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Heuristics for Minimum Cost Steady-State Gas Transmission Networks

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Abstract

In this paper we propose and present two heuristics for the problem of minimizing fuel cost on steady-state gas transmission problems on looped networks. One of the procedures is based on a two-stage iterative procedure, where, in a given iteration, gas flow variables are fixed and optimal pressure variables are found via dynamic programming in the first stage. In the second stage, the pressure variables are fixed and an attempt is made to find a set of flow variables that improve the objective function by exploiting the underlying network structure. The other proposed heuristic adapts some concepts from generalized reduced gradient methods to attempt to find the direction step. This work focuses on looped network topologies, that is, networks with at least one cycle containing two or more compressor stations. These type of topologies posse the highest degree of difficulty in real-world problems.

Keywords : natural gas, transmission networks, fuel minimization, heuristics, generalized reduced gradient

1 Introduction

A gas transmission network for delivering natural gas involves a broad variety of physical components such as pipes, regulators, and compressor stations to name a few. As the gas travels through the pipe, gas pressure is lost due to friction with the pipe wall. Some of this pressure is added back at compressor stations, which raise the pressure of the gas passing through them. In a gas transmission network, the overall operating cost of the system is highly dependent upon the operating cost of the compressor stations in a network. A compressor station's operating cost, however, is generally measured by the fuel consumed at the compressor station. Hence, the goal is to minimize the total fuel consumption used by the stations while satisfying specified delivery requirements throughout the system.

Gas transmission network problems differ from traditional network flow problem in some fundamental aspects. First, in gas networks, a pressure variable is defined at every node in addition to the flow variables representing mass flow rates through each pipe. Second, in addition to the network flow conservation constraint set, there exist two other type of constraints: (1) a nonlinear equality constraint on each pipe, which represent the relationships between the pressure drop and the flow; and (2) a nonlinear non-convex set for each compressor station, which represents the feasible operating limits for pressure and flow within the station.

The problem is very difficult due to the presence of non-convexities in both the objective function and the set of feasible solutions. Optimization algorithm (most of them based on dynamic programming) for non-looped gas network topologies are in a relatively well developed stage. However, effective algorithms for looped topologies are practically non-existent.

In this paper we propose two heuristics for the fuel cost minimization on gas transmission systems with a looped network topology, that is, networks with at least one cycle containing two or more compressor station arcs. The network based heuristic (NBH) is based on a two-stage iterative procedure. In a particular iteration, at a first stage, gas flow variables are fixed and optimal pressure variables are found via dynamic programming (DP). At the second stage, the pressure variables are fixed and an attempt is made to find a set of flow variables that improve the objective function by exploiting the underlying network structure. The GRG based heuristic (GBH) is based in the generalized reduced gradient and attempts to generate descent directions and cope with the infeasibility issue at the same time.

The organization of this paper is as follows. In Section 2 we introduce the problem and present the mathematical model. Our proposed heuristics NBH and GBH are presented in Sections 3 and 4, respectively. We wrap up with a discussion of the direction of this work in Section 5

2 Problem Statement and Mathematical Formulation

Let $G = (N, L, M)$ be a directed network defined by a set N of n nodes, a set L of l pipes, and a set M of m compressor stations. The mass flow rate on a pipe $(i,j) \in L$ is represented by u_{ij}, and the mass flow rate through a compressor station $(i,j) \in M$ is represented by v_{ij}. Note that each compressor station is represented by a special pipe which connects a pair of nodes $(i,j) \in M$, where i and j are the corresponding suction and discharge nodes, respectively. Let u, v be the vectors of u_{ij}'s and v_{ij}'s, i.e., $u = \{u_{ij}, (i,j) \in L\}, v = \{v_{ij}, (i,j) \in M\}$, and let w be the vector defined by $w = (u,v)^T$. Let $p = (p_1, ..., p_n)^T$ be the pressure vector with p_i the pressure at node i. Let $s = (s_1, ..., s_n)^T$ be the source vector with s_i the source at node i. If s_i is positive (negative), this corresponds to the gas supply limit (demand requirement) at node i. For the steady-state model, the sum of the sources is assumed to be zero, i.e., $\sum_{i=1}^{n} s_i = 0$.

The flow balance equation at a node has the following meaning: the sum of flows coming out of the node is equal to the sum of the flow entering the node. It can be represented as

$$\sum_{j:(i,j)\in L\cup M} w_{ij} - \sum_{j:(j,i)\in L\cup M} w_{ji} = s_i, \quad \forall \ i \in N, \tag{1}$$

where w_{ij} represents either u_{ij} if $(i,j) \in L$ or v_{ij} if $(i,j) \in M$.

The physical law that relates the flow in the pipe to the difference of pressure at its two ends for high-pressure networks is given, as discussed in (Osiadacz, 1987), by the Weymouth's formula:

$$p_i^2 - p_j^2 = k_{ij} u_{ij}^2, \qquad \forall \ (i,j) \in L, \tag{2}$$

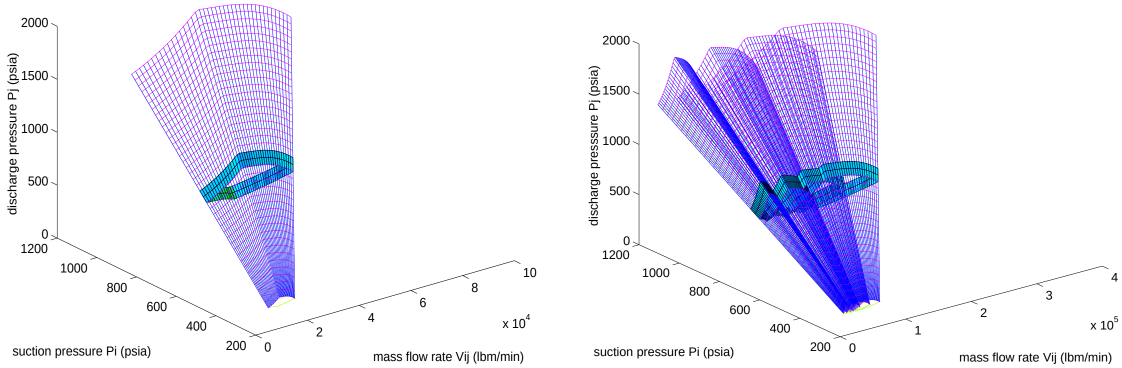
where k_{ij} is a constant whose value depends on the pipe physical properties.

The physical operational limits at each compressor station is another set of constraints, which includes the maximum/minimum compressor speed ratio, the maximum/minimum allowable volumetric flow rate. A compressor station is typically of many compressor units (which in turn can be of many types) arranged in different configurations settings. Let us assume that each compressor station (i,j) has k centrifugal compressor units hooked up in parallel.

Let D_{ij}^k denote the feasible compressor domain for variables (v_{ij}, p_i, p_j), and let $g_{ij}^k(v_{ij}, p_i, p_j)$ denote its corresponding fuel cost function. Recent work by (Wu et al., 1999) contains a detailed explanation about the structure of the domain D_{ij}^k, and the behavior of the fuel consumption function g_{ij}^k. Figure 1 from (Wu et al., 1999) shows an example of domain D_{ij}^{unit} ($k = 1$ centrifugal compressor unit) and a compound domain D_{ij}^4.

The fuel cost function, g_{ij}^{unit}, in a single compressor unit is computed by

$$g_{ij}^{unit}(v_{ij}, p_i, p_j) = a_{ij} v_{ij} \left\{ \left(\frac{p_j}{p_i} \right)^m - 1 \right\}, \qquad \forall \ (v_{ij}, p_i, p_j) \in D_{ij}^{unit}, \tag{3}$$



(a) D_{ij}^{unit} for a compressor station with a single compressor unit

(b) D_{ij}^{k} for compressor station (i, j) with $k = 4$ compressor units in parallel

Figure 1: Feasible domain of the compressor station.

where a_{ij} and m are constants which are determined by the specific type of compressors involved.

In our work, we use function g'_{ij}, extended version of g_{ij}^{unit}, which is given by $g'_{ij} = a_{ij} v_{ij} \{(\frac{p_j}{p_i})^m - 1\}$, $(v_{ij}, p_i, p_j) \in D_{ij}^k$. The mathematical formulation of the problem is given by

$$\text{minimize} \quad \sum_{(i,j) \in M} g'_{ij}(v_{ij}, p_i, p_j), \tag{4a}$$

$$\text{subject to} \quad \sum_{j:(i,j) \in L \cup M} w_{ij} - \sum_{j:(j,i) \in L \cup M} w_{ji} = si, \quad \forall\, i \in N, \tag{4b}$$

$$p_i^2 - p_j^2 = k_{ij} u_{ij}^2, \quad \forall\, (i,j) \in L, \tag{4c}$$

$$(v_{ij}, p_i, p_j) \in D_{ij}^k \subset R^3,\ (i,j) \in M. \tag{4d}$$

The difficulty in solving this problem arises from the presence of non-convexity in both the set of feasible solutions and the objective function. In addition, the type of underlying network topology becomes a crucial issue. For non-looped network topologies, DP approaches have been applied with relative success. See (Ríos-Mercado, 1999) and (Carter, 1998) for details of the DP algorithms.

These procedures rely heavily on theoretical results establishing that, for this type of (non-looped) systems, the involved flow variables can be uniquely determined in advance, and thus, eliminated from the problem. For network topologies with loops, the problem becomes more difficult because the flow variables can not be uniquely determined, so they indeed have to be explicitly treated in the model. Addressing looped networks becomes the main focus of this work.

3 The Network Based Heuristic

Let $x^0 = (v^0, p^0)$ be an initial feasible solution to problem (4). For a tree structured gas transmission network, flow variables v are uniquely determined. However, for looped networks, one may obtain better a objective function by modifying the current flow setting v^0. For this purpose, we introduce the residual network concept (Ahuja et al., 1993). The residual network was originally introduced to find the optimal flow (or to prove its optimality) in minimum cost network flow problems. We define the residual network with respect to the current flow vector v° as follows. We replace each arc (i, j) in the original network by two arcs, a forward arc (i, j) and a backward arc (j, i): the arc (i, j) has cost c_{ij} and the arc (j, i) has cost $-c_{ij}$.

In our heuristic flow modification step, the costs of the residual network are approximated by the derivatives of the objective function with respect to the flow on each compressor station, that is,

$$c_{ij} \approx a_{ij} \left\{ \left(\frac{p_j}{p_i} \right)^m - 1 \right\}, \tag{5}$$

where p_i, p_j are the current solution values delivered by dynamic programming with fixed flow variables. This cost c_{ij} is assigned at each forward edge of the residual network, while $-c_{ij}$ is assigned at each backward edge.

The cycle cost τ_C, total cost of the cycle C in a residual network, is defined by

$$\tau_C = \sum_{(i,j) \in M_C} \delta_{ij}(C) \cdot c_{ij}, \tag{6}$$

where $\delta_{ij}(C)$ equals 1 if (i,j) is contained in the cycle C and (i,j) is a forward arc of $G'(v^\circ)$, -1 if $(j,i) \in C$ and (j,i) is a backward arc of $G'(v^\circ)$, and 0 otherwise, and M_C is the set of compressor stations located in the cycle C. If τ_C is negative, then we call a negative cycle and denote it as C^-.

Modification of the flow is done by augmenting flow through a negative cycle C^-. That is, if there exists a negative cost cycle C^-, then we augment positive flow through C^-, and hence update the current flow setting. This flow modification step can be represented as

$$v^{new} = v^0 + \lambda \cdot \delta(C^-), \tag{7}$$

where $\lambda > 0$ is the positive amount of flow which will be added through the cycle, and $\delta(C^-)$ is the vector of $\delta_{ij}(C^-)$, a vector representing the negative cycle C^-. The flow modification step of NBH can be viewed as a nonlinear programming algorithm in which we try to find a direction (a vector of flow modification) such that by moving λ units in this direction, the objective function decreases. In our heuristic procedure, a negative cycle vector $\delta(C^-)$ corresponds to the search direction.

The value λ is bounded below by zero and above by $\overline{\lambda}$, which can be obtained by considering the complex inequality constraint set D_{ij}, $(i,j) \in C^-$. If $\overline{\lambda} = 0$, then the algorithm stops. Otherwise, we set $\lambda = \overline{\lambda} > 0$.

For the newly obtained flow setting v^{new}, we need to find pressure variables, which requires to rerun dynamic programming with fixed flow setting v^{new}. If dynamic programming with v^{new} has no feasible solution or no improvement has been achieved, we reduce the size of λ by setting $\lambda = \gamma\lambda$, where $0 < \gamma < 1$, and run dynamic programming until we get a desirable result. The algorithm is summarized below.

Step 1: Find an initial feasible solution $x^0 = (v^0, p^0)$.

Step 2: Construct the residual network G', and find a negative cycle C^- with negative cost τ_{C^-}.

Step 3: If $|\tau_{C^-}| < \varepsilon$, where ε is a small number, stop. Otherwise, go to Step 4.

Step 4: Set $\lambda = \overline{\lambda}$. If $\lambda = 0$, stop. Otherwise,

(a) Modify the current flow v^t by $v^{t+1} = v^t + \lambda \cdot \delta(C^-)$.

(b) Calculate pressure values using dynamic programming with modified flow v^{t+1}.
If dynamic programming yields infeasible solution, or $g^{t+1} - g^t > 0$, then set $\lambda = \gamma\lambda$, with $0 < \gamma < 1$, and go to (a). Otherwise, go to Step 2.

4 The GRG Based Heuristic

Procedure NBH, presented in the previous section, can be viewed in the context of nonlinear programming in which the descent direction contains information only on flow variables. Moreover, if we partition the set of flow variables into basic and nonbasic, and set any flow variable in the compressor station as basic and the rest of the flow variables as nonbasic, then NBH can be interpreted as a sectioning procedure (Reklaitis et al., 1983) within the space of the single nonbasic variables.

Now consider the case we want to find the descent direction but with the information on the pressure variables along with flow variables. That is, when partitioning the variables into basic and nonbasic variables, we not only consider the flow variables, but also the pressure variables defined at each suction or discharge node of the compressor station. Within this framework, the heuristic method could potentially be improved by changing all nonbasic variables simultaneously, which is the motivation of our GRG based heuristic (GBH).

Applying nonlinear methods such as GRG directly to problem (4) is mainly limited by the fact that the direction of the movement is not feasible due to the presence of nonlinear equality constraints (4c). So an extra projection step is necessary to maintain feasibility and this makes the computational effort very high. In GBH, we attempt to generate a feasible descent direction, by adapting some of the fundamentals from GRG.

Now we consider problem (4), and apply GRG. The two equality constraint sets, (4b) and (4c), are represented in vector form as

$$\begin{cases} A\,w \;=\; s, \\ A_l^T\,p^2 \;=\; \phi(u), \end{cases} \tag{9}$$

where $A = (A_l|A_m)$, A_l and A_m are the $n \times l$ node-pipe and the $n \times m$ node-compressor station incidence matrices, respectively, $p^2 = (p_1^2, ..., p_n^2)^T$, and $\phi(u)$ is the vector of $\phi_{ij}(u_{ij})$'s, with $\phi_{ij}(u_{ij}) = k_{ij}u_{ij}^2$, $(i,j) \in L$.

Suppose a feasible point $x^{(k)} = (w,p)$ is available for the problem along with a partition $x = (\hat{x}, \overline{x})^T$, where $\hat{x}$ and $\overline{x}$ correspond to the sets of basic and nonbasic variables, respectively. Accordingly, we also partition the objective function gradient vector ∇g into $\nabla\hat{g}$ and $\nabla\overline{g}$. The corresponding Jacobian matrix (matrix of partial derivatives) is partitioned into B and N, for basic and nonbasic columns, respectively. The index sets of basic and nonbasic variables are denoted by S_B and S_N, respectively. Let J be the gradient matrix of nonlinear pressure drop constraint set (4c) evaluated at $x^{(k)}$.

At $x^{(k)}$, given a choice of basis, the $\nabla\tilde{g}(x^{(k)})$ vector associated with the nonbasic variables is given by

$$\nabla\tilde{g}(x^{(k)}) = \nabla\overline{g}(x^{(k)}) - \nabla\hat{g}(x^{(k)})B^{-1}N, \tag{10}$$

where B and N are given by

$$B = \begin{pmatrix} A'_{S_B} \\ J_{S_B} \end{pmatrix} = \begin{pmatrix} A'_{S_B} \\ \nabla\hat{h}_1 \\ \nabla\hat{h}_2 \\ \vdots \\ \nabla\hat{h}_l \end{pmatrix}, \quad N = \begin{pmatrix} A_{S_N} \\ J_{S_N} \end{pmatrix} = \begin{pmatrix} A_{S_N} \\ \nabla\overline{h}_1 \\ \nabla\overline{h}_2 \\ \vdots \\ \nabla\overline{h}_l \end{pmatrix},$$

with $A'_{S_B} = (A_{S_B} \mid \mathit{0})$, $\mathit{0}$ being the $(n-1) \times l$ zero matrix. Here, $\nabla\overline{h}_e = (\partial h_e/\partial\overline{x})$, $\nabla\hat{h}_e = (\partial h_e/\partial\hat{x})$, where $h_e(x)$ corresponds to the e-th nonlinear equality constraint (4c), and $\nabla\overline{g} = (\partial g/\partial\overline{x})$, $\nabla\hat{g} = (\partial g/\partial\hat{x})$ denote the partial derivatives of g with respect to $\overline{x}$ and $\hat{x}$, respectively.

Necessary conditions for a local minimum are

$$\nabla\tilde{g} \geq 0, \tag{11}$$

$$\text{and} \qquad \overline{x}_i(\nabla\tilde{g}_i) = 0, \qquad \text{for } i \in S_N, \tag{12}$$

Since both conditions (11) and (12) must be satisfied at a local minimum, the nonbasic variable direction vector $\overline{d}$ can be defined as follows

$$\overline{d}_i = \begin{cases} -\nabla\tilde{g}_i, & \text{if } \nabla\tilde{g}_i \leq 0, \\ -\overline{x}_i\nabla\tilde{g}_i, & \text{if } \nabla\tilde{g}_i \geq 0, \end{cases} \tag{13}$$

where $i \in S_N$.

Thus, if $\nabla \tilde{g}_i < 0$, nonbasic variable i is increased; while if $\nabla \tilde{g}_i > 0$ and $\overline{x}_i > 0$, then nonbasic variable i is decreased. This definition ensures that when $\overline{d}_i = 0$ for all i, the necessary condition will be satisfied. When $\overline{d}$, the change in the nonbasic variables, is calculated using (13), then the change in the basic variables must be calculated using

$$\hat{d} = -B^{-1} N \overline{d}. \tag{14}$$

Note, however, that $d = (\overline{d}, \hat{d})$ is not a feasible direction. More precisely, $\overline{d}$ is a descent direction in the space of the nonbasic variables $\overline{x}$, but the composite direction vector $d = (\overline{d}, \hat{d})^T$, where $\hat{d}$ is calculated via the linear equation (14), yields infeasible points. Since $\overline{d}$ has desirable properties but the addition of $\hat{d}$ makes d infeasible, it seems reasonable that the computation of $\hat{d}$ should be revised.

In a GRG framework, for instance, we first calculate $\overline{d}$ using the linearization. Then, rather than calculating $\hat{d}$ using equation (14), we apply a direction correction step, which usually is an iterative procedure for solving sets of nonlinear equations.

In our proposed heuristic, we take similar steps as in GRG. That is, we take the nonbasic direction $\overline{d}$, which contains not only nonbasic flow variables, but also nonbasic pressure variables, along with $\hat{d}_v$ as initial movement, while setting $\hat{d}_p = 0$. Therefore, in GBH, we consider not only the direction of the movement for flow variables, but for the nonbasic pressure variables. Note that the addition of $\hat{d}_v = -A_{S_B}^{-1} N \overline{d}$ guarantees the flow conservation law being satisfied after the modification. Hence the direction of the movement of GBH is given by:

$$d = \begin{pmatrix} \overline{d} \\ \hat{d}_v \\ \hat{d}_p \end{pmatrix} = \begin{pmatrix} -\nabla \tilde{g} \\ -A_{S_B}^{-1} N \overline{d} \\ 0 \end{pmatrix} \tag{15}$$

After the modification along this direction d, the feasible pressure variables are obtained by changing the basic pressure variables using equations (4c). This calculation is done very effectively because each nonbasic pressure variable has its basic pressure variable connected by a single ordinary pipe, and hence (4c) can be used to get the value of the pressure at one end of the pipe once we know the pressure at the other end of the pipe.

The step size λ is chosen in a different way than that of NBH. In NBH, the step size λ is interpreted as the amount of augmenting flow through the cycle. In GBH, however, λ regulates both the flow amount and the pressure values. The computation of λ, therefore, needs further consideration.

Once λ is chosen, the feasible pressure variables are obtained by simple calculation. We now provide a step-by-step summary of the algorithm.

Step 1 Find an initial feasible solution $x^0 = (v^0, p^0)$ and set $k = 0$.

Step 2 Choose a partition of x into $\overline{x}$ and $\hat{x}$ such that B has nonzero determinant, and compute the reduced gradient

$$\nabla \tilde{g}(x^k)$$

Step 3 If $|\nabla \tilde{g}| < \varepsilon$, where ε is a small number, stop. Otherwise, set

$$\begin{aligned} \overline{d} &= -(\nabla \tilde{g})^T, \\ \hat{d} &= (\hat{d}_v, \hat{d}_p)^T = (-A_{S_B}^{-1} N \overline{d}, 0)^T, \\ \text{and} \quad d &= (\hat{d}, \overline{d})^T \end{aligned}$$

Step 4 Set $\lambda = \overline{\lambda}$. If $\lambda = 0$, stop. Otherwise,

(a) Modify the current solution x^k by $x^{k+1} = x^k + \lambda \cdot d$.

(b) Calculate pressure values using $(4c)$.
If new solution x^{t+1} is infeasible, or $g^{t+1} - g^t > 0$, then set $\lambda = \gamma\lambda$, with $0 < \gamma < 1$, and go to (a). Otherwise, update $t \leftarrow t + 1$ and go to Step 2.

5 Conclusion

In this paper we have presented the design of two heuristics for the fuel cost minimization on natural gas transmission networks in steady-state. The algorithms focus on addressing looped network topologies. Our current ongoing research involves the computational implementation of the heuristics so we can provide an empirical evaluation. Among other issues that remain to be investigated are a criteria for choosing the step size λ, and proof of convergence.

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