UNIVERSIDAD AUTÓNOMA DE NUEVO LEÓN
FACULTAD DE INGENIERÍA MECÁNICA Y ELÉCTRICA
TIPO DE EXAMEN Y/O EVALUACIÓN:
EXTRAORDINARIO (*Extra Exam*)

MATERIA/UNIDAD DE APRENDIZAJE: *Temas Selectos de Optimización*

LEARNING UNIT: Selected Topics on Optimización (in English)
SEMESTER: January – June 2024 (Spring)
ACADEMY: Statistics and Operations Research (*Estadística e Investigación de Operaciones*).
INSTRUCTOR: Dr. Roger Z. Ríos Mercado (ID 090969)

DIRECTIONS.- Answer the following questions and/or exercises in the answer sheet. Do not write in this sheet

SECTION 1: QUESTIONS (50 POINTS)

Answer and justify your answer.

1. [*UT1: Combinatorial optimization*; 5 pts] Define a combinatorial optimization problem.
2. [*UT1: Combinatorial optimization*; 5 pts] Define and explain what a brute-force enumeration method is for solving a combinatorial optimization problem.
3. [*UT2: Heuristics*; 5 pts] Define and explain what a heuristic method is for solving combinatorial optimization problems.
4. [*UT2: Constructive heuristics;*5 pts] What is a constructive heuristic?
5. [*UT2: Local search heuristics;* 5 pts] Explain clearly what a local search heuristic is for a combinatorial optimization problem.
6. [*UT2: Constructive heuristics;*5 pts] Describe in detail the nearest neighbor heuristic for solving the Traveling Salesman Problem. Illustrate your idea with an example or drawing.
7. [*UT2: Local search heuristics for the TSP;* 5 pts] Describe in detail the nearest insertion heuristic for solving the Traveling Salesman Problem. Illustrate your idea with an example or drawing.
8. [*UT1: Combinatorial optimization*; 5 pts] Is it true that the Traveling Salesman Problem is hard to solve? Explain.
9. [*UT2: Constructive heuristics;*5 pts] Do constructive heuristics guarantee to find a feasible solution to a given combinatorial optimization problem? Justify your answer.
10. [*UT2: Local search heuristics;* 5 pts] Explain the difference between the “first found” and “best found” strategies employed in local search heuristics.

SECTION 2: PROBLEMS (50 POINTS)

1. The p-Dispersion Problem (pDP) is defined as follows. Given a collection of points in the plane, $V = \{1, 2, ..., n\}$, where the distance d_{ij} between every pair of points i, j in V is known, and given a known positive integer number p, the goal is to decide a subset of p points of V, in a such a way that these points are as disperse (far away from each other) as possible. In other words, find a subset X of cardinality p such that a dispersion objective function is maximized. For a feasible solution given by $X = \{ v_1, v_2, ..., v_p \}$, the dispersion function is computed as:

$$f(X) = \sum_{i,j \in X} d_{ij}$$

that is, the sum of distances among all pairs of points in subset X. The problem consists of finding the subset X that maximizes function $f(X)$, that is, that maximizes the sum of pair-wise distances in set X. In Figure 1 below, there is an instance with 13 points and corresponding distance matrix D.

The following questions refer to the pDP instance described in Figure 1, assuming $p = 3$.

(a) [*UT1: Combinatorial optimization;* 5 pts] Is $X^{(1)} = \{3, 6, 9, 11\}$ a feasible solution? Justify your answer.

(b) [*UT1: Combinatorial optimization;* 5 pts] Is $X^{(2)} = \{1, 5, 12\}$ a feasible solution? Justify your answer.

(c) [*UT1: Combinatorial optimization;* 8 pts] Among the following three solutions, sort them from best to worst. Justify your answer.
$X^{(3)} = \{3, 6, 8\}$,
$X^{(4)} = \{6, 7\}$,
$X^{(5)} = \{2, 5, 11\}$.

(d) [*UT2: Constructive heuristics;* 10 pts] Starting from scratch, design a constructive heuristic for finding a feasible solution to the pDP with n points and p dispersion points. Show very clearly and precision each step of your heuristic either in pseudocode or flow chart.

(e) [*UT2: Constructive heuristics;* 6 pts] Illustrate how your heuristic works by applying it step by step in the example (Figure 1) to build a feasible solution to the problem. Was this solution better than solution $X^{(3)}$ from (c)?

(f) [*UT2: Local search heuristics;* 10 pts] Given a feasible solution to the pDP, design a local search for the problem. It is sufficient to describe **very clearly** how you define your move/neighborhood.

(g) [*UT2: Local search heuristics;* 6 pts] Illustrate how your local search works starting from the following feasible solution $X^{(3)} = \{3, 6, 8\}$. Do at least one **complete** iteration under the best found strategy showing all computational details. Did the solution improve?

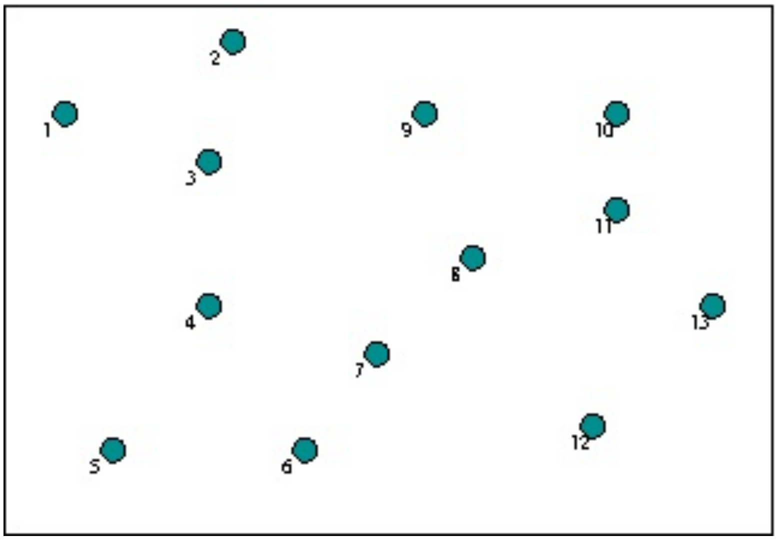

Figure 1: Original set of points V in the plane. Each point is labeled by a number. Distance matrix D is given below.

$D =$

–	1	2	3	4	5	6	7	8	9	10	11	12	13
1	0	15	13	20	28	34	33	36	30	46	47	51	56
2		0	10	22	35	35	29	27	17	33	35	44	46
3			0	12	25	25	21	23	18	34	34	39	45
4				0	16	14	15	22	24	38	35	34	42
5					0	16	23	34	38	50	47	40	51
6						0	10	21	30	38	33	24	36
7							0	11	20	28	23	20	28
8								0	13	17	13	17	20
9									0	16	18	30	29
10										0	8	26	18
11											0	18	11
12												0	14
13													0

Figure 2: Distance matrix D of example. Since D is symmetric, only the upper triangular part is shown. Just as an example, if $p = 3$, one possible solution could be subset X = {1, 2, 3}. For this subset, $f(X) = d_{12} + d_{13} + d_{23} = 15 + 13 + 10 = 38$.