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Computational experience with heuristics for the Maximal Covering Location Problem {

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Introducción {

The Maximal Covering Location Problem (MCLP) represents a fundamental class of optimization problems in facility theory. First introduced by Church an ReVelle in 1974, MCL addresses the strategic placement of a limited number of facilities to maximize coverage of demand points within a specified service distance.

Aplications

- Emergency service placement
- Healthcare facility location
- Telecommunications network design
- Environmental infrastructure planning
- ...

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Why is it important?

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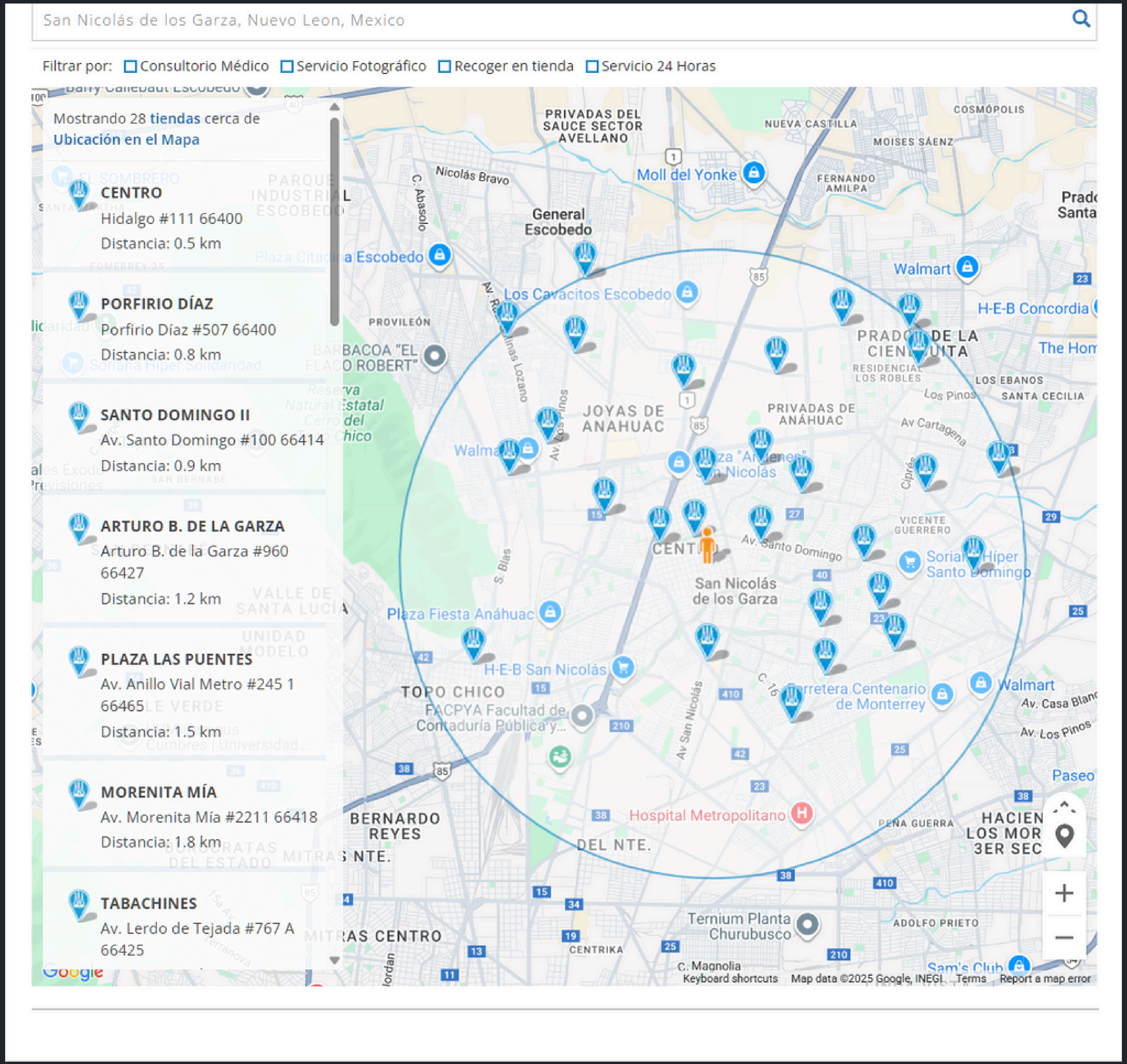
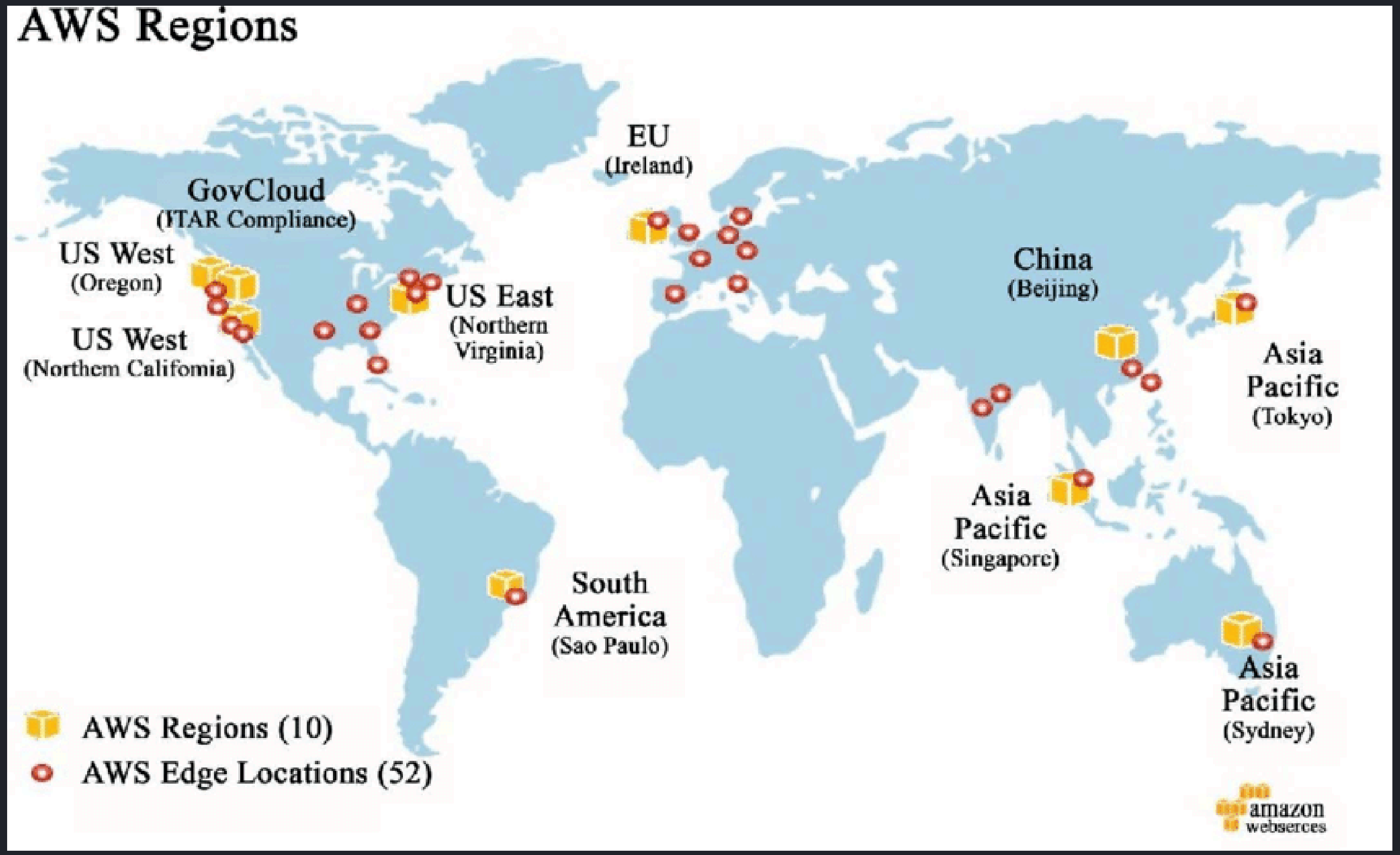


Why is it important?



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Formal definition {

THE MCLP CAN BE FORMALLY DEFINED AS FOLLOWS

Data (input information):
- A set of potential facility locations I.
- A set of demand points J, each with an associated population to be served aj.
- A maximum number of facilities to be located p.
- A coverage radius or maximum service distance S.

Decisions (variables):
- Decide which facility locations $i \in$ I will be located.
- Decide which demand points are covered by the chosen facilities.

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Formal definition {

Optimization objective:
- Maximize the number of people served or covered within the desired service distance.

Constraints:
- At most p facilities can be located.
- A demand point is considered covered if it is within distance S of at least one located facility.

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Mathematical Formulation{

Model:

$$\text{Maximize} Z = \sum_{j \in J} a_j y_j$$

Subject to:

$$\sum_{i \in N_j} x_i \geq y_j \quad \forall j \in J$$

$$\sum_{i \in I} x_i = p$$

$$x_i, y_j \in \{0, 1\} \quad \forall i \in I, j \in J$$

Where:

I = Set of potential facility locations

J = Set of demand points

aj = Demand (weight/population) at node *j*

p = Number of facilities to be located

dij = Distance between facility *i* and demand point *j*

S = Maximum service distance allowed to consider a demand point covered

Nj = { *i* ϵ I | *dij* ≤ *S*} = Set of facility sites that can cover demand node *j*

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xi = 1 if a facility is located at site i, 0 otherwise.

yj = 1 if demand node j is covered, 0 otherwise.

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Application context {

Recycling Center Location in Nuevo León

Nuevo León, México, faces significant challenges in waste management infrastructure. With increasing urbanization and population growth in metropolitan areas like Monterrey, the strategic placement of recycling centers becomes crucial for sustainable development. The region requires efficient coverage while considering:

- Population density distribution across municipalities
- Transportation accessibility
- Environmental impact minimization
- Budget constraints for facility establishment

}

Problem Parameters {

01

Demand points

Population centers across Nuevo León’s municipalities.

02

Potential sites

Suitable locations for recycling centers.

03

Coverage criterion

Maximum service distance of 20 km

04

Facility constraint

Limited number of centers due to budget restrictions

}

Constructive Heuristic: Greedy Adding {

Algorithm description

The Greedy Adding heuristic represents a myopic approach that sequentially selects facilities offering the maximum marginal coverage improvement. This method belongs to the class of greedy algorithms known for their computational efficiency and reasonable solution quality.

Pseudocoude

Matriz de Cobertura (numpy array vari°
C [i,j] = 1, si e punto de demanda i es cubetra l
por el sitio potencial j d o en caso contrario
P: Número de facilidades
W: Pesos de la Demanda (de punto

Salida:
Conjunto de sitios seleccionados

Inicialización:
N_sites = Númera de sirios $pootenciales$
S = Lista vacia de
R = Lista de sitios restanes inicialmente $[0, 1, \ldots, N_{sites} - 1]$

Bucle de Selección
mientras $|S| < P$
$Mejor\_Sitio$ = NULL
$Mejor\_Cobertura\ Adicional = -1$
Para cada j en R (sitio cand.
$Cobertura\_Actual$ = Calcul $Cobertura(S, C, W)$
$Siemporal = S \cup \{j\}$
$Nueva\_Cobertura$ = Calcul_$Cobertura(S, C, W)$
$Cobertura\_Adicional = Nueva\_Cobertura$ – Coberrut
if $Cobertura\_Adicional > Mejor\_Cobertura$
$Adicional = Cobertura\_Adicional$
$Mejor\_Sitio = j$
Actuarizar solución: o NULO (se es e rontio werera)
$Mejor\_Sitio$ a a S a S
Remover Mejor_Sitio a l
Si no compeia un sitio arranor a tceitoras
de apl oauo un R
Returnar S

Detalle de la Subrutina

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Local Search: Swap Neighborhood {

Improovement Strategy

Local search serves as an effective post-processing procedure to enhance solutions obtained from constructive heuristics. The swap neighborhood examines pairwise exchanges between selected and unselected facilities, accepting improvements until local optimality is reached.

Pseudocoude

```
Function: Swap_Move(S_current, C, W)

Inputs:
    S_current:  The current solution (list of selected site
                indices)
    C:          Coverage Matrix
    W:          Demand Weights

Output:         S_best: The best solution found after local
                search

Initialization:
    N_sites ← Number of potential sites
    S best ← S current (Copy of the initial solution)
    Coverage_best ← Calculate_Coverage(S_current, C, W)
    Improved ← TRUE
    Main Loop (While Improved is TRUE):
        Improved = FALSE
        For each i in [0,1,... |S_current| − 1] (
          (position of the faciiity to remove in S'current,
          For each j in [0,1,.., N_sites − 1]
            if j is not in S current:
              S new ← S current (Copy)
              Replace the site at position i n S new with
              site j Coveragenew ← Coverage_best
              If Coverage_new > Coverage_new
                S_best ← S new (Copy)
              Improved ← TRUE
              Break the loop over i (an improvement was found
          If Improved is TRUE
              S current = S best ( Restart the search from s_cur
```

}

Computational results {

We conducted extensive computational experiments across three problem scales:

- Set 1: 100 demand points, 30 potential sites, 8 facilities
- Set 2: 1,000 demand points, 100 potential sites, 15 facilities
- Set 3: 10,000 demand points, 200 potential sites, 25 facilities

Each set contains 20 randomly generated instances with geographical distributions and coverage constraints.

}

Performance Summary {

Set 1 Results (Small instances)

=== RESULTS SET_100 ===

Inst	Cov Const	% Cov	Time Const	Cov LS	% Cov LS	Time LS	Abs Imp	Rel Imp (%)	Swaps
1	381193	84.5%	0.0040	381193	84.5%	0.0023	0	0.00%	0
2	405276	83.5%	0.0039	405276	83.5%	0.0023	0	0.00%	0
3	433155	81.0%	0.0030	433155	81.0%	0.0018	0	0.00%	0
4	442231	84.5%	0.0029	442231	84.5%	0.0018	0	0.00%	0
5	408989	82.8%	0.0029	412109	83.5%	0.0033	3120	0.76%	2
6	447809	83.3%	0.0029	461300	85.8%	0.0044	13491	3.01%	3
7	440313	87.7%	0.0030	440313	87.7%	0.0018	0	0.00%	0
8	465284	89.0%	0.0029	465284	89.0%	0.0018	0	0.00%	0
9	406644	80.5%	0.0029	411442	81.5%	0.0025	4798	1.18%	1
10	421783	86.3%	0.0029	421783	86.3%	0.0018	0	0.00%	0
11	414596	87.3%	0.0029	415641	87.5%	0.0030	1045	0.25%	1
12	434934	85.1%	0.0029	447274	87.5%	0.0052	12340	2.84%	4
13	427525	84.6%	0.0029	427525	84.6%	0.0018	0	0.00%	0
14	476030	85.9%	0.0029	476030	85.9%	0.0018	0	0.00%	0
15	376651	77.1%	0.0029	402431	82.3%	0.0110	25780	6.84%	7
16	402335	80.6%	0.0029	402335	80.6%	0.0018	0	0.00%	0
17	431499	86.6%	0.0029	431499	86.6%	0.0018	0	0.00%	0
18	384830	78.2%	0.0029	384830	78.2%	0.0018	0	0.00%	0
19	433937	87.6%	0.0029	434798	87.8%	0.0027	861	0.20%	1
20	405840	83.9%	0.0029	410733	84.9%	0.0046	4893	1.21%	3

}

Performance Summary {

Set 2 Results (Medium Instances)

=== RESULTS SET_1000 ===

Inst	Cov Const	% Cov	Time Const	Cov LS	% Cov LS	Time LS	Abs Imp	Rel Imp (%)	Swaps
1	4005127	82.6%	0.1215	4027286	83.1%	0.1454	22159	0.55%	3
2	4136094	82.4%	0.1217	4153770	82.8%	0.1281	17676	0.43%	3
3	4165183	82.4%	0.1205	4174647	82.6%	0.1662	9464	0.23%	1
4	4059236	81.5%	0.1214	4060536	81.5%	0.1101	1300	0.03%	1
5	4054094	82.1%	0.1211	4054094	82.1%	0.0687	0	0.00%	0
6	4083274	81.3%	0.1202	4083274	81.3%	0.0682	0	0.00%	0
7	4134250	82.6%	0.1203	4134250	82.6%	0.0687	0	0.00%	0
8	4121540	81.7%	0.1206	4161150	82.4%	0.0826	39610	0.96%	2
9	4143994	82.8%	0.1205	4147733	82.9%	0.1067	3739	0.09%	1
10	4185270	81.5%	0.1207	4185270	81.5%	0.0689	0	0.00%	0
11	4310814	82.5%	0.1225	4320759	82.7%	0.1889	9945	0.23%	2
12	4278127	81.8%	0.1222	4278127	81.8%	0.0693	0	0.00%	0
13	4110852	82.6%	0.1209	4110852	82.6%	0.0685	0	0.00%	0
14	4059921	81.0%	0.1210	4114414	82.1%	0.2256	54493	1.34%	3
15	4104500	83.3%	0.1208	4104500	83.3%	0.0687	0	0.00%	0
16	4234104	82.5%	0.1205	4234104	82.5%	0.0689	0	0.00%	0
17	4190223	82.0%	0.1214	4223984	82.6%	0.0970	33761	0.81%	1
18	4070444	80.6%	0.1214	4078168	80.7%	0.2219	7724	0.19%	2
19	4184090	81.3%	0.1210	4208985	81.8%	0.1658	24895	0.59%	3
20	4019060	82.4%	0.1201	4033877	82.7%	0.1987	14817	0.37%	3

}

Performance Summary {

Set 3 Results (Large Instances)

=== RESULTS SET_10000 ===

Inst	Cov Const	% Cov	Time Const	Cov LS	% Cov LS	Time LS	Abs Imp	Rel Imp (%)	Swaps
1	38951211	76.8%	5.3919	39048502	77.0%	9.5882	97291	0.25%	4
2	39164963	77.2%	5.3490	39243696	77.4%	14.2555	78733	0.20%	5
3	38488894	76.6%	5.3332	38671206	76.9%	12.3156	182312	0.47%	6
4	38889897	77.1%	5.3086	39069428	77.5%	24.2917	179531	0.46%	8
5	38990465	77.2%	5.2848	38990465	77.2%	2.8748	0	0.00%	0
6	39003756	77.2%	5.2293	39031457	77.3%	7.0727	27701	0.07%	2
7	39183779	76.9%	5.3696	39222305	77.0%	10.8945	38526	0.10%	4
8	38926712	77.4%	5.3681	38931232	77.4%	7.5699	4520	0.01%	2
9	38468565	76.9%	5.3847	38567817	77.1%	21.7868	99252	0.26%	8
10	38595611	77.0%	5.3396	38683451	77.2%	9.7999	87840	0.23%	4
11	39005163	77.0%	5.2528	39049843	77.0%	5.1425	44680	0.11%	2
12	38853525	76.9%	5.2300	38911700	77.1%	6.2231	58175	0.15%	2
13	39123165	77.0%	5.2250	39181473	77.2%	6.4397	58308	0.15%	1
14	38970618	76.9%	5.2193	38970618	76.9%	2.8854	0	0.00%	0
15	39010888	77.1%	5.2320	39165212	77.4%	25.3550	154324	0.40%	10
16	38862671	76.9%	5.2287	38984957	77.1%	14.0125	122286	0.31%	3
17	39069353	77.3%	5.2366	39196823	77.5%	20.5815	127470	0.33%	6
18	38979237	77.1%	5.2425	38979237	77.1%	2.8916	0	0.00%	0
19	38825937	76.9%	5.2450	38825937	76.9%	2.8748	0	0.00%	0
20	38920869	77.0%	5.2632	39077258	77.3%	14.7044	156389	0.40%	5

}

Detailed Instance Performance{

SUMMARY STATISTICS

SET	Avg % Cov CH	Avg Time Const	Avg % Cov LS	Avg Time LS	Avg Absolute Improvement	Avg Relative Improvement	Improved
Small	84%	0.003 s	84.6%	0.003 s	3316	0.81%	40% (8/20)
Medium	82%	0.1210 s	82.3%	0.1193 s	11979	0.29%	60% (12/20)
Large	77%	5.2867 s	77.2%	11.0780 s	75867	0.20%	80% (16/20)

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Thanks {

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