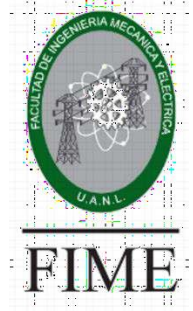




**UNIVERSIDAD AUTÓNOMA DE NUEVO  
LEÓN**

**Facultad de Ingeniería Mecánica y Eléctrica**



## **Temas Selectos de Optimización**

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### **Homework 4**

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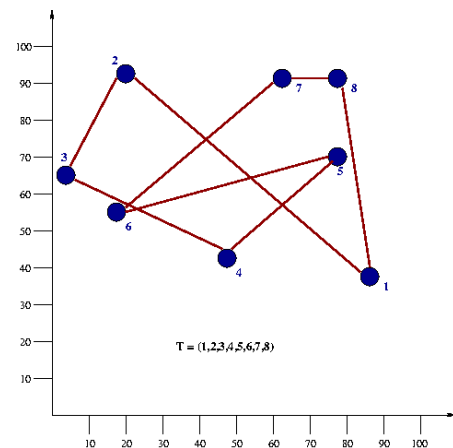
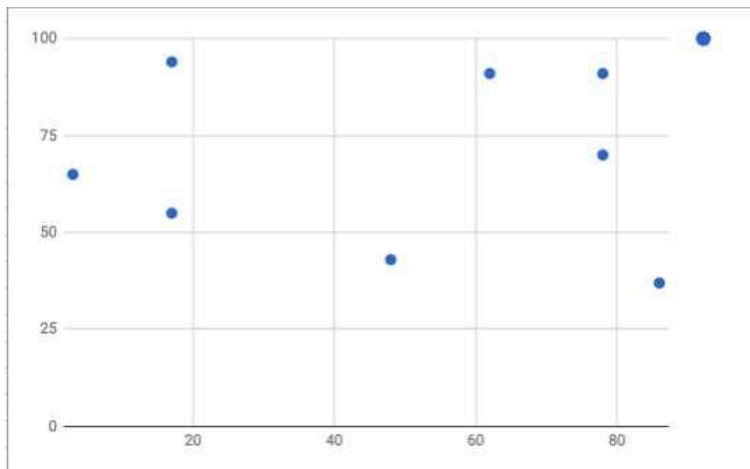
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Semester: August-June 2024

## Problem description

Use the 8-city example shown in the attached figure. The distance matrix is displayed (only the lower triangular part is shown because it is symmetric). Start the local search with the following tour  $T = (1,2,3,4,5,6,7,8)$  as depicted in the other attached figure.

	x	y		1	2	3	4	5	6	7	8
1	86	37	1	0							
2	17	94	2	89	0						
3	3	65	3	87	32	0					
4	48	43	4	38	59	50	0				
5	78	70	5	33	65	75	40	0			
6	17	55	6	71	39	17	33	62	0		
7	62	91	7	59	45	64	50	26	57	0	
8	78	91	8	54	61	79	56	21	70	16	0



## Approach to Problem-Solving

To solve this problem using the 2-OPT local search heuristic, first you would have to calculate the total distance of the selected route.

$$89 + 32 + 50 + 40 + 62 + 57 + 16 + 54 = 400$$

We will use the next formula to calculate the total distance for that specific tour:

$$\Delta f = f(x) - f(x') = d_{i,j} + d_{k,l} - (d_{i,k} + d_{j,l})$$

Knowing this information, we will solve the problem, listing the possible neighbors and applying the last formula.

Best Found strategy.

**1<sup>st</sup> Iteration**

$$\Delta f = f(1,2) - f(3,4) = 89 + 50 - (87 + 59) = -7$$

$$\Delta f = f(1,2) - f(4,5) = 89 + 40 - (38 + 65) = 26$$

$$\underline{\Delta f = f(1,2) - f(5,6) = 89 + 62 - (33 + 39) = 79}$$

$$\Delta f = f(1,2) - f(6,7) = 89 + 57 - (71 + 45) = 30$$

$$\Delta f = f(1,2) - f(7,8) = 89 + 16 - (59 + 61) = -15$$

$$\Delta f = f(2,3) - f(4,5) = 32 + 40 - (59 + 75) = -62$$

$$\Delta f = f(2,3) - f(5,6) = 32 + 62 - (65 + 17) = 12$$

$$\Delta f = f(2,3) - f(6,7) = 32 + 57 - (39 + 64) = -14$$

$$\Delta f = f(2,3) - f(7,8) = 32 + 16 - (45 + 79) = -76$$

$$\Delta f = f(2,3) - f(8,1) = 32 + 54 - (61 + 87) = -62$$

$$\Delta f = f(3,4) - f(5,6) = 50 + 62 - (75 + 33) = 4$$

$$\Delta f = f(3,4) - f(6,7) = 50 + 57 - (17 + 50) = 40$$

$$\Delta f = f(3,4) - f(7,8) = 50 + 16 - (64 + 56) = -54$$

$$\Delta f = f(3,4) - f(8,1) = 50 + 54 - (79 + 38) = -13$$

$$\Delta f = f(4,5) - f(6,7) = 40 + 57 - (33 + 26) = 38$$

$$\Delta f = f(4,5) - f(7,8) = 40 + 16 - (50 + 21) = -15$$

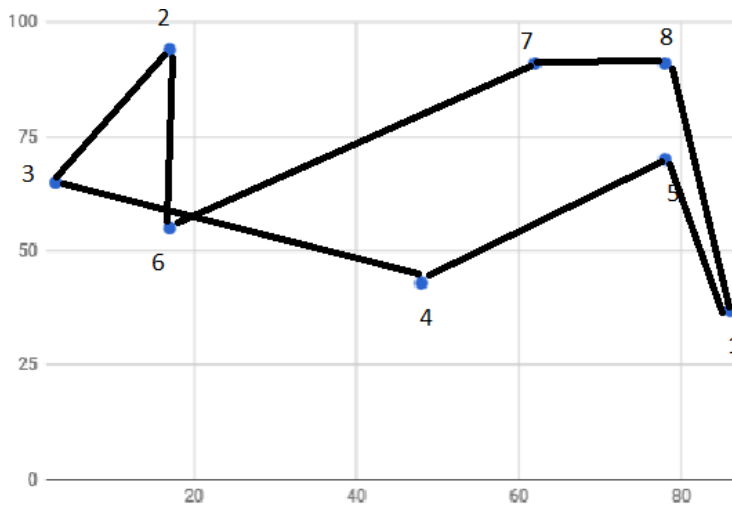
$$\Delta f = f(4,5) - f(8,1) = 40 + 54 - (56 + 33) = 5$$

$$\Delta f = f(5,6) - f(7,8) = 62 + 16 - (26 + 70) = -18$$

$$\Delta f = f(5,6) - f(8,1) = 62 + 54 - (21 + 71) = 54$$

$$\Delta f = f(6,7) - f(8,1) = 57 + 54 - (70 + 59) = -18$$

We select the biggest one and complete the tour rearranging it. The new tour would look like this:



New tour= (1,5,4,3,2,6,7,8,1)

$$F(x) = 33 + 40 + 50 + 32 + 39 + 57 + 16 + 54 = 321$$

We apply the same method with the new tour.

## 2<sup>nd</sup> Iteration

$$\Delta f = f(1,5) - f(4,3) = 33 + 50 - (38 + 75) = -30$$

$$\Delta f = f(1,5) - f(3,2) = 33 + 32 - (87 + 65) = -87$$

$$\Delta f = f(1,5) - f(2,6) = 33 + 39 - (89 + 62) = -79$$

$$\Delta f = f(1,5) - f(6,7) = 33 + 57 - (71 + 26) = -7$$

$$\Delta f = f(1,5) - f(7,8) = 33 + 16 - (59 + 21) = -31$$

$$\Delta f = f(5,4) - f(3,2) = 40 + 32 - (75 + 59) = -62$$

$$\Delta f = f(5,4) - f(2,6) = 40 + 39 - (65 + 33) = -19$$

$$\Delta f = f(5,4) - f(6,7) = 40 + 57 - (62 + 50) = -15$$

$$\Delta f = f(5,4) - f(7,8) = 40 + 16 - (26 + 56) = -26$$

$$\underline{\Delta f = f(5,4) - f(8,1) = 40 + 54 - (21 + 38) = 35}$$

$$\Delta f = f(4,3) - f(2,6) = 50 + 39 - (59 + 17) = 12$$

$$\Delta f = f(4,3) - f(6,7) = 50 + 57 - (33 + 64) = 10$$

$$\Delta f = f(4,3) - f(7,8) = 50 + 16 - (50 + 79) = -63$$

$$\Delta f = f(4,3) - f(8,1) = 50 + 54 - (56 + 87) = -39$$

$$\Delta f = f(3,2) - f(6,7) = 32 + 57 - (17 + 45) = 27$$

$$\Delta f = f(3,2) - f(7,8) = 32 + 16 - (64 + 61) = -77$$

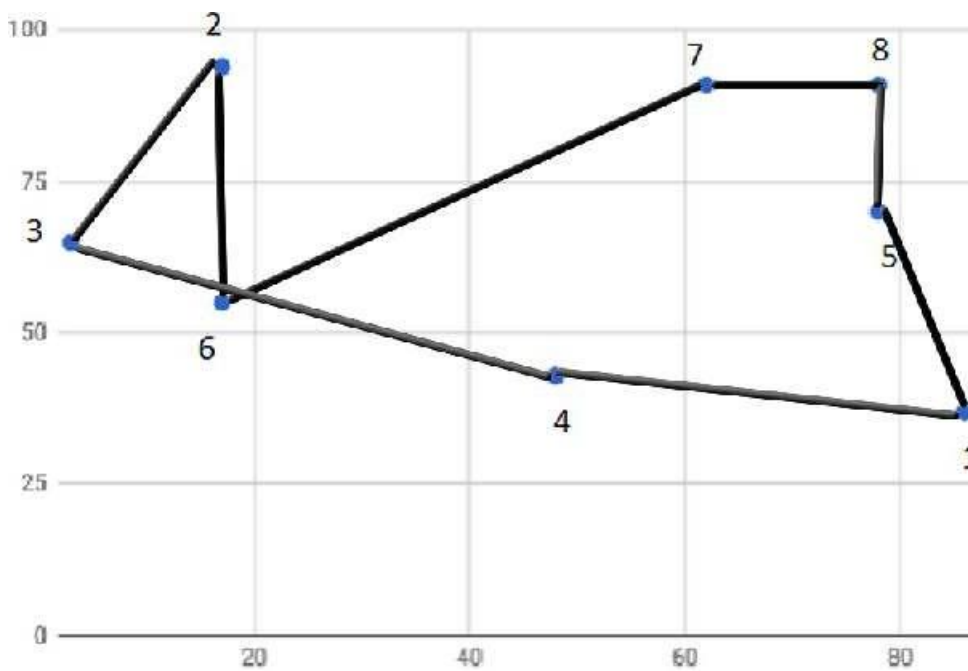
$$\Delta f = f(3,2) - f(8,1) = 32 + 54 - (79 + 89) = -82$$

$$\Delta f = f(2,6) - f(7,8) = 39 + 16 - (45 + 70) = -60$$

$$\Delta f = f(2,6) - f(8,1) = 39 + 54 - (61 + 71) = -39$$

$$\Delta f = f(6,7) - f(8,1) = 57 + 54 - (70 + 59) = -18$$

New tour:



Tour= (1,5,8,7,6,2,3,4)

$$F(x) = 33 + 21 + 16 + 57 + 39 + 32 + 50 + 38 = 286$$

### 3<sup>rd</sup> Iteration

$$\Delta f = f(1,5) - f(3,4) = 33 + 50 - (87 + 40) = -44$$

$$\Delta f = f(1,5) - f(2,3) = 33 + 32 - (89 + 75) = -99$$

$$\Delta f = f(1,5) - f(6,2) = 33 + 39 - (71 + 65) = -64$$

$$\Delta f = f(1,5) - f(7,6) = 33 + 57 - (59 + 62) = -34$$

$$\Delta f = f(1,5) - f(8,7) = 33 + 16 - (54 + 26) = -31$$

$$\Delta f = f(4,1) - f(2,3) = 38 + 32 - (59 + 87) = -76$$

$$\Delta f = f(4,1) - f(6,2) = 38 + 39 - (33 + 89) = -45$$

$$\Delta f = f(4,1) - f(7,6) = 38 + 57 - (50 + 71) = -26$$

$$\Delta f = f(4,1) - f(8,7) = 38 + 16 - (56 + 59) = -66$$

$$\Delta f = f(4,1) - f(5,8) = 38 + 21 - (40 + 54) = -35$$

$$\Delta f = f(3,4) - f(6,2) = 50 + 39 - (17 + 59) = 13$$

$$\Delta f = f(3,4) - f(7,6) = 50 + 57 - (64 + 33) = 10$$

$$\Delta f = f(3,4) - f(8,7) = 50 + 16 - (79 + 50) = -63$$

$$\Delta f = f(3,4) - f(5,8) = 50 + 21 - (75 + 56) = -60$$

$$\underline{\Delta f = f(2,3) - f(7,6) = 32 + 57 - (45 + 17) = 27}$$

$$\Delta f = f(2,3) - f(8,7) = 32 + 16 - (61 + 64) = -77$$

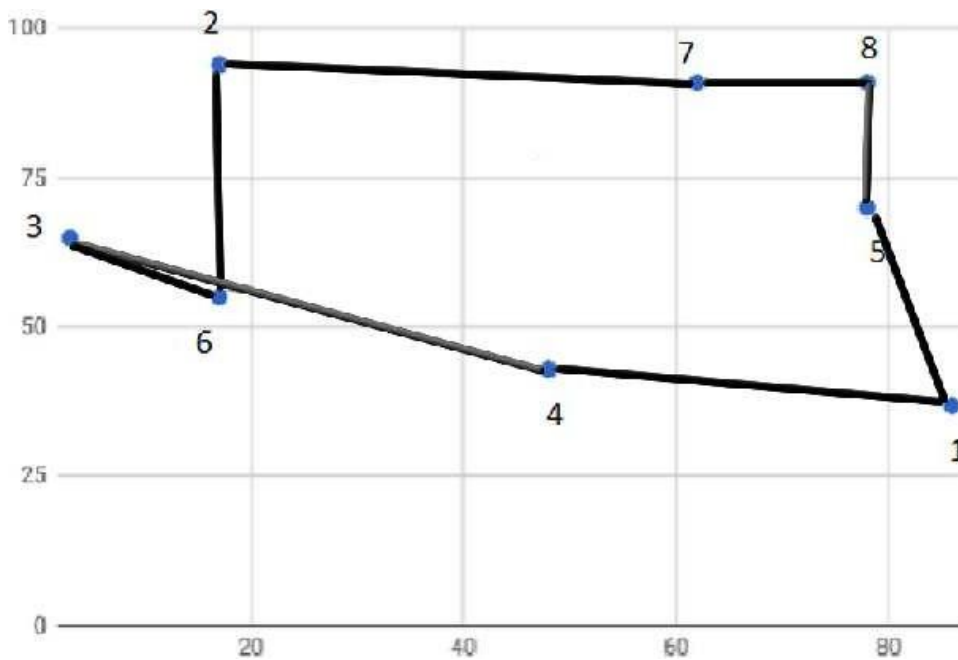
$$\Delta f = f(2,3) - f(5,8) = 32 + 21 - (65 + 79) = -91$$

$$\Delta f = f(6,2) - f(8,7) = 39 + 16 - (70 + 45) = -60$$

$$\Delta f = f(6,2) - f(5,8) = 39 + 21 - (62 + 61) = -63$$

$$\Delta f = f(7,6) - f(5,8) = 57 + 21 - (26 + 70) = -18$$

New tour= (1,5,8,7,2,6,3,4)



$$F(x) = 33 + 21 + 16 + 45 + 39 + 17 + 50 + 33 = 254$$

#### 4<sup>th</sup> Iteration

$$\Delta f = f(1,5) - f(3,4) = 33 + 50 - (87 + 40) = -44$$

$$\Delta f = f(1,5) - f(6,3) = 33 + 17 - (71 + 75) = -96$$

$$\Delta f = f(1,5) - f(2,6) = 33 + 39 - (89 + 62) = -79$$

$$\Delta f = f(1,5) - f(7,2) = 33 + 45 - (59 + 65) = -46$$

$$\Delta f = f(1,5) - f(8,7) = 33 + 16 - (54 + 26) = -31$$

$$\Delta f = f(4,1) - f(6,3) = 38 + 17 - (33 + 87) = -65$$

$$\Delta f = f(4,1) - f(2,6) = 38 + 39 - (59 + 71) = -53$$

$$\Delta f = f(4,1) - f(7,2) = 38 + 45 - (50 + 89) = -56$$

$$\Delta f = f(4,1) - f(8,7) = 38 + 16 - (56 + 59) = -61$$

$$\Delta f = f(4,1) - f(5,8) = 38 + 21 - (40 + 89) = -70$$

$$\underline{\Delta f = f(3,4) - f(2,6) = 50 + 39 - (32 + 33) = 24}$$

$$\Delta f = f(3,4) - f(7,2) = 50 + 45 - (64 + 59) = -28$$

$$\Delta f = f(3,4) - f(8,7) = 50 + 16 - (79 + 50) = -63$$

$$\Delta f = f(3,4) - f(5,8) = 50 + 21 - (75 + 56) = -60$$

$$\Delta f = f(6,3) - f(7,2) = 17 + 45 - (57 + 32) = -27$$

$$\Delta f = f(6,3) - f(8,7) = 17 + 16 - (70 + 64) = -101$$

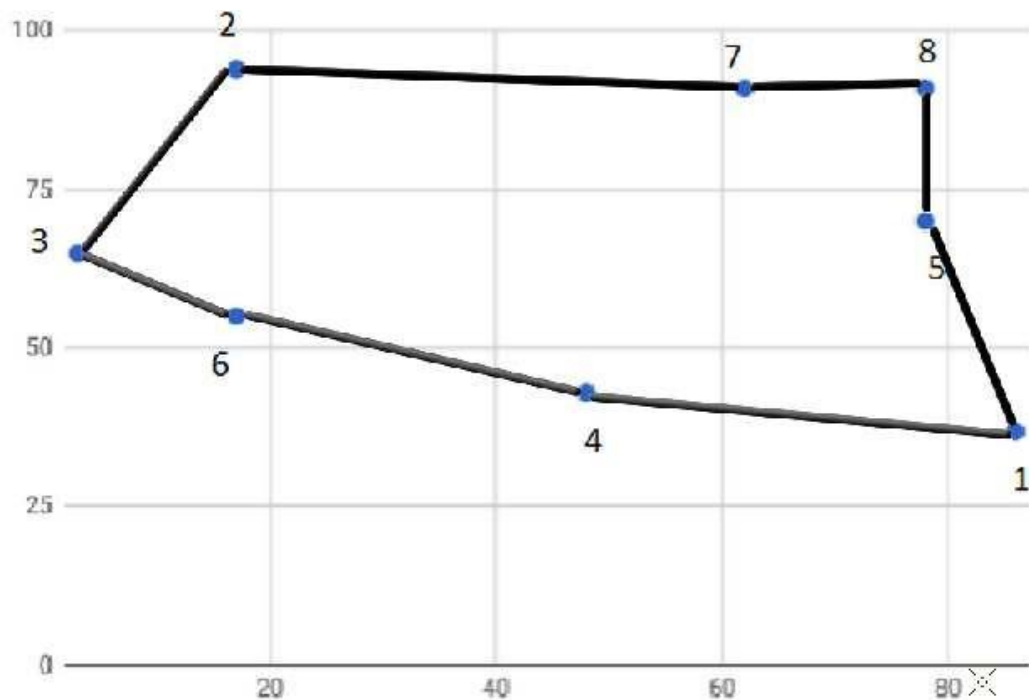
$$\Delta f = f(6,3) - f(5,8) = 17 + 21 - (62 + 79) = -103$$

$$\Delta f = f(2,6) - f(8,7) = 39 + 16 - (61 + 57) = -63$$

$$\Delta f = f(2,6) - f(5,8) = 39 + 21 - (65 + 70) = -75$$

$$\Delta f = f(7,2) - f(5,8) = 45 + 21 - (26 + 61) = -21$$

New tour= (1,5,8,7,2,3,6,4)



$$F(x) = 33 + 21 + 16 + 45 + 32 + 17 + 33 + 38 = 235$$

seeing the graphic, we can already tell that this is the optimal solution to the problem, this it is not necessary to do another iteration.



### First Found strategy.

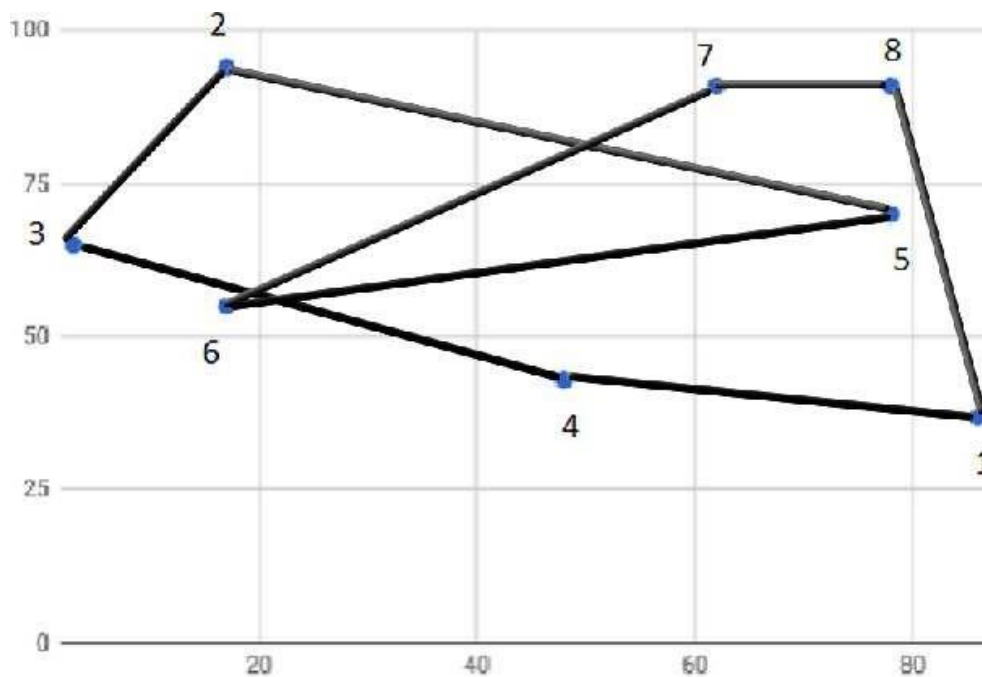
To use the first found method, we will select the first possible tour and reiterate until there are no more optimal tours.

### **1<sup>st</sup> Iteration**

$$\Delta f = f(1,2) - f(3,4) = 89 + 50 - (87 + 59) = -7$$

$$\underline{\Delta f = f(1,2) - f(4,5) = 89 + 40 - (38 + 65) = 26}$$

New tour= (1,4,3,2,5,6,7,8)



$$F(x) = 38 + 50 + 32 + 65 + 62 + 57 + 16 + 54 = 374$$

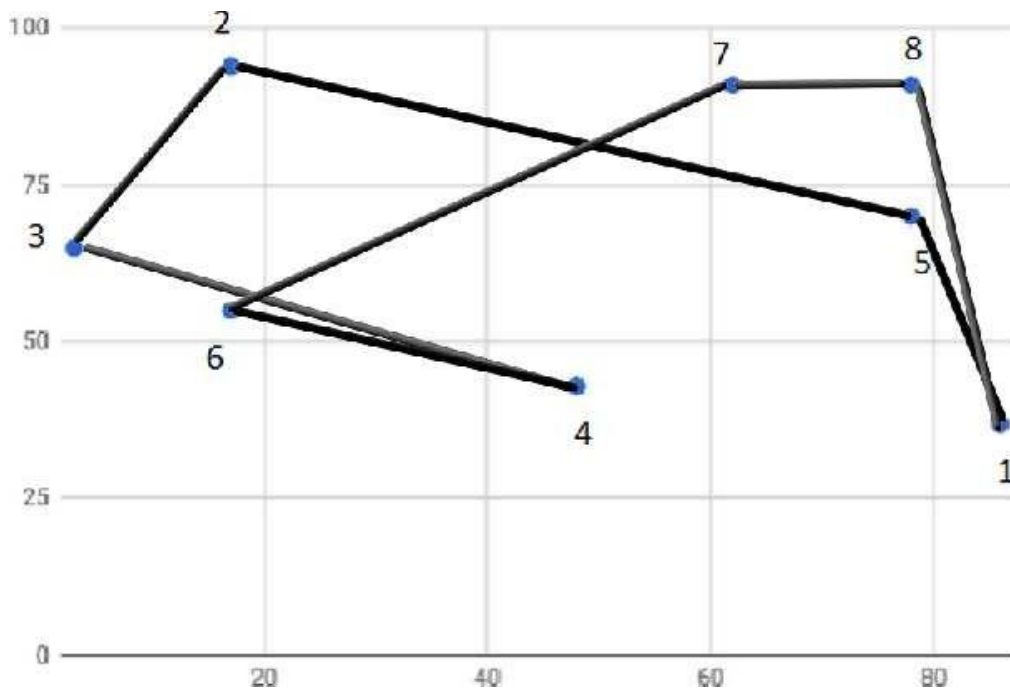
### **2<sup>nd</sup> Iteration**

$$\Delta f = f(1,4) - f(3,2) = 38 + 32 - (87 + 59) = -76$$

$$\Delta f = f(1,4) - f(2,5) = 38 + 65 - (89 + 40) = -26$$

$$\underline{\Delta f = f(1,4) - f(5,6) = 38 + 62 - (33 + 33) = 34}$$

New tour= (1,5,2,3,4,6,7,8)



$$F(x) = 33 + 65 + 32 + 50 + 33 + 57 + 16 + 54 = 340$$

### 3<sup>rd</sup> Iteration

$$\Delta f = f(1,5) - f(2,3) = 38 + 32 - (89 + 75) = -94$$

$$\Delta f = f(1,5) - f(3,4) = 38 + 50 - (87 + 40) = -39$$

$$\Delta f = f(1,5) - f(4,6) = 38 + 33 - (38 + 62) = -29$$

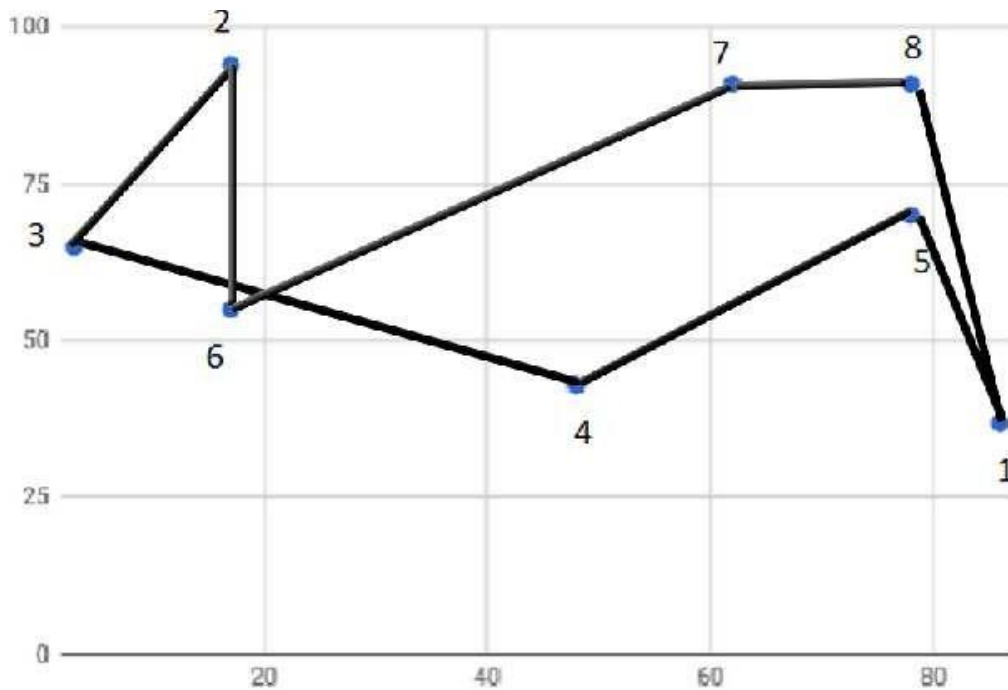
$$\Delta f = f(1,5) - f(6,7) = 38 + 57 - (71 + 26) = -5$$

$$\Delta f = f(1,5) - f(7,8) = 38 + 16 - (59 + 21) = -26$$

$$\Delta f = f(5,2) - f(3,4) = 65 + 50 - (75 + 59) = -19$$

$$\underline{\Delta f = f(5,2) - f(4,6) = 65 + 33 - (40 + 39) = 19}$$

New tour= (1,5,4,3,2,6,7,8)



$$F(x) = 33 + 40 + 50 + 32 + 39 + 57 + 16 + 54 = 321$$

#### 4<sup>th</sup> Iteration

$$\Delta f = f(1,5) - f(3,2) = 33 + 32 - (87 + 65) = -87$$

$$\Delta f = f(1,5) - f(4,3) = 33 + 50 - (38 + 75) = -30$$

$$\Delta f = f(1,5) - f(2,6) = 33 + 39 - (89 + 62) = -79$$

$$\Delta f = f(1,5) - f(6,7) = 33 + 57 - (71 + 26) = -7$$

$$\Delta f = f(1,5) - f(7,8) = 33 + 16 - (59 + 21) = -31$$

$$\Delta f = f(5,4) - f(3,2) = 40 + 32 - (75 + 59) = -62$$

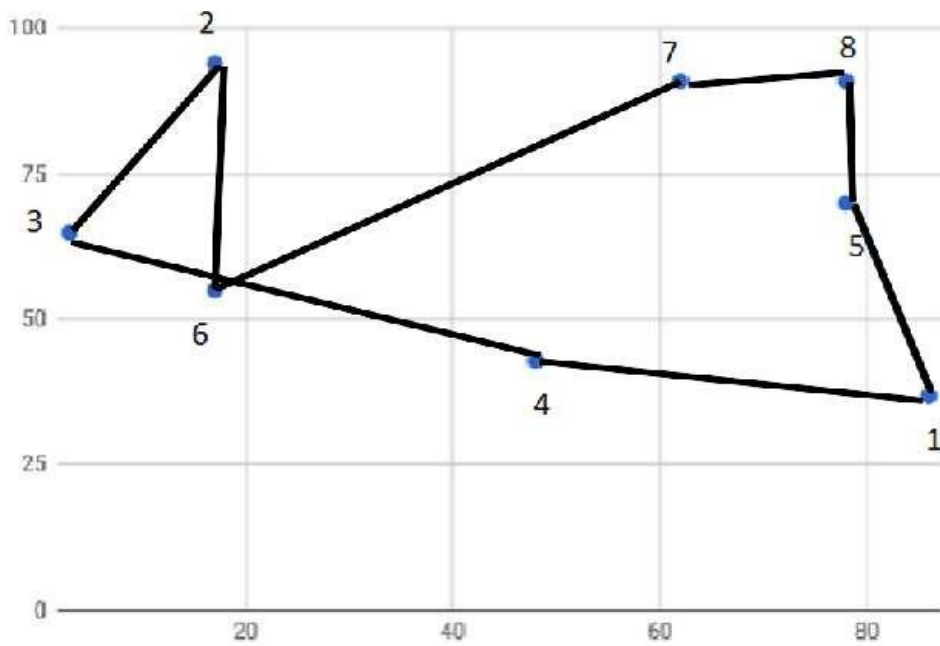
$$\Delta f = f(5,4) - f(2,6) = 40 + 39 - (65 + 33) = -19$$

$$\Delta f = f(5,4) - f(6,7) = 40 + 57 - (62 + 50) = -15$$

$$\Delta f = f(5,4) - f(7,8) = 40 + 16 - (26 + 56) = -26$$

$$\underline{\Delta f = f(5,4) - f(8,1) = 40 + 54 - (21 + 38) = 35}$$

New tour= (1,5,8,7,6,2,3,4)

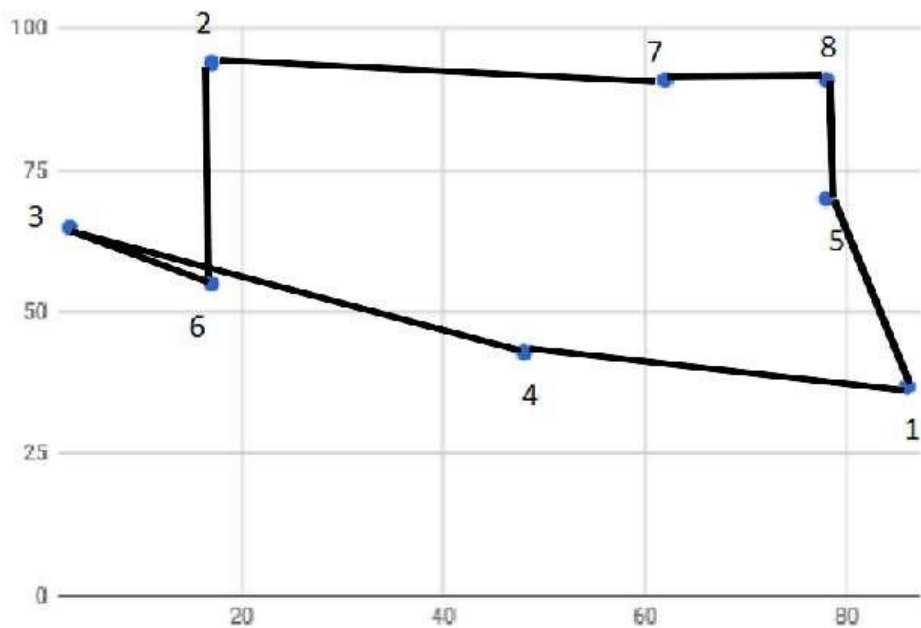


$$F(x) = 33 + 40 + 50 + 32 + 39 + 57 + 16 + 54 = 321$$

### 5<sup>th</sup> Iteration

$$\Delta f = f(2,3) - f(7,6) = 32 + 57 - (45 + 17) = 27$$

New tour= (1,5,8,7,2,6,3,4)



$$F(x) = 33 + 21 + 16 + 45 + 39 + 17 + 50 + 33 = 254$$

### 6<sup>th</sup> Iteration

$$\Delta f = f(1,5) - f(3,4) = 33 + 50 - (87 + 40) = -44$$

$$\Delta f = f(1,5) - f(6,3) = 33 + 17 - (71 + 75) = -96$$

$$\Delta f = f(1,5) - f(2,6) = 33 + 39 - (89 + 62) = -79$$

$$\Delta f = f(1,5) - f(7,2) = 33 + 45 - (59 + 65) = -46$$

$$\Delta f = f(1,5) - f(8,7) = 33 + 16 - (54 + 26) = -31$$

$$\Delta f = f(4,1) - f(6,3) = 38 + 17 - (33 + 87) = -65$$

$$\Delta f = f(4,1) - f(2,6) = 38 + 39 - (59 + 71) = -53$$

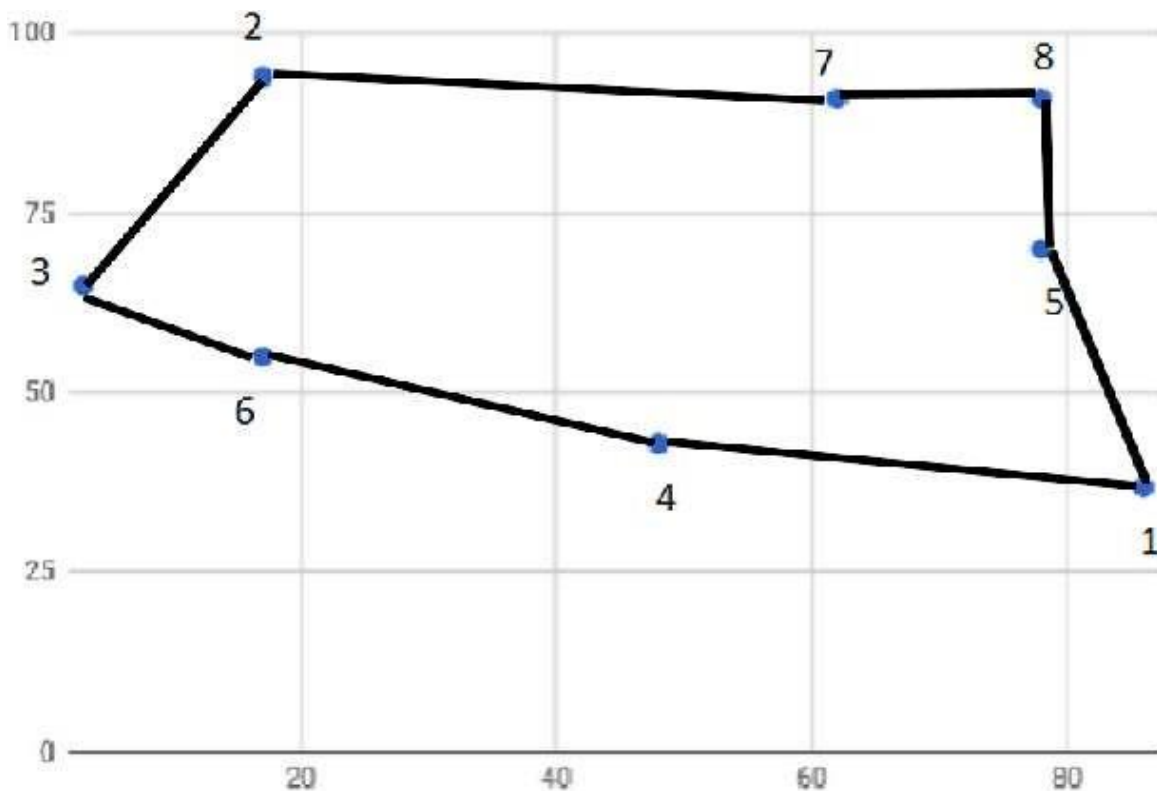
$$\Delta f = f(4,1) - f(7,2) = 38 + 45 - (50 + 89) = -56$$

$$\Delta f = f(4,1) - f(8,7) = 38 + 16 - (56 + 59) = -61$$

$$\Delta f = f(4,1) - f(5,8) = 38 + 21 - (40 + 89) = -70$$

$$\Delta f = f(3,4) - f(2,6) = 50 + 39 - (32 + 33) = 24$$

New tour= (1,5,8,7,2,3,6,4)



looks like the best-found strategy's last iteration, we already know that this is the most optimal solution for the problem by looking at the graphical representation, so we don't need to do another iteration.