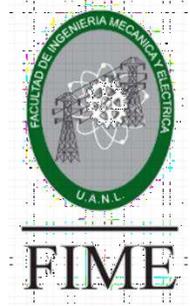




**UNIVERSIDAD AUTÓNOMA DE NUEVO
LEÓN**

Facultad de Ingeniería Mecánica y Eléctrica



Temas Selectos de Optimización

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Homework 4

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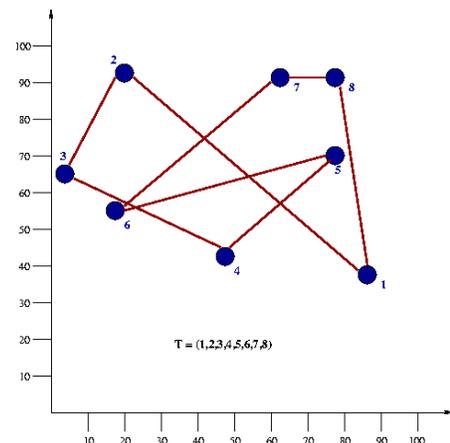
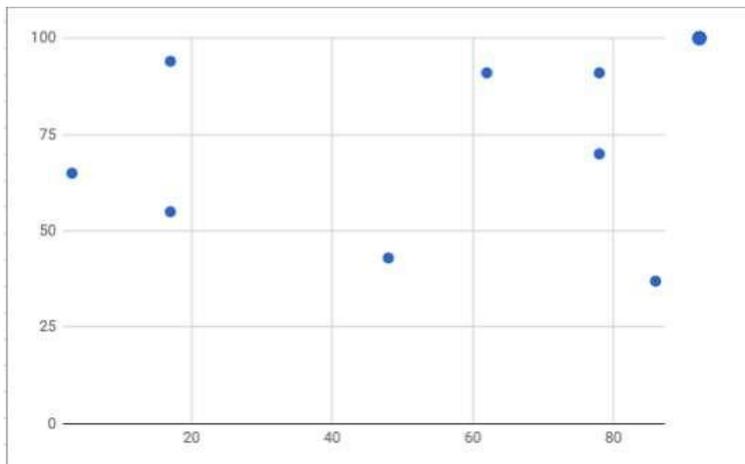
Semester: August-June 2024

Problem description

Use the 8-city example shown in the attached figure. The distance matrix is displayed (only the lower triangular part is shown because it is symmetric). Start the local search with the following tour $T = (1,2,3,4,5,6,7,8)$ as depicted in the other attached figure.

x	y	1	2	3	4	5	6	7	8
1	86	37							
2	17	94							
3	3	65							
4	48	43							
5	78	70							
6	17	55							
7	62	91							
8	78	91							

	1	2	3	4	5	6	7	8
1	0							
2	89	0						
3	87	32	0					
4	38	59	50	0				
5	33	65	75	40	0			
6	71	39	17	33	62	0		
7	59	45	64	50	26	57	0	
8	54	61	79	56	21	70	16	0



Approach to Problem-Solving

To solve this problem using the 2-OPT local search heuristic, first you would have to calculate the total distance of the selected route.

$$89 + 32 + 50 + 40 + 62 + 57 + 16 + 54 = 400$$

We will use the next formula to calculate the total distance for that specific tour:

$$\Delta f = f(x) - f(x') = d_{i,j} + d_{k,l} - (d_{i,k} + d_{j,l})$$

Knowing this information, we will solve the problem, listing the possible neighbors and applying the last formula.

Best Found strategy.

1st Iteration

$$\Delta f = f(1,2) - f(3,4) = 89 + 50 - (87 + 59) = -7$$

$$\Delta f = f(1,2) - f(4,5) = 89 + 40 - (38 + 65) = 26$$

$$\underline{\Delta f = f(1,2) - f(5,6) = 89 + 62 - (33 + 39) = 79}$$

$$\Delta f = f(1,2) - f(6,7) = 89 + 57 - (71 + 45) = 30$$

$$\Delta f = f(1,2) - f(7,8) = 89 + 16 - (59 + 61) = -15$$

$$\Delta f = f(2,3) - f(4,5) = 32 + 40 - (59 + 75) = -62$$

$$\Delta f = f(2,3) - f(5,6) = 32 + 62 - (65 + 17) = 12$$

$$\Delta f = f(2,3) - f(6,7) = 32 + 57 - (39 + 64) = -14$$

$$\Delta f = f(2,3) - f(7,8) = 32 + 16 - (45 + 79) = -76$$

$$\Delta f = f(2,3) - f(8,1) = 32 + 54 - (61 + 87) = -62$$

$$\Delta f = f(3,4) - f(5,6) = 50 + 62 - (75 + 33) = 4$$

$$\Delta f = f(3,4) - f(6,7) = 50 + 57 - (17 + 50) = 40$$

$$\Delta f = f(3,4) - f(7,8) = 50 + 16 - (64 + 56) = -54$$

$$\Delta f = f(3,4) - f(8,1) = 50 + 54 - (79 + 38) = -13$$

$$\Delta f = f(4,5) - f(6,7) = 40 + 57 - (33 + 26) = 38$$

$$\Delta f = f(4,5) - f(7,8) = 40 + 16 - (50 + 21) = -15$$

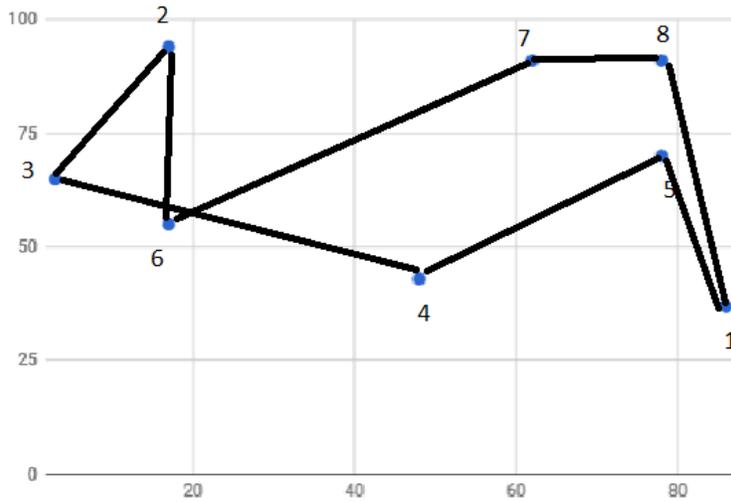
$$\Delta f = f(4,5) - f(8,1) = 40 + 54 - (56 + 33) = 5$$

$$\Delta f = f(5,6) - f(7,8) = 62 + 16 - (26 + 70) = -18$$

$$\Delta f = f(5,6) - f(8,1) = 62 + 54 - (21 + 71) = 54$$

$$\Delta f = f(6,7) - f(8,1) = 57 + 54 - (70 + 59) = -18$$

We select the biggest one and complete the tour rearranging it. The new tour would look like this:



New tour= (1,5,4,3,2,6,7,8,1)

$$F(x) = 33 + 40 + 50 + 32 + 39 + 57 + 16 + 54 = 321$$

We apply the same method with the new tour.

2nd Iteration

$$\Delta f = f(1,5) - f(4,3) = 33 + 50 - (38 + 75) = -30$$

$$\Delta f = f(1,5) - f(3,2) = 33 + 32 - (87 + 65) = -87$$

$$\Delta f = f(1,5) - f(2,6) = 33 + 39 - (89 + 62) = -79$$

$$\Delta f = f(1,5) - f(6,7) = 33 + 57 - (71 + 26) = -7$$

$$\Delta f = f(1,5) - f(7,8) = 33 + 16 - (59 + 21) = -31$$

$$\Delta f = f(5,4) - f(3,2) = 40 + 32 - (75 + 59) = -62$$

$$\Delta f = f(5,4) - f(2,6) = 40 + 39 - (65 + 33) = -19$$

$$\Delta f = f(5,4) - f(6,7) = 40 + 57 - (62 + 50) = -15$$

$$\Delta f = f(5,4) - f(7,8) = 40 + 16 - (26 + 56) = -26$$

$$\underline{\Delta f = f(5,4) - f(8,1) = 40 + 54 - (21 + 38) = 35}$$

$$\Delta f = f(4,3) - f(2,6) = 50 + 39 - (59 + 17) = 12$$

$$\Delta f = f(4,3) - f(6,7) = 50 + 57 - (33 + 64) = 10$$

$$\Delta f = f(4,3) - f(7,8) = 50 + 16 - (50 + 79) = -63$$

$$\Delta f = f(4,3) - f(8,1) = 50 + 54 - (56 + 87) = -39$$

$$\Delta f = f(3,2) - f(6,7) = 32 + 57 - (17 + 45) = 27$$

$$\Delta f = f(3,2) - f(7,8) = 32 + 16 - (64 + 61) = -77$$

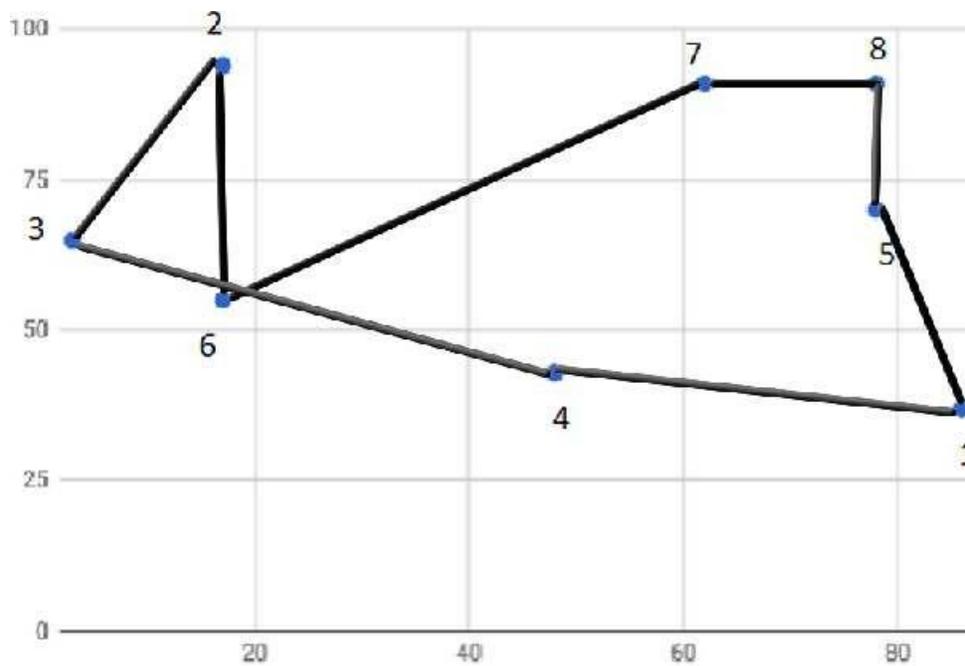
$$\Delta f = f(3,2) - f(8,1) = 32 + 54 - (79 + 89) = -82$$

$$\Delta f = f(2,6) - f(7,8) = 39 + 16 - (45 + 70) = -60$$

$$\Delta f = f(2,6) - f(8,1) = 39 + 54 - (61 + 71) = -39$$

$$\Delta f = f(6,7) - f(8,1) = 57 + 54 - (70 + 59) = -18$$

New tour:



Tour= (1,5,8,7,6,2,3,4)

$$F(x) = 33 + 21 + 16 + 57 + 39 + 32 + 50 + 38 = 286$$

3rd Iteration

$$\Delta f = f(1,5) - f(3,4) = 33 + 50 - (87 + 40) = -44$$

$$\Delta f = f(1,5) - f(2,3) = 33 + 32 - (89 + 75) = -99$$

$$\Delta f = f(1,5) - f(6,2) = 33 + 39 - (71 + 65) = -64$$

$$\Delta f = f(1,5) - f(7,6) = 33 + 57 - (59 + 62) = -34$$

$$\Delta f = f(1,5) - f(8,7) = 33 + 16 - (54 + 26) = -31$$

$$\Delta f = f(4,1) - f(2,3) = 38 + 32 - (59 + 87) = -76$$

$$\Delta f = f(4,1) - f(6,2) = 38 + 39 - (33 + 89) = -45$$

$$\Delta f = f(4,1) - f(7,6) = 38 + 57 - (50 + 71) = -26$$

$$\Delta f = f(4,1) - f(8,7) = 38 + 16 - (56 + 59) = -66$$

$$\Delta f = f(4,1) - f(5,8) = 38 + 21 - (40 + 54) = -35$$

$$\Delta f = f(3,4) - f(6,2) = 50 + 39 - (17 + 59) = 13$$

$$\Delta f = f(3,4) - f(7,6) = 50 + 57 - (64 + 33) = 10$$

$$\Delta f = f(3,4) - f(8,7) = 50 + 16 - (79 + 50) = -63$$

$$\Delta f = f(3,4) - f(5,8) = 50 + 21 - (75 + 56) = -60$$

$$\underline{\Delta f = f(2,3) - f(7,6) = 32 + 57 - (45 + 17) = 27}$$

$$\Delta f = f(2,3) - f(8,7) = 32 + 16 - (61 + 64) = -77$$

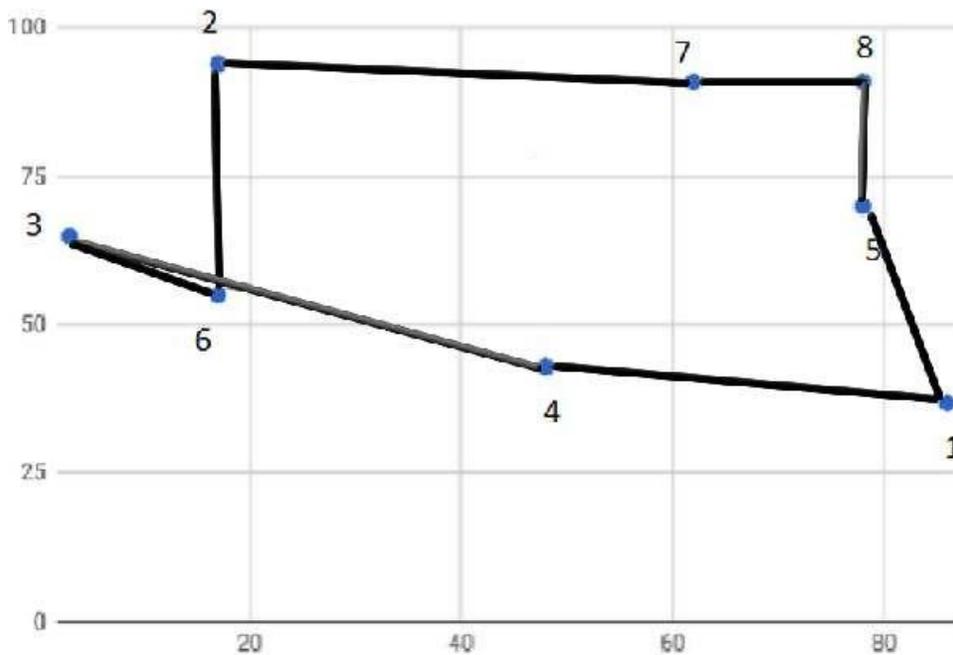
$$\Delta f = f(2,3) - f(5,8) = 32 + 21 - (65 + 79) = -91$$

$$\Delta f = f(6,2) - f(8,7) = 39 + 16 - (70 + 45) = -60$$

$$\Delta f = f(6,2) - f(5,8) = 39 + 21 - (62 + 61) = -63$$

$$\Delta f = f(7,6) - f(5,8) = 57 + 21 - (26 + 70) = -18$$

New tour= (1,5,8,7,2,6,3,4)



$$F(x) = 33 + 21 + 16 + 45 + 39 + 17 + 50 + 33 = 254$$

4th Iteration

$$\Delta f = f(1,5) - f(3,4) = 33 + 50 - (87 + 40) = -44$$

$$\Delta f = f(1,5) - f(6,3) = 33 + 17 - (71 + 75) = -96$$

$$\Delta f = f(1,5) - f(2,6) = 33 + 39 - (89 + 62) = -79$$

$$\Delta f = f(1,5) - f(7,2) = 33 + 45 - (59 + 65) = -46$$

$$\Delta f = f(1,5) - f(8,7) = 33 + 16 - (54 + 26) = -31$$

$$\Delta f = f(4,1) - f(6,3) = 38 + 17 - (33 + 87) = -65$$

$$\Delta f = f(4,1) - f(2,6) = 38 + 39 - (59 + 71) = -53$$

$$\Delta f = f(4,1) - f(7,2) = 38 + 45 - (50 + 89) = -56$$

$$\Delta f = f(4,1) - f(8,7) = 38 + 16 - (56 + 59) = -61$$

$$\Delta f = f(4,1) - f(5,8) = 38 + 21 - (40 + 89) = -70$$

$$\underline{\Delta f = f(3,4) - f(2,6) = 50 + 39 - (32 + 33) = 24}$$

$$\Delta f = f(3,4) - f(7,2) = 50 + 45 - (64 + 59) = -28$$

$$\Delta f = f(3,4) - f(8,7) = 50 + 16 - (79 + 50) = -63$$

$$\Delta f = f(3,4) - f(5,8) = 50 + 21 - (75 + 56) = -60$$

$$\Delta f = f(6,3) - f(7,2) = 17 + 45 - (57 + 32) = -27$$

$$\Delta f = f(6,3) - f(8,7) = 17 + 16 - (70 + 64) = -101$$

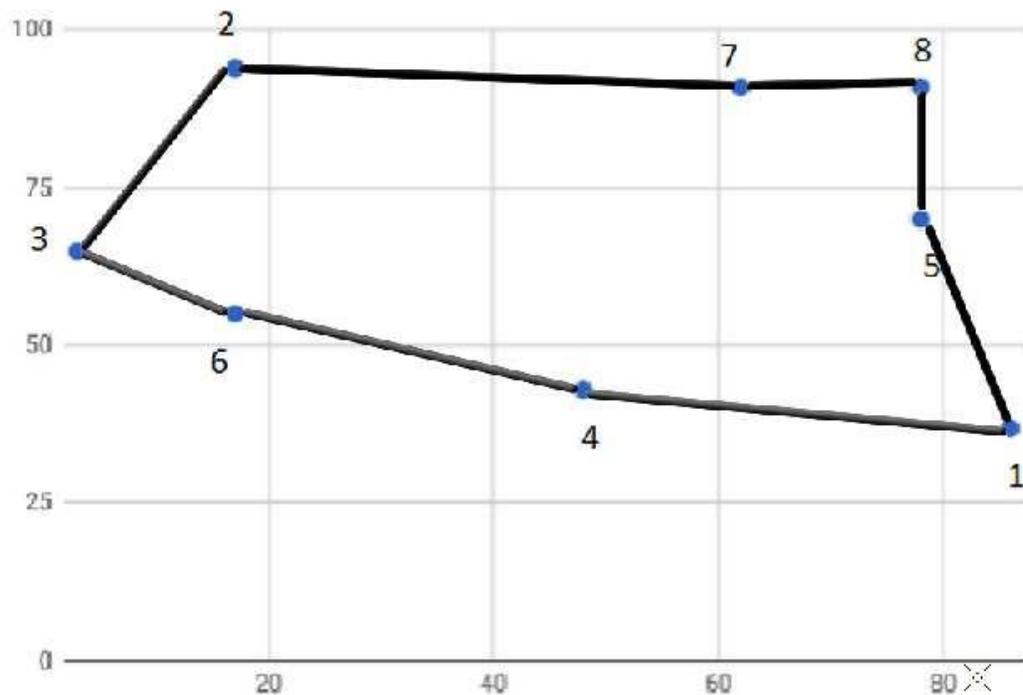
$$\Delta f = f(6,3) - f(5,8) = 17 + 21 - (62 + 79) = -103$$

$$\Delta f = f(2,6) - f(8,7) = 39 + 16 - (61 + 57) = -63$$

$$\Delta f = f(2,6) - f(5,8) = 39 + 21 - (65 + 70) = -75$$

$$\Delta f = f(7,2) - f(5,8) = 45 + 21 - (26 + 61) = -21$$

New tour= (1,5,8,7,2,3,6,4)



$$F(x) = 33 + 21 + 16 + 45 + 32 + 17 + 33 + 38 = 235$$

seeing the graphic, we can already tell that this is the optimal solution to the problem, this it is not necessary to do another iteration.

First Found strategy.

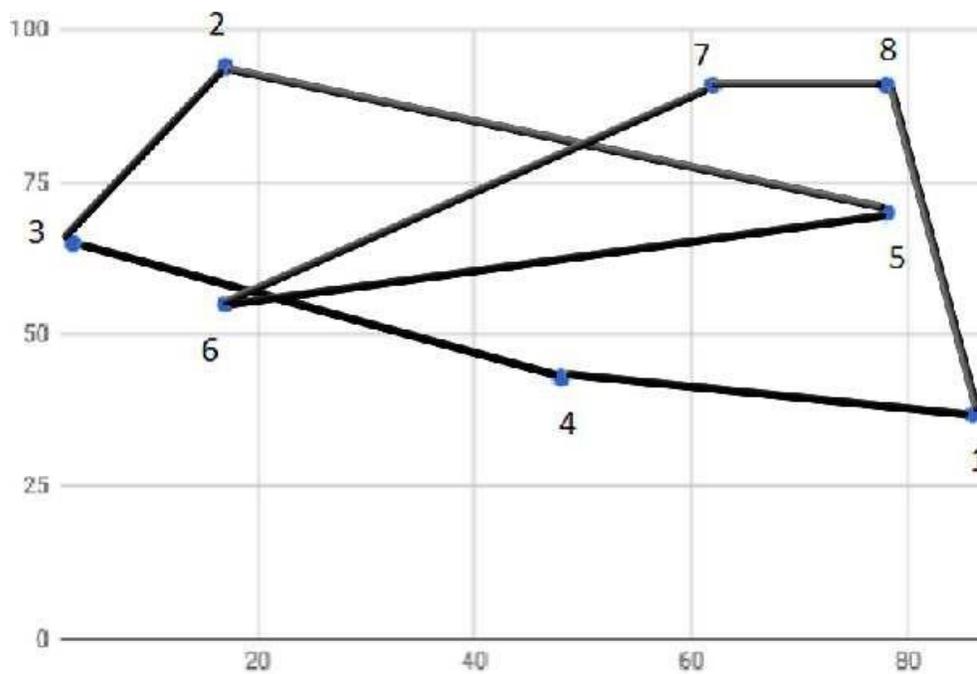
To use the first found method, we will select the first possible tour and reiterate until there are no more optimal tours.

1st Iteration

$$\Delta f = f(1,2) - f(3,4) = 89 + 50 - (87 + 59) = -7$$

$$\Delta f = f(1,2) - f(4,5) = 89 + 40 - (38 + 65) = 26$$

New tour= (1,4,3,2,5,6,7,8)



$$F(x) = 38 + 50 + 32 + 65 + 62 + 57 + 16 + 54 = 374$$

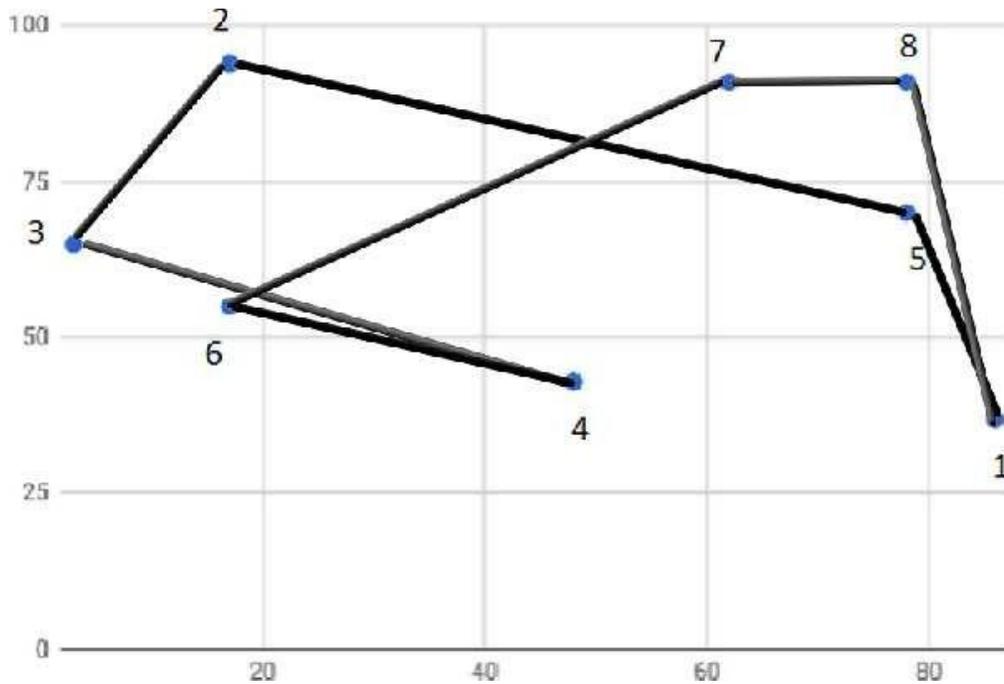
2nd Iteration

$$\Delta f = f(1,4) - f(3,2) = 38 + 32 - (87 + 59) = -76$$

$$\Delta f = f(1,4) - f(2,5) = 38 + 65 - (89 + 40) = -26$$

$$\Delta f = f(1,4) - f(5,6) = 38 + 62 - (33 + 33) = 34$$

New tour = (1,5,2,3,4,6,7,8)



$$F(x) = 33 + 65 + 32 + 50 + 33 + 57 + 16 + 54 = 340$$

3rd Iteration

$$\Delta f = f(1,5) - f(2,3) = 38 + 32 - (89 + 75) = -94$$

$$\Delta f = f(1,5) - f(3,4) = 38 + 50 - (87 + 40) = -39$$

$$\Delta f = f(1,5) - f(4,6) = 38 + 33 - (38 + 62) = -29$$

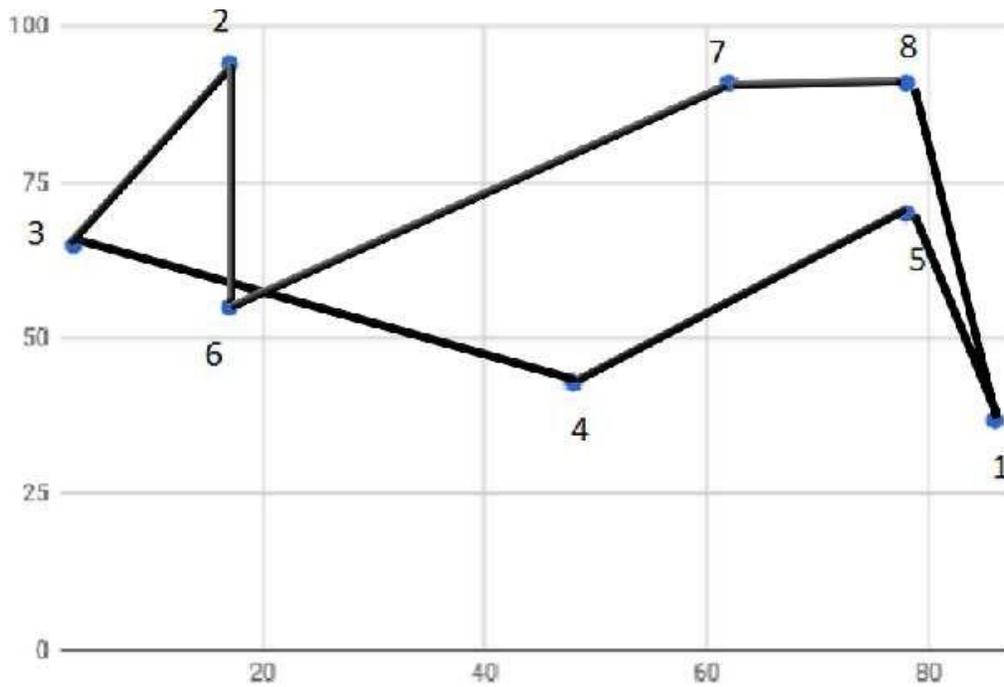
$$\Delta f = f(1,5) - f(6,7) = 38 + 57 - (71 + 26) = -5$$

$$\Delta f = f(1,5) - f(7,8) = 38 + 16 - (59 + 21) = -26$$

$$\Delta f = f(5,2) - f(3,4) = 65 + 50 - (75 + 59) = -19$$

$$\underline{\Delta f = f(5,2) - f(4,6) = 65 + 33 - (40 + 39) = 19}$$

New tour= (1,5,4,3,2,6,7,8)



$$F(x) = 33 + 40 + 50 + 32 + 39 + 57 + 16 + 54 = 321$$

4th Iteration

$$\Delta f = f(1,5) - f(3,2) = 33 + 32 - (87 + 65) = -87$$

$$\Delta f = f(1,5) - f(4,3) = 33 + 50 - (38 + 75) = -30$$

$$\Delta f = f(1,5) - f(2,6) = 33 + 39 - (89 + 62) = -79$$

$$\Delta f = f(1,5) - f(6,7) = 33 + 57 - (71 + 26) = -7$$

$$\Delta f = f(1,5) - f(7,8) = 33 + 16 - (59 + 21) = -31$$

$$\Delta f = f(5,4) - f(3,2) = 40 + 32 - (75 + 59) = -62$$

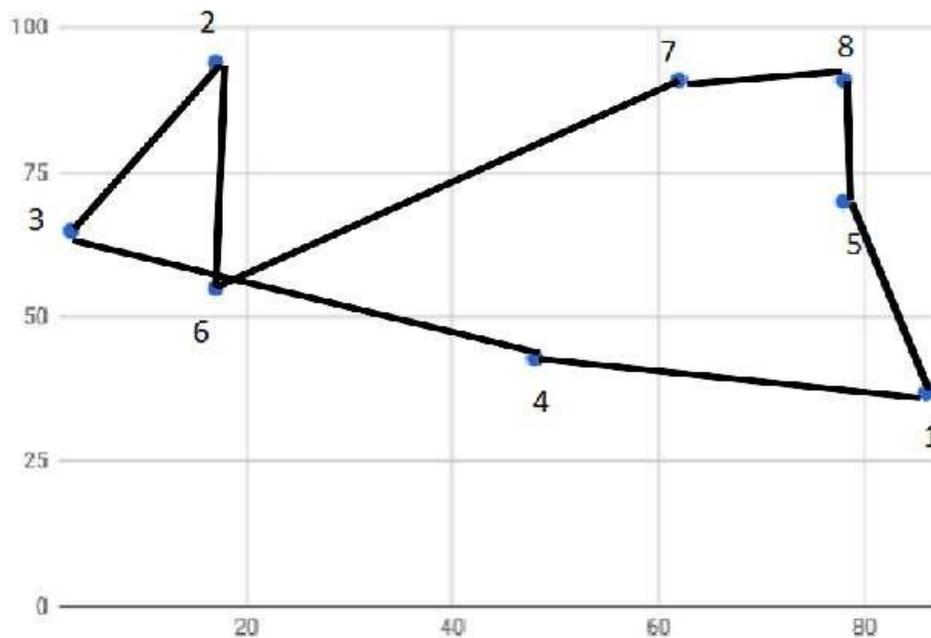
$$\Delta f = f(5,4) - f(2,6) = 40 + 39 - (65 + 33) = -19$$

$$\Delta f = f(5,4) - f(6,7) = 40 + 57 - (62 + 50) = -15$$

$$\Delta f = f(5,4) - f(7,8) = 40 + 16 - (26 + 56) = -26$$

$$\underline{\Delta f = f(5,4) - f(8,1) = 40 + 54 - (21 + 38) = 35}$$

New tour= (1,5,8,7,6,2,3,4)

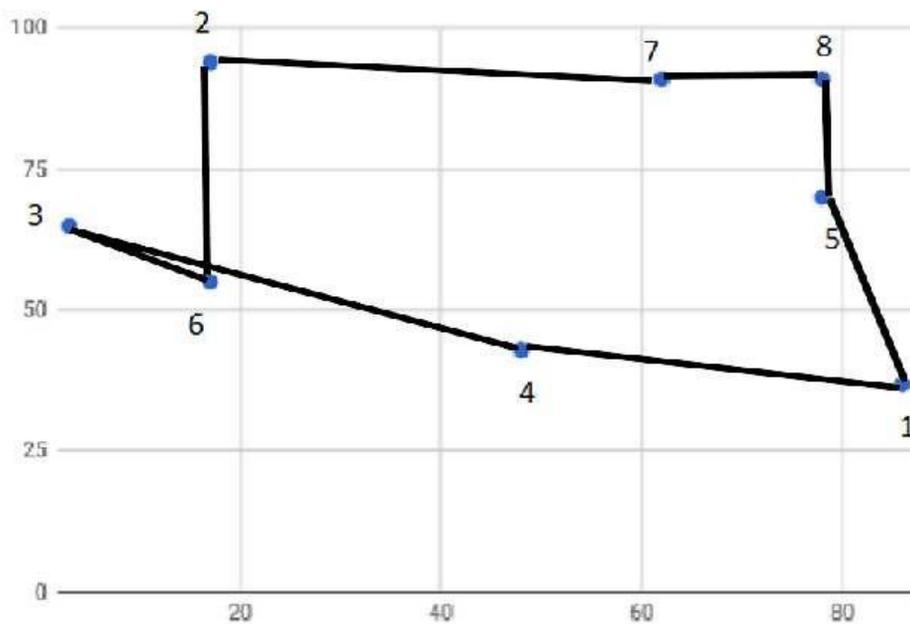


$$F(x) = 33 + 40 + 50 + 32 + 39 + 57 + 16 + 54 = 321$$

5th Iteration

$$\Delta f = f(2,3) - f(7,6) = 32 + 57 - (45 + 17) = 27$$

New tour= (1,5,8,7,2,6,3,4)



$$F(x) = 33 + 21 + 16 + 45 + 39 + 17 + 50 + 33 = 254$$

6th Iteration

$$\Delta f = f(1,5) - f(3,4) = 33 + 50 - (87 + 40) = -44$$

$$\Delta f = f(1,5) - f(6,3) = 33 + 17 - (71 + 75) = -96$$

$$\Delta f = f(1,5) - f(2,6) = 33 + 39 - (89 + 62) = -79$$

$$\Delta f = f(1,5) - f(7,2) = 33 + 45 - (59 + 65) = -46$$

$$\Delta f = f(1,5) - f(8,7) = 33 + 16 - (54 + 26) = -31$$

$$\Delta f = f(4,1) - f(6,3) = 38 + 17 - (33 + 87) = -65$$

$$\Delta f = f(4,1) - f(2,6) = 38 + 39 - (59 + 71) = -53$$

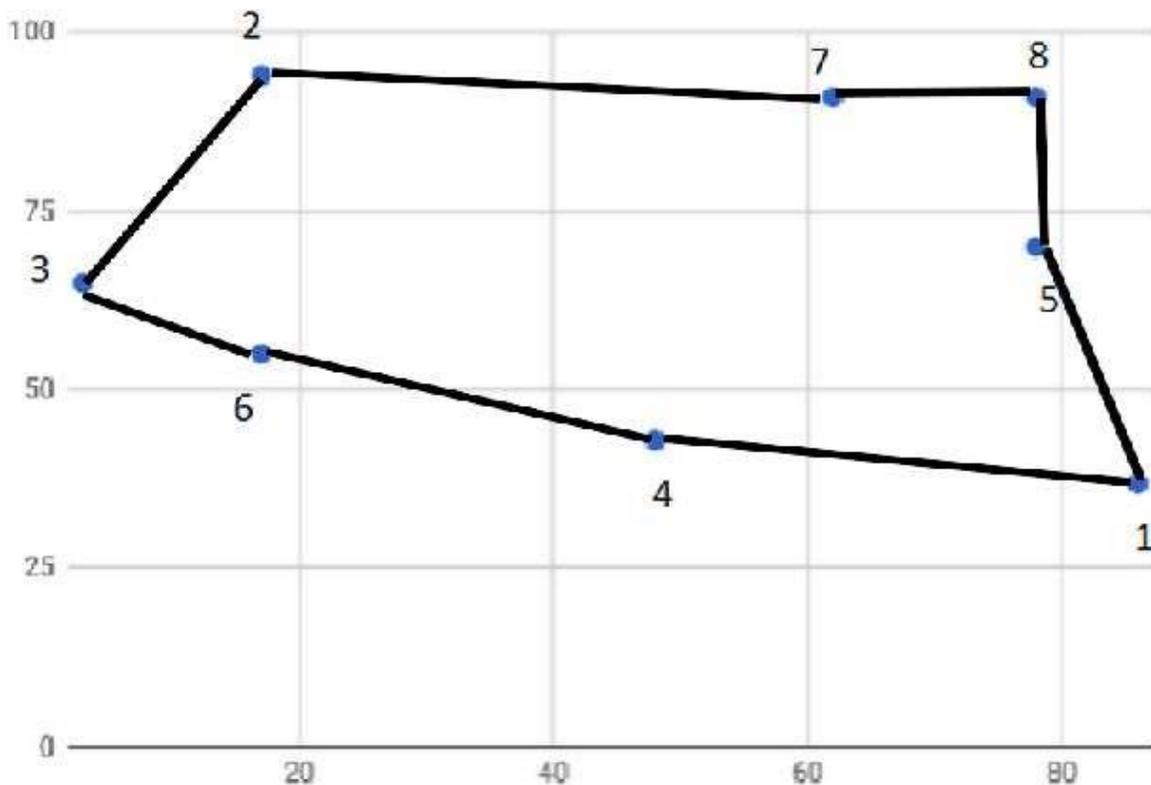
$$\Delta f = f(4,1) - f(7,2) = 38 + 45 - (50 + 89) = -56$$

$$\Delta f = f(4,1) - f(8,7) = 38 + 16 - (56 + 59) = -61$$

$$\Delta f = f(4,1) - f(5,8) = 38 + 21 - (40 + 89) = -70$$

$$\underline{\Delta f = f(3,4) - f(2,6) = 50 + 39 - (32 + 33) = 24}$$

New tour= (1,5,8,7,2,3,6,4)



looks like the best-found strategy's last iteration, we already know that this is the most optimal solution for the problem by looking at the graphical representation, so we don't need to do another iteration.