

Improving the Operation of Pipeline Systems
on Cyclic Structures by Tabu Search

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Abstract

In the paper the problem of how to efficiently operate a natural gas transmission network under steady-state assumptions is considered. The problem is modeled as a nonlinear network optimization problem where the decision variables are mass flow rate in each arc and gas pressure in each node. The objective function to be minimized is the total amount of fuel consumed in the system by the compressor stations. In the past, several techniques ranging from classical gradient-based procedures to dynamic programming, for solving this difficult nonconvex problem have been applied with limited success, particularly when applied to cyclic network topologies. A cyclic system is defined as a network containing at least one cycle involving two or more compressor stations. In this paper we propose a hybrid metaheuristic procedure that efficiently exploits the problem structure. This hybrid procedure combines very effectively a nonsequential dynamic programming algorithm for finding an optimal set of pressure variables for a fixed set of mass flow rate variables, and short-term memory tabu search procedure for guiding the search in the flow variable space. The proposed procedure represents an improvement to the best existing approach to the best of our knowledge. In addition, empirical evidence over a number of instances supports the effectiveness of the proposed procedure outperforming a multi-start GRG method both in terms of solution quality and feasibility. Furthermore, to assess the quality of the solutions obtained by the algorithm, a lower bound is derived. It is found that the solution quality obtained by the proposed procedure is relatively good.

Keywords: steady state, natural gas, transmission networks, cyclic systems, nonconvex problem, dynamic programming, tabu search

1 Introduction

In this paper, we address the problem of minimizing the fuel consumption incurred by compressor stations in a natural gas pipeline transmission system. During this process, energy and pressure are lost due to both friction between the gas and the pipes' inner wall, and heat transfer between the gas and the environment. To keep the gas flowing through the system, it is necessary to periodically increase its pressure, so compressor stations are installed through the network. It is estimated that compressor stations typically consume about 3 to 5% of the transported gas. This transportation cost is significant because the amount of gas being transported in large-scale systems is huge. In the other hand, even a marginal improvement in gas operations can have a significant positive impact from the economic standpoint, so this provides the main motivation from the practical side of the proposed work.

The problem is represented by a network, where arcs correspond to pipelines and compressor stations, and nodes correspond to their physical interconnection points. We consider two types of continuous decision variables: mass flow rates through each arc, and gas pressure level at each node. So, from the optimization perspective, this problem is modeled as a nonlinear program (NLP), where the cost function is typically nonlinear and nonconvex, and the set of constraints is typically nonconvex as well. It is well known that nonconvex NLP is NP-hard (Horst et al. 1995). This motivates the choice of the proposed heuristic approach.

The state of the art on research on this problem reveals a few important facts. First, there are two fundamental types of network topologies: noncyclic and cyclic. We would like to emphasize that, the former is a type of topology that has received most of the attention during the past 30 years. Several methods of solution have been developed, most of them based on Dynamic Programming (DP), which were focused on noncyclic networks.

In particular, as far as handling cyclic topologies is concerned, gradient search and DP ap-

proaches have been applied with little or limited success. The main limitation of the former is its local optimality status. The drawback of the latter, is that its application is limited to problems where the flow variables are fixed, so the final solution is “optimal” with respect to a prespecified set of flow variables. This is because cyclic topologies are a lot harder to solve.

In this paper, we proposed a novel solution methodology for addressing the problem of how to optimally operate the compressor stations in a natural gas pipeline system, focusing in cyclic topologies. The proposed technique combines a nonsequential DP technique (originally proposed by Carter (1998)) within a Tabu Search (TS) framework. For the past few years, TS has established its position as an effective metaheuristic guiding the design and implementation of algorithms for the solution of combinatorial optimization problems in a number of different areas (Glover and Laguna 1997). In this case, even though we are dealing with a continuous optimization problem, the high nonconvexity of the objective function and the versatility of TS to overcome local optimality make TS, with an appropriate discrete solution space, a very attractive choice.

Empirical evidence over several instances with data taken from industry shows the efficiency of the proposed approach. A comparison with a multi-start GRG-based method demonstrates the significant superiority of the proposed procedure. The method represents an improvement over existing state-of-the-art approaches. Furthermore, in order to assess the quality of the solutions delivered by the algorithm, a lower bound was derived. It is shown that the optimality gaps found by our technique are less than 16%, most of them less than 10%, which represents a significant progress to the current state of the art in this area. The scientific contribution of this work is providing the best technique known to date, to the best of our knowledge, for addressing this type of problem in cyclic topologies.

The rest of this paper is organized as follows. In Section 2, we formally introduce the fuel consumption minimization problem (FCMP), describing its main features, modeling assumptions, and important properties. Then, in Section 3, we present a review of earlier approaches for this

problem, highlighting the most related to our work, and how we attempt to exploit some of them. The proposed methodology is outlined in Section 4. A computational evaluation of the heuristic, including comparison with a multi-start GRG method, is presented in Section 5. Finally, we wrap up this work with the conclusions and directions for future research in Section 6.

2 Problem Description

Pipeline system models can be mainly classified into steady-state and transient systems. Like all those previous works (reviewed in Section 3), here we assume a steady-state model. That is, our model provides solutions for systems that have been operating for a relatively large amount of time, which is a common practice in industry. Transient analysis has been done basically by descriptive models, because transient models are a highly intractable from the optimization perspective. Optimization for transient systems remains as one of the great research challenges in this area. We also assume we work with a deterministic model, that is, each parameter is known with certainty. In terms of the compressor stations, we assume we work with centrifugal compressor units, which are the most commonly found in industry. As far as the network model is concerned, we assumed the network is balanced, that is, no gas is lost, and that each arc in the network has a prespecified direction.

The Model

This model was originally introduced by Wu et al. (2000). $V_s \subset V$ and $V_d \subset V$ are the set of supply and demand nodes, respectively. The set of arcs A is partitioned into a set of pipeline arcs A_p and a set of compressor station (or simply compressor) arcs A_c , i.e. $A_p \cup A_c$ and $A_p \cap A_c = \emptyset$. Let U_{ij} and R_{ij} the capacity and resistance of pipeline $(i, j) \in A_p$, respectively. Let P_i^L, P_i^U be the pressure lower and upper limits at node $i \in V$. Let B_i the net mass flow rate at node $i \in V$, where

$B_i > 0$ if $i \in V_s$, $B_i < 0$ if $i \in V_d$, and $B_i = 0$ otherwise. The decision variables are given by x_{ij} , the mass flow rate in arc $(i, j) \in A$, and p_i , the pressure at node $i \in V$.

Formulation FCMP

$$\text{Minimize} \quad \sum_{(i,j) \in A_c} g_{ij}(x_{ij}, p_i, p_j) \quad (1)$$

$$\text{subject to} \quad \sum_{j:(i,j) \in A} x_{ij} - \sum_{j:(j,i) \in A} x_{ji} = B_i \quad i \in V \quad (2)$$

$$x_{ij} \leq U_{ij} \quad (i, j) \in A_p \quad (3)$$

$$p_i^2 - p_j^2 = R_{ij}x_{ij}^2 \quad (i, j) \in A_p \quad (4)$$

$$p_i \in [p_i^L, p_i^U] \quad i \in V \quad (5)$$

$$(x_{ij}, p_i, p_j) \in D_{ij} \quad (i, j) \in A_c \quad (6)$$

$$x_{ij}, p_i \geq 0 \quad (i, j) \in A, i \in V \quad (7)$$

The objective function (1) represents the total amount of fuel consumption in the system. We use a function g_{ij} in the following form:

$$g(x_{ij}, p_i, p_j) = \alpha x_{ij} \left\{ \left(\frac{p_j}{p_i} \right)^m - 1 \right\}, \quad (x_{ij}, p_i, p_j) \in D_{ij},$$

where α and m are assumed constant (and known) parameters that depend on the gas physical properties. Constraints (2)-(3) are the typical network flow constraints representing node mass balance and arc capacity, respectively. Constraint (4) represents the gas flow dynamics in each pipeline under the steady-state assumption. Constraints (5) denote the pressure limits in each node. These limits are defined by the compressor physical properties. Constraint (6) represents the nonconvex feasible operating domain D_{ij} for compressor station (i, j) . The algebraic representation of D_{ij} is the result of curve fitting methods based on empirical data taken from the compressors. The details on the nature of the compressor station domain and how it is derived can be found in Wu et al. (2000). Figure 1 shows a two-dimensional shape of this domain for p_i

fixed. The precise model formulation for each of instances tested is available for download from <http://yalma.fime.uanl.mx/~roger/ftp/>. Finally, the mathematical model is bounded by nonnegative decision variables (7).

3 Previous Work

In this section, we review the most significant contributions over the last 30 years for solving the FCMP or related problems.

Methods Based on Dynamic Programming

The key advantages of DP are that a global optimum is guaranteed to be found and that nonlinearity can be easily handled. In contrast, its application is practically limited to noncyclic networks, such as linear (also known as gun-barrel) or tree topologies, and that computation increases exponentially with the dimension of the problem, commonly referred as the curse of dimensionality.

DP for pipeline optimization was originally applied to gun-barrel systems in the late 1960s. It has been one of the most useful techniques due to both its computational behavior and its versatility for handling nonlinearity on sequential systems. DP was first applied to linear systems by Wong and Larson (1968a), and then applied to tree-structured topologies by Wong and Larson (1968b). A similar approach was described by Lall and Percell (1990), who allow one diverging branch in their system.

Luongo et al. (1989) published a hierarchical approach that allowed for both cycles and branches of arbitrary complexity. This represented significant progress in terms of finally addressing the issue of real world pipeline configurations. Their technique was no longer pure DP. Basically, DP was used to optimally describe the pieces of the pipeline that were arranged in a sequential manner. This typically reduced the system to a much smaller combinatorial problem, without any possibility

of a recursive DP solution. A sufficiently small instance could be solved exactly via enumeration; otherwise it was solved inexactly using simulated annealing. This hierarchical approach worked very well for some complex pipelines, but for others the computational cost was very high.

The most significant work on cyclic networks known to date is due to Carter (1998) who developed a nonsequential DP algorithm, but limited to a fixed set of flows. This led to an interesting question of how to find the optimal setting of the flow variables and how to modify the current flow setting to obtain a better objective value. So with this in mind, recently Ríos-Mercado et al. (2006) propose a network-based heuristic for modifying the flow values. The computational results showed an improvement with respect to Carter's NDP approach. A limitation of that work, though, is that the experimental phase was done over a small number of instances. Unfortunately, that code is no longer available for research purposes. In the present work, we use Carter's ideas and incorporate them within a Tabu Search scheme for iteratively adjusting the set of flows with great success. This will be further described in Section 4.

Methods Based on Gradient Search

Percell and Ryan (1987) applied a different methodology based on a Generalized Reduced Gradient (GRG) nonlinear optimization technique for noncyclic structures. One of the advantages of GRG, when compared with DP, is that they can handle the dimensionality issue relatively well, and thus, can be applied to cyclic structures. Nevertheless, being a method based on a gradient search, there is no guarantee for a global optimal solution, especially when there are discrete decision variables. Villalobos-Morales and Ríos-Mercado (2005) evaluated preprocessing techniques for GRG, such as scaling, variable bounding, and choice of starting solution, that resulted in better results for both cyclic and noncyclic structures. More recently, Flores-Villarreal and Ríos-Mercado (2003) performed an extensive computational evaluation of the GRG method over a large set of instances on cyclic structures with relative success. No comparison to DP was done in that work, so part

of our contribution is to provide a comparison frame among Carter's NDP, GRG, and our method tested in the same set of instances.

Related Models

Discrete decisions such as number of units operating within compressor stations are incorporated into a mixed-integer nonlinear programming model (MINLP). MINLP models in pipeline optimization have been studied by Pratt and Wilson (1984) and Cobos-Zaleta and Ríos-Mercado (2002). They present satisfactory results as they were able to find local optima for many instances tested.

Optimization of individual compressor stations has been studied by Osiadacz (1980), Percell and Reet (1989), and Wu et al. (1996). Later, Wu et al. (2000) completed the analysis for the same problem, but considering several units within compressor stations. In a related work, some of the most important theoretical properties regarding pipeline networks are developed by Ríos-Mercado et al. (2002).

Carter et al. (2002) present some algorithms based on implicit filtering for a class of noisy optimization problems which also consider discrete decision variables with promising results. In a related work, Osiadacz and Górecki (1995) address a pipeline network design problem with modest success. More recently, Costa et al. (2000) use linear programming for the optimal design of pressure relief header networks.

Optimization techniques have also been applied for transient (time dependent) models. For instance, Larson and Wismer (1971) propose a hierarchical control approach for a transient operation of a gunbarrel pipeline system. Osiadacz and Bell (1986) suggest a simplified algorithm for the optimization of the transient gas transmission network, which is based on a hierarchical control approach. The hierarchical control approach for transient models can be found in Anglard and David (1988), Osiadacz (1994), and Osiadacz and Swierczewski (1994). Some degree of success has been reported from these approaches as far as optimizing the compressor station subproblem.

However, these approaches have limitations in globally optimizing the minimum cost.

See Ríos-Mercado (2002) for more references on optimization techniques applied to gas pipeline problems. It is important to mention that optimization approaches developed to date work well under some general assumptions; however, as the problems become more complex, the need arises for further research and effective development of algorithms from the optimization perspective.

4 Solution Procedure

The proposed methodology (depicted in Figure 2) proceeds as follows. In Step 1, a preprocessing phase is performed both to refine the feasible operating domain given by tightening decision variable bounds, and to reduce the size of the network by a reduction technique (motivated by the work of Ríos-Mercado et al. (2002)). Then, in Step 2, a set of initial feasible flows (x) is found by two different methods: a classic assignment technique and a reduced graph algorithm.

In Step 3, a set of optimal pressures (p), for the specified flow obtained before is found by applying a nonsequential DP (NDP) algorithm. At this point, we have an initial feasible solution (x, p) which enters a tabu search (TS) local search procedure.

Within the TS, there are two main components for neighbor generation: a flow modification component and a pressure computation component. In the former, an attempt is made to find a different set of flows, and in the latter, a corresponding set of optimal pressure values is found by NDP. The TS is performed until a stopping criteria (in the case, a number of iterations) is met. As we know from theoretical properties of pipeline networks (Ríos-Mercado et al. 2002), the flow modification step is unnecessary for noncyclic topologies because there exists a unique set of optimal flow values which can be determined in advance at preprocessing. So, here we focus on cyclic topologies. For finding the optimal set of pressures, we implemented a NDP technique motivated by the work of Carter (1998). The overall procedure is called NDPTS. The methods

employed in Steps 1 and 2 have been fairly well documented in our previous work (Borraz-Sánchez and Ríos-Mercado 2004b), so, in the reminder of this section, we assume we have an initial feasible flow and provide a description of the NDP and the TS components, which is the core of the proposed work.

Nonsequential Dynamic Programming

We include in this section a brief description of the essence of the NDP algorithm. The details can be found in Borraz-Sánchez and Ríos-Mercado (2004a). Starting with a feasible set of flow variables, the NDP algorithm searches for the optimal set of nodal pressure values associated to that prespecified flow. Rather than attempting to formulate DP as a recursive algorithm, at a given iteration, the NDP procedure grabs two connected compressors and replace them by a “virtual” composite element that represents the optimal operation of both compressors. These two elements can be chosen from anywhere in the system, so the idea of “recursion” in classical DP does not quite apply here. After performing this step, the system has been replaced by an equivalent system with one less compressor station. The procedure continues until only one virtual element, which fully characterizes the optimal behavior of the entire pipeline system, is left. Afterwards, the optimal set of pressure variables can be obtained by a straight-forward backtracking process. The computational complexity of this NDP technique is $O(|A_c|N_p^2)$, where N_p is the maximum number of elements in a pressure range discretization.

Tabu Search

We start the procedure with a given feasible solution (x, p) . We define the nature of a feasible solution based on three basic components which are directly related with a cyclic network topology: (a) *static component*, a mass flow rate value not belonging to any cycle; (b) *variable component*, a mass flow rate value belonging to a cycle; and (c) *search component*, all pressure variables in

the network. These components are depicted in Figure 3. The search space employed by TS is defined by the flow variables x_{ij} only because once the flow rates are fixed, the pressure variables are optimally found by NDP. Furthermore, we do not need to handle the entire set of flow variables, but only one per cycle. This is so because once you fix a flow rate in a cycle, the rest of the flows can be uniquely determined. Thus, a given state is represented by a vector $\bar{x} = (x_{\alpha_1}, \dots, x_{\alpha_m})$, where α_w is an arc that belongs to a selected cycle w . Note that this set of arcs is arbitrarily chosen, and that converting a flow from x to and from \bar{x} is straightforward, so in the description x and \bar{x} are used interchangeably.

Then a neighborhood $V(\bar{x})$ of a given solution \bar{x} is defined as the set of solutions reachable from \bar{x} via a slight modification of Δ_x units in each of its components. This is given by

$$V(\bar{x}) = \{x' \in R^m \mid x'_w = \bar{x}_w \pm j\Delta_x, j = 1, 2, \dots, Nsize/2, w = 1, \dots, m\} \quad (8)$$

where $Nsize$ is the predefined neighborhood size, and Δ_x accounts for the mesh size. Note that, for a given solution, we do not store the entire solution but only the flow in the selected arc to be modified. Note also that once this value is set, the rest of the flow variables in the cycle are easily determined, so in this sense, it is precisely this mass flow rate which becomes the attribute. Then the best $x' \in V(\bar{x})$ which is non-tabu is chosen and the corresponding subsets are updated accordingly. A tabu list (TL) stores recently used attributes, in our case, values of the x variables. The size of the TL (*tabu tenure*) controls the number of iterations a particular attribute is kept in the list. The search terminates after *iter_max* iterations.

5 Computational Evaluation

The purpose of designing and setting up a database with problem instances is twofold. First, it is necessary for testing our proposed algorithms. Second, it aims at providing a common framework for benchmarking different algorithms. As far as we know, there is no such database for this type

of problems. So this becomes an important contribution of this work.

There are three different kinds of network topologies: (a) linear or gun-barrel, (b) tree or branched, and (c) cyclic. Technically, the procedure for making this classification is as follows. In a given network, the compressor arcs are temporarily removed. Then each of the remaining connected components are merged into a big supernode. Finally, the compressor arcs are put back into their place. This new network is called the associated reduced network.

Linear topology: This corresponds to a linear arrangement of the compressor station arcs, that is, when the reduced network is a single path.

Tree topology: This occurs when the compressors are arranged in branches through the system, that is, when the reduced network is a tree.

Cyclic topology: This happens when compressors are arranged forming cycles with other compressor stations. That is, it refers to a cyclic reduced network.

As stated before, linear and tree topologies can be solved by dynamic programming since it has been shown that the flow variables in every arc can be uniquely determined. So in this work, our focus is on addressing cyclic topologies. Figure 4 shows examples of cyclic topologies. A stripped node (shown with an ingoing arrow next to it) represents a supply node, a black node (shown with an outgoing arrow next to it) represents a demand noid, and a white node is a transshipment node. A single directed arc joining two nodes represents a pipeline, and a directed arc with a black trapezoid joining two nodes represents a compressor arc.

So in the instance database tested below, a name $\text{net-}x\text{-mcn}$ represents an instance of type x , $x \in \{a, b, c\}$, with m nodes and n compressor arcs. In addition, a suffix $\text{-}Cy$ present means the instance uses compressors type y , where y is one of nine different type of compressors used in industry. This database is available at: <http://yalma.fime.uanl.mx/~roger/ftp/>, or directly from the authors upon request. Each instance is given as a GAMS file.

The proposed TS was developed in C++ and run on a Sun Ultra 10 workstation under Solaris 7. All of the compressor-related data, described in Villalobos-Morales et al. (2003), was provided by a consulting firm in the pipeline industry. For the tabu list size and the neighborhood size, several preliminary experiments were done using values of $\{5, 8, 10\}$ and $\{20, 30, 40\}$, respectively. For the experiments we use the following values for the algorithmic parameters which were found to produce the best results in preliminary fine-tuning computations: Iteration limit ($iter_max = 100$), discretization size in $V(x)$ ($\Delta_x = 5$), discretization size for pressure variables ($\Delta_p = 20$), tabu tenure ($T_{tenure} = 8$), and neighborhood size ($Nsize = 20$). In order to assess the effectiveness of the proposed procedures, we apply the algorithms to solving several instances under different cyclic network topologies on the same platform.

It is evident that the proposed NDPTS approach dominates NDP. This has been verified empirically, where the NDPTS has reported improvements in solution quality of up to 27% with respect to NDP. This comparison between NDPTS and NDP is presented in (Borraz-Sánchez and Ríos-Mercado 2005). In this paper, we present a comparison between the proposed NDPTS and the best GRG-based implementation known to date. In a second experiment, we provide evidence of the quality of the solution reported by NDPTS by comparing to a lower bound.

Table 1 shows a comparison between the GRG and NDPTS on cyclic networks. For the GRG we used the implementation by Flores-Villarreal and Ríos-Mercado (2003) within a multi-start strategy. That is, given that GRG is basically a local search method, the idea is to apply GRG from multiple different starting solutions for an amount of time equal to the time used by the NDPTS in each instance. In preliminary work, it was also observed that the multi-start GRG produced better results than the single application of the GRG as expected.

The first column shows the instances tested. The second column shows the total number of iterations employed by the multi-start GRG method. The third and fifth column show the GRG and NDPTS solution, respectively. The fourth column shows the running time of both methods.

The last column shows the relative improvement of NDPTS over GRG, given by

$$RI = \frac{g_{GRG} - g_{NDPTS}}{g_{NDPTS}} \times 100\%,$$

where g_Z denote the objective function value found by method Z , and Z being any of GRG or NDPTS.

First, the NDPTS obtained solutions to all instances tested, whereas GRG failed for four of these, that is, for the four harder instances the GRG could not find feasible solutions. The results indicate that NDPTS procedure outperforms GRG in terms of solution quality. In the instances where both procedures found feasible solutions, NDPTS obtains solution of significantly better quality than those obtained by GRG as can be observed from the relative improvement of NDPTS over GRG. In terms of computational effort, both procedures employed the same amount of time in a range of 270-400 seconds.

To assess the quality of the solutions delivered by the algorithm it is necessary to derive a lower bound. Now, deriving lower bounds for a nonconvex problem can become a very difficult task. Obtaining convex envelopes can be as difficult as solving the original problem. However, for this problem we note two important facts that lead us to an approximate lower bound. First, by relaxing constraint (4) in model FCMP the problem becomes separable in each compressor station. That is, the relaxed problem consists of optimizing each compressor station individually. Now, this is still a nonconvex problem, however, we exploit the fact that in each compressor, the objective is a function of three variables only, so we build a three-dimensional grid on these three variables and perform an exhaustive evaluation for finding the global optimum of the relaxed problem (for a specified discretization).

Table 2 shows these results. The first column displays the instances tested, the second and third columns show the lower bound and the best value found by the heuristic, respectively, and the last column shows the relative optimality gap obtained by NDPTS. As can be seen from the table, all

of the tested instances have a relative optimality gap of less than 17%, 7 out of 11 instances tested have a relative gap of less than 10%, and 3 of these observed an optimality gap of less than 1%. This shows the effectiveness of the proposed approach. Finally, although our NDPTS algorithm finds better solutions than the GRG method or the simple NDP, it is more computationally expensive. In general, any additional time leading to even small improvements can be easily justified since the costs involved in natural gas transportation are relatively huge.

Figure 5 show the convergence of the NDPTS algorithm on instance net-c-10c3-C1. It can be seen how, at some iterations, the solution deteriorates but then it improves to a better solution, which illustrates how getting stuck at a local optimum is overcome by the TS mechanism. This figure also shows that the algorithm often did not improve beyond fifty iterations. In fact, we have observed similar behavior in all other tested instances.

6 Conclusions

In this work we have proposed a hybrid heuristic based on NDP and TS for a very difficult problem arising in the natural gas pipeline industry. The NDPTS implementation, based primarily in a short-term memory strategy, proved very successful in the experimental work as it was able to obtained solutions of much better quality than those delivered by earlier GRG-based approaches when tested on a number of instances with data taken from industry. In addition, the way the method operates clearly produces better solutions than those found by Carter's NDP method. This represents, to the best of our knowledge, a significant contribution to the state of the art in this area of work. Other contributions include the evaluation of a simple lower bounding scheme and a data set collection which can be used for benchmarking.

There are still many areas for forthcoming research. The proposed procedure is a basic short-term memory tabu search. It would be interesting to incorporate advanced TS strategies such as

intensification and diversification. In addition, one of the great challenges in the industry is to address time-dependent systems from the optimization perspective.

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Table 1: Comparison between GRG and NDPTS.

Instance	Iterations	GRG	Time	NDPTS	RI (%)
net-c-6c2-C1	8712	2,312,548.24	271.72	2,288,252.53	1.06
net-c-6c2-C4	8535	1,393,061.12	270.02	1,393,001.99	0.004
net-c-6c2-C7	9637	1,210,687.22	272.33	1,140,097.39	6.19
net-c-10c3-C2	7581	5,811,713.60	288.92	4,969,352.82	16.95
net-c-10c3-C4	7633	4,751,940.94	283.61	2,237,507.93	112.37
net-c-15c5-C2	5040	6,219,045.57	228.32	4,991,453.59	24.59
net-c-15c5-C4	5377	3,554,598.11	317.26	3,371,985.41	5.41
net-c-15c5-C5	10040	Not found	334.01	7,962,687.43	N/A
net-c-17c6-C1	9654	Not found	368.12	8,659,890.72	N/A
net-c-19c7-C4	8906	Not found	393.45	8,693,003.78	N/A
net-c-19c7-C8	18574	Not found	398.72	7,030,280.45	N/A

Table 2: Solution quality.

Instance	LB	NDPTS	Gap (%)
net-c-6c2-C1	2,287,470.58	2,288,252.53	0.03
net-c-6c2-C4	1,392,354.29	1,393,001.99	0.05
net-c-6c2-C7	949,909.48	1,140,097.39	16.68
net-c-10c3-C2	4,303,483.50	4,969,352.82	13.40
net-c-10c3-C4	2,015,665.98	2,237,507.93	9.91
net-c-15c5-C2	4,955,752.90	4,991,453.59	0.72
net-c-15c5-C4	3,103,697.48	3,371,985.41	7.96
net-c-15c5-C5	6,792,248.08	7,962,687.43	14.69
net-c-17c6-C1	8,129,730.11	8,659,890.72	6.12
net-c-19c7-C4	7,991,897.18	8,693,003.78	8.06
net-c-19c7-C8	5,897,768.92	7,030,280.45	16.10

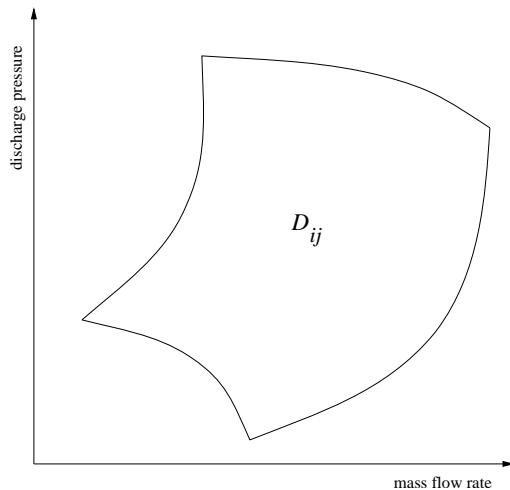


Figure 1: 2-D compressor station feasible domain for p_i (suction pressure) fixed.

```
function NDPTS()
```

Input: An instance of the FCMP.

Output: A feasible assigment (x, p) .

```
1  Preprocessing();  
2   $x \leftarrow \text{FindInitialFlow}();$   
3   $p \leftarrow \text{NDP}(x);$   
4   $(x, p) \leftarrow \text{TS}((x, p));$   
5  return  $(x, p);$   
end NDPTS
```

Figure 2: Pseudocode of NDPTS.

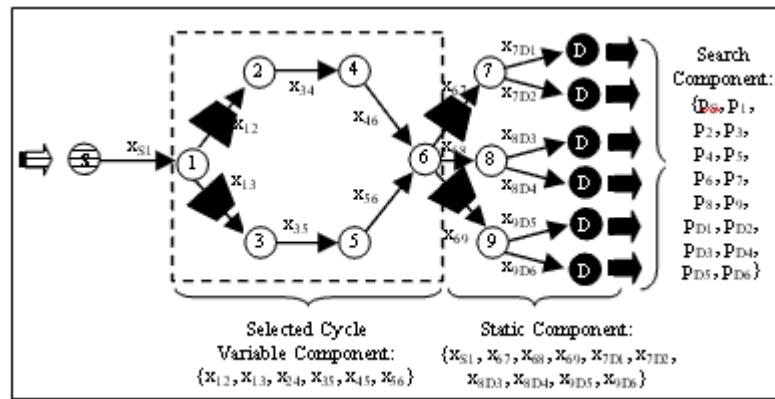


Figure 3: Basic components of a feasible solution on a cyclic topology.

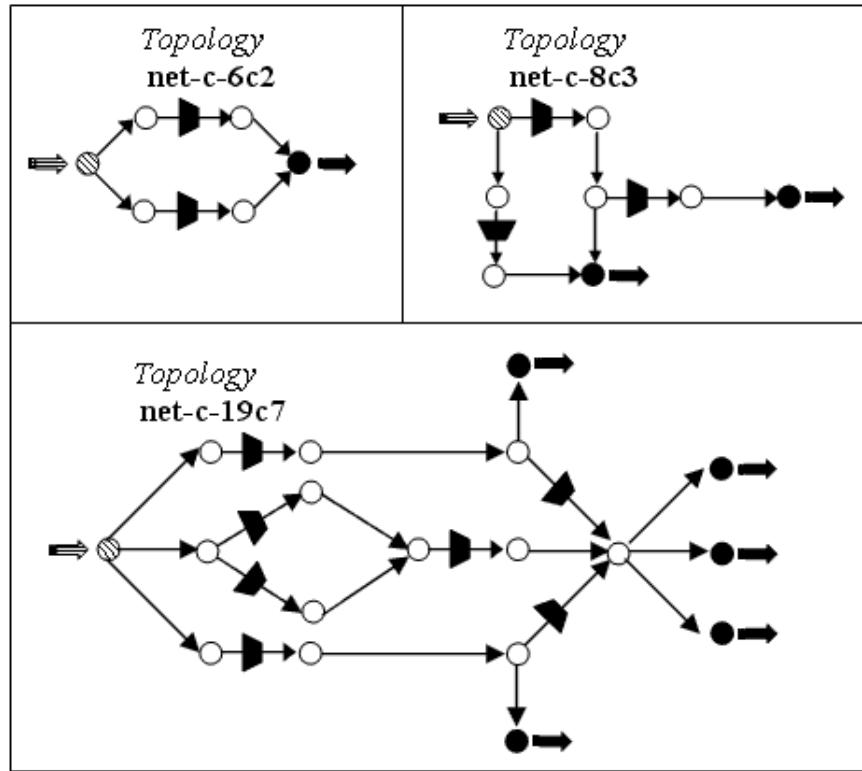


Figure 4: Cyclic topology instances.

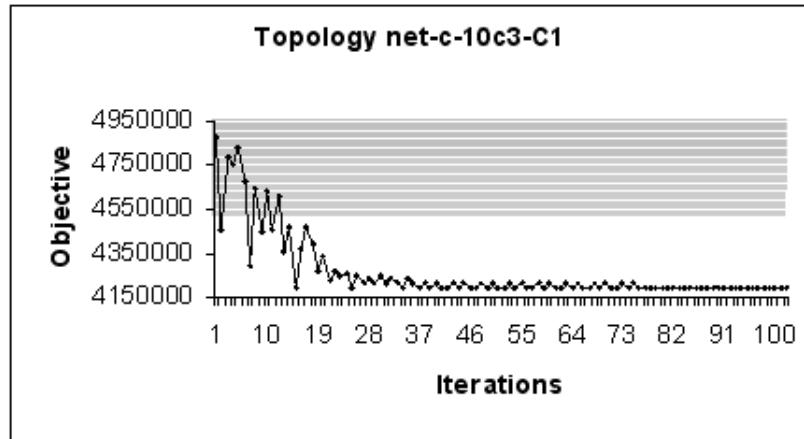


Figure 5: Convergence on instance net-c-10c3-C1.