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Computational Experience with Heuristics for the Generalized Assignment Problem (GAP) applied to Vehicle Routing and Assignment.

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INTRODUCTION

The Generalized Assignment Problem (GAP) is a combinatorial optimization challenge that arises in various real-world scenarios where resources need to be allocated efficiently to tasks or jobs. In this problem, a set of resources, each with specific capacities, must be assigned to a set of tasks, each with varying requirements and associated costs. The primary objective of the Generalized Assignment Problem is to minimize the total cost of assigning resources to tasks while satisfying the capacity constraints of each resource.

Unlike the classical assignment problem, where each task is assigned to exactly one resource, the GAP allows for multiple tasks to be assigned to a single resource, subject to its capacity limitations. This flexibility reflects the complex nature of many practical applications, such as production planning, project scheduling, and workforce management.

The Generalized Assignment Problem finds applications in diverse fields, including manufacturing, telecommunications, transportation, and project management. Its versatility makes it a valuable tool for addressing resource allocation challenges that involve multiple tasks and resources with varying capacities and costs.

Practical applications of GAP

Manufacturing and Production Planning: Allocating machines to different production tasks to minimize production costs. Assigning workers to specific manufacturing operations considering their skills and capacity.

Facility Location and Assignment: In logistics and supply chain management, GAP can be applied to decide the optimal location for facilities (such as warehouses or distribution centers) and assign tasks related to order fulfillment to these facilities.

Vehicle Routing and Assignment: In transportation and logistics, GAP can be part of solving vehicle routing problems where tasks or deliveries need to be assigned to a fleet of vehicles, considering factors like distance, vehicle capacity, and transportation costs.

Applied to vehicle routing.

The Vehicle Routing Problem (VRP) is a fundamental problem in the field of combinatorial optimization, with wide-ranging applications in transportation, logistics, and distribution management. Initially formulated in the 1950s, VRP has since garnered significant attention due to its practical relevance and computational complexity. This problem involves determining optimal routes for a fleet of vehicles to serve a set of customers while minimizing overall costs or distances traveled.

The importance of the VRP lies in its ability to address crucial challenges faced by businesses and organizations involved in transportation and distribution activities. By optimizing vehicle routes, companies can achieve various operational objectives, including cost reduction, improved resource utilization, enhanced customer satisfaction, and environmental sustainability.

In the context of package delivery services, such as those provided by courier companies like UPS and FedEx, efficient vehicle routing is essential for minimizing fuel consumption, reducing vehicle wear and tear, and ensuring timely deliveries to customers. Similarly, in public transportation systems, such as bus and shuttle services, effective route optimization leads to improved service reliability, reduced congestion, and enhanced passenger satisfaction.

Moreover, VRP finds applications in diverse domains beyond transportation. For instance, in waste collection and management, municipal authorities utilize VRP to optimize garbage collection routes, thereby minimizing operational costs and environmental impact. In inventory management and supply chain logistics, VRP helps optimize the distribution of goods from warehouses to retail stores, minimizing transportation costs and ensuring timely replenishment of inventory.

Practical applications of VRP:

The practical applications of VRP are numerous and varied, spanning industries such as retail, healthcare, e-commerce, and manufacturing. Some prominent applications include:

Package Delivery: Companies like UPS and FedEx use VRP to optimize the delivery routes for their drivers, ensuring timely deliveries while minimizing fuel consumption and vehicle wear and tear.

Public Transportation: Bus and shuttle services utilize VRP to optimize routes, schedules, and vehicle assignments, thereby improving service reliability and passenger satisfaction.

Waste Collection: Municipalities employ VRP to optimize garbage collection routes, reducing fuel usage and operational costs while ensuring timely waste removal.

Inventory Management: VRP can be applied in inventory replenishment scenarios to optimize the routes of trucks delivering goods from warehouses to retail stores, minimizing stockouts and excess inventory holding costs.

NO!!

PROBLEM DESCRIPTION

The vehicle routing problem (VRP) can be formally defined as follows:

Data: Input data consists of:

- **n** customers, each with a demand for goods/services.
- **m** vehicles, each with the capacity to meet customer demands.
- A distance or travel time matrix that indicates the distance or travel time between each pair of customers and between the customers and the warehouse.

Decisions: Decisions to be made include:

- Designate which vehicle serves each client.
- Determine the sequence of customers visited by each vehicle.

Optimization: The goal is to minimize the total distance traveled (or time spent) by all vehicles while meeting all customer demands and any other constraints.

Constraints:

- Each vehicle must start and end its journey at the depot.
- The capacity of each vehicle must not be exceeded.
- Optionally, time windows must be respected, that is, each client must be served within its specified time window.

<u>Mathematical Model:</u>

Let:

$\mathbf{n}$ be the number of customers.

$\mathbf{m}$ be the number of vehicles.

$\mathbf{q_i}$ be the demand of customer i.

$\mathbf{Q}$ be the capacity of each vehicle.

$\mathbf{d_{ij}}$ be the cost (distance or time) of traveling between customers **i** and **j**.

$\mathbf{x_{ijk}}$ be a binary variable indicating whether vehicle **k** travels directly from customer **i** to customer **j** ($\mathbf{x_{ijk}}$ =1 if traveled 0 otherwise).

$\mathbf{u_i}$ be the cumulative demand up to customer **i**.

The VRP can be mathematically modeled as follows:

Minimize:

$$\sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{m} d_{ij} * \mathbf{x}_{\mathbf{ijk}}$$

Subject to:

$$\sum_{i=1}^{n} x_{ijk} = 1, \forall \mathbf{k}$$

$$\sum_{i=1}^{m} x_{ijk} = 1, \wedge \mathbf{i}$$

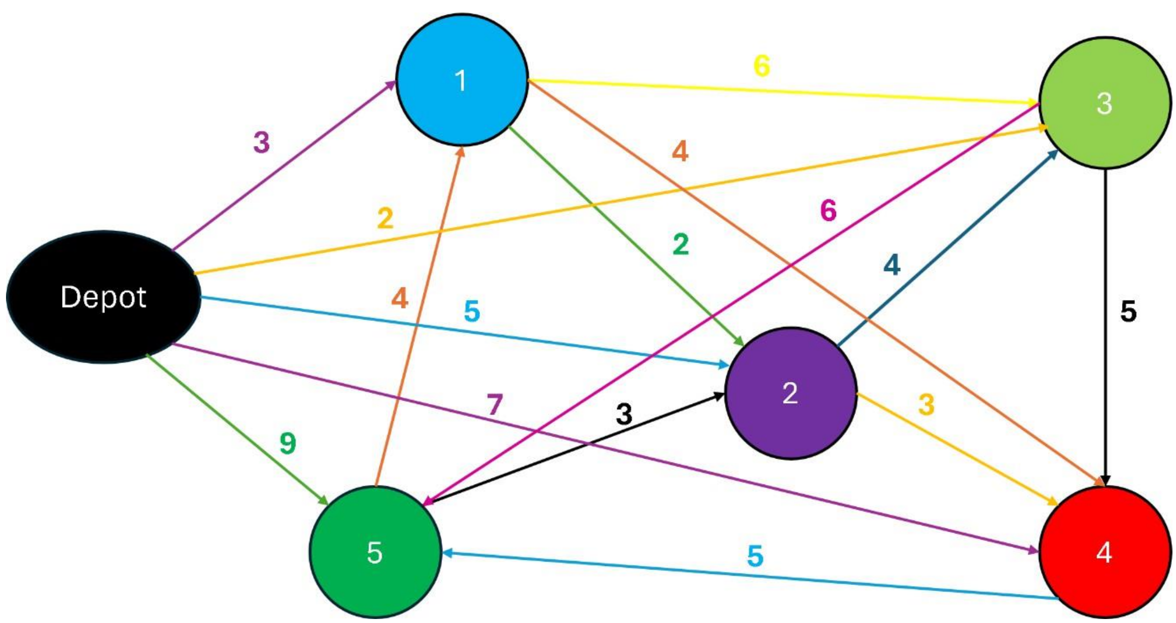
$$\sum_{i=1}^{n} q_i * \mathbf{x}_{\mathbf{ijk}} \leq \mathbf{Q}, \forall \mathbf{k}$$

$$\mathbf{u}_{\mathbf{i}} \geq q_i, \forall \mathbf{i}$$

$$\mathbf{u}_{\mathbf{i}} \geq 0, \forall \mathbf{i}$$

PROBLEM EXAMPLE

Consider a VRP instance with the following data:



- Number if costumers (**n**):5
- Number of vehicles (**m**):2
- Vehicle capacity (**Q**):10
- Demand of costumers ($\mathbf{q_i}$): [4, 2, 5, 3, 4]
- Distance travel matrix ($\mathbf{d_{ij}}$):

	$Depot$	1	2	3	4	5
$Depot$	0	3	5	2	7	9
1	3	0	2	6	4	4
2	5	2	0	4	3	3
3	2	6	4	0	5	6
4	7	4	3	5	0	5
5	9	4	3	6	5	0
$Demand$	$-$	4	2	5	3	4

Suppose the capacity and time window constraints are met. A feasible solution could be:

- Vehicle 1 visits customers 1, 2, and 5, in that order.
- Vehicle 2 visits customers 3 and 4, in that order.

Explanation:

Vehicle 1:

- Start at the depot.
- Visit customers 1, 2, and 5 (total demand: 4 + 2 + 4 = 10 <=Q).
- Return to the depot.
- Feasible solution.

Vehicle 2:

- Start at the depot.
- Visit customers 3 and 4 (total demand: 5 + 3 = 8 <= Q).
- Return to the depot.
- Feasible solution.

The objective function for this solution would be the total travel time of both vehicles.

$$\sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{m} d_{ij} * \mathbf{x}_{\mathbf{ijk}} = 3 + 2 + 3 + 9 + 2 + 5 + 7 = 31$$

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