UNIVERSIDAD AUTÓNOMA DE NUEVO LEÓN
FACULTAD DE INGENIERÍA MECÁNICA Y ELÉCTRICA
TIPO DE EXAMEN Y/O EVALUACIÓN:
FINAL ORDINARIO (*Final Exam*)

MATERIA/UNIDAD DE APRENDIZAJE: Temas Selectos de Optimización

LEARNING UNIT: *Selected Topics on Optimization* (in English)
SEMESTER: January – June 2024 (Spring)
ACADEMY: *Statistics and Operations Research* (Estadística e Investigación de Operaciones).
INSTRUCTOR: Dr. Roger Z. Ríos Mercado (ID 090969)

DIRECTIONS.- Answer the following questions and/or exercises in the answer sheet. Do not write in this sheet

SECTION 1: QUESTIONS (40 POINTS)

Answer and justify your answer.

1. [*UT2: Heuristics*; 5 pts] Explain when it is appropriate to use heuristics for solving a combinatorial optimization problem.
2. [*UT1: Combinatorial optimization*; 5 pts] When do we say a combinatorial optimization problem is "hard" to solve? Elaborate.
3. [*UT1: Combinatorial optimization*; 5 pts] Are there "easy" combinatorial optimization problems? Justify your answer.
4. [*UT2: Constructive heuristics;*5 pts] Do constructive heuristics guarantee to find a feasible solution to a given combinatorial optimization problem? Justify your answer.
5. [*UT2: Constructive heuristics;*5 pts] Explain clearly what a construction heuristic is for a combinatorial optimization problem.
6. [*UT2: Local search heuristics;* 5 pts] Explain clearly what a local search heuristic is for a combinatorial optimization problem.
7. [*UT2: Local search heuristics for the TSP;* 5 pts] Describe in detail the 2-OPT heuristic for the Traveling Salesman Problem. You may additionally illustrate your idea with an example or drawing.
8. [*UT2: Heuristics;* 5 pts] Explain in detail how would you compare two different heuristics for a given combinatorial optimization problem, that is, how would you determine which one is better.

SECTION 2: PROBLEMS (60 POINTS)

9. The Clustering Problem (CP) is defined as follows. Given a set n objects $V = \{1, 2, \dots, n\}$, a number of clusters p, and a dissimilarity or "distance" measure d_{ij} between each pair of objects $i, j \in V$, which can also be represented by a matrix $D = (d_{ij})$, we must find a p-partition or p clusters (subsets of V) such that the objects within each cluster are the most similar possible according to this dissimilarity measure. For instance, if a given feasible solution is given by $X = (X_1, X_2, \dots, X_p)$, where each X_k represents a subset of objects from V, then the objective function, or dissimilarity, is given by $f(X) = \sum_{k=1}^{p} d(X_k)$, where the dissimilarity of each subset is measured as $d(X_k) = \sum_{i,j \in X_k} d_{ij}$, that is, the sum of all the dissimilarity or distance values among all the elements in that subset. It is understood that if a subset is comprised of a single object, then its dissimilarity is zero. Recall that a p-partition $X = (X_1, X_2, \dots, X_p)$ is valid or feasible if and only if the union of all subsets is equal to V and each element of V belongs to only one subset X_k. As an example, Figure 1 shows a dissimilarity or distance matrix for a set of 10 objects. Figure 2 shows how the dissimilarity function is computed for a given subset.

   The following questions refer to the CP instance depicted in Figure 1, assuming $p = 3$.

   (a) [*UT1: Combinatorial optimization;* 6 pts] Is $X^{(1)} = (\{3,6,8\}, \{1,4,9\}, \{2,5,10\})$ a feasible solution? Justify your answer.

   (b) [*UT1: Combinatorial optimization;* 6 pts] Is $X^{(2)} = (\{2,4,6,8,10\}, \{1,5,9\}, \{3,7\})$ a feasible solution? Justify your answer.

   (c) [*UT1: Combinatorial optimization;* 6 pts] Among the following three solutions, sort them from best to worst. Justify your answer.
   $X^{(3)} = (\{1,5,9\}, \{2,4,6,8,10\}, \{3,7\})$,
   $X^{(4)} = (\{2,5,9\}, \{1,3,4,8\}, \{6,10\})$,
   $X^{(5)} = (\{1,3,7,8\}, \{2,5,10\}, \{4,6,9\})$.

   (d) [*UT2: Constructive heuristics;* 14 pts] Starting from scratch, design a constructive heuristic for finding a feasible solution to the CP with n objects and p clusters. Show very clearly and precision each step of your heuristic either in pseudocode or flow chart.

   (e) [*UT2: Constructive heuristics;* 8 pts] Illustrate how your heuristic works by applying it step by step in the example (Figure 1) to build a feasible solution to the problem. Was this solution better than solution $X^{(3)}$ from (c)?

   (f) [*UT2: Local search heuristics;* 12 pts] Given a feasible solution to the CP, design a local search heuristic for the problem. It suffices with describing **very clearly** how you define your move/neighborhood.

   (g) [*UT2: Local search heuristics;* 8 pts] Illustrate how your local search works starting from the following feasible solution $X^{(3)} = (\{2,4,6,8,10\}, \{1,5,9\}, \{3,7\})$. You may do just one **complete** iteration by Best Found strategy. Did the solution improve?

$$
D = \left(\begin{array}{c|cccccccccc}
 & v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 & v_8 & v_9 & v_{10} \\
\hline
v_1 & 0 & 12 & 26 & 14 & 31 & 8 & 46 & 35 & 20 & 10 \\
v_2 & & 0 & 41 & 9 & 19 & 33 & 28 & 39 & 14 & 30 \\
v_3 & & & 0 & 25 & 9 & 15 & 37 & 20 & 47 & 23 \\
v_4 & & & & 0 & 29 & 12 & 34 & 8 & 15 & 18 \\
v_5 & & & & & 0 & 45 & 31 & 16 & 11 & 27 \\
v_6 & & & & & & 0 & 13 & 29 & 40 & 32 \\
v_7 & & & & & & & 0 & 36 & 9 & 21 \\
v_8 & & & & & & & & 0 & 17 & 46 \\
v_9 & & & & & & & & & 0 & 23 \\
v_{10} & & & & & & & & & & 0
\end{array}\right)
$$

Figure 1: Example matrix D. Only the upper-diagonal part is shown because D is symmetric.

$$
D = \left(\begin{array}{c|cccccccccc}
 & v_1 & v_2 & \boxed{v_3} & v_4 & v_5 & \boxed{v_6} & v_7 & \boxed{v_8} & v_9 & v_{10} \\
\hline
v_1 & 0 & 12 & 26 & 14 & 31 & 8 & 46 & 35 & 20 & 10 \\
v_2 & & 0 & 41 & 9 & 19 & 33 & 28 & 39 & 14 & 30 \\
\boxed{v_3} & & & 0 & 25 & 9 & \boxed{15} & 37 & \boxed{20} & 47 & 23 \\
v_4 & & & & 0 & 29 & 12 & 34 & 8 & 15 & 18 \\
v_5 & & & & & 0 & 45 & 31 & 16 & 11 & 27 \\
\boxed{v_6} & & & & & & 0 & 13 & \boxed{29} & 40 & 32 \\
v_7 & & & & & & & 0 & 36 & 9 & 21 \\
\boxed{v_8} & & & & & & & & 0 & 17 & 46 \\
v_9 & & & & & & & & & 0 & 23 \\
v_{10} & & & & & & & & & & 0
\end{array}\right)
$$

Figure 2: Computation of dissimilarity function for subset $X_1=\{v_3, v_6, v_8\}$ is shown in the boxed cells. $d(X_1) = d_{36} + d_{38} + d_{68} = 15 + 20 + 29$.