Title: Metaheuristics for Natural Gas Pipeline Network Optimization

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Summary. In this chapter, an overview on metaheuristic algorithms that have been very successful on tackling a particular class of natural gas pipeline network optimization problems is presented. In particular, the problem of minimizing fuel consumption incurred by the compressor stations driving natural gas in pipeline networks is addressed. This problem has been studied from different angles over the past few years by virtue of its tremendous economical impact. First, a general mathematical framework for this class of problems is presented. Then, the most relevant model properties and fundamental network topologies are thoroughly discussed. It is established how these different network topologies play a very important role on choosing an appropriate solution technique. This is followed by a presentation of current state-of-the-art metaheuristics for handling different versions of this problem. A discussion on metaheuristics developed to address related problems is included. Finally, some of the most relevant and important challenges of this very exciting area of research in natural gas transportation networks is highlighted.

Keywords: Natural gas transmission systems; Pipeline optimization; Nonlinear programming; Mixed-integer nonlinear programming; Tabu search; Ant colony optimization.

Introduction

There are many interesting decision-making problems in the natural gas industry that have been studied over the years. These include fields such as pipeline design, gas storage, gathering, transporation, and marketing, to name a few. An important class of these problems, referred to as the minumum fuel consumption problem (MFCP), deals with how to operate a natural gas transportation network for delivering the gas from storage facilities to local distribution companies so as to minimize the fuel consumption employed by the compressor stations moving the gas along the network. Efficient design and operation of these complex networks can substantially reduce air-borne emissions, increase safety, and decrease the very high daily operating costs due to the large amounts of fuel per day needed to operate the compressor stations driving the gas.

This type of networks are very complex and highly nonlinear since the relationship between the flow variables in every arc and the pressure values at the interconnection points is represented by nonlinear equations, and, in some cases, by partial differential equations. Thus, in general the class of MFCPs is very challenging due to the presence of nonlinearities and nonconvexities in the models representing such problems. These problems have been studied since the late 1960s from many differeny angles, most of them based on classical hierarchical control and mathematical programming approaches.

It was until very recently that metaheuristics techniques were introduced for addressing some of these problems. One of the great advantages of metaheuristic algorithms over existing approaches is that the former do not depend on gradient-based information so they can handle the nonlinear and nonconvex nature of the problems with relative ease. Furthermore, they can be combined with existing mathematical programming approaches in intelligent ways to derive hybrid metaheuristic methods.

The purpose of this chapter is to introduce the reader with an important class of challenging optimization problems in natural gas transportation networks and to give a detailed discussion on how metaheuristics have been successfully applied on addressing these problems. There are of course other important decision-making problems in the natural gas industry for which optimization techniques have made important contributions. A survey by Zheng et al [65] surveys optimization models in the natural gas industry, focusing on natural gas production, transportation, and marketing. Ríos-Mercado and Borraz-Sánchez [50] present an extensive review on on classical techniques for fuel consumption minimization on transmission systems, including gathering, transmission, and local distribution. Schmidt et al [53] present stationary nonlinear programming models of gas networks that are primarily designed to include detailed nonlinear physics in the final optimization steps for mid term planning problems. Farrokhifar et al [19] present a comprehensive survey of literature in the contexts of coordinated planning of both natural gas and electricity systems.

It is important to note that we are focusing in the decision-making process of operating a pipeline network assuming the network is already designed. The problem consisting of how to design such network, known as a pipeline network design problem, is also an optimization problem where the decision variables are the diameter choices of the pipes, the flows, the potentials, and the states of various network components [25; 31]. This design problem is usually cast as a nonconvex mixed-integer nonlinear programming problem and it is out of scope of the present work. The distribution problem [63] which consists of delivering the gas from the demand stations to the end customers, is not considered either.

The chapter is organized as follows. The basic mathematical framework for the steady-state case, including important model properties, is presented in the first section. Then, in the following section, the existing classical approaches for handling different versions of this problem including steady-state and transient models are briefly highlighted. This is followed by two sections where both a Tabu Search algorithm for handling a nonlinear programming model and an Ant Colony Optimization algorithm for handling a mixed-integer nonlinear programming model are described. Other metaheuristic approaches for related problems in the natural gas industry are reviewed next. Final remarks and discussion on future research trends about metaheuristic techniques in natural gas optimization problems are given in the last section.

Problem Description and Modeling Framework

Background

Basically, the main purpose of a natural gas transmission network is to transport gas from storage facilities to local distribution companies. The gas is moved by pressure, and pressure is lost due to the friction of the gas flow with the inner wall of the pipelines. Thus, to keep the gas moving, compressor stations, whose primary role is to increase gas pressure, are needed. In turn, every compressor station is composed of several compressor units. These units may be identical or non-identical and hook-ed up in different ways. The most typical configuration, which is assumed throughout this chapter, is that of identical compressor units hooked-up in parallel. It is well known that most of the operating costs in a pipeline network are due the amount of fuel consumed at the compressor stations.

When operating a natural gas transmission system aiming at minimizing fuel consumption, there are two main groups of decision variables that must be taken: (i) the mass flow rate through every pipe and compressor stations, and (ii) gas pressure values in each interconnection point. Additionally, decisions such has how many individual compressor units to operate within stations may be taken. Hence, the objective for a transmission network is to minimize the total fuel consumption of the compressor stations while satisfying specified delivery flow rates and minimum pressure requirements at the delivery terminals. The MFCP is typically modeled as a nonlinear or mixed-integer nonlinear network optimization problem. It is of course assumed that the network is given, that is, this is not a design problem.

Depending on how the gas flow changes with respect to time, we distinguish between systems in steady state and transient state. A system is said to be in steady state when the values characterizing the flow of gas in the system are independent of time. In this case, the system constraints, particularly the ones describing the gas flow through the pipes, can be described using algebraic nonlinear equations. In contrast, transient analysis requires the use of partial differential equations (PDEs) to describe such relationships. This makes the problem considerably harder to solve from the optimization perspective. In fact, optimization of transient models is one of the most challenging ongoing research areas. In the case of transient optimization, variables of the system, such as pressures and flows, are functions of time.

Gas transmission network problems differ from traditional network flow problems in some fundamental aspects. First, in addition to the flow variables for each arc, which in this case represent mass flow rates, a pressure variable is defined at every node. Second, besides the mass balance constraints, there exist two other types of constraints: (i) a nonlinear equality constraint on each pipe, which represents the relationship between the pressure drop and the flow; and (ii) a nonlinear nonconvex set which represents the feasible operating limits for pressure and flow within each compressor station. The objective function is given by a nonlinear function of flow rates and pressures. The problem is very difficult due to the presence of a nonconvex objective function and a nonconvex feasible region.

Description of Basic Model

Let $G = (V, A)$ be a directed graph representing a natural gas transmission network, where V is the set of nodes representing interconnection points, and A is the set of arcs representing either pipelines or compressor stations. Let V_s and V_d be the set of supply and demand nodes, respectively. Let $A = A_p \cup A_c$ be partitioned into a set of pipeline arcs A_p and a set of compressor station arcs A_c. That is, $(u, v) \in A_c$ if and only if u and v are the input and output nodes of compressor station (u, v), respectively.

Two types of decision variables are defined: Let x_{uv} denote the mass flow rate at arc $(u, v) \in A$, and let p_u denote the gas pressure at node $u \in V$. The following parameters are assumed known: B_u is the net mass flow rate in node u, and P_u^L and P_u^U are the pressure limits (lower and upper) at node u. By convention, $B_u > 0$ ($B_u < 0$) if $u \in V_s$ ($u \in V_d$), and $B_u = 0$ otherwise.

The basic mathematical model of the minimum fuel cost problem (MFCP) is given by:

$$\text{Minimize} \quad g(x, p) = \sum_{(u,v)\in A_c} g_{uv}(x_{uv}, p_u, p_v) \tag{1}$$

$$\text{subject to} \quad \sum_{v:(u,v)\in A} x_{uv} - \sum_{v:(v,u)\in A} x_{vu} = B_u \qquad u \in V \tag{2}$$

$$(x_{uv}, p_u, p_v) \in D_{uv} \qquad (u, v) \in A_c \tag{3}$$

$$x_{uv}^2 = R_{uv}(p_u^2 - p_v^2) \qquad (u, v) \in A_p \tag{4}$$

$$p_u \in [P_u^L, P_u^U] \qquad u \in V \tag{5}$$

$$x_{uv} \geq 0 \qquad (u, v) \in A \tag{6}$$

The objective function (1) measures the total amount of fuel consumed in the system, where $g_{uv}(x_{uv}, p_u, p_v)$ denotes the fuel consumption cost at compressor station $(u, v) \in A_c$. For a single compressor unit the following function is typically used:

$$g^{(1)}(x_{uv}, p_u, p_v) = \frac{\alpha x_{uv}}{\eta} \left\{ \left(\frac{p_v}{p_u} \right)^m - 1 \right\},$$

where α and m are assumed constant and known parameters that depend on the gas physical properties, and η is the adiabatic efficiency coefficient. This adiabatic coefficient is a function of (x_{uv}, p_u, p_v), that is, in general, a complex expression, implicitly defined. A function evaluation of η requires solving a linear system of algebraic equations. In practice, though, polynomial approximation functions that fit the function relatively well and are simpler to evaluate are employed. In other cases, when the fluctuations of η are small enough, η can be assumed to be a constant.

For a compressor station (u, v) with n_{uv} identical compressor units hooked-up in parallel which is very commonly found in industry, the fuel consumption is given by:

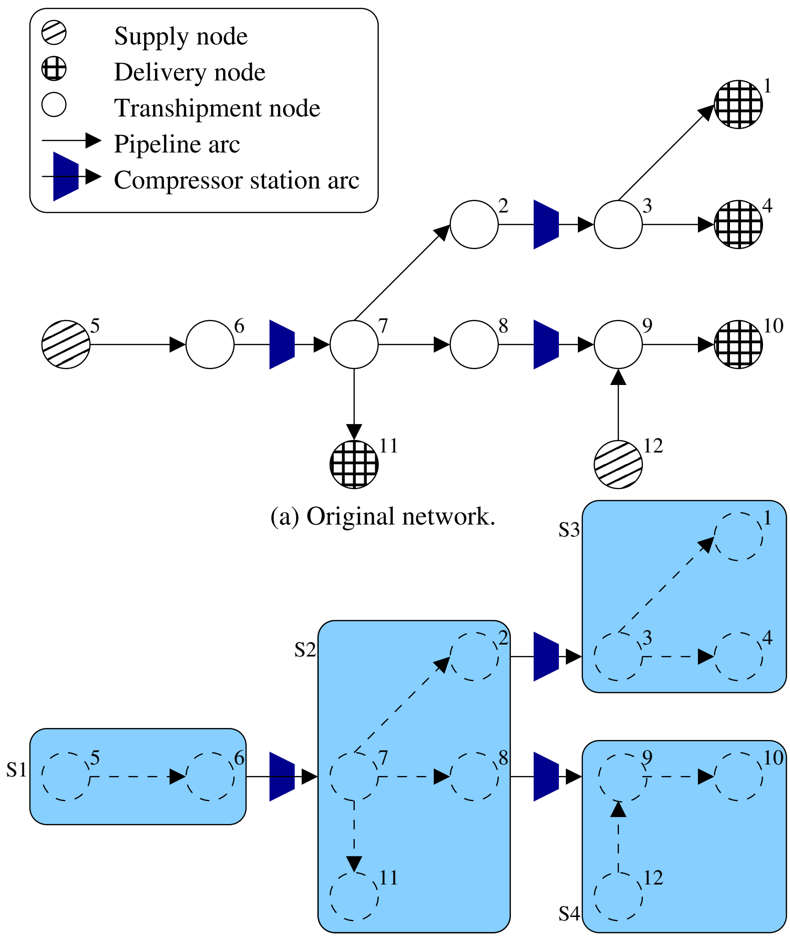
$$g_{uv}(x_{uv}, p_u, p_v) = n_{uv} g^{(1)}(x_{uv}/n_{uv}, p_u, p_v). \tag{7}$$

When all n_{uv} units are fixed and operating we have a nonlinear programming (NLP) model. Treating n_{uv} as decision variables, leads to mixed-integer nonlinear programming (MINLP) models.

Constraints (2) establish the mass balance at each node. Constraints (3) denote the compressor operating limits, where D_{uv} denote the feasible operating domain for compressor $(u, v) \in A_c$. Equations (4) express the relationship between the mass flow rate through a pipe and its pressure values at the end points under isothermal and steady-state assumptions, where R_{uv} (also known as the pipeline resistance parameter) is a parameter that depends on both the physical characteristics of the pipeline and gas physical properties. When the steady-state assumption does not hold, this relationship is a time-dependent partial differential equation which leads to transient models. Constraints (5) set the lower and upper limits of the pressure value at every node, and (6) set the non-negativity condition of the mass flow rate variables. Further details of this model can be found in Wu et al [61] and Ríos-Mercado [49].

Network Topology

There are three different kinds of network topologies: (a) linear or gun-barrel, (b) tree or branched, and (c) cyclic. Technically, the procedure for making this classification is as follows. In a given network, the compressor arcs are temporarily removed. Then each of the remaining connected components is merged into a big super-node. Finally, the compressor arcs are put back into their place. This new network is called the *associated reduced network*. Figure 1 illustrates the associated reduced network for a 12-node, 11-arc example. As can be seen, the reduced network has 4 supernodes (labeled S1, S2, S3, S4) and 3 arcs (the compressor station arcs from the original network).



(a) Original network.

(b) Associated reduced network.

Fig. 1. Illustration of a reduced network.

Types of network topologies:

Linear topology: Reduced network is a single path.

Tree topology: Reduced network is a tree.

Cyclic topology: Reduced network has cycles (either directed or undirected).

These different types of network topologies are shown in Figure 2, were the original network is represented by solid line nodes and arcs, and the reduced network by dotted super nodes. Note that even though networks in Figure 2(a) and 2(b) are not acyclic from a strict network definition, they are considered as non-cyclic pipeline network structures.

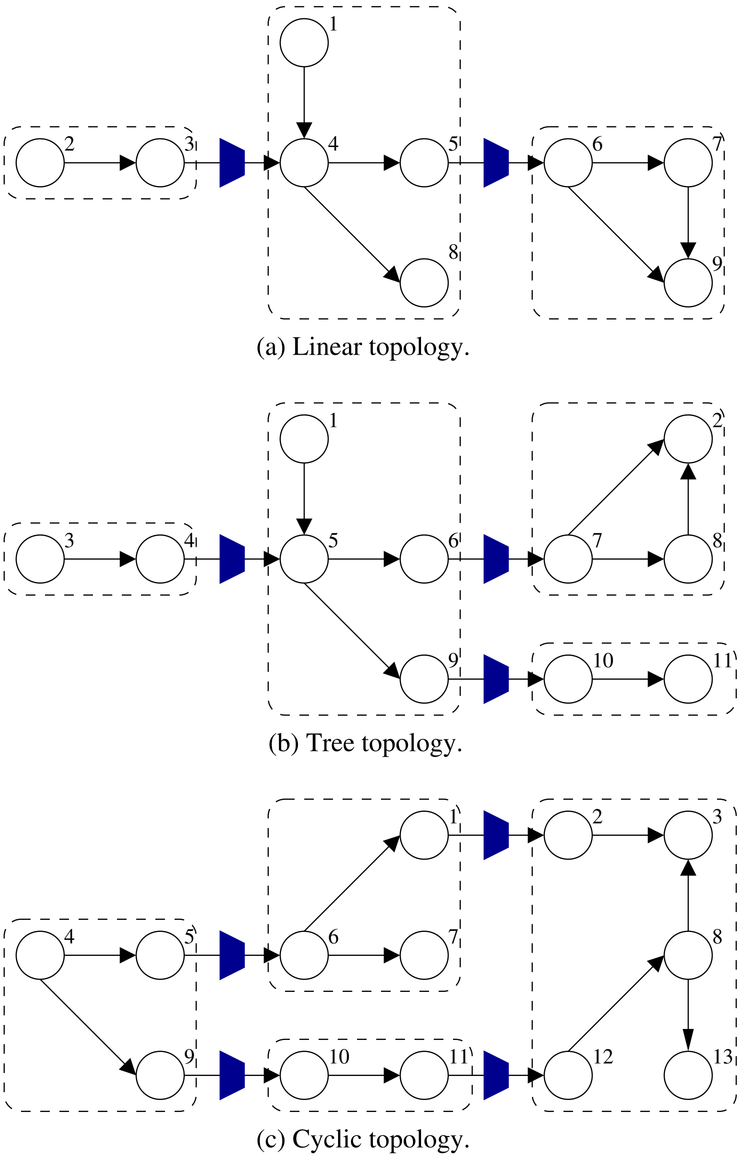


Fig. 2. Different kinds of pipeline network topologies.

Importance of Network Topology on Solution Algorithms

Let us consider the MFCP model given by (1)-(6), that is, the number of compressor units operating in each compressor station is known and fixed in advance. This is a nonconvex NLP. The network topology plays an important role, particular in staeady-state topologies, when deriving algorithms for finding the optimal set of variables. Current state of the art on solution techniques for this staedy-state MFCP reveals these important facts:

- There are theoretical results indicating that in non-cyclic systems, the values of the flow variables can be uniquely determined and fixed beforehand [51]. Therefore, the problem reduces to finding out the optimal set of pressure variables at each node in the network. Of course, the problem is still hard to solve, but it reduces its dimension in terms of the decision variables.
- As a direct consequence of this, there exists successful implementations mostly based on dynamic programming (DP) that efficiently solve the problem in non-cyclic instances by appropriately discretizing the pressure variables.
- When in a cyclyc system, we impose the limitation of fixing the flow variables in each arc, a nonsequential dynamic programming (NDP), developed by Carter [8], can been successfully applied for finding the optimal set of pressure variables. Although this algorithm has the limitation of narrowing the set of solutions to those subject to a fixed set of flows, it can be used within other flow-modification based approaches.

Solution Techniques: Classical Approaches

There is certainly a number of different optimization techniques that have been tried in the past to address problems in fuel cost minimization of natural gas transportation networks .

Steady-State NLP models: Most of the work for nonconvex NLP models has been based on steady-state models. One can find work on dynamic programming based techniques [5; 8; 15; 26; 29; 34; 52; 59; 60], including attempts to handle non-identical compressor units [15], methods based on gradient search [20; 23; 47; 45], global optimization methods [24; 26], linearization techniques [12; 22], interior-point methods [21], exploiting model properties and lower bounding schemes [7; 51; 61], multi-objective optimization models [14], and multi-criteria approaches [40].

Steady-State MINLP models: There has also been studies on developing optimization methods for addressing MINLP models. In most of these models, integer variables for deciding which individual compressor units must be opearting within a compressor station are introduced. Solution methodologies include mainly successive branch and bound [48; 56], outer approximation with augmented penalty [11; 66], sequential linear programming [23] and linearization techniques [38].

Transient models: Transient models are more challenging as the governing PDEs associated to the dynamics of the gas system must be taken into consideration. Efforts on addressing this class of very difficult problems include hierarchical control techniques [3; 30; 41; 42; 43; 46] in the early years, and, more recently, mathematical programming approaches [1; 16; 17; 28; 32; 36; 37; 44; 57; 64], multi-objective optimization methods [10], and deep learning methods [2].

For a complete literature review and detailed discussion of some of these techniques the reader is referred to the surveys of Zheng et al [65], Ríos-Mercado and

Borraz-Sánchez [50], and Arya et al [4]. In the following sections we review the most successful metaheuristic techniques applied to variations of the MFCP. Bear in mind that we are not including in the review below some papers on metaheuristics for natural gas pipeline optimization when the authors do not provide a full description of the algorithm (e.g. [58]).

Tabu Search: An Approach for NLP Models

For the past few years, Tabu Search (see Chapter "Tabu Search") has established its position as an effective metaheuristic guiding the design and implementation of algorithms for the solution of hard combinatorial optimization problems in a number of different areas. A key reason for this success is the fact that the algorithm is sufficiently flexible to allow designers to exploit prior domain knowledge in the selection of parameters and subalgorithms. Another important feature is the integration of memory-based components.

When addressing the MFCP, even though we are dealing with a continuous optimization problem, Tabu Search (TS), with an appropriate discrete solution space, is a very attractive choice due to the the non-convexity of the objective function and the versatility of TS to overcome local optimality.

We now describe the TS-based approach of Borraz-Sánchez and Ríos-Mercado [6, 7], which is regarded as the most successful implementation of a metaheuristic for the MFCP. This TS takes advantage of the particular problem structure and properties and in fact can be regarded as a hybrid metaheuristic or matheuristic (see Chapter "Matheuristics.")

Let us consider the MFCP model given by (1)-(6), that is, the number of compressor units operating in each compressor station is known and fixed in advance. As established earlier, this is a nonconvex NLP.

Recall, from last section, that, in a cyclyc system, if we fix the flow variables in each arc, NDP [8] can been successfully applied for finding the optimal set of pressure variables. Although this algorithm has the limitation of narrowing the set of solutions to those subject to a fixed set of flows, it can be used within flow-modification based approaches such as the TS presented here.

It is clear that the TS approach is aiming at finding high-quality solutions for cyclic systems. It exploits the fact that for a given set of flows an optimal set of pressure values can be efficiently found by NDP.

Nonsequential Dynamic Programming

We include in this section a brief description of the essence of the NDP algorithm. Further details can be found in [6]. Starting with a feasible set of flow variables, the NDP algorithm searches for the optimal set of node pressure values associated to that pre-specified flow. Rather than attempting to formulate DP as a recursive algorithm, at a given iteration, the NDP procedure grabs two connected compressors and replace them by a "virtual" composite element that represents the optimal operation of both compressors. These two elements can be chosen from anywhere in the system, so the idea of "sequential recursion" in classical DP does not quite apply here. After performing this step at a stage t, the system with t compressor stations has been replaced by an equivalent system with $t-1$ stations. The procedure continues until only one virtual element, which fully characterizes the optimal behavior of the entire pipeline system, is left. Afterwards, the optimal set of pressure variables can be obtained by a straight-forward backtracking process. The computational complexity of this NDP technique is $O(|A_c|N_p^2)$, where N_p is the maximum number of elements in a pressure range discretization.

The Tabu Search Approach

```
begin TS()
Input: An instance of the MFCP.
Output: A feasible solution (X, P).
(X, P)^best ← ∅;
TabuList ← ∅;
X̄ ← FIND_INITIAL_FLOW( );
while ( stopping criteria not met ) do
    for ( X ∈ V(X̄) such that X ∉ TabuList ) do
        P ← NDP( X );
    end-for
    Choose best (non-tabu) solution (X, P);
    if (|TabuList| == TabuTenure) then
        Remove oldest element from TabuList;
    end-if
    TabuList ← TabuList ∪ X;
    (X, P)^best ← Best( (X, P), (X, P)^best );
end-while
return (X, P)^best;
end TS
```

Fig. 3. Pseudocode of Procedure TS.

The main steps of the algorithm are shown in Figure 3. Here, a solution $Y = (X, P)$ is partitioned into its set of flow variables X and set of pressure variables P. First note that the search space employed by TS is defined by the flow variables X only because once the flow rates are fixed, the corresponding pressure variables are optimally found by NDP. Furthermore, we do not need to handle the entire set of flow variables, but only one per cycle. This is so because once you fix a flow rate in a cycle, the rest of the flows can be uniquely determined. Thus, a given state is represented by a vector $\hat{X} = (X_{\alpha_1}, \ldots, X_{\alpha_m})$, where α_w is an arc that belongs to a selected cycle w. Note that this set of arcs is arbitrarily chosen, and that converting a flow from X to and from $\hat{X}$ is straightforward, so in the description X and $\hat{X}$ are used interchangeably. This situation is illustrated in Figure 4. The network represents the associated reduced network. It is clear that given a specified amount of net flow entering at node 1, only one arc in each cycle is needed to uniquely determine the flows in each arc of the network. In this case, the bold arcs (5,6) and (10,11), one per cycle, suffice.

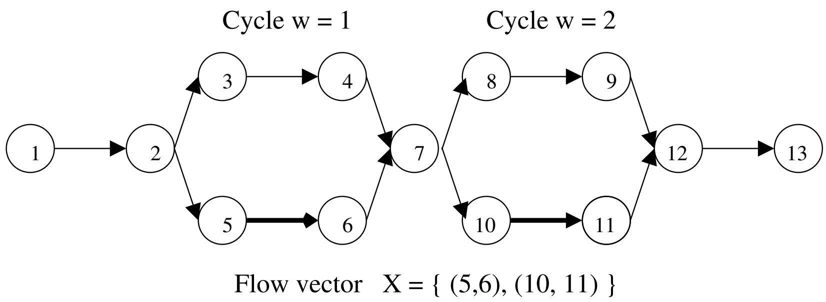


Fig. 4. Flow components of a feasible solution on a cyclic topology.

We now describe each component.

- *Initial set of flows:* First, in Step 3, in initial set of feasible flows is found. Here, different methods such as classical assignment techniques can be applied in a straightforward manner.
- *Neighborhood definition:* In Step 5, a neighborhood $V(\bar{X})$ of a given solution $\bar{X} = \{\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_m\}$ is defined as the set of solutions reachable from $\bar{X}$ via a modification of the current flow in each arc by Δ_x units in each of its components. This is given by

$$V(\bar{x}) = \{X' \in R^m \mid x'_w = \bar{x}_w \pm k\Delta_x, k = 1, 2, \ldots, N/2,\ w = 1, \ldots, m\}$$

where N is the pre-defined neighborhood size, Δ_x accounts for the mesh size, and the index w refers to the w-th cyclic component. Note that, for a given solution, the entire solution does not need to be stored but only the flow in the selected arc component to be modified. Note also that once this value is set, the rest of the flow variables in the cycle are easily determined, so in this sense, it is precisely this mass flow rate which becomes the attribute of the solution.

- *Optimal pressure values:* In Steps 6-8, the corresponding set of pressure values for the given flow is found by invoking the NDP algorithm only for those flow values that are non-tabu.
- *Tabu list:* Then in Step 9 the best $X' \in V(\bar{X})$ which is non-tabu is chosen and the corresponding subsets are updated accordingly. A tabu list (TabuList) stores recently used attributes, in our case, values of the X variables. The size of the TabuList (*TabuTenure*) controls the number of iterations a particular attribute is kept in the list.
- *Stopping criterion:* The search usually terminates after a given number of iterations , or when no significant change has been found in certain number of iterations. iterations.

As we know from theoretical properties of pipeline networks [51], the flow modification step is unnecessary for noncyclic topologies because there exists a unique set of optimal flow values which can be determined in advance at preprocessing.

Empirical Performance

To illustrate the effectiveness of the TS, Table 1 shows the performance of the TS approach when it is compared with NDP. The table indicates the objective function value obtained by both methods and the relative improvement (RI) of the TS over the NDP solution when applied to eleven cyclic real-world size instances of up to 19 super-nodes and 7 compressor stations. As we can see, the TS method significantly outperformed NDP in terms of solution quality. The running times pof the TS were about 220 and 400 CPU seconds. The running times of NDP were less than 20 CPU seconds. Although the NDP runs faster, the TS obtains solutions in less than 7/8 minutes, which is still very reasonable. Thus, it clearly pays off to spend additional computational effort because the improvement in solution quality leads to substantial economical savings in real-world instances.

Table 1. Comparison between NDP and TS.

Instance	NDP	TS	RI (%)
net-c-6c2-C1	2,317,794.61	2,288,252.53	1.27
net-c-6c2-C4	1,394,001.99	1,393,001.99	0.07
net-c-6c2-C7	1,198,415.69	1,140,097.39	4.86
net-c-10c3-C2	6,000,240.25	4,969,352.82	17.18
net-c-10c3-C4	2,533,470.72	2,237,507.93	11.68
net-c-15c5-C2	6,006,930.42	4,991,453.59	16.90
net-c-15c5-C4	3,669,976.44	3,371,985.41	8.11
net-c-15c5-C5	8,060,452.17	7,962,687.43	1.21
net-c-17c6-C1	9,774,345.45	8,659,890.72	11.40
net-c-19c7-C4	12,019,962.22	8,693,003.78	27.67
net-c-19c7-C8	8,693,003.78	7,030,280.45	19.12

Ant Colony Optimization: An Approach for MINLP Models

Let us consider now the problem where, in addition to the flow variables in each arc and the pressure variables in each node, the decision process involves determining the number of operating units in each compressor as well. This leads to a MINLP model. In this section, the Ant Colony Optimization (ACO) algorithm by Chebouba et al [9] for this version of the MFCP is described.

Ant Colony Optimization (see Chapter "Ant Colony Optimization") is a relatively new evolutionary optimization method that has been successfully applied to a number of combinatorial optimization problems. ACO is based on the communication of a colony of simple agents (called ants), mediated by (artificial) pheromone trails. The main source of ACO is a pheromone trail laying and following behavior of real ants which use pheromones a communication medium. The pheromone trails in ACO serve as distributed, numerical information which the ants use to probabilistically construct solutions to the problem being solved and which the ants adapt during the algorithm's execution to reflect its search experience.

Regarding natural gas pipeline network optimization, Chebouba et al [9] present an ACO metaheuristic for the MFCP with a variable number of compressor units within a compressor station. They focus on the linear topology case. As it was mentioned earlier, solving the MFCP on linear topologies has been successfully addressed by dynamic programming approaches when the number of compreessor units is fixed and known; however, when the number of individual compressor units is variable and part of the decision process it leads to a MINLP that has a higher degree of difficulty.

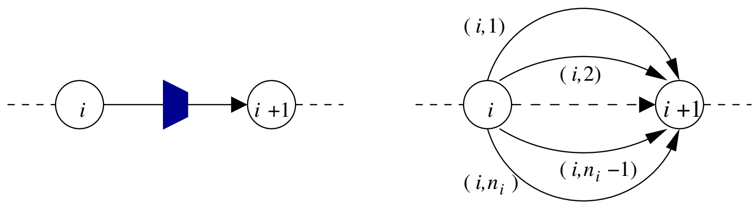


Fig. 5. Modeling compressor unit choices as a multigraph.

Consider the MINLP given by objective function (7) subject to constraints (2)-(6). When the number of individual compressor units within a compressor station are identical and hooked-up in parallel, the linear system, as depicted in Figure 2(a), can be represented by a multigraph with the compressor stations aligned sequentially where the i-th compressor station (compressor arc $(i, i+1)$ in the figure) is modeled by a set of n_i arcs between suction node i and discharge pressure $i+1$ (see Figure 5). Here, n_i is the number of individual compressor units and each of the multi arcs $(i, i+1)$ represents a decision on how many units are used in that particular station. Each multi arc in the i-th station is denoted by $(i, i+1, r)$ (or simply (i, r)), where r identifies the number of individual compressor stations to be used in a particular solution. Let L be the set of edges in this multi-graph given by $L = \{(i, r) : i \in \{1, \ldots, n\}, r \in \{1, \ldots, n_i\}\}$. In this case, the cost of arc (i, r) given by c_{ir} depends on the values of the pressure variables p_i and p_{i+1}. This will be determined during the construction of the solution. Following equation (7), the cost is then given by

$$c_{ir} = rg^{(1)}(x_{i,i+1}/r, p_i, p_{i+1}).$$

where it can be seen in a straightforward manner that, in the case of linear systems with known supply/demand values, the flow variables $x_{i,i+1}$ through the entire network can be determined and fixed beforehand. Furthermore, this cost is heuristically estimated once at the start of the procedure.

At the start of the algorithm, m ants are placed at the starting node. Ants build a solution while moving from node to an adjacent node by choosing one of the multi arcs and by randomly generating values of the pressure variables for correct computation of the arc cost. During iteration t, each ant k carries out a partial path $T^k(t)$, and in this step, the choice of arc (i, r) depends on both the cost c_{ir} and the concentration of pheromone $\tau_{ir}(t)$ on arc (i, r) at iteration t. The pheromone trail takes into account the ant's current history performance. This pheromone amount is intended to represent the learned desirability of choosing the r-th edge at node i. The pheromone trail information is changed during problem solution to reflect the experience acquired by ants during problem solving.

First, the algorithm introduces a transition rule depending on parameter $q_0 \in [0,1]$, which determines the relative importance of intensification/diversification trade-off: evert time an ant at node i chooses arc (i, r) according to the following transition rule:

$$r = \begin{cases} \arg\max_u (\tau_{iu}(t))^{\alpha}/(c_{iu})^{\beta} & \text{if } q \leq q_0, \\ s & \text{otherwise.} \end{cases}$$

where q is random variable uniformly distributed in $[0, 1]$ and s is a random variable chosen according to the following probability function:

$$p_{is}^k(t) = \frac{(\tau_{is}(t))^{\alpha}/(c_{is})^{\beta}}{\sum_u (\tau_{iu}(t))^{\alpha}/(c_{iu})^{\beta}}$$

As can be seen, low values of q_0 lead to diversification and high values of q_0 stimulates intensification. Parameters α and β control the relative importance of the pheromone trail and greedy construction value. The main steps of the algorithm are shown in Figure 6.

The pheromone trail is changed both locally (Step 7) and globally (Step 10) as follows.

- *Local updating:* Every time arc (i, r) is chosen by an ant, the amount of pheromone changes by applying this local trail update:

  $$\tau_{ir}(t) \leftarrow (1 - \rho)\tau_{ir}(t) + \rho\tau_0$$

  where τ_0 is the initial pheromone value and ρ the evaporation rate.

- *Global updating:* Upon completion of a solution by every ant in the colony, the global trail updating is done as follows. The best ant (solution) from this finished iteration is chosen according to the best objective function value g^*. Then, in each arc $(i, i + 1, r)$ used by this best ant, the trail is updated as:

  $$\tau_{ir}(t + 1) \leftarrow (1 - \rho)\tau_{ir}(t) + \frac{\rho}{g^*}$$

```
begin ACO()
Input: An instance of the MFCP.
Output: A feasible solution X.
t ← 0;
while ( stopping criteria not met ) do
    t ← t + 1;
    X^best ← ∅;
    for (k = 1, . . . , m) do
        Build solution X;
        Apply local updating rule along path of X;
        X^best ← Best(X, X^best);
    end-for
    Apply global updating rule along path of X^best;
end-while
return X^best;
end ACO
```

Fig. 6. Pseudocode of Procedure ACO.

Empirical Performance

To illustrate the usefulness of the ACO, the algorithm was tested on the Hassi R'mell-Arzew real-world pipeline network in Argelia consisting of 5 pipes, 6 nodes, 5 compressor stations, and 3 units in each compressor. The authors also built three additional cases with up to 23 compressor stations, and 12 compressor units in each compressor. We must point out that these are non-cyclic instances. The empirical evaluation includes a comparison with DP. Although these are non-cyclic systems, the number of individual compressor units within each compressor station is considered a decision variable as well, thus, the DP may not guarantee an exact solution.

Table 2 displays the results in terms of objective function value (OFV) and running time when both DP and ACO are applied to three instances.The last column (RD) indicates the relative difference between the DP solution and the ACO solution. As we can see, the DP gives a slightly better solution but it takes considerable more running time than the ACO. In the other hand, the difference in solution quality is very slim (less than 0.36 %).

Table 2. Comparison between DP and ACO.

	DP		ACO		
Instance	OFV	Time	OFV	Time	RD (%)
Cs11–Nb6	49,114	10,332	49,217	721	0.21
Cs17–Nb9	81,050	28,262	81,052	1251	0.13
Cs23–Nb12	112,985	55,168	113,390	1869	0.36

We conclude that ACO method performs reasonably well on these type of non-cyclic networks. A great advantage is its relatively ease of implementation. The issue on how this algorithm can be modified so as to handle non-cyclic systems remains an interesting topic for further investigation along this area.

Metaheuristic Approaches to Related Problems

In this section, we review some other related optimization problems in natural gas pipeline networks that have been addressed by metaheuristic methods.

Particle Swarm Optimization for Non-isothermal Systems

Wu et al [62] address a variation of the problem where, rather than minimizing fuel consumption, the focus is on maximizing a weighted combination of the maximum operation benefit and the maximum transmission amount. The operation benefit is defined as the sales income minus the costs. These costs include gas purchasing cost, pipeline's operation cost, management cost, and compressors running cost. The transmission amount is defined as the total gas volume that flows into the pipeline. In addition, a non-isothermal model is considered, that is, the authors consider the dynamics of the pipes being a function of temperature. Most of the literature focus on the isothermal case. They develop a Particle Swarm Optimization (PSO) metaheuristic enhanced by an adaptive inertia weight strategy to adjust the weight value dynamically. In a PSO implementation (see Chapter "Particle Swarm Methods,") the inertia weight parameter is used to balance the global and local search ability. If the weight has a large value, the particle will search in a broader solution space. Ifthe weight has a small value, the evolution process will focus on the space near to the local best particle. Thus, the global and local optimization performances of the algorithm can be controlled by dynamically adjusting the inertia weight value. This method adjusts the inertia weight adaptively based on the distance from the particles to the global best particle [55] .

They tested their metaheuristic (named IAPSO) in the Sebeie-Ningxiae-Lanzhou gas transmission pipeline in China. Nine stations along the pipeline distribute gases to sixteen consumers. There are four compressor stations with eight compressors to boost the gas pressure. The results show that IAPSO has fast convergence, obtaining reasonably good balances between the gas pipeline's operations benefit and its transportation amount.

Simulated Annealing for Time-Dependent Systems

As mentioned earlier, the previous two chapters addressed steady-state systems. However, when the steady-state assumption does not hold, the constraints that describe the physical behavior through a pipeline cannot be represented in the simplifying form as in (4). On the contrary, this behavior is governed by partial differential equations with respect to both flow and time. Therefore, to handle this situation, a discretization over the time variable must be done resulting in a highly complex optimization problem.

The resulting model is a mixed-integer nonlinear problem where now both, flow variables and pressure variables are also a function of time; that is, we now have x_{ij}^t and p_i^t variables for every arc $(i, j) \in A$ and time step $t \in T$, where T is the set of time steps.

Although some efforts have been made to address transient systems, one of the most successfull techniques for handling this problem is the Simulated Annealing (SA) algorithm of Mahlke et al [35] (see Chapter "Simulated Annealing.") In that work, the authors use the following main ideas. First, they relax the equations describing the gas dynamic in pipes by adding these constraints combined with appropriate penalty factors to the objective function. The penalty factor is dynamically updated ressembling a strategic oscillation strategy. This gives the search plenty of flexibility. Then, they develop a suitable neighborhood structure for the relaxed problem where time steps as well as pressure and flow of the gas are decoupled. Their key idea of the neighborhood generation is a small perturbation of flow and pressure variables in the segments and nodes, respectively. An appropriate cooling schedule, an important feature of each SA implementation, is developed. They tested their metaheuristic on data instances provided by the German gas company E.ON Ruhrgas AG. The proposed SA algorithm yields feasible solutions in very fast running times.

Simulated Annealing for Integrated Preventive Maintenance in Natural Gas Transmission Networks

An interesting work proposing a framework for optimizainng a natural gas pipeline including preventive maintenance scheduling operations is due to [39]. In his dissertation, the author proposes a simulated annealing metaheuristic to mininize compressor fuel cost in a natural gas transmission network through the integration of an annual maintenance plan for compressors in the network. The underlying multi-period mathematical model is an extension of the typical single-period MFCP models. Its solution is an operational/maintenance plan based on the best set of values uncovered for the operational and maintenance decision variables which results in the approximate overall minimisation of the fuel consumed by the compressors in the network.

The proposed solution approach is applied to three different case studies from the literature which were developed for the evaluation of single-period MFCP models. The first case study involves a linear transmission network, the second a tree transmission network, and the third a cyclic transmission network. The time horizon of each case study is 52 weeks. Before execution of the SA algorithm, a set of experiments were carried out in order to determine appropriate combinations of model parameter values for each of the three case studies. The numerical results indicate that the algorithm is capable of finding high-quality solutions to all three instances.

NSGA-II for Multi-objective Optimization

The non-dominated sorting genetic algorithm II (NSGA-II) [13] is a multi-objective evolutionary heuristic that is used to tackle optimization problems with multiple objectives. This algorithm has been successfully applied to a variety of real-world multi-objective optimization problems, in particular with nonlinear objective functions. See Chapter "Multi-Objective Optimization.")

The solution delivered by the NSGA-II is a set of non-dominated solutions which is a trade-off between the two objectives. Generally, every non-dominated solution is an acceptable solution. However, a decision maker has to choose a single solution from the optimal set by incorporating practical information and experience which significantly improves the operation of the system.

Kashani and Molaei [27] present a study that aims to find optimum values of three conflicting objective functions namely maximum gas delivery flow, maximum line pack, and minimum operating cost (sum of fuel consumption and carbon dioxide emission costs), simultaneously, for a natural gas pipeline network. They apply NSGA-II tom solve a small-scale case study with 5 compressor stations, eight pipelines and 14 nodes.

Demissie et al [14] present a nonlinear multi-objective optimization model for optimizing the operation of natural gas pipeline networks in steady state. They consider linear, tree, and cyclic topologies. Their bi-objective optimization model aims at both minimizing fuel consumption and maximizing gas delivery flow rate. To address the problem they apply a NSGA-II heuristic. They tested their algorithm in the data set instances by Wu et al [61].

In a related work, Su et al [54] use NSGA-II to solve a bi-objective optimization problem that assesses the trade-off between reliability and power demand in natural gas pipeline networks under steady state. The model considers the uncertainties of the supply conditions and customer consumptions The effectiveness of the algorithm is tested on two typical pipeline networks, a tree-topology network and a loop-topology network. The results show that the developed optimization model is able to find solutions which effectively compromise the need of minimizing gas supply shortage risk and minimizing energyu cost.

Conclusion

In this paper we have presented a description of successful metaheuristic implementations for handling very difficult optimization problems in fuel cost minimization of natural gas transportation networks. Compared to existing approaches, metaheuristics have the great advantage of not depending on gradient-based information such that they can handle nonlinearities and nonconvexities with relatively ease.

Nonetheless, metaheuristics have been widely applied mostly to discrete linear optimization problems, and not to fully extent to handle the nasty problems within the natural gas industry. Therefore, there is a tremendous area of opportunity from the metaheuristic perspective in this very important field. One must have in mind that these are real-world problems where even a marginal improvement in the objective function value represent a significant amount of money to be saved given the total flow operation of these networks throughout the year. Therefore, further research in this area is justified and needed from the practical and scientific perspective.

Important research issues such as how to derive new metaheuristics or how the developed metaheuristics can be applied, extended, modified, so as to handle MFCPs under different assumptions (e.g., non-isothermal models, non-identical compressor units, non-transient models, uncertainty) remain to be investigated. In these lines we have seen some preliminary efforts citing for instance the work of Mahlke et al [35] who present a Simulated Annealing (see Chapter “Simulated Annealing”) algorithm for addressing a MFCP under transient conditions. However, further work is needed. We know that advanced concepts in metaheuristic optimization research such as reactivity, adaptive memory, intensification/diversification strategies, or strategic oscillation, are worthwhile investigating. Furthermore, as we have seen in this paper, these models have a rich mathematical structure that allow for hybridization where part of the problem can be solved with mathematical programing techniques while being guided within a metaheuristic framework.

Over the past few years, we have also seen an increased interest in using machine learning techniques for handling certain variations of natural gas pipeline networks. One interesting feature of machine learning techniques such as deep reinforcement learning (DRL) is that they employ a model-free mechanism to optimize the system. For instance, Fan et al [18] propose a method based on Bayesian networks and DRL to optimize the reliability of gas supply in natural gas pipeline networks. More recently, Liu et al [33] propose an optimization framework for natural gas transportation pipeline networks based on DRL. The mathematical simulation model is derived from mass balance, hydrodynamics principles of gas flow, and compressor characteristics. The optimization control problem in steady state is formulated into a one-step Markov decision process and solved by DRL. The decision variables are selected as the discharge ratio of each compressor. This technique was empirically tested on very small scale non-cyclic systems (gun-barrel and tree topologies). Although the technique was compared with DP and a GA, a step forward on this direction would be to investigate its applicability on the more challenging cyclic systems on larger real-world instances.

We hope we can stimulate the interest of the scientific community, particularly from the metaheuristic optimization field, to contribute to advance the state of the art in this very challenging research area.

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Cross-references

- Ant Colony Optimization
- Matheuristics
- Multi-objective Optimization
- Particle Swarm Methods
- Simulated Annealing
- Tabu Search

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