A Divide-and-Conquer Approach to Commercial Territory Design

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December 2010

Abstract

In this paper, a commercial territory design problem with compactness maximization criteria subject to multiple territory balancing and connectivity constraints is addressed. A divide-and-conquer approach attempting to exploit recent results on successful solution of related integer quadratic models is developed with the aim of obtaining good quality solutions to large problem instances. The procedure consists of a successive dichotomy process where at each iteration a given subproblem is divided into two smaller subproblems by solving an associated territory design problem with two territories. This division process is applied until the subproblem has a tractable size and can be solved exactly by means of an integer quadratic programming model. The proposed heuristic is the first developed, to the best of our knowledge, for this particular territory design problem. Computational results showed that this proposed procedure is an attractive technique for obtaining locally optimal solutions for large instances that are intractable by using exact optimization methods.

Keywords: Territory design; Combinatorial optimization; Heuristics; Integer quadratic programming; Divide and Conquer.

1 Introduction

The problem addressed in this work is motivated by a real-world application from a beverage distribution firm in the city of Monterrey, Mexico. The problem consists of finding a partition of the entire set of city blocks (basic units, BUs) into p territories, such that a measure of territory compactness is maximized. Additionally, it is required to find territories that are connected and balanced (similar in size) with respect to the number of customers and product demand. A territory is connected if the set of BUs belonging to it induces a connected subgraph.

This problem can be found in every distribution firm before the routing plan takes place. Having shorter routes in product distribution is a direct consequence of having compact territories in the design stage. In addition, it is well established by the firm that compact territories reduce the number of unsatisfied customers caused by different deals offered to their customers.

The first related work that appears in the literature is that one studied by [21]. In this work, a reactive GRASP procedure is developed in order to minimize a dispersion measure (based on the p-Center objective) that is subject to multiple balancing constraints (number of customers, product demand, and workload). Caballero-Hernandez et al. [5] studied a related model by considering BU joint-assignment constraints. They develop a GRASP including a pre-processing phase that first satisfies the joint-assignment constraints and then a construction phase based on a territory merging mechanism with relatively good results.

Salazar-Aguilar et al. [22] present an exact optimization framework for solving small- to medium-size instances of the problem. This method is successfully applied to both p-Median and p-Center objective models. In addition, the authors propose new integer quadratic programming models that allowed to efficiently solve larges instances by commercial MINLP solvers such as DICOPT and AlphaECP. These reported results motivate the solution procedure proposed in this work.

In this work, we proposed a divide-and-conquer heuristic aiming at solving large instance of the commercial territory design problem based on the p-Median Objective for measuring dispersion. This work can be seen as an extension of the work by Salazar-Aguilar et al. [22] focusing on exact methods for small- and medium-size instances of the problem.

In particular, our proposed heuristic follows a successive dichotomies' idea where at each iteration a given subproblem is partitioned into two smaller subproblems by solving an associated territory design problem with two territories. When a given subproblem is small enough, it solved exactly by means of an integer quadratic programming model.

The proposed procedure (IQPHTDP) was evaluated over a set of randomly generated instances based on real-world data. Results revealed that IQPHTDP procedure is a very attractive technique that allows to obtain good quality solutions for large instances in reasonable times.

The remainder of this paper is organized as follows. Section 2 is devoted to the description of the problem. Section 3 highlights relevant works on the territory design/districting literature. The proposed procedure is described in Section 4. Computational results are presented in Section 5, followed by conclusions in Section 6.

2 Problem Statement

Territory design or districting consists of dividing a set of basic units (typically city blocks, zip-codes or individual customers) into subsets or groups according to specific planning criteria. These groups are known as territories or districts. Diverse applications from different areas require the territories creation. For instance, school districts, political districting, and sales territory design (see Kalcsics et al. [14]). There are few works related to is this commercial territory design problem. The first work related to this problem was introduced by Ríos-Mercado and Fernández [21]. Different versions of this problem have been studied by Caballero-Hernández et al. [5] and Salazar-Aguilar et al. [22].

Specifically, the firm wants to partition the basic units (blocks) of the city into a specific number of disjoint territories that are suitable according to their logistic, marketing and planning requirements. The company wishes to create a specific number of territories (p) that are balanced with respect to each of two attributes (number of customers and product demand). Additionally, each territory needs to be connected, so basic units (BUs) in the same territory can reach each other without leaving the territory. Territory compactness is required to guarantee that customers within a territory are relatively close to each other. The problem is modeled by a graph $G = (V, E)$, where V is the set of nodes (city blocks) and E is the set of edges that represents adjacency between blocks. That is, a block or BU j is associated with a node, and an arc connecting nodes i and j exists if BUs i and j are located in adjacent blocks. Multiple attributes such as geographical coordinates (c_x, c_y), number of customers and product demand are associated to each node $j \in V$. It is required that each node is assigned to only one territory (exclusive assignment). In particular, the firm seeks perfect balance among territories, it means each territory must have around the same number of customers and product demand associated. Let $A = \{1, 2\}$ be the set of node activities, where 1 refers to the number of customers and 2 refers to product demand. We define the size of territory V_k with respect to activity a as $w^{(a)}(V_k) = \sum_{i \in V_k}(w_i^{(a)})$, where $a \in A$ and $w_i^{(a)}$ is the value associated to activity a in the node $i \in V$. Hence, the target value is given by $\mu^{(a)} = \sum_{j \in V} \frac{w_j^{(a)}}{p}$. Another important constraint is that of connectivity, i.e., for each pair of nodes i, j that belong to the same territory, there must exist a path between them such that it is totally contained in the territory. In addition, in each territory the BUs must be relatively close to each other (compactness).

Depending on how the dispersion is measured, different models can be obtained. In this work we consider a dispersion measures based on the p-Median Problem. Full description of this model can be found in [22]. For completeness, we include here the combinatorial formulation of the MPTDP model studied in this work. Let Π be the set of all possible p-partitions of V. For a particular territory B_k, $c(k)$ is a territory center and d_{ij} is the Euclidian distance between nodes i and j, $i, j \in B_k$. A territory center is computed as

$$c(k) = \arg\min_{j \in B_k} \sum_{i \in B_k} d_{ij}$$

$$
\begin{aligned}
\text{(MPTDP)} \quad \min_{B\in\Pi} \quad f(B) &= \sum_{k=1,\ldots p} \sum_{i\in B_k} d_{ic(k)} && (1)\\
\text{Subject to :} & \\
w^{(a)}(B_k) &\in \left[(1-\tau^{(a)})\mu^{(a)}, (1+\tau^{(a)})\mu^{(a)}\right], a\in A; k=1,\ldots,p && (2)\\
G &= (B_k, E(B_k)) \quad \text{is connected} \quad \forall k=1,\ldots,p && (3)
\end{aligned}
$$

In this model, the objective is to find a p-partition of V, such that the dispersion (1) on each territory B_k is minimized. Constraints (2) establish that the territory size (number of customers and product demand) should be between the range allowed by the tolerance parameter $\tau^{(a)}$. In addition, each territory should induce a connected subgraph (3). It has been shown that MDTDP is NP-hard [23].

3 Related Work

Tables 1 and 2 contain a summary of the most important work on territory design that have been developed in diverse fields such as political districting, sales districting, and public services. These tables illustrate the main features included on these applications. Planning criteria (third column) as balancing, connectivity, and fixed number of territories are shown as 'B', 'C', and 'F', respectively. In those works where the number of territory is not fixed, the capital letter 'F' is replaced by 'V', and '-' appears in the cases where connectivity is not a constraint. In the fourth column, 'Single($\sum$)' means that two or more criteria were placed together in a weighted sum objective function.

This survey reveals that there are only a few works addressing the commercial territory design problem. Furthermore, among those works, the only studying p-Median based dispersion measures focus on exact methods for small- and medium-size instances. Therefore, the contribution of our work is to present a heuristic for solving large instances of the commercial TDP with p-Median based objective function.

4 Proposed Divide-and-Conquer Procedure

Algorithm 1 shows schematically the proposed solution procedure. It consists basically of solving a series of IQP models in such a way that those problems with more BUs than the number allowed by the *maxN* parameter are solved using a p value equal to 2. It means, given a TDP instance, if $|V| > maxN$ the algorithm carries out a dichotomy of this instance by solving the problem with $p = 2$. For example, suppose the size to the original instance is given by $(n, p) = (1000,49)$, let $S = (V_1, V_2)$ be the solution of the original instance with $p = 2$. Then the target size for V_1 should be equal to the size determined by the target value of 24 territories from the original instance, and the target size for V_2 should be equal to the target value of 25 territories from the original instance. That is, the dichotomy yields two smaller subproblems and each of them is analyzed to determine if

Table 1: Summary of territory design applications, part 1.

Author	**Application**	**Criteria**	**Objective**	**Solution Technique**
Hess and Weaver [12]	Political	B,C,F	Single	Location-allocation
Garfinkel and Nemhauser [9]	Political	B,C,F	Single	Exact procedure
Hess and Samuels [11]	Sales	B,-,F	Single	Location-allocation
Bertolazzi et al. [2]	Services	B,-,F	Single	Exact procedure
Marlin [15]	Services	B,-,F	Single	Location-allocation
Pezzella et al. [18]	Services	B,C,F	Single	Location-allocation
Fleischman and Paraschis [8]	Sales	B,-,F	Single	Location-allocation
Hojati [13]	Political	B,C,F	Single	Location-allocation
Mehrotra [16]	Political	B,C,V	Single	Heuristic based on Branch & Price
Drexl and Haase [7]	Sales	B,C,V	Single	Heuristic
Guo et al. [10]	Political	B,C,F	Bi-objective	MOZART
Muyldermans et al. [17]	Services	B,C,F	Single($\sum$)	Heuristic of two phases
Blais et al. [3]	Services	B,C,F	Single($\sum$)	Tabu search

Table 2: Summary of territory design applications, part 2.

Author	**Application**	**Criteria**	**Objective**	**Solution Technique**
Bozkaya et al. [4]	Political	B,C,F	Single($\sum$)	Tabu search and adaptive memory
Ricca and Simeone [19]	Political	B,C,F	Single($\sum$)	Old bachelor acceptance
Bong and Wang [26]	Political	B,C,F	Three-objective	Tabu search and scatter search
Bação et al. [1]	Political	B,C,F	Single	Genetic algorithms
Chou et al. [6]	Political	B,C,F	Single($\sum$)	Simulated annealing and genetic algorithms
Tavares and Figueira [25]	Services	B,C,F	Bi-objective	Evolutionary algorithm with local search
Caballero-Hernández et al. [5]	Commercial	B,C,F	Single	GRASP
Segura-Ramiro et al. [24]	Commercial	B,C,F	Single	Location-allocation
Ricca and Simeone [20]	Political	B,C,F	Single($\sum$)	Descent, tabu search, old bachelor acceptance, and simulated annealing
Ríos-Mercado and Fernández [21]	Commercial	B,C,F	Single	Reactive GRASP
Salazar-Aguilar [22]	Commercial	B,C,F	Single	Exact procedure

another dichotomy is required or not. If the instance (subproblem) given by V_1 is such that $|V_1| < maxN$, then the subproblem is solved by using $p = 24$, and the target value for each territory is given by the target value $\mu^{(a)}$ and the tolerance $\tau^{(a)}, a \in A$ (obtained from the original instance). Else, the iterative process of successive dichotomies continues until all subproblems are solved with $|V| < maxN$. The final solution is obtained by joining all partitions obtained from solving the smaller subproblems.

Note that IQPHTDP requires a TDP instance, $maxN$, and ρ as input. The control parameter ρ helps to keep balanced partitions as much as possible. It is required because if the initial dichotomy produces a partition with high relative deviation with respect to the average (target value), in the following dichotomies this value affects in such a way that the final subproblems could not have a feasible solution with respect to the average size in the original instance.

An Illustrative Example

Suppose that IQPHTDP is used for solving an instance I with (n, p) = (1999,50) and input parameters $maxN = 300$, and $\rho = 0.8$. Figure 1 shows the dichotomies process. Note that in the first dichotomy each partition V_1' and V_2' contains half the total number of required territories (thus 25 out of 50) and the number of BUs on each of them is greater than $maxN$, thus another dichotomy is needed. Partitions V_1' and V_2' are used to generate two subproblems of TDP $((G_1' = (V_1', E(V_1'))) \subset G$, and $(G_2' = (V_2', E(V_2'))) \subset G$, respectively) which are solved using $p = 2$. In Figure 1, (V_3', V_4') corresponds to the 2-partition of V_1', and (V_5', V_6') is a 2-partition of V_2'. These partitions V_3', V_4', V_5', and V_6' contain more BUs than the allowed by $maxN$, so the dichotomies process is applied on each of them until the last obtained partitions $V_l' : l = 7, ..., 14$ contain less BUs than the limit value (given by $maxN$). The latter are solved using the number of territories contained on each partition. For instance, the subproblem given by V_7' is solved for $p_7' = 6$ and the subproblem given by V_8' is solved for $p_8' = 6$. The upper and lower balancing requirements are taken from the original instance I. Note that the balancing requirements for dichotomies are computed using the control parameter ρ and the number of territories contained on each sub-instance (see Algorithm 1).

The final solution for instance I is computed by putting together all partitions obtained for solving the small subproblems (in the example the small subproblems are those generated by V_l' : $l = 7, ..., 14$). Figure 2 shows the final solution obtained for instance I by applying IQPHTDP.

Some small subproblems can be infeasible with respect to the balancing constraints, so the solution for the original instance will be infeasible. This can be avoided by selecting a suitable value for the ρ parameter. In any other case, a simple local search procedure can be applied to the final solution given by IQPHTDP in order to reach a feasible solution.

Algorithm 1 IQPHTDP(I, $maxN$, ρ)

Input:

I:= Instance of TDP

$maxN$:= Maximum number of BUs for solving the IQP model

ρ:= Control parameter

Output: $S = (V_1, ..., V_p)$: Solution, p-partition of V

$I_0(n_0, p_0, V_0, w_i^{(a)}, \tau^{(a)}) = I$:= Original instance

$L = \emptyset$:= Subproblems list

$L = L \cup I_0$, $c = 0$

while (L has instances to be solved) **do**

  Take $I_c \in L$

  if ($n_c > maxN$) **then**

    The target value for those territories contained in V_c is given by

$$\mu_{V_c}^{(a)} = \frac{\sum_{1 \in V_c} w_i^{(a)}}{p_c}$$

    if (p_c) is pair **then**

      $p'_{c1} = \frac{p_c}{2}$; $p'_{c2} = \frac{p_c}{2}$

    else

      $p'_{c1} = \frac{p_c+1}{2}$; $p'_{c2} = \frac{p_c-1}{2}$

    end if

    Solve I_c for $p = 2$

    Let $S_c = (V'_{c1}, V'_{c2})$ be the obtained solution from I_c

    the size for V'_{c1} should be,

$$p'_{c1}(1 - \rho\tau^{(a)})\mu_{V_c}^{(a)} \geq \sum_{i \in V_c} z_{i1} \leq p'_{c1}(1 + \rho\tau^{(a)})\mu_{V_c}^{(a)}$$

    and the size for V'_{c2} should be,

$$p'_{c2}(1 - \rho\tau^{(a)})\mu_{V_c}^{(a)} \geq \sum_{i \in V_c} z_{i2} \leq p'_{c2}(1 + \rho\tau^{(a)})\mu_{V_c}^{(a)}, a \in A$$

    Add the instances defined by V'_{c1} and V'_{c2} in L. It is,

    $L = L \cup Instance(V'_{c1}) \cup Instance(V'_{c2})$

  else

    Solve I_c for $p = p_c$ and $\mu^{(a)}$ {*It uses the target value associated to the instance I*}

  end if

  $c = c + 1$

end while

Put together all partitions obtained from instances with $n_c < maxN$

return $S = (V_1, ..., V_p)$

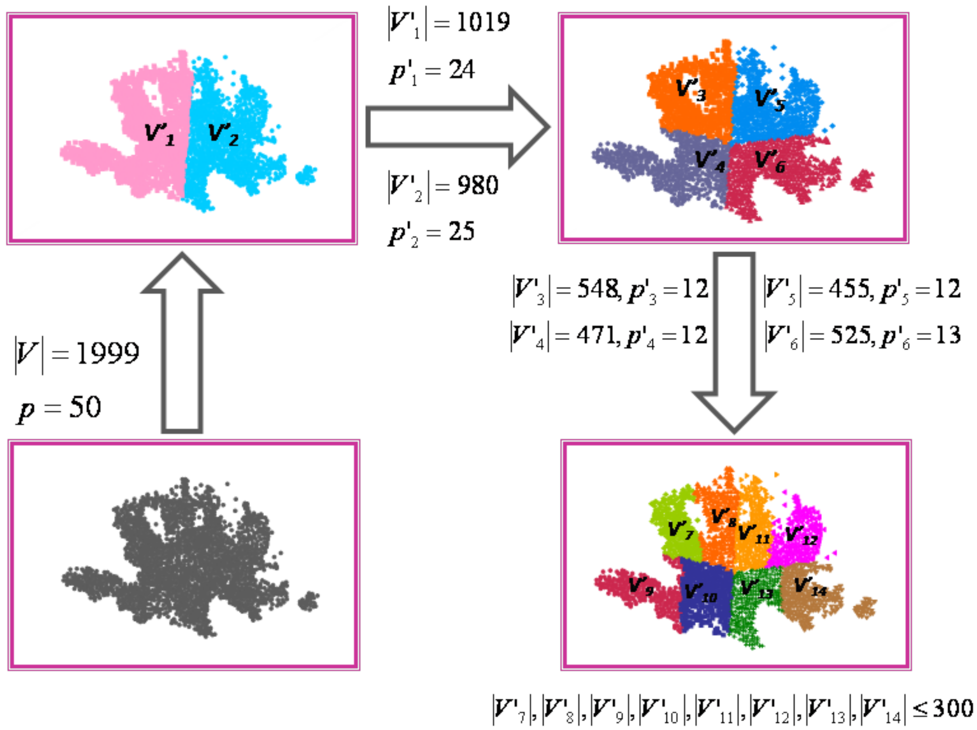

Figure 1: Successive dichotomies process for solving instance I.

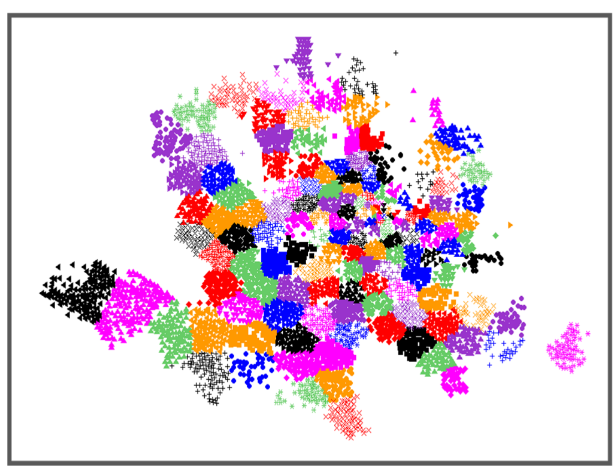

Figure 2: Final solution for instance I (using IQPHTDP).

Table 3: Best dispersion values (p-Median) for instances from (2000, 50).

Instance	$\rho = 1.0$	$\rho = 0.1$	$\rho = 0.2$
DU2k-1	Infeas	Infeas	54423.02
DU2k-2	Infeas	54337.56	54487.95
DU2k-3	Infeas	Infeas	55111.29
DU2k-4	Infeas	55642.04	54963.38
DU2k-5	Infeas	54616.84	55122.05
DU2k-6	Infeas	54145.92	55070.89
DU2k-7	Infeas	54813.34	Infeas
DU2k-8	Infeas	53048.47	54722.55
DU2k-9	Infeas	54968.87	55402.97
DU2k-10	Infeas	Infeas	55085.06

5 Experimental Work

The procedure was coded in C++, and compiled with with the Sun C++ compiler workshop 8.0 under the Solaris 9 operating system and run on a SunFire V440. Each integer quadratic subproblem is solved by calling GAMS/DICOPT MINLP solver. The data sets were taken from the library developed by [21]. These data set contains randomly generated instances based on real-world data provided by the firm. The experimental work was carried out over two instance sets $(n, p) \in \{(1000,50), (2000,50)\}$ with $\tau^{(a)} = 0.05$. For each of them 10 instances were generated.

Different values of ρ were used in order to determine the effect produced by this parameter in the final solution reported by the IQPHTDP procedure. In Table 3, the first column contains the instance name and each of the following columns show the objective value reported by the IQPHTDP for $\rho \in \{1.0, 0.1, 0.2\}$. Appropiate selection of parameter ρ is very important for the successful of the proposed procedure. If $\rho = 1.0$ it means that the balancing deviation in all IQP subproblems is given by $\tau^{(a)}$. This implies that, when the size of a partition is really close to the balancing bounds, subsequent partitions created from this partition may be very unbalanced with respect to the target value in the original instance. Hence, the final solution reported by IQPHTDP is no feasible with respect to the balance constraints in the original problem. In contrast, if the ρ value is very restrictive, some subproblems can not be solved with feasibility (see $\rho = 0.1$) and then, we do not have the way to obtain feasible solutions to the real instance. When $\rho = 0.2$ was set it allowed to solve more instances than $\rho = 0.1$. Similar behavior was observed for those instances with $(1000, 50)$. However, for these instances, $\rho = 0.1$ was a good choice for getting feasible solutions, see Table 4.

To the best of our knowledege, there is not a heuristic procedure that allows to obtain solutions for the problem addressed in this work. Until now, an available heuristic procedure developed for solving large instances of a similar problem is that one introduced by [21]. It is important to mention that the reactive GRASP procedure uses a dispersion measure based in the p-Center and the IQPHTDP uses a dispersion measure based in the p-Median. We just adapted the GRASP

Table 4: Best dispersion values for instances from (1000,50).

Instance	$\rho = 1$	$\rho = 0.1$
DU1k-1	Infeas	25679.38
DU1k-2	Infeas	26455.53
DU1k-3	Infeas	25965.95
DU1k-4	Infeas	26286.99
DU1k-5	Infeas	26522.25
DU1k-6	Infeas	26180.19
DU1k-7	Infeas	26325.41
DU1k-8	Infeas	27022.62
DU1k-9	Infeas	26347.22
DU1k-10	Infeas	26896.69

Table 5: Comparison with GRASP-RF. Instances from (1000,50).

Instance	p-Median		p-Center	
	IQPHTDP	GRASP-RF	IQPHTDP	GRASP-RF
DU1K-01	25679.38	31541.49	**71.89**	74.68
DU1K-02	26455.53	30289.81	82.13	69.38
DU1K-03	25965.95	30350.12	73.56	72.77
DU1K-04	26286.99	31084.62	**68.1**	69.87
DU1K-05	26522.25	30154.66	72.79	67.54
DU1K-06	26180.19	Infeas	**68.47**	Infeas
DU1K-07	26325.41	29173.25	**64.28**	71.04
DU1K-08	27022.61	Infeas	**69.78**	Infeas
DU1K-09	26347.22	30048.23	70.09	67.07
DU1K-10	26896.69	29369.11	77.31	62.17

procedure for using two balancing constraints. Therefore, to obtain insight about the performance of the proposed procedure. We solved the instances set by using both GRASP and IQPHTDP procedures. When, when we have a solution reported by IQPHTDP we compute its corresponding value of the p-Center measure. Conversely, when we have a solution reported by GRASP, we compute the corresponding value of the p-Median measure. Tables 5 and 6 show a summary of this test.

To the best of our knowledge, there is not a heuristic procedure that allows to obtain solutions for the problem addressed in this work. In [21], Ríos-Mercado and Fernández develop a GRASP procedure for a similar problem. In that work, they present a GRASP for the commercial TDP that uses a dispersion measure based in the p-Center Problem objective. Nevertheless, as both methods attempt to minimize a measure of dispersion we now present a comparison when our method and the GRASP of Ríos-Mercado and Fernández (named GRASP-RF) are applied to the same set of instances to obtain insight about the heuristic performance. GRASP-RF was adapted for handling two balancing constraints. When we have a solution reported by IQPHTDP we compute

Table 6: Comparison with GRASP-RF. Instances from (2000,50).

Instance	p-Median		p-Center	
	IQPHTDP	GRASP-RF	IQPHTDP	GRASP-RF
DU2K-01	54423.02	58909.07	76.69	66.07
DU2K-02	54487.96	61133.65	85.41	63.39
DU2K-03	55111.29	58654.13	75	63.85
DU2K-04	54963.32	58916.57	67.73	62.3
DU2K-05	55122.05	58676.64	67.71	61.15
DU2K-06	55070.89	59558.59	81.36	65.72
DU2K-07	Infeas	62371.46	Infeas	68.38
DU2K-08	54722.55	59908.42	80.83	67.55
DU2K-09	55402.97	58590.57	74.74	66.58
DU2K-10	55085.06	58560.103	77.37	60.55

its corresponding value of the p-Center measure. Conversely, when we have a solution reported by GRASP-RF, we compute the corresponding value of the p-Median measure. Tables 5 and 6 show a summary of this experiment.

Tables 5 and 6 show that IQPHTDP is a competitive procedure that allows obtaining good solutions to large instances of the problem. Moreover, IQPHTDP is a very attractive procedure given that it can be easily implemented and the quadratic subproblems are solved in short time. In addition, we observed that for instances from $(1000, 50)$ the GRASP-RF did not report feasible solutions for 2 out of 10 instances tested and even though p-Center is the objective function minimized by GRASP-RF, there were 5 out of 10 instances where the solution reported by IQPHTDP was better than the solution obtained by GRASP-RF.

6 Conclusions

A novel heuristic procedure based on the divide-and-conquer paradigm for territorial design called IQPHTDP has been proposed and described. This procedure allows to obtain locally optimal solutions for large instances (1000 and 2000 BUs) in short time. These instances were intractable by using existing exact methods. However, the performance of this procedure depends on the choice of the control parameter ρ. As we showed in the experimental work, the best ρ value was 0.02 for those instances with 2000 BUs and 0.01 for instances with 1000 BUs. Bad values of ρ may yield highly infeasible solutions with respect to the balancing requirements. Therefore, when the final solution is infeasible, the IQPHTDP procedure can be applied by using another ρ value, however, this change does not guarantee that the new solution will be feasible and the time increases for each trial-and-error attempt of the ρ value. An extension of this work could be the implementation of a local search procedure to reach feasibility in those cases where IQPHTDP is not able to find feasible solutions.

Acknowledgements
This work was supported by the Mexican National Council for Science and Technology under grants SEP-CONACYT 48499–Y and SEP-CONACYT 61903, Universidad Autónoma de Nuevo León under its Scientific and Technological Research Support, grant UANL-PAICYT CE012–09, and Tecnológico de Monterrey under research grant CAT128. The research of the first author has been funded by a doctoral fellowship from Universidad Autónoma de Nuevo León, grant NL–2006–C09–32652.

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