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Computational Experience with Heuristics for the p-Median Problem

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Selected Topics on Optimization

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Introduction

The p-Median problem is a fundamental optimization challenge with practical implications spanning urban planning, logistics, and healthcare management. This problem revolves around determining the optimal locations for a specified number of facilities, such as warehouses or hospitals, to efficiently serve a dispersed population.

Research efforts have yielded various methodologies, from mathematical models to computational algorithms, aimed at solving this problem. Noteworthy contributions include the seminal works of Hakimi (1964) and Church and ReVelle (1974), which have laid the groundwork for subsequent advancements in this field.

The significance of the p-Median problem lies in its ability to inform decision-making processes across diverse domains. In urban planning, it aids in the strategic placement of public amenities and emergency services to enhance accessibility and alleviate congestion. Similarly, in logistics and supply chain management, it facilitates the optimization of distribution networks and inventory management strategies, thereby improving operational efficiency.

Moreover, the p-Median problem finds practical applications in healthcare delivery, guiding the strategic siting of medical facilities to ensure equitable access and enhance patient outcomes. By identifying optimal locations for clinics, hospitals, and specialized care centers, healthcare providers can better allocate resources and address community needs effectively.

In essence, the p-Median problem serves as a valuable tool for making informed decisions that impact societal well-being and resource allocation in various sectors.

Keywords: p-Median, optimization, mathematical model, algorithm, urban planning, logistics.

Problem description

The p-Median problem involves making strategic decisions regarding the placement of facilities to efficiently serve a dispersed population. This problem can be defined by four fundamental components:

1. Data

The input consists of:

- The location of potential facility sites.
- The spatial distribution of the population or demand points that these facilities are intended to serve.
- The costs or distances associated with providing service from each facility site to every demand point.

2. Decisions

The decision involves selecting p locations out of the potential facility sites where facilities will be placed.

3. Optimization

The objective is to minimize the total cost or distance required to provide service from the chosen facility locations to all demand points. Mathematically, this can be represented as minimizing the sum of the distances (or costs) between each demand point and its assigned facility, where the assignment is based on proximity.

4. Constraints

The feasible solution is defined by the requirement that each demand point must be assigned to exactly one facility. Additionally, exactly p facilities must be selected from the potential facility sites.

Mathematical method

The p-Median problem can be formulated as an integer linear programming (ILP) model. Let's denote:

- n as the number of potential facility sites.
- m as the number of demand points.
- p as the number of facilities to be selected.

We can use binary decision variables x_i to indicate whether facility i is selected or not, and binary decision variables y_{ij} to represent whether demand point j is assigned to facility i. The objective function can be formulated as minimizing the total distance or cost, which is the sum of the distances (or costs) multiplied by the assignment variables y_{ij}. The constraints ensure that each demand point is assigned to exactly one facility and that exactly p facilities are selected.

The ILP formulation of the p-Median problem can be expressed as follows:

Minimize:

$$\sum_{i=1}^{n}\sum_{j=1}^{m} c_{ij} y_{ij}$$

Subject to:

$$\sum_{i=1}^{n} x_i = p$$

$$\sum_{i=1}^{n} y_{ij} = 1, \forall_j = 1,\ldots, m$$

$$y_{ij} \leq x_i, \forall_i = 1,\ldots, n, \forall_j = 1,\ldots, m$$

$$x_i \in \{0, 1\}, \forall_i = 1,\ldots, n$$

$$y_{ij} \in \{0, 1\}, \forall_i = 1,\ldots, n, \forall_j = 1,\ldots, m$$

Problem example

Let's consider a simplified scenario to illustrate the p-Median problem. Suppose we have a small town with 5 potential locations for a new healthcare clinic (facilities) and 8 demand points representing households in the town that need medical services. We want to select 2 clinic locations out of the 5 potential locations to minimize the total distance traveled by residents to access healthcare services.

Data

- Potential clinic locations (facilities): A, B, C, D, E
- Demand points (households): 1, 2, 3, 4, 5, 6, 7, 8
- Distance matrix (in kilometers) between potential clinic locations and demand points:

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
A	3	5	2	7	4	6	5	8
B	6	8	4	9	3	7	6	10
C	5	7	3	8	2	6	4	9
D	4	6	2	7	3	5	3	8
E	2	4	1	6	2	4	2	7

Problem illustration

- ***Facility selection:*** Suppose we select clinics at locations B and D
- ***Assignment of demand points:*** Assign demand points to the nearest clinic locations based on the distance matrix

Demand Point	**Nearest**
1	D
2	D
3	D
4	D
5	B
6	D
7	D

8	D

- ***Objective Function Evaluation:*** The objective function evaluates the total distance traveled by residents to access healthcare services from the selected clinic locations. Let's calculate this for our example:
  Total distance = 6 + 8 + 3 + 2 + 3 + 2 + 3 + 7 = 34 kilometers
  So, for this feasible solution, the total distance traveled by residents is 34 kilometers.

References

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