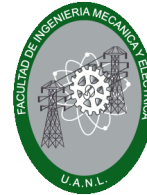




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UNIVERSIDAD AUTÓNOMA DE NUEVO LEÓN



FIME

FACULTAD DE INGENIERÍA MECÁNICA Y ELÉCTRICA

**Universidad Autónoma de Nuevo León
Facultad de Ingeniería Mecánica y Eléctrica**

**Selected Topics on Optimization
HW4 2-OPT Local Search Heuristic**

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Group 002

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Tuesday classes

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Data

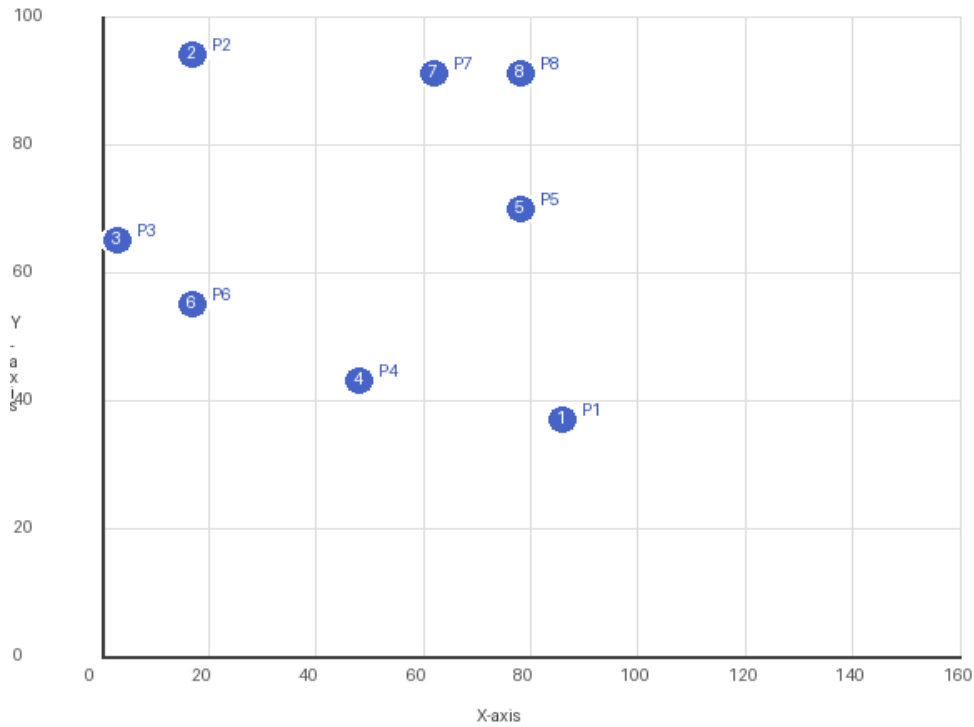
The problem consists of 8 cities with the following (x, y) coordinates and distance matrix:

Node	x	y
1	86	37
2	17	94
3	3	65
4	48	43
5	78	70
6	17	55
7	62	91
8	78	91

Distance Matrix:

	1	2	3	4	5	6	7	8
1	0							
2	89	0						
3	87	32	0					
4	38	59	50	0				
5	33	65	75	40	0			
6	71	39	17	33	62	0		
7	59	45	64	50	26	57	0	
8	54	61	79	56	21	70	16	0

Traveling Salesman Problem



Initial Tour

$T = (1, 2, 3, 4, 5, 6, 7, 8, 1)$

Distance traveled: $89 + 32 + 50 + 40 + 62 + 57 + 16 + 54 = 400$

Edge	Distance	Edge	Distance
(1→2)	89	(5→6)	62
(2→3)	32	(6→7)	57
(3→4)	50	(7→8)	16
(4→5)	40	(8→1)	54

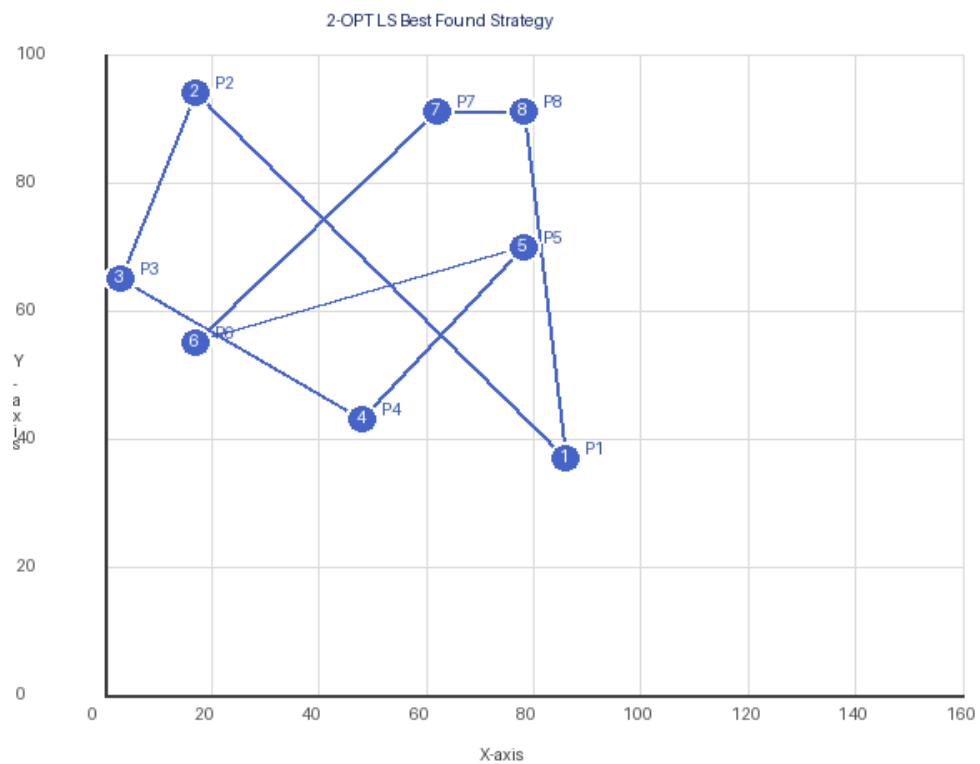
Move definition:

$e_1 = (i, j)$ — first removed edge

$e_2 = (k, l)$ — second removed edge

$\text{move}(e_1, e_2) = \text{swap edges NOT adjacent}$

$\Delta z = \text{cost new} - \text{cost old}$



Iteration 1

Current Tour: $T = (1, 2, 3, 4, 5, 6, 7, 8)$ $L = 400$

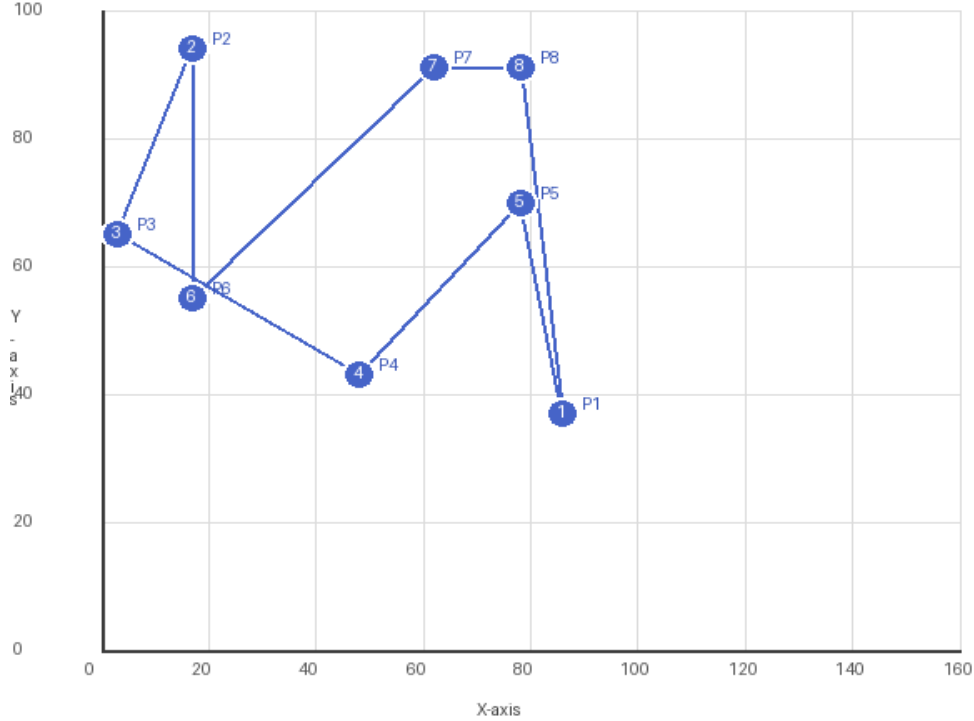
e_1	e_2	New Edge 1	New Edge 2	Cost New e_1	Cost New e_2	Cost old e_1	Cost old e_2	Δz
(1,2)	(3,4)	(1,3)	(2,4)	87	59	89	50	+7
(1,2)	(4,5)	(1,4)	(2,5)	38	65	89	40	-26
(1,2)	(5,6)	(1,5)	(2,6)	33	39	89	62	-79
(1,2)	(6,7)	(1,6)	(2,7)	71	45	89	57	-30
(1,2)	(7,8)	(1,7)	(2,8)	59	61	89	16	+15
(2,3)	(4,5)	(2,4)	(3,5)	59	75	32	40	+62
(2,3)	(5,6)	(2,5)	(3,6)	65	17	32	62	-12
(2,3)	(6,7)	(2,6)	(3,7)	39	64	32	57	+14
(2,3)	(7,8)	(2,7)	(3,8)	45	79	32	16	+76
(2,3)	(8,1)	(2,8)	(3,1)	61	87	32	54	+62
(3,4)	(5,6)	(3,5)	(4,6)	75	33	50	62	-4
(3,4)	(6,7)	(3,6)	(4,7)	17	50	50	57	-40
(3,4)	(7,8)	(3,7)	(4,8)	64	56	50	16	+54
(3,4)	(8,1)	(3,8)	(4,1)	79	38	50	54	+13
(4,5)	(6,7)	(4,6)	(5,7)	33	26	40	57	-38
(4,5)	(7,8)	(4,7)	(5,8)	50	21	40	16	+15
(4,5)	(8,1)	(4,8)	(5,1)	56	33	40	54	-5
(5,6)	(7,8)	(5,7)	(6,8)	26	70	62	16	+18
(5,6)	(8,1)	(5,8)	(6,1)	21	71	62	54	-24
(6,7)	(8,1)	(6,8)	(7,1)	70	59	57	54	+18

Decision: Edges (1,2) and (5,6) are replaced with edges (1,5) and (2,6)

$T = (1, 5, 4, 3, 2, 6, 7, 8, 1)$

Distance traveled: $33 + 40 + 50 + 32 + 39 + 57 + 16 + 54 = 321$

2-OPT LS Best Found Strategy



Iteration 2

Current Tour: $T = (1, 5, 4, 3, 2, 6, 7, 8)$ $L = 321$

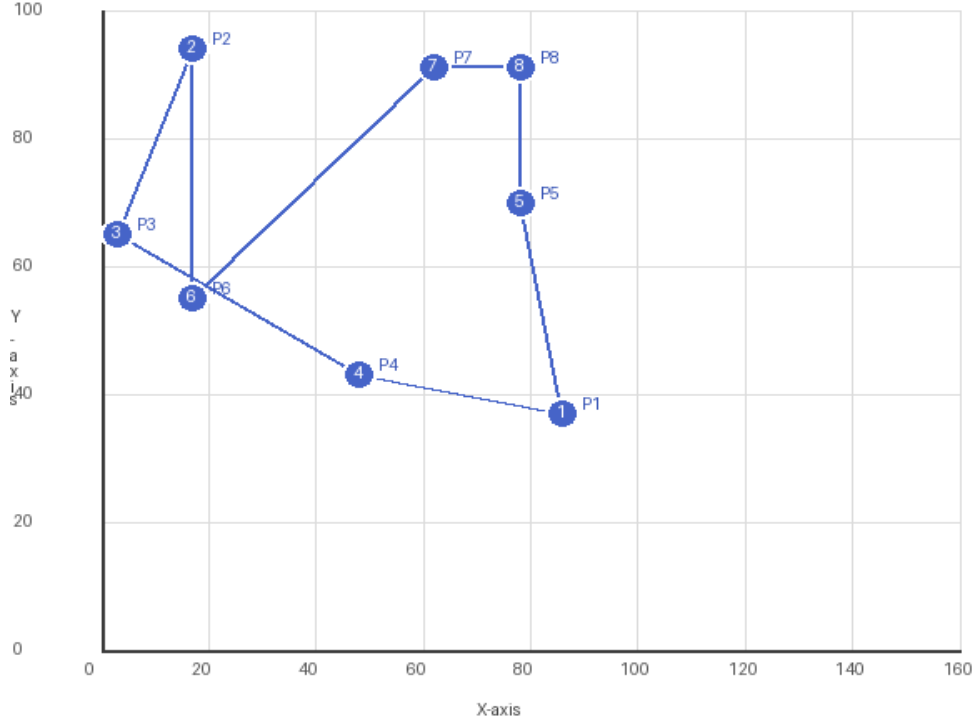
e_1	e_2	New Edge 1	New Edge 2	Cost New e_1	Cost New e_2	Cost old e_1	Cost old e_2	Δz
(1,5)	(4,3)	(1,4)	(5,3)	38	75	33	50	+30
(1,5)	(3,2)	(1,3)	(5,2)	87	65	33	32	+87
(1,5)	(2,6)	(1,2)	(5,6)	89	62	33	39	+79
(1,5)	(6,7)	(1,6)	(5,7)	71	26	33	57	+7
(1,5)	(7,8)	(1,7)	(5,8)	59	21	33	16	+31
(5,4)	(3,2)	(5,3)	(4,2)	75	59	40	32	+62
(5,4)	(2,6)	(5,2)	(4,6)	65	33	40	39	+19
(5,4)	(6,7)	(5,6)	(4,7)	62	50	40	57	+15
(5,4)	(7,8)	(5,7)	(4,8)	26	56	40	16	+26
(5,4)	(8,1)	(5,8)	(4,1)	21	38	40	54	-35
(4,3)	(2,6)	(4,2)	(3,6)	59	17	50	39	-13
(4,3)	(6,7)	(4,6)	(3,7)	33	64	50	57	-10
(4,3)	(7,8)	(4,7)	(3,8)	50	79	50	16	+63
(4,3)	(8,1)	(4,8)	(3,1)	56	87	50	54	+39
(3,2)	(6,7)	(3,6)	(2,7)	17	45	32	57	-27
(3,2)	(7,8)	(3,7)	(2,8)	64	61	32	16	+77
(3,2)	(8,1)	(3,8)	(2,1)	79	89	32	54	+82
(2,6)	(7,8)	(2,7)	(6,8)	45	70	39	16	+60
(2,6)	(8,1)	(2,8)	(6,1)	61	71	39	54	+39
(6,7)	(8,1)	(6,8)	(7,1)	70	59	57	54	+18

Decision: Edges (5,4) and (8,1) are replaced with edges (5,8) and (4,1)

$T = (1, 5, 8, 7, 6, 2, 3, 4, 1)$

Distance traveled: $33 + 21 + 16 + 57 + 39 + 32 + 50 + 38 = 286$

2-OPT LS Best Found Strategy



Iteration 3

Current Tour: $T = (1, 5, 8, 7, 6, 2, 3, 4)$ $L = 286$

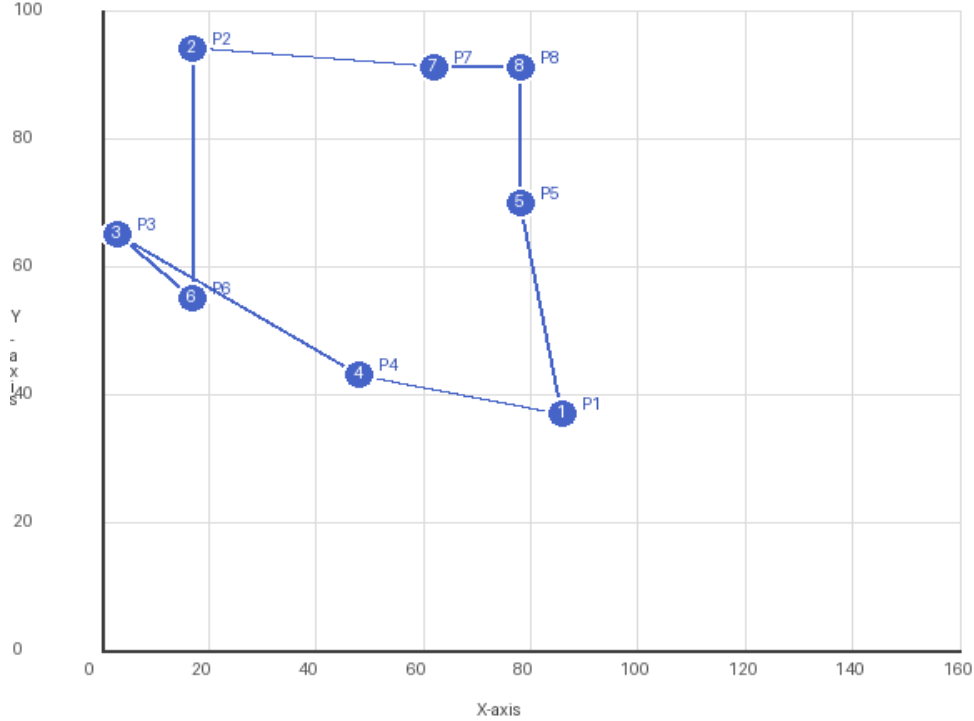
e_1	e_2	New Edge 1	New Edge 2	Cost New e_1	Cost New e_2	Cost old e_1	Cost old e_2	Δz
(1,5)	(8,7)	(1,8)	(5,7)	54	26	33	16	+31
(1,5)	(7,6)	(1,7)	(5,6)	59	62	33	57	+31
(1,5)	(6,2)	(1,6)	(5,2)	71	65	33	39	+64
(1,5)	(2,3)	(1,2)	(5,3)	89	75	33	32	+99
(1,5)	(3,4)	(1,3)	(5,4)	87	40	33	50	+44
(5,8)	(7,6)	(5,7)	(8,6)	26	70	21	57	+18
(5,8)	(6,2)	(5,6)	(8,2)	62	61	21	39	+63
(5,8)	(2,3)	(5,2)	(8,3)	65	79	21	32	+91
(5,8)	(3,4)	(5,3)	(8,4)	75	56	21	50	+60
(5,8)	(4,1)	(5,4)	(8,1)	40	54	21	38	+35
(8,7)	(6,2)	(8,6)	(7,2)	70	45	16	39	+60
(8,7)	(2,3)	(8,2)	(7,3)	61	64	16	32	+77
(8,7)	(3,4)	(8,3)	(7,4)	79	50	16	50	+63
(8,7)	(4,1)	(8,4)	(7,1)	56	59	16	38	+61
(7,6)	(2,3)	(7,2)	(6,3)	45	17	57	32	-27
(7,6)	(3,4)	(7,3)	(6,4)	64	33	57	50	-10
(7,6)	(4,1)	(7,4)	(6,1)	50	71	57	38	+26
(6,2)	(3,4)	(6,3)	(2,4)	17	59	39	50	-13
(6,2)	(4,1)	(6,4)	(2,1)	33	89	39	38	+45
(2,3)	(4,1)	(2,4)	(3,1)	59	87	32	38	+76

Decision: Edges (7,6) and (2,3) are replaced with edges (7,2) and (6,3)

$T = (1, 5, 8, 7, 2, 6, 3, 4, 1)$

Distance traveled: $33 + 21 + 16 + 45 + 39 + 17 + 50 + 38 = 259$

2-OPT LS Best Found Strategy



Iteration 4

Current Tour: $T = (1, 5, 8, 7, 2, 6, 3, 4)$ $L = 259$

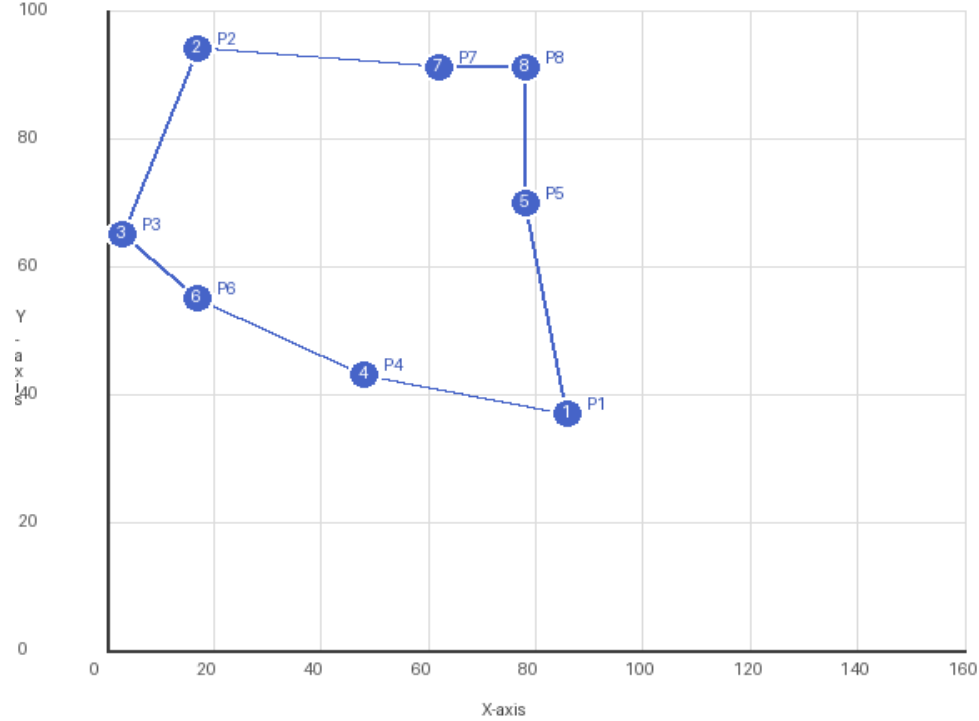
e_1	e_2	New Edge 1	New Edge 2	Cost New e_1	Cost New e_2	Cost old e_1	Cost old e_2	Δz
(1,5)	(8,7)	(1,8)	(5,7)	54	26	33	16	+31
(1,5)	(7,2)	(1,7)	(5,2)	59	65	33	45	+46
(1,5)	(2,6)	(1,2)	(5,6)	89	62	33	39	+79
(1,5)	(6,3)	(1,6)	(5,3)	71	75	33	17	+96
(1,5)	(3,4)	(1,3)	(5,4)	87	40	33	50	+44
(5,8)	(7,2)	(5,7)	(8,2)	26	61	21	45	+21
(5,8)	(2,6)	(5,2)	(8,6)	65	70	21	39	+75
(5,8)	(6,3)	(5,6)	(8,3)	62	79	21	17	+103
(5,8)	(3,4)	(5,3)	(8,4)	75	56	21	50	+60
(5,8)	(4,1)	(5,4)	(8,1)	40	54	21	38	+35
(8,7)	(2,6)	(8,2)	(7,6)	61	57	16	39	+63
(8,7)	(6,3)	(8,6)	(7,3)	70	64	16	17	+101
(8,7)	(3,4)	(8,3)	(7,4)	79	50	16	50	+63
(8,7)	(4,1)	(8,4)	(7,1)	56	59	16	38	+61
(7,2)	(6,3)	(7,6)	(2,3)	57	32	45	17	+27
(7,2)	(3,4)	(7,3)	(2,4)	64	59	45	50	+28
(7,2)	(4,1)	(7,4)	(2,1)	50	89	45	38	+56
(2,6)	(3,4)	(2,3)	(6,4)	32	33	39	50	-24
(2,6)	(4,1)	(2,4)	(6,1)	59	71	39	38	+53
(6,3)	(4,1)	(6,4)	(3,1)	33	87	17	38	+65

Decision: Edges (2,6) and (3,4) are replaced with edges (2,3) and (6,4)

$T = (1, 5, 8, 7, 2, 3, 6, 4, 1)$

Distance traveled: $33 + 21 + 16 + 45 + 32 + 17 + 33 + 38 = 235$

2-OPT LS Best Found Strategy



Iteration 5

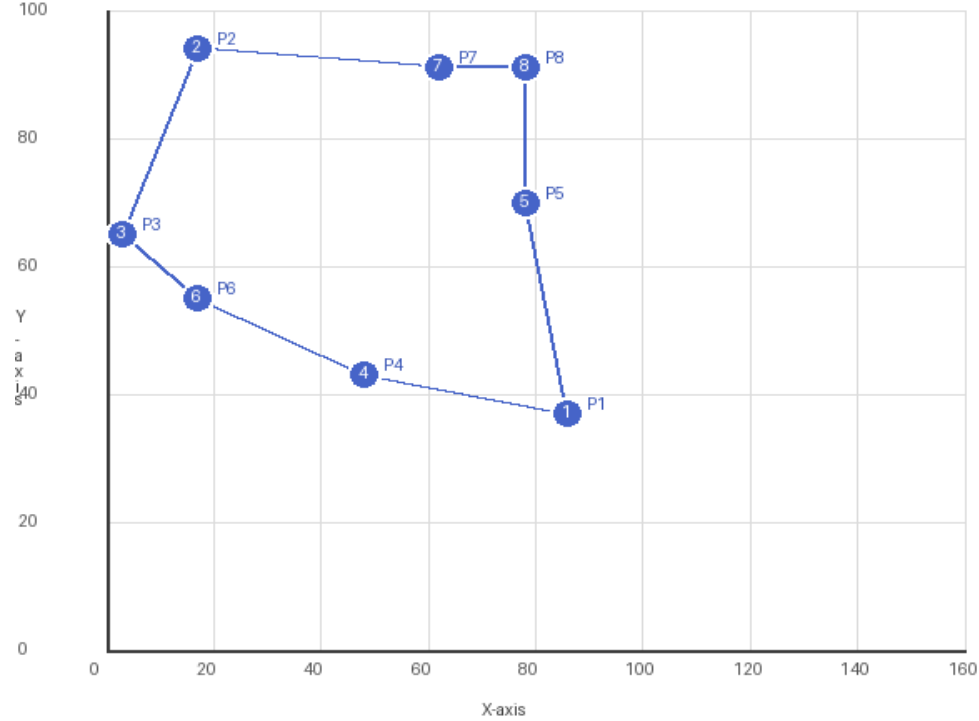
Current Tour: T = (1, 5, 8, 7, 2, 3, 6, 4) L = 235

e_1	e_2	New Edge 1	New Edge 2	Cost New e_1	Cost New e_2	Cost old e_1	Cost old e_2	Δz
(1,5)	(8,7)	(1,8)	(5,7)	54	26	33	16	+31
(1,5)	(7,2)	(1,7)	(5,2)	59	65	33	45	+46
(1,5)	(2,3)	(1,2)	(5,3)	89	75	33	32	+99
(1,5)	(3,6)	(1,3)	(5,6)	87	62	33	17	+99
(1,5)	(6,4)	(1,6)	(5,4)	71	40	33	33	+45
(5,8)	(7,2)	(5,7)	(8,2)	26	61	21	45	+21
(5,8)	(2,3)	(5,2)	(8,3)	65	79	21	32	+91
(5,8)	(3,6)	(5,3)	(8,6)	75	70	21	17	+107
(5,8)	(6,4)	(5,6)	(8,4)	62	56	21	33	+64
(5,8)	(4,1)	(5,4)	(8,1)	40	54	21	38	+35
(8,7)	(2,3)	(8,2)	(7,3)	61	64	16	32	+77
(8,7)	(3,6)	(8,3)	(7,6)	79	57	16	17	+103
(8,7)	(6,4)	(8,6)	(7,4)	70	50	16	33	+71
(8,7)	(4,1)	(8,4)	(7,1)	56	59	16	38	+61
(7,2)	(3,6)	(7,3)	(2,6)	64	39	45	17	+41
(7,2)	(6,4)	(7,6)	(2,4)	57	59	45	33	+38
(7,2)	(4,1)	(7,4)	(2,1)	50	89	45	38	+56
(2,3)	(6,4)	(2,6)	(3,4)	39	50	32	33	+24
(2,3)	(4,1)	(2,4)	(3,1)	59	87	32	38	+76
(3,6)	(4,1)	(3,4)	(6,1)	50	71	17	38	+66

Decision: There is no swap that improves the cost, so we do not move anything.

T = (1, 5, 8, 7, 2, 3, 6, 4, 1) ← LOCAL OPTIMUM

2-OPT LS Best Found Strategy



Final Summary

The 2-OPT local search heuristic achieved a 41.25% improvement over the initial tour:

Iteration	Move Applied	Δz^*	Tour Length L
Start	T = (1,2,3,4,5,6,7,8)	—	400
1	Remove (1,2),(5,6) → Add (1,5),(2,6)	-79	321
2	Remove (5,4),(8,1) → Add (5,8),(4,1)	-35	286
3	Remove (7,6),(2,3) → Add (7,2),(6,3)	-27	259
4	Remove (2,6),(3,4) → Add (2,3),(6,4)	-24	235
5 (Stop)	All $\Delta z \geq 0$ — Local optimum reached	—	235 ✓

Optimal Local Tour: $T^* = (1 \rightarrow 5 \rightarrow 8 \rightarrow 7 \rightarrow 2 \rightarrow 3 \rightarrow 6 \rightarrow 4 \rightarrow 1)$

Total Length: $L^* = 235$

