Optimizing Supply Chain Sales Territories through Incremental Brownfield Design

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Abstract Territory design or districting consists of assigning basic geographic units to territories or clusters according to some relevant criteria. Districting is a tactical decision-making process that has become a fundamental factor within the supply chain. In districting problems, we refer to approaches that construct territory designs from scratch without considering existing plans as *Greenfield* approaches. However, in business environement, analytics practitioners rarely have the luxury of starting from scratch. Most businesses operate on *Brownfield* designs-legacy maps that may be decades old but serve as the organization's playbook. These maps contain nuances, historical relationships, and tribal knowledge that Greenfield approaches cannot fully capture. In other words, the focus is on redistricting decisions, that is, find better plans trying not to deviate too much from the existing plan. This redistricting issue has been studied in some applications such as political redistricting; however, in sales districting, Brownfield approaches are practically non-existent. In this chapter, we present a Brownfield approach to a sales redistricting problem. A solution algorithm based on a series of incremental, high-impact moves is presented. This includes empirical evidence over a set of real-world instances, and a comparison with a Greenfield approach. Through our empirical evaluation of four distinct business regions, we demonstrated that an Incremental Brownfield Algorithm provides significant territory improvements with a minimal number of geographical unit reassignments.

Keywords Integer programming · Sales redistricting · Local search heuristics

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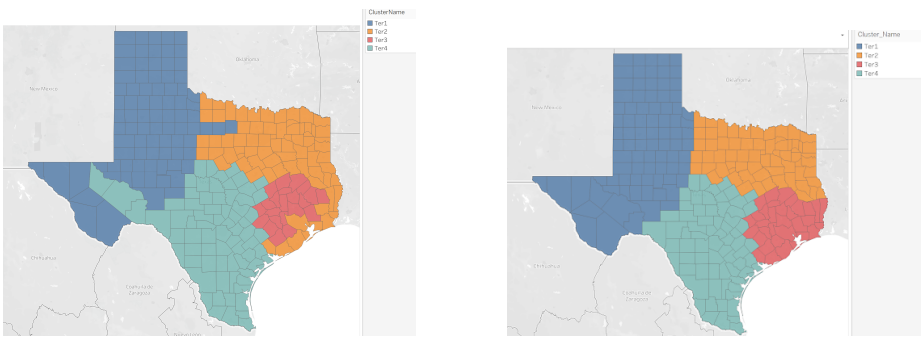
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1 Introduction

A sales territory is defined as a geographical area for which an individual salesperson or a sales team is responsible. The territories determine the workload and opportunity, since salespeople are often compensated based on the total revenue they generate. Since territories directly dictate a salesperson's workload and earning potential, the task of designing them is a high-stakes challenge, particularly for firms in business-to-business (B2B) markets. Ineffective territory design can lead to manager burnout and turnover, unfair distribution of opportunity within the sales team, missed sales, and loss of market share. Moreover, sales territories are often aligned with the firm's supply chain network design, further amplifying their impact by causing unbalanced distribution centers and under- and over-utilized production capacities. Developing an effective sales territory design therefore has wide-ranging impacts throughout the firm, both operationally and financially.

An effective sales territory design balances the salesforce effort across territories while creating compact and clear territory assignments. An effective design supports the productivity of the sales force and establishes clear customer connections. Consider a hypothetical example using the state of Texas displayed in Figure 1. The territory design on the left is less effective than the one on the right because the light blue and dark blue territories are not as compact and have highly irregular boundaries. This leads to more travel time for salespeople and potential confusion for customers and employees. The red territory is completely embedded within the orange and light blue territories, which results in further difficulties. However, the territory design on the right provides more compact, clearly defined, and well-balanced territories. In Section 3, we introduce metrics to quantify and control these features.

Fig. 1: Examples of sales territory design.



(a) Less effective design

(b) More effective Design

The analysis and decision-making processes for sales territory design can be quite challenging. The decision process involves many people with conflicting objectives and is executed on varying timelines. Local and regional sales managers compete for

prime accounts and areas, and the process must be carefully managed by vice-presidents or other senior leaders to ensure the final decisions support the firm's success rather than individual self-interests. A structured quantitative data-driven approach to this process helps ensure proper alignment of sales territory changes with company goals. Although firms typically have an annual or periodic timeline for reviewing and changing territories, intermediate updates are required by changing market and organizational conditions, such as the acquisition of a competitor or a major customer account. A number of different quantitative metrics or key performance indicators (KPIs) are typically used to evaluate potential designs and changes. Both existing sales volume and potential sales are important considerations that help balance the sales force workload between supporting existing customer accounts and acquiring new ones. Sometimes, population or other data can be used as indirect proxies for these metrics. Distance and travel time can also be important considerations in sales territory design. While the technology supporting remote meetings has dramatically improved in recent years, there is still value in personal face-to-face meetings and in being able to put "eyes" directly on a situation or problem. Furthermore, the computational complexity of the problem is significant having an exponential number of possible solutions to consider. All of these factors contribute to the challenge and complexity of designing sales territories.

Most approaches to tackling this problem in the literature focus on finding a global or near optimal solution to a structured model, and may not address all of the true complexities of the problem. In the quote commonly attributed to George Box, "All models are wrong, but some are useful." Along that line of thinking, many firms have an existing sales territory design with well-established relationships between customers and sales managers. Pivoting to a drastically different territory design risks disrupting those important relationships and potentially losing key customer accounts and successful sales managers, even when the new design satisfies the optimality criteria of a well-structured analytical model. We refer to such solutions that ignore the existing territory design as *Greenfield* approaches and advocate for *Brownfield* approaches that consider the current territory design while recommending changes which improve on that existing design. These approaches are also referred to in the literature as *districting* and *redistricting*, respectively.

The objective of this chapter is to challenge the Greenfield bias in districting literature by providing a robust framework for Brownfield territory optimization. While traditional models focus on the global optimum of a blank slate, this work prioritizes the path of least resistance to make it easier to implement the changes. The specific aims of this chapter are:

- To advocate for the Brownfield mindset: Shifting the goal from reaching a theoretical maximum to identifying targeted reassignments that provide best results with low organizational friction.
- To formalize a multi-objective optimization stability function: Integrating "Change Fatigue" as a mathematical constraint in addition to traditional geographic and volume-based metrics.
- To provide an actionable implementation guide: Using a US county-clustering case study to show how practitioners can negotiate between the "Optimal" and the "Implementable".

The rest of the chapter is organized as follows. Section 2 presents a discussion of related works in the literature, focusing mainly on sales districting and redistricting research. This is followed by Section 3, where the problem under study is described in detail. The solution methodology is presented in Section 4. The empirical results are shown in Section 5. We wrap up with the conclusions in Section 6.

2 Literature Review

Territory design or districting has been the subject of a vast amount of research since the 1960s. Applications come from political districting [20], school districting [6, 19], commercial districting [22], police patrol districting [16], and healthcare and emergency response districting [27], to name a few of the most important. The literature covers different models, algorithms, and applications, providing theoretical, methodological, and practical contributions (see, e.g., Duque et al. [10], Kalcsics and Ríos-Mercado [15], and Campos-Valdés et al. [5], for comprehensive surveys of different territory design problems).

Here, we focus our review on sales districting and redistricting. Sales territory alignment problems have been studied since the 1970s. Early works focused on developing profit-maximization models [17, 25, 28, 3, 26]. Later models emphasized the salesman workload balance criterion [11]

Drexl and Haase [9] studied a more general context of sales force deployment involving the simultaneous resolution of four interrelated subproblems: sales force sizing, salesman location, sales territory alignment, and sales resource allocation. In their work, the authors present a nonlinear mixed-integer programming model that covers all four subproblems simultaneously. In a follow-up work, Haase and Müller [12] proposed an improved method based on branch-and-price for the same sales force deployment problem, yielding better results. Chen et al. [7] considered a bi-objectice optimization sales territroy design problem where territory compactness and balance are simultaneously optimized. More recently, sales districting research has addressed multi-period models [2, 1] and models including scheduling and routing decisions [13, 14]. The reader can find extensive surveys on sales districting models in the works by Zoltners and Sinha [29] and Campos-Valdés et al. [5] (Section 3.2).

Now, redistricting models have been studied in other districting applications. Redistricting takes into account the existing plan and tries to come up with a new plan typically trying not to depart too much from the existing plan. In Bozkaya et al. [4], the authors addressed a political redistricting problem that arises in Edmonton, Canada. The authors developed a tabu search metaheuristic. They optimized a scalarized multi-objective function, where one of the terms takes into account similarity with the existing plan. To do this, they developed a specific measure in order to compare the similarity of a proposed solution X with the existing plan. This measure computes, for each district k of the existing plan, the largest overlay $O_k(X)$ with a district contained in a new solution x. Formally, the similarity index is defined as $f_{\text{sim}} = 1 - \sum_{k \in K} O_k(X)/A$, where A is the entire area and K is the set of districts. This index always takes a value between 0

and 1 and can be used even if the old and new plans do not contain the same number of districts. This index considers the actual shape of the individual basic units.

Caro et al. [6] considered a school redistricting problem. They presented an integer programming formulation, and introduced a constraint that ensures that only a certain proportion of blocks should be reallocated. This is be done by imposing constraints:

$$\sum_{(i,n)\in R(k)} x_{ikn} \geq (1-P)|R(k)|,$$

where x_{ikn} is a binary decision variable equal to 1 if grade k students of block (basic unit) i are assigned to school n, and 0 otherwise; $R(k)$ is the set of (i,n) pairs representing the current block-school allocations for each grade k; and $(l-P)\%$ of these pairs must be kept. Thus, these constraints say that, for each grade, at least a proportion $(l-P)$ of blocks must remain assigned to their current school.

Ríos-Mercado and López-Pérez [23], López-Pérez and Ríos-Mercado [18] studied a commercial redistricting problem from a bottle beverage distribution company. They presented an integer programming model, where they penalized deviations from existing plan with a term in the objective function. The authors used the term

$$\sum_{i\in V}\sum_{j\in F^i} q_{ij}(1-x_{ij}),$$

where x_{ij} is a binary decision variable equal to 1 if basic unit j is assigned to district with center in i, and 0 otherwise; V is the set of basic units; F^i is the set of basic units that are assigned to territory with center in i under the current design; and q_{ij} is the weight of assigning unit j to center i.

In de Assis et al. [8], the authors addressed a redistricting problem applied to meter reading in power distribution networks. They presented an integer programming model and used the following constraint to limit the number of customers that can change district:

$$\sum_{k\in K}\sum_{i\in V} l_{ki}(1-x_{ki}) \leq L,$$

where x_{ki} is a binary decision variable equal to 1 if basic unit i is assigned to district k, and 0 otherwise; V is the set of basic units; K is the set of districts; l_{ki} represents the number of customers in basic unit i originally assigned to district k; and L is the maximum number of customers that can change from their original district.

As illustrated above, several approaches can be used to maintain similarity with an existing plan. For instance, in integer programming models, sometimes it is by adding a term in the objective function, sometimes we have some constraints. It also depends on how the decision variables are defined. Other times, such as in Bozkaya et al. [4], we do not have an integer programming model, but a combinatorial description.

3 Problem Description

We now describe a version of the sales redistricting problem which we will use to illustrate key points and insights with a numerical example later in the paper. The necessary inputs for the problem are as follows. First of all, we need a listing of the basic geographical units (census tracts, counties, *etc.*) which we will refer to as basic units (BUs) along with their relevant volume measures such as population, current sales, or potential market size. The data must also include the list of BUs adjacent to each individual BU to ensure the connectivity or contiguity of the territories. The travel distance or time between each pair of BUs are needed to quantify the geographic dispersion of each territory. The number and list of current territories along with the current BU-territory assignments are also key inputs to the analysis. Finally, we need to know the median or anchor BU for each territory to compute a proxy for the total travel distance or time of a salesperson assigned to the territory. These territory medians may be provided as inputs or computed from the other data. Table 1 defines the mathematical notation we will use for this input data.

Table 1: Definition of sets, indices, parameters, and decision variables.

Symbol	Description
V	= set of basic geographic units (basic units or BUs).
i, j	= indices of set V.
E	= set of edges (i, j) connecting adjacent BUs i and j.
G	= (V, E); graph of basic units and adjacency edges.
T	= set of territories.
N^i	= $\{j \in V : (i, j) \in E\}$; set of BUs adjacent to i, its *neighborhood*.
d_{ij}	= distance or travel time between BUs i and j; $i, j \in V$.
w_i	= volume or weight of BU i; $i \in V$.
β	= percentage threshold of maximum number of allowed changes of BUs.
p	= $\lvert T \rvert$; number of territories.
x_{ij}	= Binary decision variable equal to 1 if BU j is assigned to territory having its median in BU i, and 0 otherwise; $i, j \in V$.

We use binary decision variables in the model to determine the territory assignment for each basic unit. Define $x_{ij} \in \{0, 1\}\ \forall i \in V$ and $\forall j \in V$, where $x_{ij} = 1$ if basic unit j is assigned to the territory having its median at BU i and $x_{ij} = 0$, otherwise. When $x_{ii} = 1$, BU i is the median of a territory, and other BUs can be assigned to i only if it is selected to be a territory median: $x_{ij} \leq x_{ii}, \forall i$ and $j \in V$. Clearly, each BU must be assigned to a territory, which is enforced using the constraints $\sum_{i \in V} x_{ij} = 1, \forall j \in V$. We also need a constraint ensuring there are exactly $p = |T|$ territories which we can easily implement using $\sum_{i \in V} x_{ii} = p$.

One of the metrics we want to minimize is the travel time required for the salesperson assigned to a territory. We measure this characteristics using the average distance or travel time from each BU to the median of its assigned territory, which can be computed as

$$D(x) = \frac{1}{|V|} \sum_{i \in V} \sum_{j \in V} d_{ij} x_{ij}.$$

We also want to balance the workload across the sales territories which we accomplish by minimizing the variation of the total volume or weight of the BUs assigned to each territory. We accomplish this leveling by minimizing the ratio of the standard deviation of the territory weights divided by the average territory weight, which can be expressed as

$$B(x) = \frac{1}{\bar{w}\,|T|} \sqrt{\sum_{i \in V} (w(i) - \bar{w})^2},$$

where $\bar{w} = \sum_{i \in V} w_i / |V|$ is the average BU weight and $w(i) = \sum_{j \in V} w_j x_{ij}$ is the total weight of the BUs assigned to the territory with median i.

The third component of the objective function is a measure of the "smoothness" of the territories. This metric penalizes rough, jagged edges along the boundary of each territory which can increase salesforce workloads and confuse customer perceptions of assigned sales responsibilities. The smoothness metric, $S(x)$, computes the ratio of the number of edges that cross territory boundaries, which we call "cuts", to the total number of edges in the problem. Let $t_j(x) = i$ where $x_{ij} = 1$, that is, $t_j(x)$ is the territory median to which BU j is assigned in the solution x. Using this notation, we can define our smoothness metric as

$$S(x) = \frac{1}{|E|} \sum_{(j,k) \in E} 1\{t_j(x) \neq t_k(x)\}.$$

We will use a linear weighted combination of these three expressions, $D(x)$, $B(x)$, and $S(x)$ as the objective function which we minimize in the model.

Finally, we need a set of constraints that ensure each territory is connected or contiguous. There should be no "islands" of BUs in a territory that are disconnected from the other BUs in the same territory. We introduce some additional notation to facilitate the development of these contiguity constraints. Let $S \subset V$ be a proper subset of the BUs, and define $N(S) = \{j \in N^i \text{and} \notin S, \forall i \in S\}$, that is, $N(S)$ is the neighborhood of the subset S which contains the BUs adjacent to S. Using this notation, the contiguity constraints can be expressed as

$$\sum_{j \in N(S)} x_{ij} - \sum_{j \in S} x_{ij} \geq 1 - |S|, \forall i \in V \text{ and } \forall S \subset V \setminus (\{i\} \cup N^i).$$

These constraints ensure that, for every proper subset S of BUs that are all assigned to i and don't include i or its neighborhood, at least one neighboring BU of S is also assigned to i. There are admittedly a combinatorial number of these constraints which grows exponentially with the number of BUs in the problem instance. However, a cut generation procedure can be used to iteratively add only the active constraints to the model, and many researchers have noted that only a few of these constraints are typically needed to find an optimal solution, for example Drexl and Haase [9].

We can now express the complete formulation of our problem as the following nonlinear integer programming problem, where $\alpha_1 + \alpha_2 + \alpha_3 = 1$:

$$\begin{aligned}
\min \quad & \alpha_1 B(x) + \alpha_2 D(x) + \alpha_3 S(x) && && (1)\\
\text{s.t.} \quad & x_{ij} \le x_{ii} && \forall\, i, j \in V && (2)\\
& \sum_{i \in V} x_{ij} = 1 && \forall\, j \in V && (3)\\
& \sum_{i \in V} x_{ii} = p && && (4)\\
& \sum_{j \in N(S)} x_{ij} - \sum_{j \in S} x_{ij} \ge 1 - |S| && \forall\, i \in V \text{ and } \forall\, S \subset V \setminus (\{i\} \cup N^i) && (5)\\
& C(x) \le \lfloor |V|\beta \rfloor && && (6)\\
& x_{ij} \in \{0, 1\} && \forall\, i, j \in V. &&
\end{aligned}$$

As said before, (1) represents the weighted objective function. Constraints (2) ensure that a BU j can be assigned to the territory centered at i only if the territory is open. Constraints (3) establish that every BU j must be assigned to a territory. Constraints (4) ensure p territories are built. As discussed before, connectivity requirements are met by Constraints (5). In Constraints (6), $C(x)$ represents the number of BUs that changed to a different territory with respect to the existing plan. If, for instance, $\beta = 0.10$ and $|V| = 105$, then it means that the maximum number of changes allowed is at most 10. Now that we have defined a mathematical formulation for the problem, the next section will describe two strategically different approaches to tackling this problem.

4 Solution Methodologies

A traditional approach to solving integer programming problems like the one described in the previous section is by algorithms such as branch and bound that seek the global optimal solution. Other exact optimization algorithms such as branch and cut and branch and price might also be applied. Some successful cases applied to districting related problems can be found in Salazar-Aguilar et al. [24] for commercial districting, or Ríos-Mercado and Bard [21] for recollection territories of waste electric and electronic equipment, to name a few. However, given the inherent computational complexity of the redistricting problem, these exact methods are inapplicable for handling middle- to large-scale instances like the ones intended in our particular application. As a consequence, one has to resort to heuristic or metaheuristic solution algorithms that do not guarantee to find a global optimum, but a solution for good quality in reasonable time. Most of these heuristic approaches, particularly for sales districtting, do not consider current territory assignments along with the potentially adverse effect on existing B2B relationships from changing those existing assignments. Furthermore, the modeling assumptions and data may not fully capture all the nuances in the relationships between the territory

assignments, operating costs, and sales revenues. We refer to the approaches that ignore the existing plan and focus on finding an optimal design from scratch as the Greenfield version of the problem because they treat the problem as an open field with no existing solution structure already in place.

Algorithm 1 IBA(X) - Incremental Brownfield Algorithm

Input: $X = (X^1, \ldots, X^P)$: A districting plan
Output: X^*: An improved solution
1: $X^* \leftarrow X$
2: $LIMIT \leftarrow \lfloor |V|\beta \rfloor$
3: $nchanges \leftarrow 0$
4: $improved \leftarrow TRUE$
5: **while** ($improved = TRUE$ **AND** $nchanges < LIMIT$) **do**
6: Generate $N(X^*)$, neighborhood of X^*
7: $X^{\text{best}} \leftarrow$ Best solution in $N(X^*)$
8: **if** (X^{best} is better than X^*) **then**
9: $X^* \leftarrow X^{\text{best}}$
10: $nchanges \leftarrow nchanges + 1$
11: **else**
12: $improved \leftarrow FALSE$
13: **end if**
14: **end while**
15: **return** X^*

To overcome the limitations of these traditional approaches, we propose an alternative Brownfield approach that iteratively changes the existing territory assignments to generate incremental improvements while avoiding radical and risky changes. The Incremental Brownfield Algorithm (IBA) is depicted in Algorithm 1. In Step 5, to generate $N(X)$, the procedure first identifies all BUs that are adjacent to another BU assigned to a different territory, that is, the selected BU lies on the boundary of two territories. This operates like a local search where the move is defined as flipping a BU from its current territory to an adjacent different territory. Thus $N(X)$ is formed by all posiible moves. The IBA then evaluates the entire $N(X)$ to find the best possible move, in terms of its objective function value. The algorithm selects, In Step 6, the BU assignment change that generates the greatest benefit in terms of the objective function (1). If this improves the incumben solution then it makes the assignment change, and then updates the list of boundary BUs. The algorithm terminates when either none of the potential assignment changes generates a reduction in the objective function value or maximum allowable number of changes (set in (6)) has been reached. The IBA approach identifies improvements to the existing sales territory design without overwhelming executives and managers with wholesale changes that would be difficult to implement or gain buy-in from the sales force.

5 Empirical Evaluation

Next, we present a case study analysis that reveals how our Incremental Brownfield Algorithm produces significant improvements with a limited number of territory assignment changes. As we will see, a traditional Greenfield optimization approach typically requires considerably more changes, which would likely face significant organizational resistance. These results demonstrate how a "minimal change, maximal impact" approach generates a solution that is much easier to implement.

5.1 Current Territory Design Overview

To understand the real-life implications of each approach, we will analyze four regions of a fictitious company:

- Texas: currently split into three territories,
- Mid-Atlantic that consists of New York, New Jersey, and Pennsylvania: currently split into three territories,
- Carolinas that consists of North and South Carolina: currently split into two territories,
- Florida: currently split into four territories.

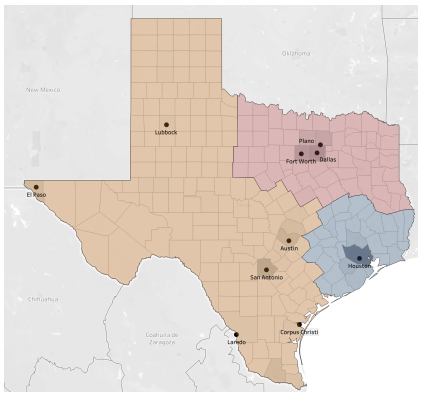
See Figure 2 for a detailed visualization of each region, its current territory assignments, and major cities. The senior leadership acknowledges the shortcomings of the current territory design and is looking for ways to improve it. These four regions have been selected as examples for their varying degree of complexity: the shapes of the regions, the volume differences between the counties that constitute the territories, the geographical sparsity of the region, and how imbalanced the current territories are.

Tables 2 and 3 below provide more statistical details for the regions and territories. To measure how balanced the current territory design is, we will take the percentage delta between the largest and the smallest territory by volume. Of the four regions analyzed, Texas is the most balanced one and Florida is the least balanced one, with the average value for all four regions at 20.32%, which is viewed to be very high by management. The goal is to half the current average imbalance to stay under 10%. Next, we will use the Incremental Brownfield approach to balance the territory volume within each region, and then we will apply the Greenfield approach to create balanced territories from scratch. For both solution methods, we use $\alpha_1 = 0.35$, $\alpha_2 = 0.25$, and $\alpha_3 = 0.40$ as the weights for the three components of the objective function (1).

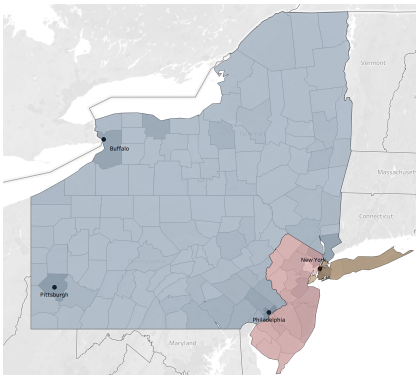
5.2 Incremental Brownfield Approach

For our IBA approach we used $\beta = 0.15$ as our change limit threshold, that is, we allowed a maximum of 15% change of BUs. We observed that in every case, this limit was not reached. We can see in Table 4 the results vary a lot by region. The imbalance

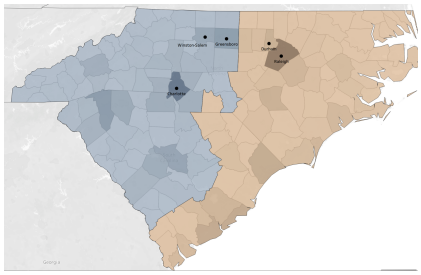
Fig. 2: Heat map of the four region examples.



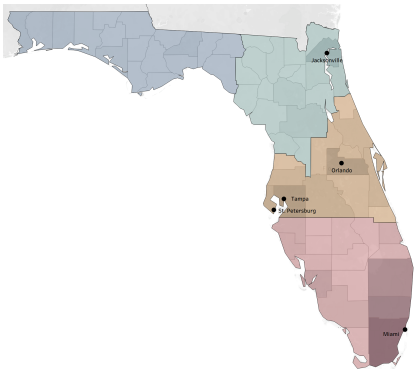
(a) Texas region



(b) Mid-Atlantic region



(c) Carolinas region



(d) Florida region

in Texas and Carolinas can be essentially removed while Florida shows only a slight improvement. This happens due to the fact that the IBA cannot move the center of the shape as doing so would cause a lot of county changes. On average, the imbalance has been reduced from 20.32% to 9.94%, meeting the “under 10%” target. This is achieved by switching a minimum of 6 and a maximum of 16 counties, so approximately 1 county change brings a 1% improvement in the balancing. This can also be appreciated in Figure 3.

Table 2: Data on the four current regional territory assignments.

Region	**Territory**	**Population**	**Current %**
Texas	Texas1	8,983,375	29%
Texas	Texas2	11,474,826	37%
Texas	Texas3	10,832,630	35%
Florida	Florida1	1,633,124	7%
Florida	Florida2	8,788,053	38%
Florida	Florida3	9,793,723	42%
Florida	Florida4	3,157,315	14%
Mid-Atlantic	Mid-Atlantic1	21,539,580	51%
Mid-Atlantic	Mid-Atlantic2	11,406,419	27%
Mid-Atlantic	Mid-Atlantic3	9,500,851	22%
Carolinas	Carolinas1	9,091,553	55%
Carolinas	Carolinas2	7,433,302	45%

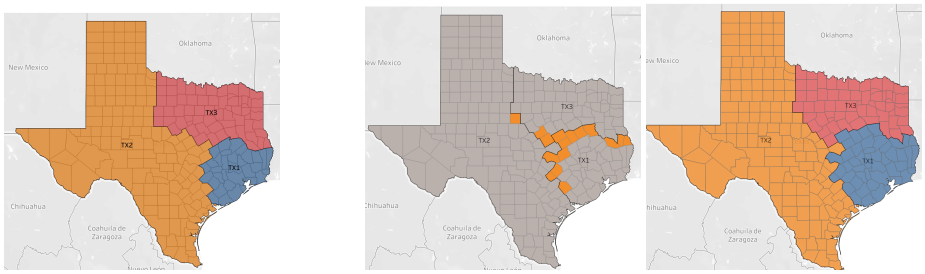
Table 3: Percent volume range of territories in each region.

Region	**Current Max-Min**
Texas	7.96%
Florida	34.92%
Mid-Atlantic	28.36%
Carolinas	10.03%
Average	20.32%

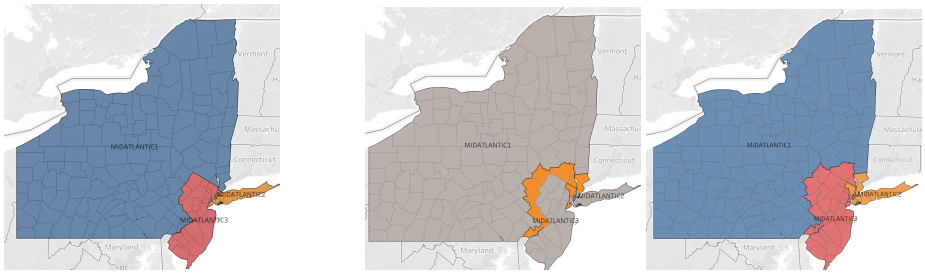
Table 4: IBA results summary.

Region	**Current Max-Min**	**New Max-Min**	**Changes**
Texas	7.96%	0.04%	16
Florida	34.92%	31.34%	10
Mid-Atlantic	28.36%	8.29%	11
Carolinas	10.03%	0.08%	6
Average	20.32%	9.94%	11

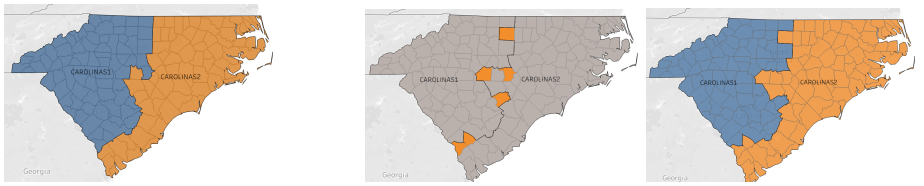
Fig. 3: Maps of IBA results.



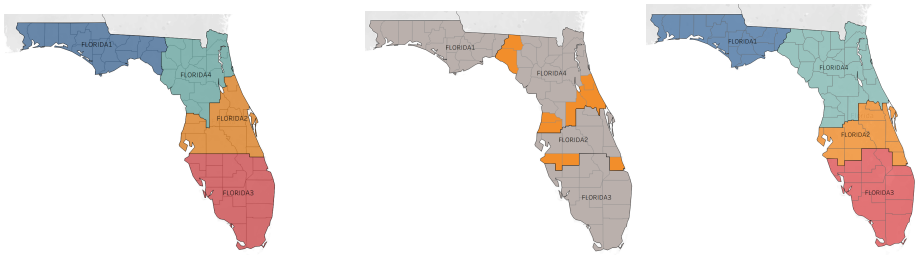
(a) Texas current (b) Texas changes (c) Texas new



(d) Mid-Atlantic current (e) Mid-Atlantic changes (f) Mid-Atlantic new



(g) Carolinas current (h) Carolinas changes (i) Carolinas new



(j) Florida current (k) Florida changes (l) Florida new

5.3 Greenfield Approach

For the Greenfield approach, we used complete enumeration to find global optimal designs. Naturally, under this aproach, Constraints (6) were relaxed. Table 5 indicates the number of combinations for each problem under the Greenfield approach. Each region is treated separately, and the problem complexity for the Greenfield approach varies a lot, depending on the count of counties and the count of territories. This approach requires more computational resources compared to the IBA. Unlike IBA, the Greenfield approach created almost perfectly balanced territories, with the average max-min volume under 2%; however, the average number of county changes is 48. Specifically, under the Greenfield approach, we can see from Table 6 that the number of changes found for Texas, Florida, Mid-Atlantic and Carolinas was 54, 33, 78, and 27, representing 21.2%, 49.2%, 52.0%, and 18.5%, respectively. As you can see from the maps in Figure 4, two of the regions end up with a design that resembles the IBA result (Carolinas, Texas), but for the other two regions, it results in a complete restructuring. The empirical results ultimately demonstrate that a "minimal change, maximal impact" approach, such as the IBA, allows for both significantly lower computation times and immediate organizational implementation.

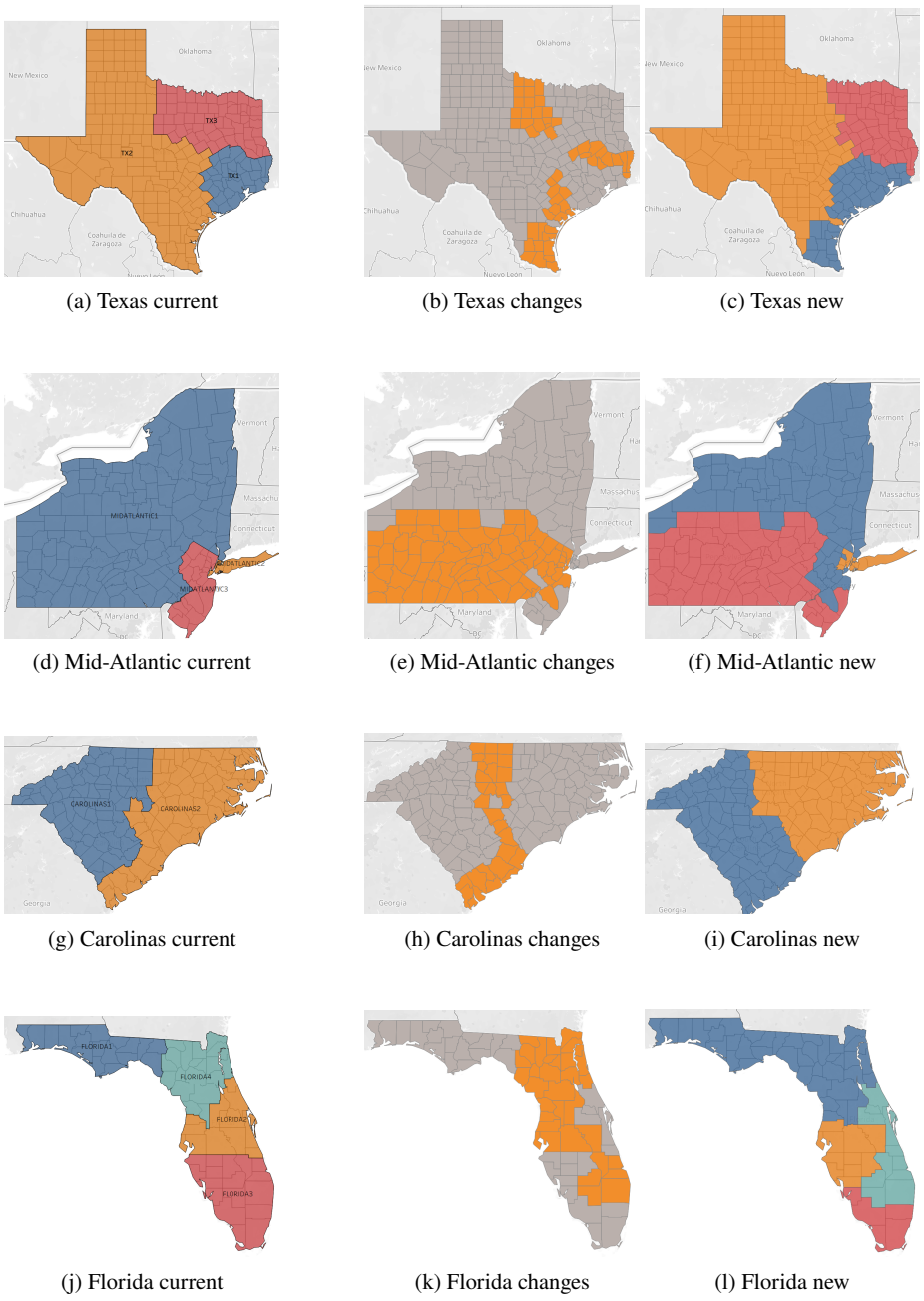
Table 5: Greenfield problem size.

Region	**Territories**	**Counties**	**Combinations**
Texas	3	254	2,699,004
Florida	4	67	766,480
Mid-Atlantic	3	150	551,300
Carolinas	2	146	10,585

Table 6: Greenfield results summary.

Region	**Current Max-Min**	**New Max-Min**	**Changes**
Texas	7.96%	2.76%	54
Florida	34.92%	2.76%	33
Mid-Atlantic	28.36%	0.29%	78
Carolinas	10.03%	0.49%	27
Average	20.32%	1.57%	48

Fig. 4: Maps of Greenfield results.



(a) Texas current (b) Texas changes (c) Texas new

(d) Mid-Atlantic current (e) Mid-Atlantic changes (f) Mid-Atlantic new

(g) Carolinas current (h) Carolinas changes (i) Carolinas new

(j) Florida current (k) Florida changes (l) Florida new

6 Conclusions

In supply chain and sales operations, the districting problem has traditionally been treated as a theoretical mathematical exercise in clustering. Conventional algorithms often rely on Greenfield approaches that ignore existing territory structures. However, in business reality, analytics practitioners rarely operate on a blank slate. Implementing drastically different territory designs risks disrupting established B2B relationships and can lead to lost market share and manager burnout. Moreover, a proposal to completely redo the existing sales territories will inevitably be perceived as a high-risk move, and the management would be less likely to authorize this initiative, regardless of the anticipated benefits.

This chapter has advocated for a Brownfield mindset toward territory optimization, shifting the goal from reaching a theoretical maximum to identifying targeted reassignments that offer the path of least organizational resistance. By integrating Change Fatigue as a mathematical penalty term alongside traditional volume-based and geographic metrics, we formalized a multi-objective optimization framework that respects the legacy operational structures of a business.

Through our empirical evaluation of four distinct business regions, we demonstrated that an Incremental Brownfield Algorithm provides significant territory improvements with a minimal number of geographical unit reassignments. Specifically, the IBA successfully reduced the average territory volume imbalance from 20.32% to 9.94% by switching an average of only 11 counties. In contrast, a Greenfield approach achieved a near-perfect balance with an average volume gap of 1.57%, but required an average of 48 county changes.

In business reality, getting from a 20% gap to a 1.5% gap by changing 48 counties represents a massive disruption that is often dead on arrival. Alternatively, reaching an acceptable 10% gap with only 11 assignment changes offers a "minimal change, maximal impact" solution that leadership and sales forces can readily adopt. Ultimately, the most effective data-driven improvements in supply chain management are achieved not by completely redrawing the map, but through a series of incremental, targeted realignments that successfully negotiate between theoretical optimality and practical execution.

References

[1] M. Bender and J. Kalcsics. Multi-period service territory design. In R. Z. Ríos-Mercado, editor, *Optimal Districting and Territory Design*, volume 284 of *International Series in Operations Research and Management Science*, chapter 7, pages 129–152. Springer, Cham, Switzerland, 2020.

[2] M. Bender, J. Kalcsics, S. Nickel, and M. Pouls. A branch-and-price algorithm for the scheduling of customer visits in the context of multi-period service territory design. *European Journal of Operational Research*, 269(1):382–396, 2018.

[3] C. A. Beswick. Allocating selling effort via dynamic programming. *Management Science*, 23(7):667–678, 1977.

[4] B. Bozkaya, E. Erkut, and G. Laporte. A tabu search heuristic and adaptive memory procedure for political districting. *European Journal of Operational Research*, 144(1):12–26, 2003.

[5] C. Campos-Valdés, E. Álvarez-Miranda, J. Pereira, R. Z. Ríos-Mercado, and D. L. Huerta-Muñoz. Sixty years of research on districting problems: Models, algorithms, and applications. Preprint DIE-RR26-01, Graduate Program in Electrical Engineering, Universidad Autonoma de Nuevo Leon, San Nicolas de los Garza, Mexico, March 2026.

[6] F. Caro, T. Shirabe, M. Guignard, and A. Weintraub. School redistricting: Embedding GIS tools with integer programming. *Journal of the Operational Research Society*, 55(8):836–849, 2004.

[7] P. Chen, X. Wang, X. Shu, Y. Liu, and P. Wang. A districting system for sales territory design. In *WWW'25: Companion Proceedings of the ACM on Web Conference 2025*, pages 143–151, Sydney, Australia, May 2025. ACM. ISBN: 979-8-4007-1331-6.

[8] L. S. de Assis, P. M. Franca, and F. L. Usberti. A redistricting problem applied to meter reading in power distribution networks. *Computers & Operations Research*, 41:65–75, 2014.

[9] A. Drexl and K. Haase. Fast approximation methods for sales force deployment. *Management Science*, 45(10):1307–1323, 1999.

[10] J. C. Duque, R. Ramos, and J. Suriñach. Supervised regionalization methods: A survey. *International Regional Science Review*, 30(3):195–220, 2007.

[11] B. Fleischmann and J. N. Paraschis. Solving a large scale districting problem: A case report. *Computers & Operations Research*, 15(6):521–533, 1988.

[12] K. Haase and S. Müller. Upper and lower bounds for the sales force deployment problem with explicit contiguity constraints. *European Journal of Operational Research*, 237(1):677–689, 2014.

[13] L. Hervert-Escobar and V. Alexandrov. Iterative projection approach for solving the territorial business sales optimization problem. *Procedia Computer Science*, 122:1069–1076, 2017.

[14] L. Hervert-Escobar and V. Alexandrov. Territorial design optimization for business sales plan. *Journal of Computational and Applied Mathematics*, 340:501–507, 2018.

[15] J. Kalcsics and R. Z. Ríos-Mercado. Districting problems. In G. Laporte, S. Nickel, and F. Saldanha da Gama, editors, *Location Science*, chapter 25, pages 703–741. Springer, Cham, Switzerland, 2nd edition, 2019.

[16] F. Liberatore, M. Camacho-Collados, and B. Vitoriano. Police districting problem: Literature review and annotated bibliography. In R. Z. Ríos-Mercado, editor, *Optimal Districting and Territory Design*, volume 284 of *International Series in Operations Research and Management Science*, chapter 2, pages 9–29. Springer, Cham, Switzerland, 2020.

[17] L. M. Lodish. Sales territory alignment to maximize profit. *Journal of Marketing Research*, 12(1):30–36, 1975.

[18] J. F. López-Pérez and R. Z. Ríos-Mercado. Embotelladoras ARCA uses operations research to improve territory design plans. *Interfaces*, 43(3):209–220, 2013.

[19] A. Ozel, K. Smilowitz, and L. K. S. Goldstein. Community-engaged school district design: A stream-based approach. *Operations Research*, 73(4):1916–1931, 2025.

[20] F. Ricca and A. Scozzari. Mathematical programming formulations for practical political districting. In R. Z. Ríos-Mercado, editor, *Optimal Districting and Territory Design*, volume 284 of *International Series in Operations Research and Management Science*, chapter 6, pages 105–128. Springer, Cham, Switzerland, 2020.

[21] R. Z. Ríos-Mercado and J. F. Bard. An exact algorithm for designing optimal districts in the collection of waste electric and electronic equipment through an improved reformulation. *European Journal of Operational Research*, 276(1): 259–271, 2019.

[22] R. Z. Ríos-Mercado and E. Fernández. A reactive GRASP for a commercial territory design problem with multiple balancing requirements. *Computers & Operations Research*, 36(3):755–776, 2009.

[23] R. Z. Ríos-Mercado and J. F. López-Pérez. Commercial territory design planning with realignment and disjoint assignment requirements. *Omega*, 41(3):525–535, 2013.

[24] M. A. Salazar-Aguilar, R. Z. Ríos-Mercado, and M. Cabrera-Ríos. New models for commercial territory design. *Networks and Spatial Economics*, 11(3):487–507, 2011.

[25] R. J. Shanker, R. E. Turner, and A. A. Zoltners. Sales territory design: An integrated approach. *Management Science*, 22(3):309–320, 1975.

[26] B. Skiera and S. Albers. COSTA: Contribution optimizing sales territory alignment. *Marketing Science*, 17(3):196–213, 1998.

[27] S. Yanık and B. Bozkaya. A review of districting problems in health care. In R. Z. Ríos-Mercado, editor, *Optimal Districting and Territory Design*, volume 284 of *International Series in Operations Research and Management Science*, chapter 3, pages 31–55. Springer, Cham, Switzerland, 2020.

[28] A. A. Zoltners. Integer programming models for sales territory alignment to maximize profit. *Journal of Marketing Research*, 13(4):426–430, 1976.

[29] A. A. Zoltners and P. Sinha. Sales territory design: Thirty years of modeling and implementation. *Marketing Science*, 24(3):313–331, 2005.