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UNIVERSIDAD AUTÓNOMA DE NUEVO LEÓN
FACULTAD DE INGENIERÍA MECÁNICA Y ELÉCTRICA
TIPO DE EXAMEN Y/O EVALUACIÓN:
FINAL ORDINARIO (*Final Exam*)

MATERIA/UNIDAD DE APRENDIZAJE: Temas Selectos de Optimización

LEARNING UNIT: Selected Topics on Optimization (in English)
SEMESTER: January – June 2024
ACADEMY: Statistics and Operations Research (*Estadística e Investigación de Operaciones*).
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ANSWER SHEET

Section 1

1.Its recommendable to use heuristics when we work with a optimization problem that has a lot of data, the brute force method can be effective but the heuristic will save us a lot amount of time.

2.When the time required to be solved is not exponential, meaning that the problem in small instances would not take a lot of time but in bigger instances it may take a bunch of time even with heuristics.

3.We consider an easy combinatorial optimization problem the problem which has a heuristic that solves the problem in a polynomial amount of time.

4.Yeah that's their main goal, give us a solution in a proportional amount of time, it's possible that this solution may not be the optimal solution but at least it will be feasible.

5.Its a heuristic that use a greedy function to solve a problem, the optimal of the solution it would be proportional to the greedy function meaning that if we use a good greedy function, we may get better results.

6.its a heuristic that starts from a feasible solution and try to improve it with small perturbation of that solution, we used to name those perturbations as moves.

7. the 2-OPT it's a local search heuristic that in the TSP use the edges of a feasible solution to improve the solution and get the local optimal of that solution, the heuristic have one simple condition, the edges removed it and reconnected MUST NOT BE ADJACENT, the moves are describe as follow:

2OPT1(**e1, e2**)

2OPT1(**i**, **j; k, l**) = remove edges (**i,j**) and (**k,l**) and reconnect to form a tour (**i,k**) + (**j,l**) **NO ADJACENT** edges allowed.

$$\Delta z = -\boldsymbol{d_{i,j}} - \boldsymbol{d_{k,l}} + \boldsymbol{d_{i,k}} + \boldsymbol{d_{j,l}}$$

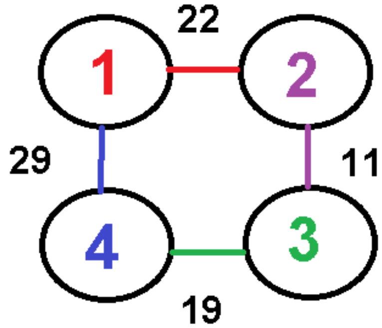
To know if a neighbor its improving the solution, our Z factor must follow the next conditions:

If the Z factor of a neighbor its more than 0, discard that neighbor $\Delta z > 0$ **DISCARD**

If the Z factor of a neighbor its less than 0, consider that neighbor $\Delta z < 0$ **CONSIDER**

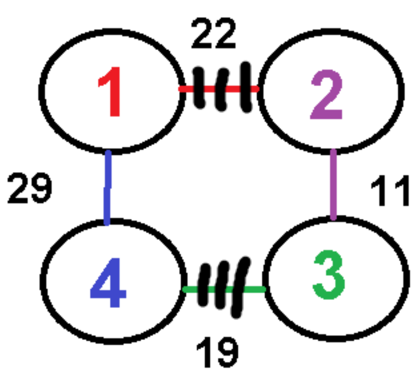
The best neighbor is the one with the lowest value.

This is what 2OPT do in a graphical view: given the next feasible solution: T= (1,2,3,4)

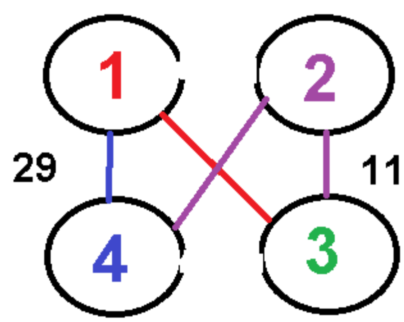


2OPT1(**1**, **2; 3, 4**) = remove edges (**1,2**) and (**3,4**) and reconnect to form a tour (**1,2**) + (**3,4**)

We remove the edges.



We reconnected to form a tour.



Our new solution will be T= (1,3,2,4)

8.When we are comparing heuristics, we need to consider two major factors time and optimality, the best heuristics will be the one who have the lowest time and the optimal objective function, so if I compared two heuristic I would choose the one who have the better results in a low amount of time, there might be some cases where the two heuristic just have one optimal factor, in those cases all depends of my needs if I need a solution and the time is crucial, i would pick the fastest one and if i need a solution but the time doesn't matter I would pick the optimal one.

Section 2

9a. It's not feasible because it doesn't meet all the constraints, in this solution there's not repeated elements but when I checked the union which of the subsets which is $X_{(1)} = \{1,2,3,4,5,6,8,9,10\}$ the element 7 is missing making this solution not feasible because it doesn't meet the constrain that every element of V must be included.

9b. It's feasible because it meets all the constraints, it doesn't have repeated elements and when we check the union of the subsets which is $X_{(2)} = \{1,2,3,4,5,6,7,8,9,10\}$ all the elements of V are in so the solution its feasible,

9c. <u>*Ranking*</u>:

1-$X_{(5)}$ = ({1,3,7,8}, {2,5,10}, {4,6,9}) **BEST**
2-$X_{(3)}$ = ({1,5,9}, {2,4,6,8,10}, {3,7}) **GOOD**
3- $X_{(4)}$ = ({2,5,9}, {1,3,4,8}, {6,10}) **WORST**

<u>Justification.</u>

What I did fist was to check the feasibility of all the solutions given $X_{(3)}$ and $X_{(5)}$ were feasible, because they meet all the constrain but in the case of $X_{(4)}$ when I did the union of the subset the element 7 was missing, making it by consequence the WORST solution of the three because its not even feasible, so now that we discard one of three solutions I evaluate the two remaining to find the best solution which is the one with the lowest dissimilarity.

Dissimilarity of $X_{(3)}$ = ({1,5,9}, {2,4,6,8,10}, {3,7})
d {1,5,9} = $\boldsymbol{d}_{1,5} + \boldsymbol{d}_{1,9} + \boldsymbol{d}_{5,9} = 31 + 20 + 11 = 62$
d {2,4,6,8,10} = $\boldsymbol{d}_{2,4} + \boldsymbol{d}_{2,6} + \boldsymbol{d}_{2,8} + \boldsymbol{d}_{2,10} + \boldsymbol{d}_{4,6} + \boldsymbol{d}_{4,8} + \boldsymbol{d}_{4,10} + \boldsymbol{d}_{6,8} + \boldsymbol{d}_{6,10} + \boldsymbol{d}_{8,10} = 9 + 33 + 39 + 30 + 19 + 28 + 23 + 29 + 27 + 21 = 258$
d {3,7} = $\boldsymbol{d}_{3,7} = 37$
Total dissimilarity: 62+258+37 = 357

Dissimilarity of $X_{(5)}$ = ({1,3,7,8}, {2,5,10}, {4,6,9})
d {1,3,7,8} = $\boldsymbol{d}_{1,3} + \boldsymbol{d}_{1,7} + \boldsymbol{d}_{1,8} + \boldsymbol{d}_{3,7} + \boldsymbol{d}_{3,8} + \boldsymbol{d}_{7,8} = 26 + 46 + 35 + 37 + 20 + 36 = 200$
d {2,5,10} = $\boldsymbol{d}_{2,5} + \boldsymbol{d}_{2,10} + \boldsymbol{d}_{5,10} = 33 + 30 + 27 = 90$
d {4,5,9} = $\boldsymbol{d}_{4,6} + \boldsymbol{d}_{4,9} + \boldsymbol{d}_{6,9} = 29 + 15 + 17 = 61$
Total dissimilarity: 62+258+37 = 351 so because this one has the lowest dissimilarity of both this is the best solution of the three.

9d. The lowest dissimilarity heuristic:

This heuristic consists in creating clusters calculate their dissimilarity and pick the ones with the lowest dissimilarity increase, to get the optimal solution, the greedy function of this heuristic is the dissimilarity its crucial to minimize it as possible and obviously its needs to be feasible.

<u>Flow chart.</u>
Step 1: START
Step 2: We created a p empty list for each cluster.
Step 3: We sort elements V, the ones having the largest sums being considered first.

Step 4: We compute the potential increase in dissimilarity for adding v element to each cluster Xi.
Step 5: We assign the element to cluster Xi that results in the smallest increase in dissimilarity.
Step 6: We repeat until all elements V are included and all the cluster are non-empty.
Step 7: Calculated the total dissimilarity of the obtained solution.
Step 8: END

9e. This is how it look the heuristic working:

1. Initialize clusters: X = ({}, {}, {})
2.We Sort V: V=(1,2,3,4,5,6,7,8,9,10)
3.Add v1 to the cluster with minimum dissimilarity: X = ({1}, {}, {})
4.We repeat this until all elements are included.
4a.Add v2: X = ({1}, {2}, {})
4b.Add v3: X = ({1}, {2}, {3})
4c.Add v4: X = ({1,4}, {2}, {3})
4d.Add v5: X = ({1,4}, {2,5}, {3})
4e.Add v6: X = ({1,4}, {2,5}, {3,6})
4f.Add v7: X = ({1,4,7}, {2,5}, {3,6})
4g.Add v8: X = ({1,4,7}, {2,5,8}, {3,6})
4g.Add v9: X = ({1,4,7,9}, {2,5,8}, {3,6})
4h.Add v10: X = ({1,4,7,9,10}, {2,5,8}, {3,6})
5.We calculated the total dissimilarity of the obtained solution:
Total dissimilarity *of X*= ({1,4,7,9,10}, {2,5,8}, {3,6}) =

d{1,4,7,9,10} =$\boldsymbol{d}_{1,4} + \boldsymbol{d}_{1,7} + \boldsymbol{d}_{1,9} + \boldsymbol{d}_{1,10} + \boldsymbol{d}_{4,7} + \boldsymbol{d}_{4,9} + \boldsymbol{d}_{4,10} + \boldsymbol{d}_{7,9} + \boldsymbol{d}_{7,10} + \boldsymbol{d}_{9,10}$ = 14+46+20+10+34+15+18+11+27+46= 241

d{2,5,8} = $\boldsymbol{d}_{2,5} + \boldsymbol{d}_{2,8} + \boldsymbol{d}_{5,8} = 33 + 39 + 16 = 88$

d{3,6} = $\boldsymbol{d}_{3,6} = 15$

Total dissimilarity = 241+88+15 = 344.

The solution was better by 14 units so this is a considerable improvement.

9f. Local search: Element move.

For the creation the creation of this local search heuristic I take in mind what I did in my final project, this local search consist in move a element of a cluster to another to see if the solution is decreased, The move of this local search is the next one:

move1(**i**,**j**;**k**) = move element **i** **from cluster j** to cluster **k**

$$\Delta z = -\mathrm{J}_{\boldsymbol{i},\mathrm{v1}\ldots} - \mathrm{J}_{\boldsymbol{i},\mathrm{vn}} + \mathrm{K}_{\boldsymbol{i},\mathrm{v1}\ldots} + \mathrm{K}_{\boldsymbol{i},\mathrm{vn}}$$

$\Delta z > 0$ **DISCARD**

$\Delta z < 0$ **CONSIDER**

The best neighbor is the one with the lowest value.

9g. Taking in mind the next feasible solution: $X_{(3)}$ = ({2,4,6,8,10}, {1,5,9}, {3,7}).

Iteration 1:

move1(**2,1;2**) = - $\boldsymbol{d}_{2,4}$ - $\boldsymbol{d}_{2,6}$ - $\boldsymbol{d}_{2,8}$- $\boldsymbol{d}_{2,10} + \boldsymbol{d}_{1,2} + \boldsymbol{d}_{2,5} + \boldsymbol{d}_{2,9}$ = 9-33-39-30+12+33+14 = -52 **YES**

move1(**2**,**1;2**) = move element **2 from cluster 1** to cluster **2**

our new solution will be $X_{(3)}$ = ({4,6,8,10}, {2,1,5,9}, {3,7}).

New total dissimilarity = 302

For this case in the first neighbor we find an improvement to the objective function which means that the local search its efficient.