

UNIVERSIDAD AUTÓNOMA DE NUEVO LEÓN  
FACULTAD DE INGENIERÍA MECÁNICA Y ELÉCTRICA

Homework 4

Due date: Mon 3--2025 22:00

Course: Selected Topics on Optimization

Semester: Fall 2025

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Objective: Minimize cost

Best-found Strategy

Move = 2OPT Remove 2 nonadjacent edges and reconnect to form a move

procedure TwoOpt(route)

best\_distance  $\leftarrow$  Distance(route)

improved  $\leftarrow$  true

while improved do

improved  $\leftarrow$  false

for i  $\leftarrow$  1 to length(route) - 2 do

for j  $\leftarrow$  i + 1 to length(route) - 1 do

new\_route  $\leftarrow$  TwoOptSwap(route, i, j)

new\_distance  $\leftarrow$  Distance(new\_route)

if new\_distance < best\_distance then

route  $\leftarrow$  new\_route

best\_distance  $\leftarrow$  new\_distance

improved  $\leftarrow$  true

end if

end for

end for

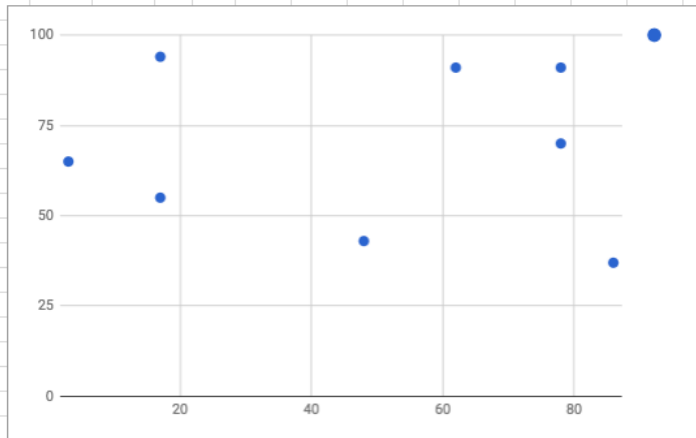
end while

return route

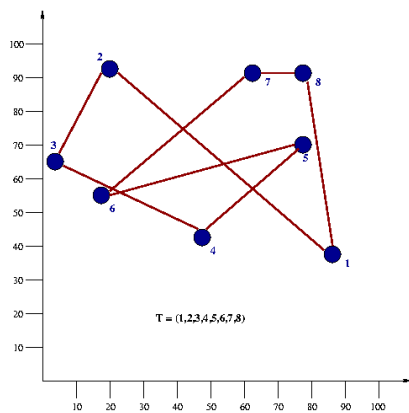
end procedure

Matrix:

	coordenadas										
	x	y		1	2	3	4	5	6	7	8
1	86	37	1	0							
2	17	94	2	89	0						
3	3	65	3	87	32	0					
4	48	43	4	38	59	50	0				
5	78	70	5	33	65	75	40	0			
6	17	55	6	71	39	17	33	62	0		
7	62	91	7	59	45	64	50	26	57	0	
8	78	91	8	54	61	79	56	21	70	16	0



Initial solution:


$$T = (1, 2, 3, 4, 5, 6, 7, 8)$$

$$\text{Cost} = 89 + 32 + 50 + 40 + 62 + 57 + 16 + 54 = 400$$

Edges:

1-2	A
2-3	B
3-4	C
4-5	D
5-6	E
6-7	F
7-8	G
8-1	H

Moves:

1. AC

$$\Delta z = d(1,2) + d(3,4) - d(2,4) - d(1,3)$$

$$\Delta z = 89 + 50 - 59 - 87 = -7$$

2. AD

$$\Delta z = d(1,2) + d(4,5) - d(1,4) - d(2,5)$$

$$\Delta z = 89 + 40 - 38 - 65 = 26$$

3. AE

$$\Delta z = d(1,2) + d(5,6) - d(1,5) - d(2,6)$$

$$\Delta z = 89 + 62 - 33 - 39 = 79$$

4. AF

$$\Delta z = d(1,2) + d(6,7) - d(2,7) - d(1,6)$$

$$\Delta z = 89 + 57 - 45 - 71 = 30$$

5. AG

$$\Delta z = d(1,2) + d(7,8) - d(2,8) - d(1,7)$$

$$\Delta z = 89 + 16 - 61 - 59 = -15$$

6. BD

$$\Delta z = d(2,3) + d(4,5) - d(2,4) - d(3,5)$$

$$\Delta z = 32 + 40 - 59 - 75 = -62$$

7. BE

$$\Delta z = d(2,3) + d(5,6) - d(2,5) - d(3,6)$$

$$\Delta z = 32 + 62 - 59 - 75 = -40$$

8. BF

$$\Delta z = d(2,3) + d(6,7) - d(2,6) - d(3,7)$$

$$\Delta z = 32 + 57 - 59 - 75 = -45$$

9. BG

$$\Delta z = d(2,3) + d(7,8) - d(2,7) - d(3,8)$$

$$\Delta z = 32 + 16 - 45 - 79 = -76$$

10. BH

$$\Delta z = d(2,3) + d(8,1) - d(2,8) - d(3,1)$$

$$\Delta z = 32 + 40 - 59 - 75 = -62$$

11. CE

$$\Delta z = d(3,4) + d(5,6) - d(3,5) - d(4,6)$$

$$\Delta z = 50 + 62 - 75 - 33 = 4$$

12. CF

$$\Delta z = d(3,4) + d(6,7) - d(3,6) - d(4,7)$$

$$\Delta z = 50 + 57 - 17 - 50 = 40$$

13. CG

$$\Delta z = d(3,4) + d(7,8) - d(3,7) - d(4,8)$$

$$\Delta z = 50 + 16 - 17 - 56 = -7$$

14. CH

$$\Delta z = d(3,4) + d(8,1) - d(3,8) - d(4,1)$$

$$\Delta z = 50 + 54 - 79 - 38 = -13$$

15. DF

$$\Delta z = d(4,5) + d(6,7) - d(4,6) - d(5,7)$$

$$\Delta z = 40 + 57 - 33 - 50 = 14$$

16. DG

$$\Delta z = d(4,5) + d(7,8) - d(4,7) - d(5,8)$$

$$\Delta z = 40 + 16 - 50 - 21 = -15$$

17. DH

$$\Delta z = d(4,5) + d(8,1) - d(4,8) - d(5,1)$$

$$\Delta z = 40 + 54 - 56 - 33 = 5$$

18. EG

$$\Delta z = d(5,6) + d(7,8) - d(5,7) - d(6,8)$$

$$\Delta z = 62 + 16 - 26 - 70 = -18$$

19. EH

$$\Delta z = d(5,6) + d(8,1) - d(5,8) - d(6,1)$$

$$\Delta z = 62 + 54 - 21 - 71 = 24$$

20. FH

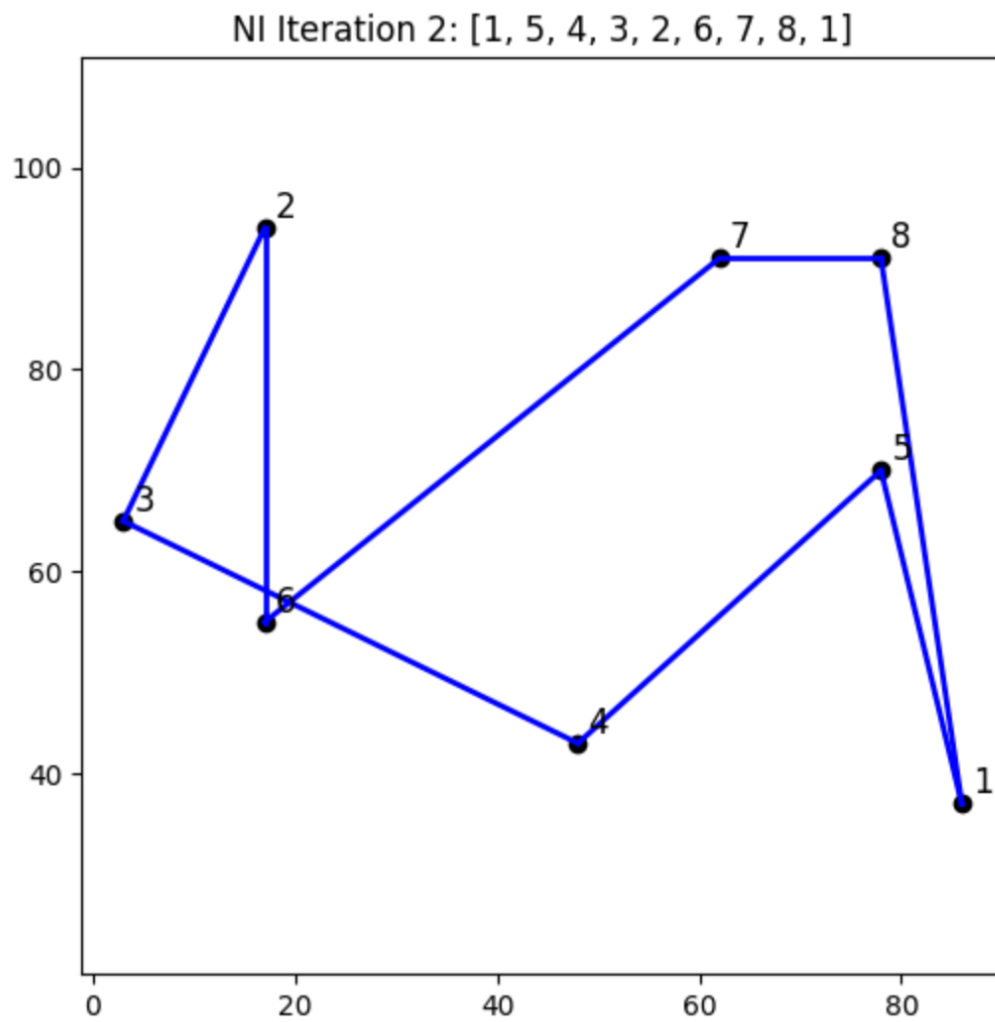
$$\Delta z = d(6,7) + d(8,1) - d(6,8) - d(7,1)$$

$$\Delta z = 57 + 54 - 70 - 59 = 18$$

BEST FOUND: AE

$T=[1,5,4,3,2,6,7,8,1]$

Total Cost =  $400 - 79 = 321$



Edges:

1-5	A
5-4	B
4-3	C
3-2	D
2-6	E
6-7	F
7-8	G
8-1	H

1. AC

$$\begin{aligned}\Delta z &= d(1,5) + d(4,3) - d(1,4) - d(5,3) \\ \Delta z &= 33 + 57 - 17 - 50 = 23\end{aligned}$$

2. AD

$$\begin{aligned}\Delta z &= d(1,5) + d(3,2) - d(1,3) - d(5,2) \\ \Delta z &= 33 + 32 - 87 - 65 = -87\end{aligned}$$

3. AE

$$\begin{aligned}\Delta z &= d(1,5) + d(2,6) - d(1,2) - d(5,6) \\ \Delta z &= 33 + 39 - 89 - 62 = -79\end{aligned}$$

4. AF

$$\begin{aligned}\Delta z &= d(1,5) + d(6,7) - d(1,6) - d(5,7) \\ \Delta z &= 33 + 57 - 71 - 26 = -7\end{aligned}$$

5. AG

$$\begin{aligned}\Delta z &= d(1,5) + d(7,8) - d(1,7) - d(5,8) \\ \Delta z &= 33 + 16 - 59 - 21 = -31\end{aligned}$$

6. BD

$$\begin{aligned}\Delta z &= d(5,4) + d(3,2) - d(5,3) - d(4,2) \\ \Delta z &= 40 + 32 - 75 - 59 = -62\end{aligned}$$

7. BE

$$\begin{aligned}\Delta z &= d(5,4) + d(2,6) - d(5,2) - d(4,6) \\ \Delta z &= 40 + 39 - 65 - 33 = -19\end{aligned}$$

8. BF

$$\begin{aligned}\Delta z &= d(5,4) + d(6,7) - d(5,6) - d(4,7) \\ \Delta z &= 40 + 57 - 62 - 50 = -15\end{aligned}$$

9. BG

$$\begin{aligned}\Delta z &= d(5,4) + d(7,8) - d(5,7) - d(4,8) \\ \Delta z &= 40 + 16 - 26 - 56 = -26\end{aligned}$$

10. BH

$$\begin{aligned}\Delta z &= d(5,4) + d(8,1) - d(5,8) - d(4,1) \\ \Delta z &= 40 + 54 - 21 - 38 = 35\end{aligned}$$

11. CE

$$\begin{aligned}\Delta z &= d(4,3) + d(2,6) - d(4,2) - d(3,6) \\ \Delta z &= 50 + 39 - 59 - 17 = 13\end{aligned}$$

12. CF

$$\begin{aligned}\Delta z &= d(4,3) + d(6,7) - d(4,6) - d(3,7) \\ \Delta z &= 50 + 57 - 33 - 64 = 10\end{aligned}$$

13. CG

$$\Delta z = d(4,3) + d(7,8) - d(4,7) - d(3,8)$$

$$\Delta z = 50 + 16 - 50 - 79 = -63$$

14. CH

$$\Delta z = d(4,3) + d(8,1) - d(4,8) - d(3,1)$$

$$\Delta z = 50 + 54 - 56 - 87 = -39$$

15. DF

$$\Delta z = d(3,2) + d(6,7) - d(3,6) - d(2,7)$$

$$\Delta z = 32 + 57 - 17 - 45 = 27$$

16. DG

$$\Delta z = d(3,2) + d(7,8) - d(3,7) - d(2,8)$$

$$\Delta z = 32 + 16 - 64 - 61 = -77$$

17. DH

$$\Delta z = d(3,2) + d(8,1) - d(3,8) - d(2,1)$$

$$\Delta z = 32 + 54 - 59 - 89 = -62$$

18. EG

$$\Delta z = d(2,6) + d(7,8) - d(2,7) - d(6,8)$$

$$\Delta z = 39 + 16 - 45 - 70 = -60$$

19. EH

$$\Delta z = d(2,6) + d(8,1) - d(2,8) - d(6,1)$$

$$\Delta z = 39 + 54 - 61 - 71 = -39$$

20. FH

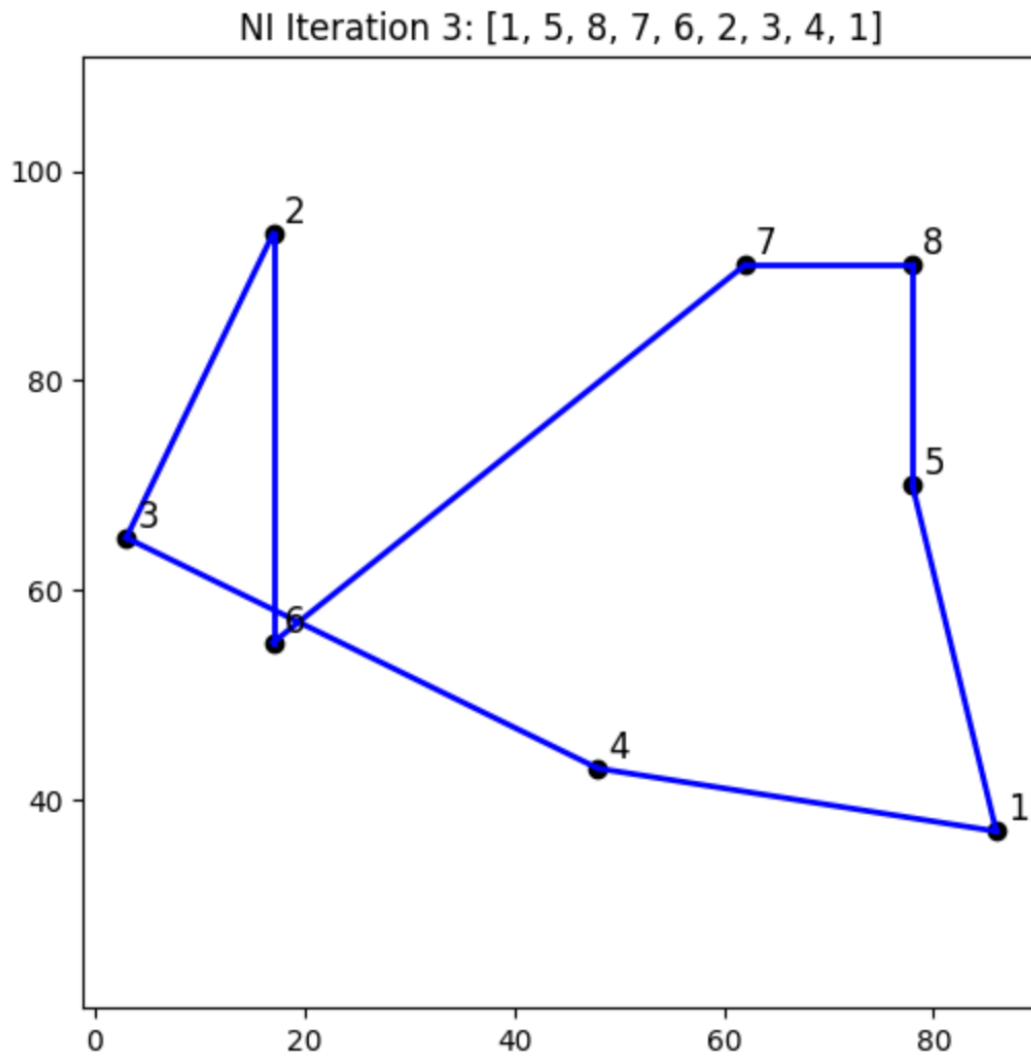
$$\Delta z = d(6,7) + d(8,1) - d(6,8) - d(7,1)$$

$$\Delta z = 57 + 54 - 70 - 59 = -18$$

Best Found: BH

T=[1,5,8,7,6,2,3,4,1]

Total cost= 321 - 35 = 286



Edges:

1-5	A
5-8	B
8-7	C
7-6	D
6-2	E
2-3	F
3-4	G
4-1	H

1. AC

$$\Delta z = d(1,5) + d(8,7) - d(1,8) - d(5,7)$$



$$\Delta z = 33 + 16 - 54 - 26 = -31$$

2. AD

$$\Delta z = d(1,5) + d(7,6) - d(1,7) - d(5,6)$$

$$\Delta z = 33 + 57 - 59 - 62 = -31$$

3. AE

$$\Delta z = d(1,5) + d(6,2) - d(1,6) - d(5,2)$$

$$\Delta z = 33 + 39 - 71 - 65 = -64$$

4. AF

$$\Delta z = d(1,5) + d(2,3) - d(1,2) - d(5,3)$$

$$\Delta z = 33 + 32 - 89 - 75 = -99$$

5. AG

$$\Delta z = d(1,5) + d(3,4) - d(1,3) - d(5,4)$$

$$\Delta z = 33 + 50 - 87 - 40 = -44$$

6. BD

$$\Delta z = d(5,8) + d(7,6) - d(5,7) - d(8,6)$$

$$\Delta z = 21 + 57 - 26 - 70 = -18$$

7. BE

$$\Delta z = d(5,8) + d(6,2) - d(5,6) - d(8,2)$$

$$\Delta z = 21 + 39 - 62 - 61 = -63$$

8. BF

$$\Delta z = d(5,8) + d(2,3) - d(5,2) - d(8,3)$$

$$\Delta z = 21 + 32 - 62 - 61 = -70$$

9. BG

$$\Delta z = d(5,8) + d(3,4) - d(5,3) - d(8,4)$$

$$\Delta z = 21 + 50 - 75 - 56 = -60$$

10. BH

$$\Delta z = d(5,8) + d(4,1) - d(5,4) - d(8,1)$$

$$\Delta z = 21 + 38 - 40 - 54 = -35$$

11. CE

$$\Delta z = d(8,7) + d(6,2) - d(8,6) - d(7,2)$$

$$\Delta z = 16 + 39 - 70 - 45 = -60$$

12. CF

$$\Delta z = d(8,7) + d(2,3) - d(8,2) - d(7,3)$$

$$\Delta z = 16 + 32 - 61 - 64 = -77$$

13. CG

$$\Delta z = d(8,7) + d(3,4) - d(8,3) - d(7,4)$$

$$\Delta z = 16 + 50 - 79 - 50 = -63$$

14. CH

$$\Delta z = d(8,7) + d(4,1) - d(8,4) - d(7,1)$$

$$\Delta z = 16 + 38 - 56 - 59 = -61$$

15. DF

$$\Delta z = d(7,6) + d(2,3) - d(7,2) - d(6,3)$$

$$\Delta z = 57 + 32 - 45 - 17 = 27$$

16. DG

$$\Delta z = d(7,6) + d(3,4) - d(7,3) - d(6,4)$$

$$\Delta z = 57 + 50 - 64 - 33 = 10$$

17. DH

$$\Delta z = d(7,6) + d(4,1) - d(7,4) - d(6,1)$$

$$\Delta z = 57 + 38 - 50 - 71 = -26$$

18. EG

$$\Delta z = d(6,2) + d(3,4) - d(6,3) - d(2,4)$$

$$\Delta z = 39 + 50 - 17 - 59 = 13$$

19. EH

$$\Delta z = d(6,2) + d(4,1) - d(6,4) - d(2,1)$$

$$\Delta z = 39 + 38 - 33 - 89 = -36$$

20. FH

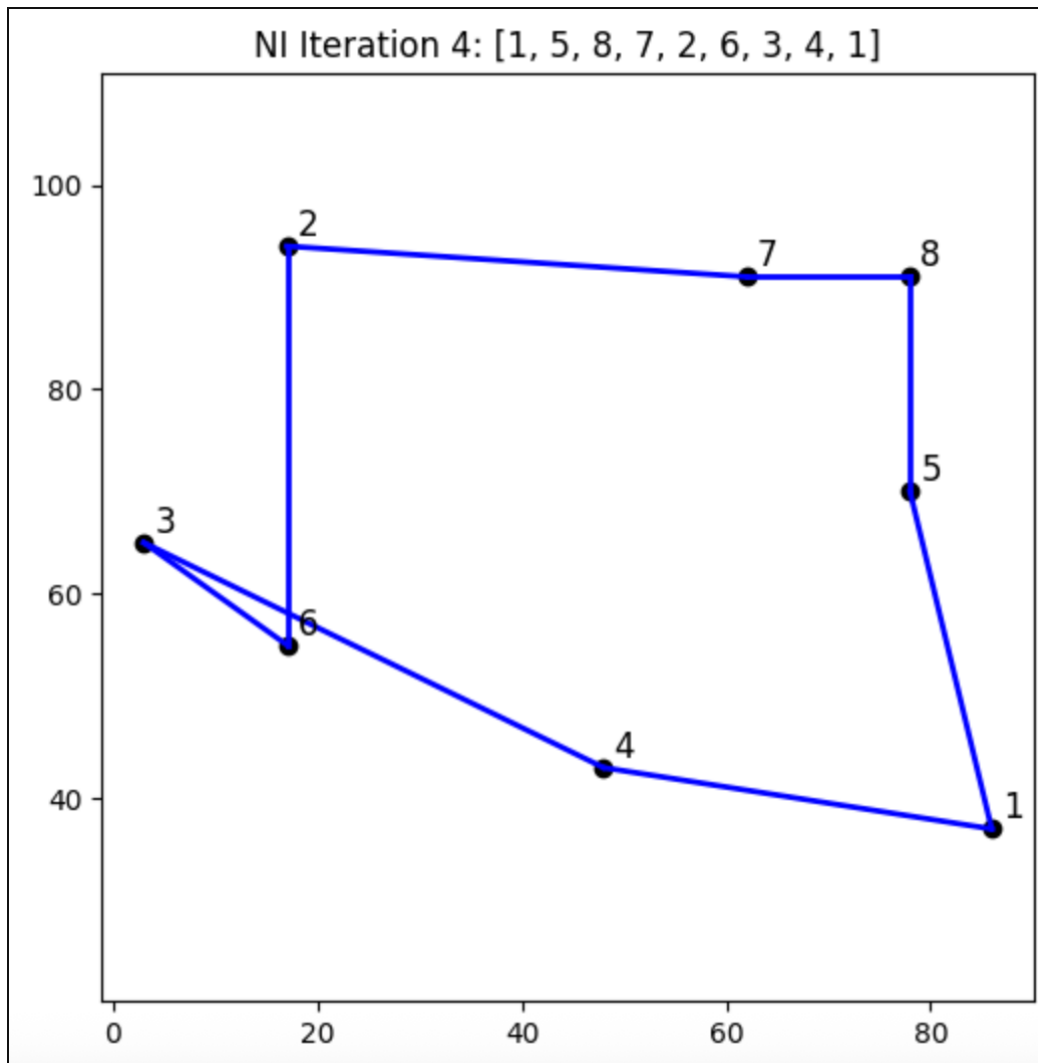
$$\Delta z = d(7,6) + d(4,1) - d(7,4) - d(6,1)$$

$$\Delta z = 57 + 38 - 50 - 71 = -26$$

Best Found: DF

T=[1,5,8,7,2,6,3,4,1]

Total cost= 286-27=259



Edges:

1-5	A
5-8	B
8-7	C
7-2	D
2-6	E
6-3	F
3-4	G
4-1	H

1. AC

$$\Delta z = d(1,5) + d(8,7) - d(1,8) - d(5,7)$$

$$\Delta z = 33 + 16 - 54 - 26 = -31$$

2. AD

$$\Delta z = d(1,5) + d(7,2) - d(1,7) - d(5,2)$$

$$\Delta z = 33 + 57 - 59 - 65 = -34$$

3. AE

$$\Delta z = d(1,5) + d(6,2) - d(1,6) - d(5,2)$$

$$\Delta z = 33 + 39 - 71 - 65 = -64$$

4. AF

$$\Delta z = d(1,5) + d(6,3) - d(1,6) - d(5,3)$$

$$\Delta z = 33 + 32 - 71 - 75 = -81$$

5. AG

$$\Delta z = d(1,5) + d(3,4) - d(1,3) - d(5,4)$$

$$\Delta z = 33 + 50 - 87 - 40 = -44$$

6. BD

$$\Delta z = d(5,8) + d(7,2) - d(5,7) - d(8,2)$$

$$\Delta z = 21 + 45 - 26 - 61 = -21$$

7. BE

$$\Delta z = d(5,8) + d(2,6) - d(5,2) - d(8,6)$$

$$\Delta z = 21 + 39 - 65 - 70 = -75$$

8. BF

$$\Delta z = d(5,8) + d(6,3) - d(5,2) - d(8,3)$$

$$\Delta z = 21 + 17 - 62 - 61 = -85$$

9. BG

$$\Delta z = d(5,8) + d(3,4) - d(5,3) - d(8,4)$$

$$\Delta z = 21 + 50 - 75 - 56 = -60$$

10. BH

$$\Delta z = d(5,8) + d(4,1) - d(5,4) - d(8,1)$$

$$\Delta z = 21 + 38 - 40 - 54 = -35$$

11. CE

$$\Delta z = d(8,7) + d(6,2) - d(8,6) - d(7,2)$$

$$\Delta z = 16 + 39 - 70 - 45 = -60$$

12. CF

$$\Delta z = d(8,7) + d(6,3) - d(8,6) - d(7,3)$$

$$\Delta z = 16 + 17 - 70 - 64 = -101$$

13. CG

$$\Delta z = d(8,7) + d(3,4) - d(8,3) - d(7,4)$$

$$\Delta z = 16 + 50 - 79 - 50 = -63$$

14. CH

$$\Delta z = d(8,7) + d(4,1) - d(8,4) - d(7,1)$$

$$\Delta z = 16 + 38 - 56 - 59 = -61$$

15. DF

$$\Delta z = d(7,2) + d(6,3) - d(7,6) - d(2,3)$$

$$\Delta z = 45 + 17 - 57 - 57 = -27$$

16. DG

$$\Delta z = d(7,2) + d(3,4) - d(7,3) - d(2,4)$$

$$\Delta z = 45 + 50 - 64 - 59 = -28$$

17. DH

$$\Delta z = d(7,2) + d(4,1) - d(7,4) - d(2,1)$$

$$\Delta z = 45 + 38 - 50 - 89 = -56$$

18. EG

$$\Delta z = d(2,6) + d(3,4) - d(2,3) - d(6,4)$$

$$\Delta z = 39 + 50 - 32 - 33 = 24$$

19. EH

$$\Delta z = d(2,6) + d(4,1) - d(2,4) - d(6,1)$$

$$\Delta z = 39 + 38 - 59 - 71 = -53$$

20. FH

$$\Delta z = d(6,3) + d(4,1) - d(6,4) - d(3,1)$$

$$\Delta z = 17 + 38 - 33 - 87 = -65$$

BEST FOUND: EG

T= [1,5,8,7,2,3,6,4,1]

TOTAL COST: 259-24=235

NI Iteration 5: [1, 5, 8, 7, 2, 3, 6, 4, 1]

