. Bagherinejad et al. [5] included the joint partial coverage when a demand point was covered by multiple facilities, developing multiple heuristics for networks up to 900 demand points. They included the gradual covering concept and the cooperative coverage in a single problem. A simulated annealing and tabu search were used to solve instances of up to 150 demand nodes. In Drezner et al. [23], the gradual covering competitive facility location problem is proposed, which captures the market share by new facilities in a continuous space. Other recent applications using a gradual function are presented by Küçükaydın and Aras [38] for the location of multi-type facilities that include customer preference, by Erkut et al. [27] for ambulance location problems that include a survival function, by Dogan et al. [19] for a multi-objective location of preventing health care facilities, and by Yücel et al. [58] for the location of mobile medical sites.

The variation of the coverage radius in a gradual covering location problem has been proposed by Drezner et al. [22] for a single facility and by Bashiri et al. [6] for multiple facilities. Eydi and Mohebi [28] introduced the MCLP with gradual coverage and variable radius over multiple periods. In their work, they assumed facilities with finite capacity and variable costs directly impacting the coverage radius. They proposed a simulated annealing algorithm to solve the problem.

Table 1: Characteristics of related works with partial coverage.

Paper	Partial coverage	Variable radius	Existing facilities	Facility types	Multi-objective	Coverage types/levels	Stochastic considerations	Allocation decisions	Consider costs/profits	Joint/cooperative coverage	Limited resources	Multiple periods	Co-location	Multiple institutions
Berman and Krass [7]	✓	✓				✓								
Karasakal and Karasakal [36]	✓													
Tavakoli and Lightner [54]	✓		✓					✓			✓			
Araz et al. [4]	✓				✓	✓	✓				✓			
Meltem and Bahar [42]	✓	✓						✓						
Chan et al. [12]	✓	✓	✓				✓							
Wang et al. [56]	✓		✓	✓		✓		✓		✓	✓			
Bagherinejad et al. [5]	✓	✓						✓		✓				
Eydi and Mohebi [28]	✓	DV			✓				✓		✓	✓		
Berman et al. [10]	✓	✓								✓			✓	
Küçükaydın and Aras [38]	✓	✓		✓			✓		✓					
Chanta and Sangsawang [13]	✓							✓	✓		✓			
SLP	✓	✓	✓											✓

DV: Decision Variable

A review of gradual coverage location problems can be found in Karatas and Eriskin [37], including features such as number of facilities, feasible space, type of model (binary/gradual), coverage type (individual/cooperative) and objective type. In our work, we include two tables. Table 1 shows the features of the most related papers with partial coverage and Table 2 shows

Table 2: Solution methods and instances sizes of related works with partial coverage.

Paper	Method	Software/Solver	Demand nodes	Candidate sites	Selected sites
Berman and Krass [7]	B&B and Greedy heuristic / LP-Relaxation	Cplex	400	400	80
Karasakal and Karasakal [36]	Heuristic: Lagrangian relaxation based solution procedure		1,000	40	24
Araz et al. [4]	Lexicographic optimization and different versions of the Fuzzy goal programming	Cplex 8.0	50	50	8
Meltem and Bahar [42]	B&B	Cplex 12.4 / Gurobi 5.0.2	81	22	20
Chan et al. [12]	B&B	Cplex 12.1	11,701	5,000	200
Wang et al. [56]	B&B	GAMS/BARON	420	17	6
Bagherinejad et al. [5]	Cooperative covering and location-allocation features	Heuristic: Simulated anneling and Tabu search	150	150	20
Eydi and Mohebi [28]	Heuristic: Simulated anneling	Cplex	100	100	5
Berman et al. [10]	B&B and Heuristics: greedy heuristic, ascent heuristic, and Tabu search.		400	400	133
Küçükaydın and Aras [38]	B&B / Lagrangian relaxation / Local search	Cplex 12.8	1,000	250	Variable
Chanta and Sangsawang [13]	B&B	OPL	104	104	15
Karatas and Eriskin [37]	B&B	Cplex 12.5.0.1	4000	2000	3-10
Haghi et al. [33]	B&C / Adaptive large neighborhood	Cplex 12.1.0	30	5	1-4
SLP	B&B	Cplex 20.1.0	47,549	3,583	1500

some information related to the solution method and the problem sizes in the case studies. The last row in both tables corresponds to the model addressed in this paper, the MCLP for a segmented system with partial coverage (SLP). Variable coverage radius, interaction with existing facilities, and multiple institutions are features considered in this problem. The proposed model in this paper is a particular application of the MCLP with gradual covering that incorporates the handling of multiple institutions where the coverage of demand points can be expanded through collaboration among institutions. A function to determine the benefit of new facilities in the demand coverage is proposed. As far as we know, the MCLP with multiple institutions (organizations) has not studied in previous works. However, other works related to health care planning problems that incorporate a segmented system are proposed by Mendoza-Gómez et al. [45] and Mendoza-Gómez et al. [44] for the planning of highly specialized health care services, and Mendoza-Gómez and Ríos-Mercado [43] for location-allocation of primary health care units.

As far of solution methods is concerned (depicted in Table 2), some of the previous models were solved by commercial solvers, indicated in the third column, and others by heuristic methods such as greedy heuristics, tabu search, and simulated annealing. The instance sizes of case studies are indicated by the number of demand points, candidate sites, and selected sites in the last three columns. The instances used in our case study are the largest concerning the number of demand points and selected sites, but they occupy second place in the number of candidate sites. However, all instances tested were optimally solved by the B&B algorithm of CPLEX. Many recent improvements have been made in the performance of exact methods to solve integer programming problems, optimize computational resources, and develop new technologies. These advances avoided the need to develop alternative heuristic methods for solving the problem addressed and the size of the instances considered in our work.

3 Formulation of the problem

3.1 Problem description

The goal of this problem is to locate new sites for installing SHCS in a system with multiple institutions maximizing the covered demand. The coverage is based on partial coverage using a decay function. Each institution operates a set of Health Care Units (HCUs) that, in addition to covering the demand of its own beneficiaries, also cover beneficiaries of other institutions with a different coverage level. The number of new sites to be installed is defined by each institution. At each demand point, there are beneficiaries from each institution. There is a network of HCUs that is already operating with the service for each institution and they cover some demand points. However, with the new sites, additional demand points can be covered and the demand coverage of some demand points can be improved. Figure 1 shows an small example of a health care system with partial coverage and three institutions: A, B and C.

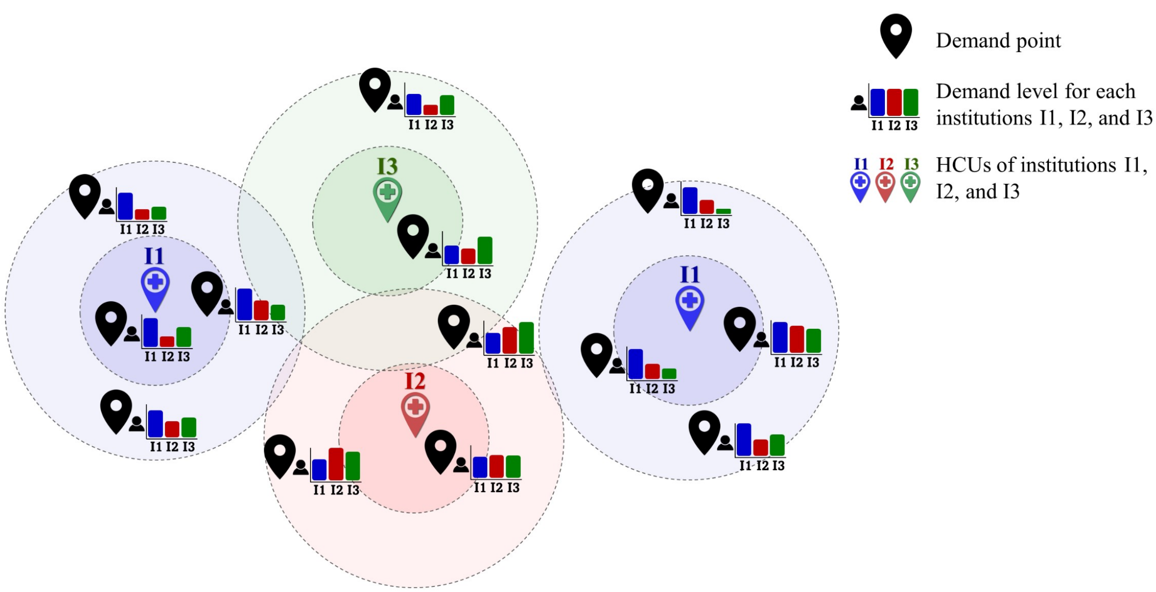


Figure 1: Example of a health care system with multiple institutions.

3.2 Mathematical model

The sets, parameters, and variables of the problem are described below:

Sets:

M Set of demand points; $i \in M$.

K Set of institutions in the system; $q, k \in K$.

G Set of HCUs with the service installed; $j \in G$.

N Set of candidate sites where the service can be installed; $j \in N$. The candidate sites for institution k is given by N^k. Clearly, $\cup_{k \in K} N^k = N$.

N_i Subset of N such that the demand point $i \in M$ is at least partially covered ($a_{ij} > 0$); $j \in N_i$.

N_i^k Subset of N_i that belongs to institution $k \in K$; $j \in N_i^k$.

G_i^k Set of HCUs with the service installed that belongs to institution $k \in K$ and cover the demand point $i \in M$ ($a_{ij} > 0$); $j \in G_i^k$.

Parameters:

h_i^k Demand in number of beneficiaries in demand point i of institution k; $i \in M$, $k \in K$.

l_j is the primary coverage radius of the candidate site j; $j \in G \cup N$.

u_j is the secondary coverage radius of the candidate site j; $j \in G \cup N$.

d_{ij} is the distance between the demand point i and the site j; $i \in M$, $j \in G \cup N$.

a_{ij} Coverage rate of the site j for the demand point i such that $0 \leq a_{ij} \leq 1$; $i \in M$, $j \in G \cup N$.

b_i^k Current coverage rate of demand point i for beneficiaries of institution k; $i \in M$, $k \in K$.

Φ_{ij}^k Benefit in the coverage level of beneficiaries of institution k at demand point i if the service is installed in site j; $k \in K$, $i \in M$, $j \in N_i$.

P^k Maximum sites number of institution k where the service can be installed; $k \in K$.

λ^q Percentage of collaboration of institution q for beneficiaries of other institutions; $q \in K$.

Decision variables:

Y_j Binary variable equal to 1 if the service is installed in site j; 0, otherwise; $j \in N$.

X_{ij}^k Binary variable equal to 1 if the beneficiaries of institution k at demand point i are covered (partially or fully) by the candidate site j, and this site has the highest benefit in the coverage rate (Φ_{ij}^k) for demand point i among all other selected sites; 0, otherwise; $k \in K$, $i \in M$, $j \in N_i$.

Note that for X_{ij}^k, the beneficiaries of institution k at demand point i could be allocated to any candidate $j \in N_i$, including other institutions.

The formulation of the problem is the following:

$$\max \quad \sum_{k\in K}\sum_{i\in M}\sum_{j\in N_i} h_i^k \Phi_{ij}^k X_{ij}^k \tag{1}$$

$$\text{s.t.} \quad \sum_{j\in N^k} Y_j \le P^k \qquad k \in K \tag{2}$$

$$\sum_{j\in N_i} X_{ij}^k \le 1 \qquad k \in K, i \in M \tag{3}$$

$$X_{ij}^k \le Y_j \qquad k \in K, i \in M, j \in N_i \tag{4}$$

$$Y_j \in \{0,1\} \qquad j \in N \tag{5}$$

$$X_{ij}^k \in \{0,1\} \qquad k \in K, i \in M, j \in N_i \tag{6}$$

Objective function (1) maximizes the benefit in the demand coverage of all institutions in all demand points. Constraints (2) impose a limit on the maximum number of new sites where the service can be installed for each institution. The beneficiaries of each institution at each demand point can be covered by one facility, either of the same institution or another, according to constraints (3). The binary variable X_{ij}^k determines the highest benefit in the objective function for the demand of each institution at each demand point. In case of a tie, one active site is randomly chosen. According to constraints (4), if the candidate site j is not selected in the solution, all the associated X_{ij}^k are equal to zero. Binary conditions regarding the decision variables are imposed by constraints (5) and (6).

Note that the maximum number of binary variables (assuming that $N_i = N$, $\forall i \in M$) is given by $|N|(|K| \times |M| + 1)$, and the maximum number of constraints is $|K|(1 + |M|(1 + |N|))$.

3.3 Determining Φ_{ij}^k

A typical coverage binary function is defined by a critical distance around each candidate site. However, this function type considers an abrupt ending coverage which may not represent a real situation. A non-increasing function is used to enlarge the coverage of a facility to a second critical distance avoiding the abrupt coverage ending [8]. In this case, the coverage level gradually decreases in the gap between these two critical distances l_{ij} and u_{ij}. The non-increasing function is given by:

$$a_{ij} = \begin{cases} 1 & \text{if} \quad d_{ij} \le l_j \\ \frac{u_j - d_{ij}}{u_j - l_j} & \text{if} \quad l_j < d_{ij} < u_j \\ 0 & \text{if} \quad d_{ij} \ge u_j. \end{cases} \tag{7}$$

In Equation (7), if the distance between a demand point i and a candidate site j is less than or equal to the primary coverage radius, this point is fully covered ($a_{ij} = 1$). The demand point is not covered if the distance is equal to or greater than the secondary coverage radius ($a_{ij} = 0$). The demand point is partially covered if the distance is between these two critical bounds ($0 < a_{ij} < 1$). In this problem, the values of the primary and secondary coverage radius are different for each site according to the population density of the place where they are located. This consideration is done because population densities vary widely from region to region and we are assuming that the coverage distance must be related to population density.

The primary coverage radius l_j can be determined by a function that considers the population density behavior as proposed in the case study in Section 4.2. For the secondary coverage radius, we propose a proportional increase of l_j (Δ) according to Equation (8). However, there are many ways in which these bounds can be determined.

$$u_j \quad = \quad (1 + \Delta)l_j \tag{8}$$

The set of HCUs where service is installed (G) is required to calculate the additional benefit in demand coverage (Φ_{ij}^{k}) of demand points that fall inside the secondary coverage radius of a candidate site. For demand points not currently covered, there is a logical benefit if the service is installed in a nearby site, but, for some demand points that are already covered, a nearer HCU with the service installed can improve the coverage level. Demand points whose coverage cannot be improved by any candidate site (i.e. $\Phi_{ij}^{k} = 0$) are not considered in the formulation because there is no impact on the objective function. For determining Φ_{ij}^{k} it is required to first compute the current coverage level of each demand point (b_i^k). This parameter indicates the current coverage for the beneficiaries of each institution at each demand point. This value takes into consideration HCU with the service installed of the same institutions ($j \in G_i^k$) and other institutions ($j \in G_i^q | q \neq k$), but these last ones are multiplied by λ^q because the collaboration percentage applies for all the HCUs that supply the service.

The proportion of collaboration is considered because each institution must prioritize its own demand; therefore, there must be a distinction between coverage of internal and external demand. This parameter must be fixed between $0 < \lambda^q < 1$. As the value of λ^q increases, the coverage level to other institutions also increases. In our work, we assume this value as a function of q only; however, there might be situations where this parameter can be defined for each site and the collaboration agreement between each pair of institutions (i.e., λ_{ij}^{qk}).

$$b_i^k = \max\{\max_{j \in G_i^k}\{a_{ij}\}, \max_{j \in G_i^q | q \neq k}\{\lambda^q a_{ij}\}\} \qquad k \in K, i \in M \tag{9}$$

The benefit of the coverage rate is calculated by subtracting b_i^k from a_{ij}. If this value is negative, the benefit is equal to zero. Demand points such that $\sum_{k \in K} \sum_{j \in N_i} \Phi_{ij}^{k} = 0$ are not considered in the formulation. The equation to calculate Φ_{ij}^{k} is the following:

$$\Phi_{ij}^k = \begin{cases} \max\left\{a_{ij} - b_i^k, 0\right\} & \text{if} \quad j \in N_i^k \\ \max\left\{\lambda^q a_{ij} - b_i^k, 0\right\} & \text{if} \quad j \in N_i^q | q \neq k. \end{cases} \tag{10}$$

Coverage rate (a_{ij})

	j		
i	A	B	C
1	0.5	0	0
2	1	0.1	0.2
3	0	0.5	0

Percentage of collaboration

λ^1	0.8
λ^2	0.6

				Additional coverage (Φ_{ij}^k)	
i	k	h_i^k	Current coverage (b_i^k)	j =A	j =B
1	1	10	0	max{0.5−0,0}=**0.5**	**0**
1	2	10	0	max{0.5(0.8)−0,0}=**0.4**	**0**
2	1	10	0.2	max{1−0.2,0}=**0.8**	max{0.1(0.6)−0.2,0}=**0**
2	2	10	0.2(0.8)=0.16	max{1(0.8)−0.16,0}=**0.64**	max{0.1−0.16,0}=**0**
3	1	10	0	**0**	max{0.5(0.6) −0,0 }=**0.3**
3	2	10	0	**0**	max{0.5−0,0}=**0.5**

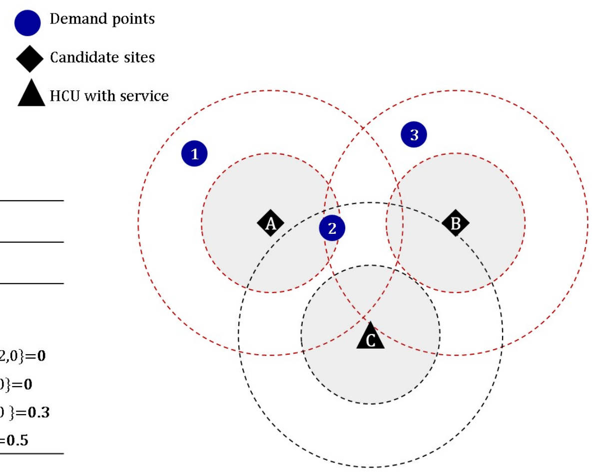


Figure 2: Determination of additional demand coverage in Example 1.

We now present a small example illustrating the model settings and how λ^q influences the objective function when there are multiple institutions. This example is displayed in Figure 2.

Example 1 *Let $M = \{1, 2, 3\}$, $K = \{1, 2\}$ $N = \{A, B\}$, and $G = \{C\}$.*

In this example, there are three demand points (i =1, 2, and 3), two institutions (k =1 and 2), two candidate sites (j =A and B), and one HCU that supplies the service (j =C). A and C belongs to institution 1, and C to institution 2. The subsets are defined as follows:

- *The sets of candidate sites that cover each demand point are: $N_1 = \{A\}$, $N_2 = \{A, B\}$, and $N_3 = \{B\}$.*
- *The sets of candidates sites of each institution that cover each demand point are: $N_1^1 = \{A\}$, $N_1^2 = \{\varnothing\}$, $N_2^1 = \{A\}$, $N_2^2 = \{B\}$, $N_3^1 = \{\varnothing\}$, and $N_3^2 = \{B\}$.*
- *The sets of HCUs that provide the service and cover each demand point are: $G_2^1 = \{C\}$, and all other sets are empty.*

Let us suppose that the number of beneficiaries of each institution is the same at each demand point ($h_i^k = 10$). In Figure 2, the left-hand side figure represents the physical locations of demand points and sites with their coverage radius. The two tables at the top right of the figure show the coverage rate for each site and the percentage of collaboration between institutions, respectively. The bottom-right table shows how parameters b_i^k and Φ_{ij}^k are computed. For example, if the candidate *site A is selected, the additional demand covered will be equal to 23.4 obtained from* $0.5(10) + 0.4(10) + 0.8(10) + 0.64(10)$. *And, if the candidate site B is selected, the additional demand covered will be equal to 8 obtained from* $0.3(10) + 0.5(10)$. *In conclusion, candidate site A is a better option than B.*

3.4 Special case: The MCLP with partial coverage

The following problem (NSLP) is a special case of SLP considering a single institution. This case could be applied in many countries with a centralized public institution or a segmented system with no collaboration between institutions ($\lambda^q = 0$, $\forall q \in K$), solving this model for each institution. In this problem, the coverage of HCUs with the service already installed is also considered.

The following parameters and the decision variables must be redefined as follows:

Parameters:

h_i Number of beneficiaries in demand point i; $i \in M$.

p Maximum number of candidate sites where the service will be installed.

a'_{ij} Benefit in the coverage level at demand point i if the candidate site j is selected; $i \in M$, $j \in N_i$.

Decision variables:

X_{ij} Binary variable equal to 1 if the candidate site j is selected and it has the highest benefit in the coverage rate (a'_{ij}) for demand point i among all other selected sites; 0, otherwise; $i \in M$, $j \in N_i$.

The increase in the coverage level of beneficiaries at each demand point for each candidate site is calculated as follows:

$$a'_{ij} = \max\{a_{ij} - \max_{l \in G_i}\{a_{il}\}, 0\} \qquad i \in M, j \in N_i. \tag{11}$$

The formulation of the problem is the following:

$$\max \quad \sum_{i \in M} \sum_{j \in N_i} h_i a'_{ij} X_{ij} \tag{12}$$

$$\text{subject to} \quad \sum_{j \in N} Y_j \le p \tag{13}$$

$$\sum_{j \in N_i} X_{ij} \le 1 \qquad i \in M \tag{14}$$

$$X_{ij} \le Y_j \qquad i \in M, j \in N_i \tag{15}$$

$$Y_j \in \{0, 1\} \qquad j \in N \tag{16}$$

$$X_{ij} \in \{0, 1\} \qquad i \in M, j \in N_i \tag{17}$$

In the case of new services in the system, the previous model can be used to replace a'_{ij} for the original coverage rate a_{ij}. A computational study with an application of this model is introduced in Section 4.8.

4 Case Study

4.1 Experimental settings

In this section, the problem is applied separately to four second-level services in the MHCS: gynecology (S1), pediatric care (S2), internal medicine (S3), trauma care and orthopedics (S4). For each service, the four leading public institutions in Mexico are considered to evaluate the model: (I1) The Ministry of Health (SSA) and IMSS-Bienestar, (I2) IMSS, and (I3) ISSSTE. The first two institutions are unified as a single one because both of them attend to uninsured populations. The main difference is that IMSS-Bienestar is located in regions with extreme poverty. I2 is an institution that provides services to formal workers in the private sector, and I3 offers services to public sector workers. There are many other institutions in the health care system, but their affiliated members represent about 4.7% of the population. According to INEGI [35], the population of Mexico was 126,014,024 inhabitants in 2020, distributed among 189,280 demand points. For each service, the demand was determined based on the number of each institution's beneficiaries obtained from the Census 2020. Demand is determined by multiplying the number of beneficiaries and the proportion of the population to which each service is intended. For gynecology, the proportion of women from 12 years old at each demand point was considered, and for pediatrics, the proportion of the population up to 18 years old. All the population was considered for internal medicine, trauma care, and orthopedic services. Table 3 shows the demand (beneficiaries) and demand points with affiliated members of each institution for each service. The sum of demand by row is lower than the total population (126,014,024) because we only consider the people that are affiliated with these institutions. In the case of S3 and S4, the demand is the same because these services are aimed at all the affiliated members of each institution. For each institution, demand points where there are no beneficiaries were discarded, for instance, I3 for S1 has 45,315 demand points with affiliated members out of 189,280 demand points of Mexico. Universal Transverse Mercator coordinates were used for demand points, current locations, and candidate sites to calculate the Euclidean distances. This metric was used as an approach to the actual distances that were out-of-reach due to the high number of calls and its high costs in web mapping platforms.

The data set is available at: `https://data.mendeley.com/datasets/s8x7nsjrgx`.

For determining sets G and N, we consider hospitals but also advanced primary HCUs where service can be installed; therefore, we refer to both sites types just as "HCUs". Table 4 shows the number of the existing HCUs of each institution that supply each service (G); they were obtained

Table 3: Demand and demand points for each service by institution.

	Total demand (No. beneficiaries)			Total demand points		
Service	I1	I2	I3	I1	I2	I3
S1	26,767,059	19,732,411	3,063,010	106,972	74,418	45,315
S2	21,036,798	13,715,433	2,060,414	104,726	73,348	45,044
S3	65,555,276	47,168,735	7,159,057	107,137	74,522	45,326
S4	65,555,276	47,168,735	7,159,057	107,137	74,522	45,326

from an official data-base of the Ministry of Health. For the case of set N, we consider that this problem can be applied to two different cases: to open a new HCU in a candidate location (N_A) and to install the service in an existing HCU (N_B). For N_A, existing HCUs of each institution were analyzed to evaluate if the service can be installed. For N_B, demand points with no HCUs and with a population density greater than or equal to 10,000 inhabitants were considered. As can be seen in the table, institutions I2 and I3 supply all services in nearly all of their HCUs. On the other hand, S1 and S2 are available in almost the same subset of HCUs for I1, S3 is available in a lower number of HCUs, and S4 is the one with the lowest number of sites where service is available.

Table 4: Number of available HCUs and candidate locations.

	Existing HCUs (G)				Candidate locations (N_A)				Candidate HCUs (N_B)			
Service	I1	I2	I3	Total	I1	I2	I3	Total	I1	I2	I3	Total
S1	634	198	89	921	650	913	1,004	2,567	120	48	17	185
S2	613	200	90	903	647	913	1,004	2,564	160	46	16	222
S3	437	205	88	730	667	913	1,004	2,584	260	30	18	308
S4	263	172	89	524	667	913	1,004	2,584	434	63	17	514

For determining set M for each service, we identify demand points such that $\sum_{j \in N_i} \Phi_{ij}^k > 0$ using Equations (9) and (10). Table 5 shows the number of demand points to be considered in the problem by institution once all demand points such that $\sum_{j \in N_i} \Phi_{ij}^k = 0$ are discarded. In this case, the largest instance was the one for S3 with 47,549 demand points as shown in Table 2.

Table 5: Demand points considered in the problem.

	Demand points			
Service	I1	I2	I3	Total
S1	8,847	6,214	3,399	18,460
S2	12,361	10,629	4,802	27,792
S3	22,878	15,414	9,257	47,549
S4	19,864	14,246	8,979	43,089

Solution method. The branch-and-bound algorithm from the CPLEX callable library, version 20.1.0, with a C++ API was used to find the optimal solution for each instance. The experiments were carried out in an Intel Core i7-5600U at 2.60GHz with 16GB of RAM under Windows 10 operating system.

4.2 The variable coverage radius for the Mexico case

We propose a function that calculates the coverage radius according to the population density of each municipality where a candidate site is located. To the best of our knowledge, this particular type of function has not been employed in this kind of problem. The population density distribution of the 2,469 municipalities (counties) of Mexico in 2020 obtained from the INEGI website (`http://www.inegi.org.mx`), is shown in Figure 3. The demand points are grouped by municipalities in Mexico, and this is the lowest level with data on population density. The municipalities on the horizontal axis are sorted by population density, and the cumulative population is shown on the vertical axis. We can note that half of the population lives in areas with a population density lower than 400 inhabitants per square kilometer (inh/km^2). Three-quarters of the population lives in a territory with a population density lower than 2,000 inh/km^2, and the remaining population (25%) lives in a territory between 2,000 to 17,624 inh/km^2. In this context, we designed a logarithmic function with high sensitivity to low population density rates (e.g. 1-2000 inh/km^2), but that includes the entire threshold values of the population density rates. The coverage radius decreases as the population density increases, but in a logarithmic decrease.

The function to estimate a variable coverage radius is presented in Equation (20). The graphical representation of the coverage radius function applied to the municipalities of Mexico is shown in Figure 4. The function is adjusted based on a minimum and a maximum coverage radius. These limits are adjusted in a range of population density rates ($\delta_{\min}$, $\delta_{\max}$). Two coefficients that depend on the previous parameters must be determined to adjust the function.

The notation in the equations is the following:

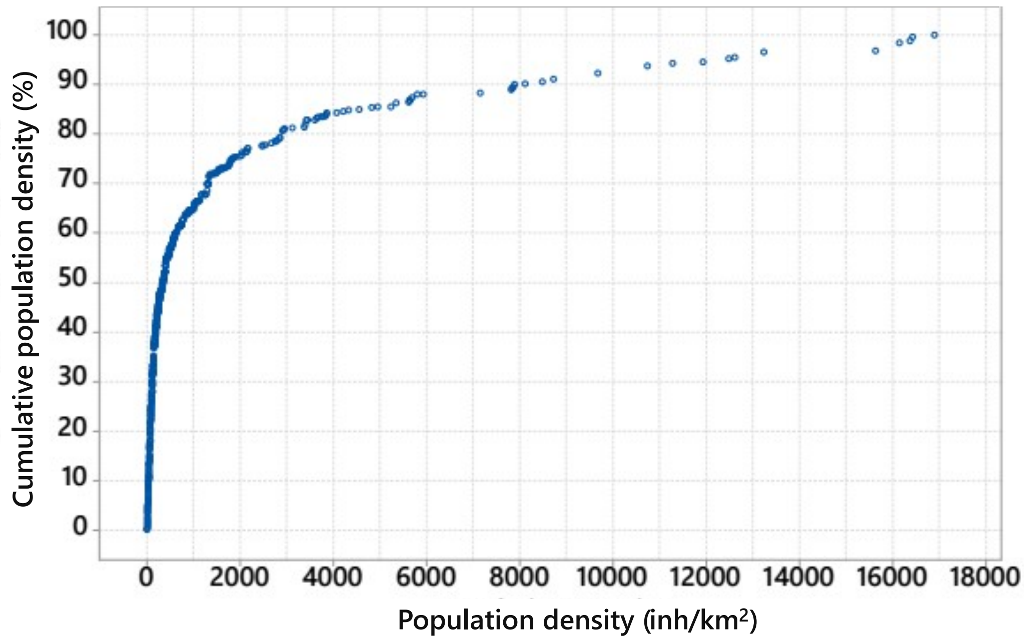


Figure 3: Classification of municipalities according to the population density.

r_j Variable coverage radius of location j.

$r_{\max}$ Maximum coverage radius.

$r_{\min}$ Minimum coverage radius.

δ_j Population density of location j.

$\delta_{\max}$ Maximum population density.

$\delta_{\min}$ Minimum population density.

α Exponent value of the logarithm calculated by Equation (18).

β Adjustment coefficient calculated by Equation (19).

$$\alpha = \frac{r_{\max} - r_{\min}}{\log_{10}(\delta_{\max}) - \log_{10}(\delta_{\min})} \tag{18}$$

$$\beta = \log_{10}(\delta_{\max})^{\alpha} + r_{\min} \tag{19}$$

$$r_j = \beta - \log_{10}(\delta_j)^{\alpha} \tag{20}$$

For experimental purposes, the values of some parameters were fixed. The minimum and maximum population densities were based on the population density of Mexico (2020). The minimum coverage radius was taken from the average distance of HCUs in Mexico City because it is the most populated city with the largest number of HCUs. The maximum coverage radius was set to

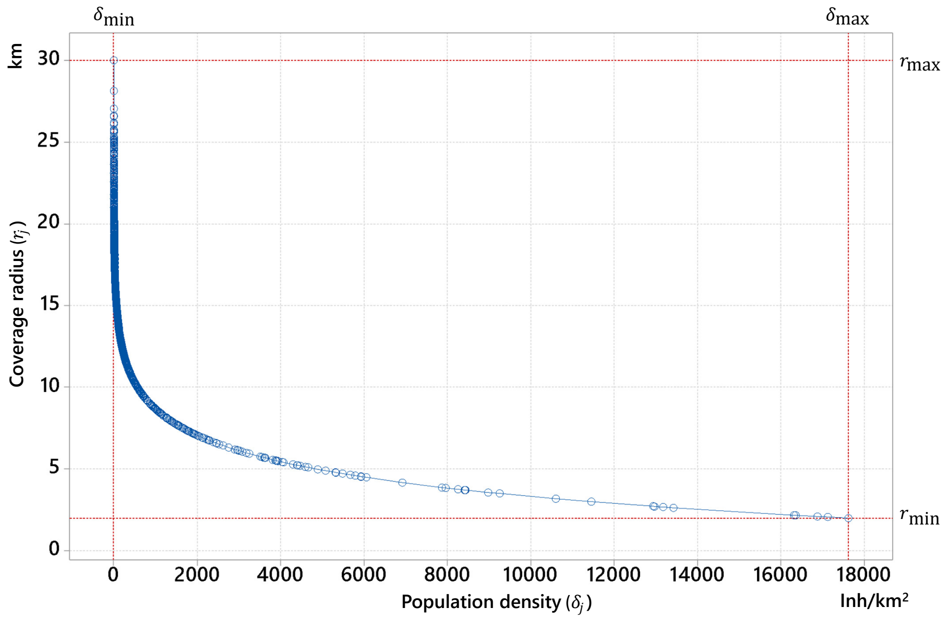


Figure 4: Graphical representation of the coverage radius.

30 km because this distance is reachable in rural areas of Mexico. The parameters α and β were calculated with Equations (18) and (19), respectively. The values of all these parameters are the following:

$$
\begin{aligned}
r_{\text{min}} &= 2\,\text{km}; & \delta_{\text{min}} &= 0.11\,\text{inh./km}^2; \\
r_{\text{max}} &= 30\,\text{km}; & \delta_{\text{max}} &= 17,624\,\text{inh./km}^2; \\
\alpha &= 5.36; & \beta &= 24.76.
\end{aligned}
$$

In the case of partial coverage, the coverage radius must be extended to a secondary coverage radius. In this case, limits were set as follows:

$$
\begin{aligned}
l_j &= r_j \\
u_j &= 2l_j.
\end{aligned}
$$

The secondary coverage radius of a HCU for the most populated areas is nearly 4 km, and for the least populated areas is 60 km.

4.3 The effect of the collaboration among institutions

Parameter λ^q is critical because it determines the coverage degree of a given site of institution q for the beneficiaries of other institutions. For instance, $\lambda^q = 0$ means that HCUs cannot cover beneficiaries of other institutions, $\lambda^q = 0.5$ means that only half of the beneficiaries of other institutions can be covered, and $\lambda^q = 1$ means that all the beneficiaries of other institutions are covered. In the following experiment, we calculated the coverage level of the existing HCUs using different values of λ^q. In this case, we fixed the same value for all the institutions. Table 6 shows these coverage levels using λ^q equal to 0, 0.25, 0.5, 0.75, and 1 for each institution. For each value of λ^q, there is a column (Gb) that represents the global demand coverage in the system. The covered demand includes all the beneficiaries of the fully covered demand points and the proportion of beneficiaries in the partially covered demand points. As we can see, the percentage of demand covered increases in all the services as the value of λ^q increases. If we compare the difference in the global demand coverage between $\lambda^q = 0$ and $\lambda^q = 1$, the coverage increase is 13%, 12%, 12%, and 13% for each service, respectively. These values represent the maximum effect on the coverage when collaboration between institutions is done. The most benefited institution is I3, with an increase of up to 46% in demand coverage for S2. Table 7 is similar to Table 6, but instead of showing the demand level, this table shows the percentage of demand points (fully or partially) covered for each value of λ^q. The percentage of additional demand points covered from $\lambda^q = 0$ to $\lambda^q = 1$ is 31%, 31%, 27%, and 20% for each service, respectively. The coverage increase is more significant in the number of demand points than the percentage of demand covered. This happens because many demand points with low demand levels are currently not covered by the own HCUs of each institution. Another observation is that, for instances with $\lambda^q = 0$, the problem to solve could be split into multiple problems equivalent to NSLP because there is no interaction between institutions. If $\lambda^q = 1$, one single problem using NSLP could be used because there is no distinction between the institution to which each HCU belongs and the others. In the case of the experiments in the following sections, we fixed $\lambda^q = 0.5$ for all the institutions.

Table 6: Percentage of demand covered with different levels of λ^q by institution.

	$\lambda^q = 0$				$\lambda^q = 0.25$				$\lambda^q = 0.5$				$\lambda^q = 0.75$				$\lambda^q = 1$			
Service	I1	I2	I3	Gb	I1	I2	I3	Gb	I1	I2	I3	Gb	I1	I2	I3	Gb	I1	I2	I3	Gb
S1	80	71	46	74	80	75	56	76	81	79	67	79	82	84	79	83	83	91	91	87
S2	75	70	45	71	76	73	55	74	76	78	66	76	77	83	78	79	78	90	91	83
S3	67	73	46	68	68	75	56	70	69	78	66	73	71	82	76	76	74	87	88	80
S4	53	68	48	59	55	70	55	61	57	74	63	64	60	78	72	68	63	83	81	72

Table 7: Percentage of demand points covered with different levels of λ^q by institution.

	$\lambda^q = 0$				$\lambda^q = 0.25$				$\lambda^q = 0.5$				$\lambda^q = 0.75$				$\lambda^q = 1$			
Service	I1	I2	I3	Gb.	I1	I2	I3	Gb.	I1	I2	I3	Gb	I1	I2	I3	Gb	I1	I2	I3	Gb
S1	78	31	17	51	80	83	85	82	80	83	85	82	80	83	85	82	80	83	85	82
S2	73	28	18	47	75	80	82	78	75	80	82	78	75	80	82	78	75	80	82	78
S3	64	30	16	43	67	73	74	70	67	73	74	70	67	73	74	70	67	73	74	70
S4	43	26	16	32	48	55	56	52	48	55	56	52	48	55	56	52	48	55	56	52

4.4 Evaluating different levels of collaboration (λ^q)

The objective of this experiment is to assess the impact of collaboration among institutions on the benefit in the demand coverage. This benefit is obtained from the objective function value of the introduced problem. In the mathematical model, collaboration is defined by parameters λ^1, λ^2, and λ^3 for institutions I1, I2, and I3, respectively. We evaluate the benefit in the demand coverage for different values of λ^1, λ^2, and λ^3. In this case, we use 0.0, 0.5, and 1.0 for each institution resulting in 27 distinct solutions. The service S1 was used for this analysis with a fixed number of new sites set at one hundred for each institution. Table 8 shows the results that correspond to the benefit in the demand covered in the system (in millions of people) for each solution. Notably, the highest benefit is observed when all parameters are set to one, representing full collaboration between institutions. In the other hand, the lowest benefit is observed when all parameters are set zero, representing no collaboration. The average of the solution for each value of λ^1 are shown in the last column, for λ^3 are shown in the last row, and for λ^2 are 20.7, 20.5, and 23.9 for 0.0, 0.5, and 1.0, respectively. Comparing these averages, we note higher benefit when λ^2 and λ^3 are close to 1.0, while the least favorable outcomes occur when these parameters are set to 0.5, particularly when λ^1 is also 0.5 or 1.0. With this experiments, we observe a clear relation between collaboration levels and the benefit in the demand coverage obtained from the model solution.

Table 8: Benefit in demand coverage for different levels of λ^q.

		λ^3			
λ^1	λ^2	0.0	0.5	1.0	Average
0.0	0.0	19.2	20.1	22.9	21.5
	0.5	19.5	20.4	23.1	
	1.0	21.6	22.5	24.2	
0.5	0.0	19.5	20.5	23.2	21.3
	0.5	19.9	16.0	23.5	
	1.0	22.0	22.8	24.5	
1.0	0.0	20.6	21.7	23.9	22.2
	0.5	21.1	16.5	24.2	
	1.0	22.9	23.7	25.1	
	Average	20.7	20.5	23.9	

As a subsequent analysis, we explored whether the contribution of each institution to the benefit in demand coverage obtained with the solutions are influenced by the collaboration level of the institutions and the interaction between them. Utilizing a factorial analysis of 3^3, we systematically investigated the effects of λ^1, λ^2, and λ^3 on the contribution (as a percentage) of each institution to the benefit in demand coverage. For example, in a given solution I1 contributes in 44% in the total value of the objective function, I2 to 42% and I3 contributes in the remaining 14%. We used the results of the previously tested instances to evaluate three factorial models, one for each institution. The factors considered were the values of λ^1, λ^2, and λ^3, while the response variable was the contribution made to the objective function by each institution. With a confidence level of 95%, our findings revealed that changes in the values of λ^1, λ^2, and λ^3 significantly impacted the contribution of I1 and I3 to the benefit in demand coverage, with similar results observed. However, changes in the values of λ^3 did not yield a significant change in the contribution of I2 to the benefit in demand coverage. Moreover, no significant interactions between factors were observed. Importantly, the analysis adhered to normality assumptions, ensuring the reliability of our conclusions. Figure 5 illustrates the effect of collaboration levels on the contribution of each institution, excluding λ^3 for I2 due to its lack of significant effect. Notably, for all the institutions, the contribution decreases as its associated λ^q increases, while the contribution increases with higher values of other institutions. We conclude that the collaboration level of each institution affects the contribution to the benefit of demand coverage in the system. However, these results should be considered with reservation since they are specific to these proven instances.

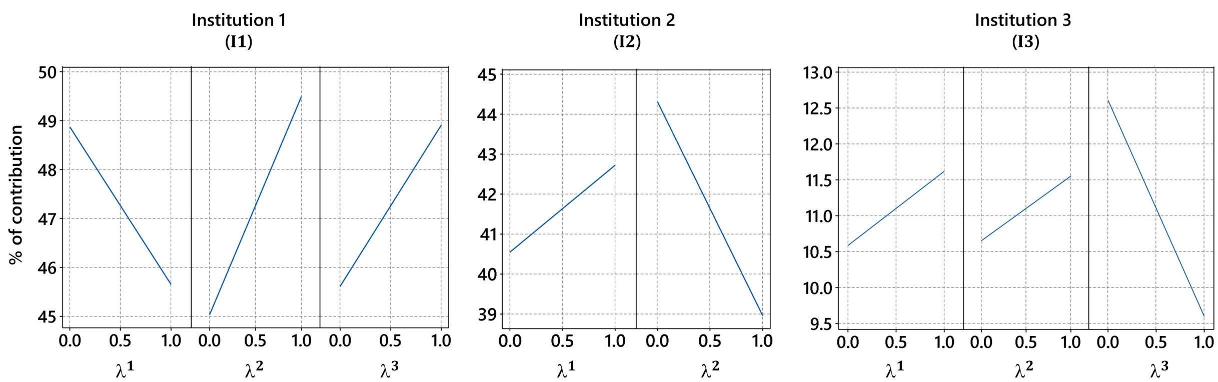


Figure 5: Average effect of λ^1, λ^2, and λ^3 in the percentages of contribution to the objective function for each institution.

4.5 Evaluating different types of candidate sites

The SLP can be used in two cases. The first case is to select between candidate locations (N_A) where new HCUs will be installed. For instance, in the case of pediatrics and gynecology, it is very frequent to build specialized HCUs where only one type of service is provided. In the other case, the service can be installed in HCUs that currently do not provide service. In this case, the candidate sites only include a set of potential HCUs (N_B) where the service can be installed. In this experiment, we evaluate these two cases for the selected services. The aim is to compare the impact on the demand covered in each case. In all the instances, the problem was optimally solved in less than one hour with an average computing time of 1,075 seconds when N_A was used and 50 seconds when N_B was used.

Table 9 shows the additional covered demand for each institution, the additional total demand covered, and its percentage of the total demand for each case. The second column shows the number of new sites selected for each institution. For instance, $P^k = 1$ means that one new site

Table 9: Additional demand coverage in thousand of inhabitants evaluating two types of candidate sites with $\lambda^q = 0.5$.

Services	P^k	With candidate locations (N_A) I1	I2	I3	Total	(%)	With candidate HCUs (N_B) I1	I2	I3	Total	(%)
S1	1	186	344	11	541	1.1	5	154	0	159	0.3
	2	233	532	50	815	1.6	65	211	2	279	0.6
	3	291	687	77	1,055	2.1	66	287	2	355	0.7
	4	351	829	104	1,284	2.6	74	327	12	413	0.8
	5	402	974	110	1,485	3.0	98	348	14	460	0.9
S2	1	159	293	9	461	1.3	56	53	2	111	0.3
	2	251	397	10	658	1.8	134	57	3	195	0.5
	3	325	498	12	835	2.3	178	68	7	252	0.7
	4	364	600	34	998	2.7	231	69	8	307	0.8
	5	414	679	36	1,129	3.1	276	69	8	353	1.0
S3	1	590	1,007	33	1,631	1.4	167	159	5	331	0.3
	2	975	1,585	66	2,627	2.2	359	238	17	614	0.5
	3	1,230	1,981	155	3,367	2.8	586	283	18	888	0.7
	4	1,455	2,271	200	3,927	3.3	788	314	22	1,124	0.9
	5	1,661	2,572	218	4,451	3.7	967	314	22	1,304	1.1
S4	1	613	1,180	35	1,829	1.5	37	312	1	350	0.3
	2	1,031	1,784	70	2,885	2.4	88	592	5	686	0.6
	3	1,518	2,118	96	3,732	3.1	256	751	11	1,018	0.8
	4	1,910	2,416	107	4,433	3.7	459	835	23	1,317	1.1
	5	2,157	2,718	174	5,049	4.2	689	885	27	1,601	1.3

was opened for each institution. As can be seen, there is a higher impact on the demand coverage when the service is installed on new locations (N_A). On average, the covered demand when the service is installed in new locations is 3.5 times greater than considering existing HCUs to install the service. These results are intuitive because most of these HCUs are installed in urban areas where other HCUs already cover the region. On the other hand, the set of candidate sites considers places where no other HCUs are already installed.

Tables 10 and 11 show detailed results about the demand covered and demand points inside the secondary coverage radius for the instances with $P^k = 5$ for each service, considering the set of candidate sites N_A. In Table 10, the second column shows the percentage of demand currently covered by the existing HCUs that supply the service. The third column shows the percentage of demand covered with the new sites. The fourth column shows demand that was not covered, but is in the secondary coverage radius of a candidate site that was not selected. Finally, the last column gives the demand that cannot be covered because there are no candidate sites near them. This demand is scattered in areas with low infrastructure requirements to build new sites. Table 11 has the same data, but refers to the number of partially or fully covered demand points. As we can see in the results, with the new sites, the percentage of demand covered ranges between 3.0%

and 4.2% for the analyzed services. The number of new demand points covered was lower, between 0.4% and 1.8%. However, there are many demand points that an existing HCU already covers, but the new ones help to improve the coverage level. More candidate sites can be considered in the problem to reduce the demand and demand points out of reach, but this increase the complexity of the problem. This is analyzed in Section 4.7.

Table 10: Classification of demand for solutions with $P^k = 5$ and $\lambda^q = 0.5$.

Service	Currently covered (%)	Newly covered (%)	Not covered (%)	Out of reach (%)
S1	79.2	3.0	9.8	8.0
S2	76.3	3.1	10.6	10.0
S3	72.7	3.7	12.5	11.2
S4	63.9	4.2	13.0	18.8

Table 11: Classification of demand points for solutions with $P^k = 5$ and $\lambda^q = 0.5$.

Service	Currently covered (%)	Newly covered (%)	Not covered (%)	Out of reach (%)
S1	82.1	0.4	11.7	5.9
S2	78.1	1.4	12.0	8.5
S3	70.4	1.1	15.4	13.1
S4	52.0	1.8	32.3	13.9

4.6 Solving the problem at different clustering levels

In the case of Mexico, the government is divided into three hierarchical levels: federal, state, and municipal authorities. Although health care institutions have federal jurisdiction, some planning decisions are just analyzed considering only local impact because some resources come from local governments. In this experiment, we evaluate the effects on the demand coverage when federal states independently do the location of new sites. In the other case, the same number of new sites is selected, but as a single territory composed of all federal states. In both cases, demand points are partially or fully covered without considering if they belong to the federal state where the site is located. In the first case, additional constraints to ensure that each institution selects one new site in each of the 32 federal states are considered, and in the second case, 32 new sites were selected by each institution, considering all the federal states. Then, another experiment was also done by dividing the country into eight regions, as shown in Figure 6. Each institution can select a new site by region in the first case, and in the second one, eight new sites can be selected as a whole by institution.

In these experiments, all instances were optimally solved in less than one hour of computing time with an average of 1,016 seconds. In Table 12, we can see the percentage of demand and demand points covered by each service in each case, considering the existing HCUs that supply the

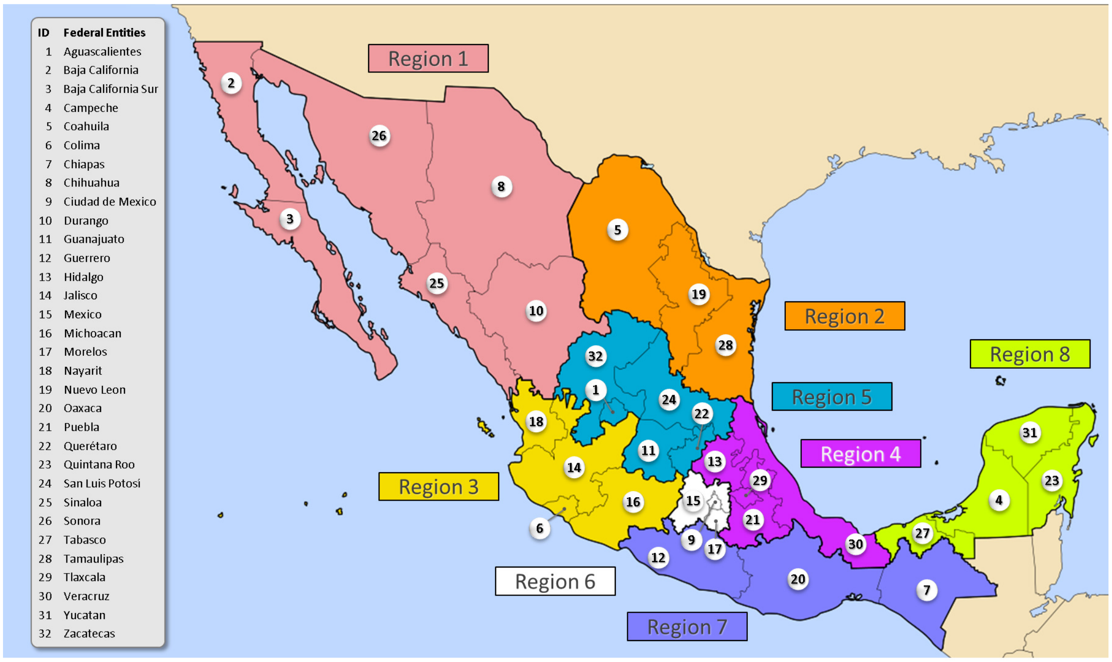


Figure 6: Map of Mexico divided into 8 regions and 32 federal states.

service. We can observe that more demand was covered in both instances where the problem was solved as a whole system. In the case of the federal states, the percentage of additional demand covered is 2.3%, 2.4%, 2.7%, and 3.4% for each service, respectively. In the case of the regions, the percentages are 0.9%, 0.8%, 1.1%, and 1.3% for each service, respectively. This consistency was not found in the number of additional demand points covered. In the case of the federal states, the number of additional demand points covered was also increased, but in the case of the regions, the number of demand points was reduced. Finally, in the case of federal states, we detected that in the second case there were no chosen sites in 9 out of 32 federal states, and 22% of the selected sites were concentrated in a single federal state. In the case of regions, 4 out of 8 regions have a selected site installed in the second case, and 42% of new sites were concentrated in a single region.

Table 12: Demand coverage for different clustering levels.

	Demand covered (%)				Demand points covered (%)			
	S1	S2	S3	S4	S1	S2	S3	S4
By federal states								
1 site by state (32 states)	84.2	81.3	78.8	70.8	85.3	82.4	76.3	58.9
32 new sites as whole	86.4	83.7	81.5	74.3	85.4	83.3	76.6	61.0
Difference	2.3	2.4	2.7	3.4	0.0	0.9	0.3	2.1
By regions								
1 site by region (8 regions)	82.3	79.4	76.4	68.2	83.2	80.3	73.4	55.1
8 new sites as whole	83.2	80.2	77.5	69.5	82.8	80.1	73.0	54.9
Difference	0.9	0.8	1.1	1.3	-0.3	-0.2	-0.4	-0.2

4.7 Handling larger problem instances

The goal of this experiment is find out the size of the instances that can be solved with the present model by using off-the-shelf branch and bound methods. To this end, we tested instances of larger size by increasing the number of candidate sites and the number of these sites that are selected for installing the service. We evaluate the performance of the branch-and-bound algorithm of CPLEX in one hour of CPU time. In Table 13, we consider three types of candidate sites for each service. The candidate sites were chosen based on the population size of each demand point. For type A, the candidate sites are demand points with 10 thousand inhabitants or higher, for type B, demand points with at least 7.5 thousand inhabitants or higher; and for type C, demand points with more than 5 thousand inhabitants. The specific number of candidate sites for each service of each type is shown in the third column. For the three types, the number of candidate sites to be selected ($\sum_{k \in K} P^k$), including all institutions, was ranged between 150 to 1500 sites, as seen in the table. The same number of site to be selected was fixed for each institution. This table shows the CPU time in hours and the optimality gap where ϵ denotes a very small value. The CPU time is lower than one hour when the optimal solution is found, and the optimality gap is higher than zero when the time limit is reached. We can observe that all instances of type A were optimally solved, 4 out of 24 instances of type B were not optimally solved, two of them with very bad solutions, and none of the instances of type C were optimally solved having very high bad solutions. In general, more candidate sites improve the demand coverage, but this leads to larger models that cannot be solved efficiently. Table 14 shows the number of demand points, decision variables, and constraints of the tested instances of type A, B, and C of each service to compare each type of instance. Clearly, our model is still useful for solving type A and mostly type B instances. For the remaining, heuristic procedures must be developed.

Table 13: Optimal solutions and relative gaps for different instance sizes.

Service	Type		CPU time (h)						Optimality gap (%)					
			Selected sites						Selected sites					
		Candidate Sites (N)	150	300	600	900	1200	1500	150	300	600	900	1200	1500
S1	A	2,567	0.2	0.2	0.3	0.2	0.2	0.2	0	0	0	0	0	0
S2	A	2,564	0.2	0.2	0.3	0.3	0.3	0.2	0	0	0	0	0	0
S3	A	2,584	0.5	0.6	0.5	0.5	0.5	0.4	0	0	0	0	0	0
S4	A	1,948	0.4	0.4	0.4	0.4	0.3	0.0	0	0	0	0	0	0
S1	B	3,566	0.7	0.7	0.6	0.6	0.7	0.6	0	0	0	0	0	0
S2	B	3,563	0.8	0.7	0.7	0.8	0.7	0.7	0	0	0	0	0	0
S3	B	3,583	0.9	0.9	1.0	1.0	1.0	1.0	0	0	0	0	ϵ	0
S4	B	2,865	1.0	1.0	1.0	1.0	1.0	0.9	94	22	90	9	0	0
S1	C	5,628	1.0	1.0	1.0	1.0	1.0	1.0	94	93	91	93	93	92
S2	C	5,625	1.0	1.0	1.0	1.0	1.0	1.0	97	97	95	94	93	91
S3	C	5,645	1.0	1.0	1.0	1.0	1.0	1.0	98	97	95	94	94	92
S4	C	4,823	1.0	1.0	1.0	1.0	1.0	1.0	98	97	97	95	94	93

Table 14: Number of decision variables and constraints.

Set type	Service	Demand points	Variables X_{ij}^k	Variables Y_j	Constraints
A	S1	66,101	1,001,927	2,584	1,135,074
	S2	39,294	916,046	2,564	1,030,551
	S3	26,266	909,109	2,567	1,015,888
	S4	112,736	824,292	1,948	972,697
B	S1	66,101	1,480,171	3,583	1,623,672
	S2	39,294	1,354,615	3,563	1,481,030
	S3	26,266	1,344,892	3,566	1,464,878
	S4	112,736	1,278,091	2,865	1,436,020
C	S1	66,101	2,528,672	5,645	2,686,454
	S2	39,294	2,334,235	5,625	2,476,849
	S3	26,266	2,314,642	5,628	2,452,506
	S4	112,736	2,325,933	4,823	2,498,808

4.8 Evaluating new services in existing HCUs

In some cases, temporary services must be activated in the HCUs networks to face sanitary emergencies, as with the Covid-19 pandemic. In this experiment, we consider only the HCUs that belong to the Ministry of Health to identify which HCUs to install a new service to maximize the demand coverage solving NSLP with CPLEX. There are, in total, 774 candidate sites to install the service. The population at each demand point was used as the demand. There are, in total, 189,280 demand points, but only 150,357 can be entirely or partially covered. Figure 7 shows the percentages of demand and demand points covered with each solution varying the number of new sites (p). These percentages were calculated based on the total population and total demand points in Mexico. For instance, with five new sites, only 9% of the population was covered, and with 100 new sites, 56% of the population was covered. If the service is installed on every candidate site, the maximum percentage of demand covered is nearly 86% of the total population. As we can see in the plot, the portion of demand covered has a logarithmic behavior. It becomes more challenging to cover the remaining demand because the best sites to improve the coverage were already selected. The set of candidate sites can cover fully or partially a maximum of 41% of demand points and the behavior of demand points covered is more linear in the range between 5 to 550 new sites. Finally, a graphical representation of the solution for $P = 100$ new sites is shown in Figure 8. In this solution, 56% of demand was covered, and 7.6% of demand points were inside the secondary coverage radius of a selected site. Figure 9 shows a detailed visualization of the central region in the left-hand side plot where most of the population is concentrated and 66 out of 100 candidate site was selected in this region. The northeast region of Mexico is shown in the right-hand side figure where only thirteen candidate sites were selected.

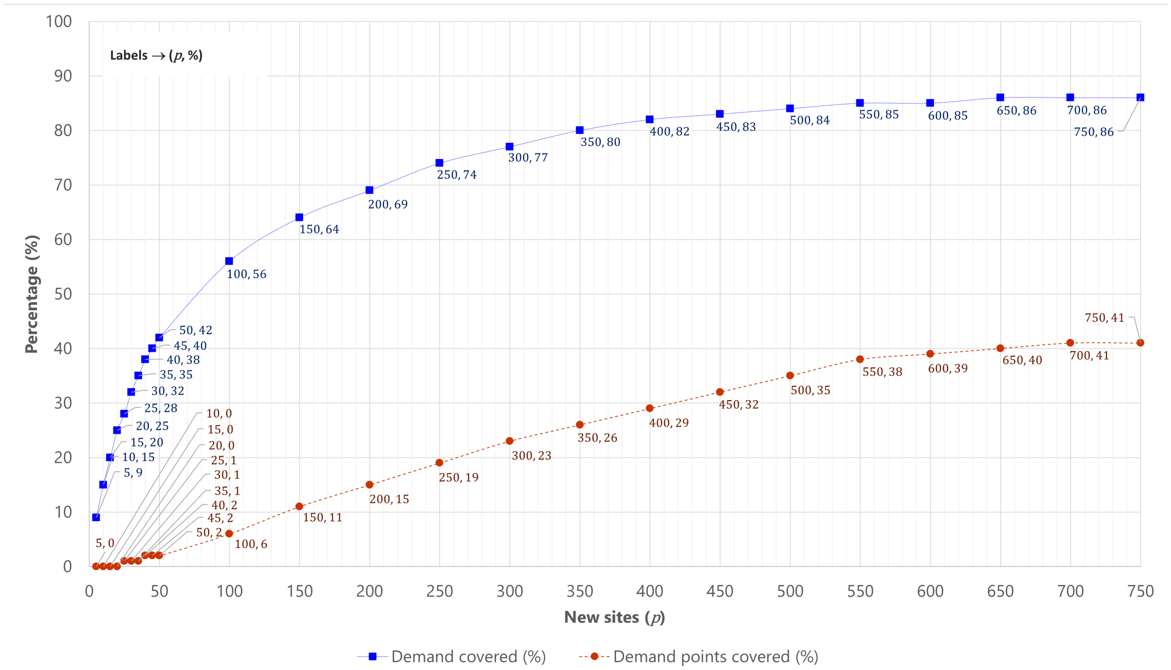


Figure 7: Demand covered according to the number of new sites.

5 Conclusions

In this paper, we revisited the MCLP with partial coverage and proposed an extension to solve the problem of locating SHCS in a multi-institutional network. This model is motivated by the need to improve access to these services in developing countries. A decay function based on two critical coverage bounds for partial coverage is employed. A logarithmic function is proposed to determine the coverage radius of each candidate site based on the population density of the area where the new facilities can be installed. Since many sites are currently operating the services, the problem is improving the demand coverage. Therefore, for each demand point, the additional benefit in the coverage must be determined before solving the problem.

The case study, based on real-world data from the Mexican Health Care System, revealed interesting results. Four second-level services were evaluated with the model to locate additional sites to improve the current coverage. One contribution is the integration of multiple institutions in the demand coverage. If the collaboration between institutions is done, the additional covered demand could be increased between 12% and 13% for the analyzed services. As the percentage of collaboration decreases, this percentage is also reduced, but it still significantly impacts the coverage and access to these services.

Two choices for installing new services were evaluated: installing services in existing HCUs that currently do not supply the service and building new facilities. We found that the additional demand covered increased to 3.5 times the demand covered when new sites were considered. Even

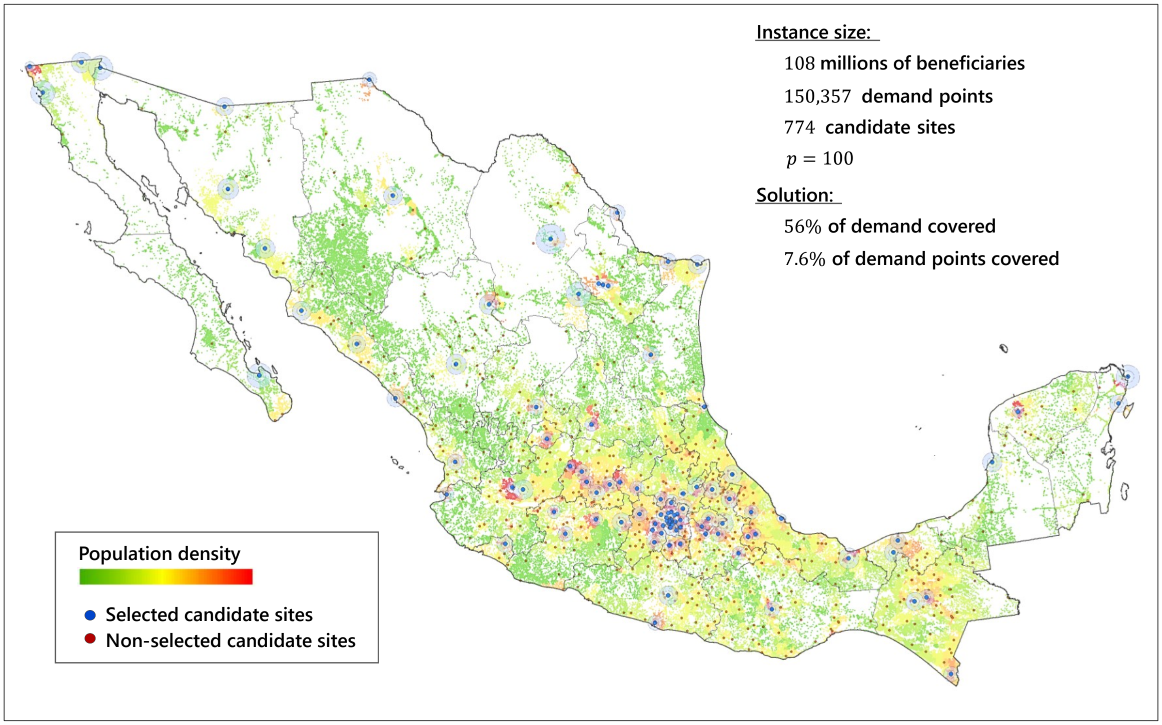


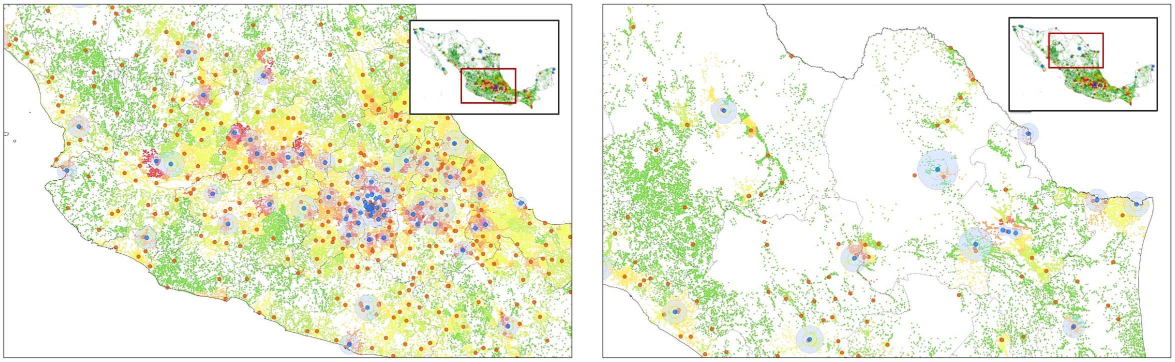
Figure 8: Map with the solution for $p = 100$.

though it is cheaper and easier to install the service in existing HCUs, there is no significant contribution from the access point of view since these HCUs are located in cities where other HCUs are already operating. Therefore, new sites are required to improve access to these services.

In the case of Mexico, this type of planning is done locally on some occasions; even the institutions have federal jurisdiction. Therefore, we evaluate the effect on the coverage when the new locations are selected by each federal state and then as a whole system. We found that the additional demand coverage is between 2.7% and 4% when the planning is done as an entire system. In a second analysis, comprehensive planning was compared against regional planning. In this case, comprehensive planning was found better, showing improvements between 1.0% and 1.9%.

The models developed in this paper can also be used to face emergency issues. For instance, opening temporary modules to deal with health emergencies or vaccination campaigns in which access is one of the most important factors for planning. In this experiment, it was observed that installing some new services in existing HCUs was advantageous. The impact on the demand coverage is higher for the first new sites chosen because the remaining uncovered demand points have lower demand levels and are scattered on the territory.

There are various possibilities for future research on this problem. As part of upcoming investigations, including capacity constraints in service provision and addressing congestion using queuing theory can significantly help in dealing with the evolving challenges in public health care service delivery. Another aspect to consider is integrating multiple second-level services into one



(a) Zoom on the central region. (b) Zoom on the north region.

Figure 9: Zoom on the solution for $p = 100$.

single location problem. Incorporating hierarchical structures into the model is essential because primary HCUs serve as the primary contact point for a substantial portion of the demand directed toward second and third-level specialized hospitals. Financial limitations in the public sector can be considered by introducing a budget constraint to the problem. Considering joint coverage among HCUs can help expand service coverage. For instance, hospitals without operating rooms may still offer specialized outpatient services. Integrating all these features into a single problem and exploring alternative solution methods like metaheuristic algorithms could be a valuable avenue for future exploration.

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