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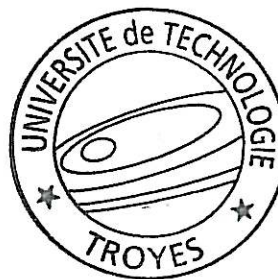Troyes

EU/MEeting 2008

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' Systèmes Industriels**
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ATTENDANCE CERTIFICATE
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TO WHOM IT MAY CONCERN:

We hereby certify the attendance of Irma Garcia to the ninth International Workshop EU/MEeting (EU/MEeting 2008) on Metaheuristics for Logistics and Vehicle Routing held in the University of Technology of Troyes, France, 23-24 October 2008.

Caroline Prodhon
Conference Chairman
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A Vehicle Routing Problem with flexibility in the delivery dates: a real case

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1 Introduction

We address a real problem proposed to the authors by a bakery company in Northern Spain. The company has to meet required known orders for a set of distribution centers over a week by a fleet of homogeneous vehicles. Each order has a predetermined number of pallets and a specific delivery day. Each trip finishes in the same day it started and the vehicles return to the depot at the end of the route. This problem shares some characteristics with the Periodic Routing Problem [1].

The way the problem is tackled by the company is by solving each day a classical VRP with those orders which require commodities that day. Nevertheless, the company perceived that the costs will be still improved.

In order to reduce the travel cost over the week, and taking into account that the commodities are sent to distributors, not to final customers, it is possible to consider a certain flexibility in the delivery date. Nevertheless, as they are dealing with perishable products, every order should be delivered in the original deadline or few days (one or two) before. Obviously, this will cause a stock cost, but it is controlled and acceptable for the company.

The proposed model is a variant of the two index flow formulation for Capacitated Vehicle Routing Problem (CVRP), described by Toth and Vigo [8], considering all the orders in the week and adding constraints regarding to the flexibility in the delivery date. To solve this model a method based on the GRASP metaheuristic is proposed.

2   The proposed model

The problem will be modeled using a complete graph. The company central depot and the distribution centers are the nodes of the graph. Let us introduce the following notation.

n = number of orders in a week
$G = (V, A)$ a complete graph
$V = \{v_0, v_1, v_2, \ldots, v_n\}$ nodes of the graph, v_0 is the depot
k = number of vehicles
Q = capacity of the vehicles
c_{ij} = cost(distance) from node i to node j.
d_i = demand at node i.
e_j = deadline of node j, that is, the day the node j needs the products
     $e_j \in \{1, 2, \ldots, 5\}$.
F = maximum number of days that an order could be served before its duedate.
     In the real application $F = 1$.

The decision variables

$$x_{ij} = \begin{cases} 1, & \text{if node } j \text{ is visited right after node } i \\ 0, & otherwise \end{cases}$$

In order to guarantee that the deadlines of all the nodes belonging to the same route do not differ in more than F days, the following constraint needs to be added: If r and s are nodes in the same route, then

$$|e_r - e_s| \leq F$$

To keep the model as a linear one, let us introduce a variable z_j for each node j, as a lower bound of the deadlines of the route. In this way, if a vehicle goes from node i to node j, then

$$z_j \leq z_i$$

This can be modeled as

$$z_j - z_i \leq M(1 - x_{ij})$$

for each $i, j \in V \backslash \{0\}$, with $M > 0$ a constant.

In addition, let w_j an upper bound of the deadlines of the route. If $x_{ij} = 1$ then

$$w_j \geq w_i$$

and we add the constraint

$$w_i - w_j \leq M(1 - x_{ij})$$

for each $i, j \in V \backslash \{0\}$. To guarantee that the deadlines of all the nodes belonging to the same route do not differ in more than F days we impose the following constraints

$$w_j - z_j \leq F$$

for each node j.

The mixed–integer linear model is as follows

$$\min \sum_{i \in V} \sum_{j \in V} c_{ij} x_{ij}$$

subject to

$$\sum_{i \in V} x_{ij} = 1, \qquad \forall j \in V \backslash \{0\} \tag{1}$$

$$\sum_{j \in V} x_{ij} = 1, \qquad \forall i \in V \backslash \{0\} \tag{2}$$

$$\sum_{i \in V} x_{i0} \leq k \tag{3}$$

$$\sum_{i \in V} x_{i0} = \sum_{j \in V} x_{0j} \tag{4}$$

$$u_i - u_j + Q x_{ij} \leq Q - d_j, \quad \forall i, j \in V \backslash \{0\}, i \neq j, \tag{5}$$
$$\text{such that } d_i + d_j \leq Q$$

$$d_i \leq u_i \leq Q, \qquad \forall i \in V \backslash \{0\} \tag{6}$$

$$z_j - z_i \leq M(1 - x_{ij}), \qquad \forall i, j \in V \backslash \{0\} \tag{7}$$

$$w_i - w_j \leq M(1 - x_{ij}), \qquad \forall i, j \in V \backslash \{0\} \tag{8}$$

$$w_j - z_j \leq F, \qquad \forall j \in V \backslash \{0\} \tag{9}$$

$$1 \leq z_j \leq e_j, \qquad \forall j \in V \backslash \{0\} \tag{10}$$

$$e_j \leq w_j \leq 5, \qquad \forall j \in V \backslash \{0\} \tag{11}$$

$$u_j, z_j, w_j \geq 0, \qquad \forall j \in V \backslash \{0\} \tag{12}$$

$$x_{ij} \in \{0, 1\}, \qquad \forall i, j \in V \tag{13}$$

Note that we have added $2n$ new continuous variables and $2n^2 + n$ constraints to the classical VRP.

Constraints (1) and (2) are the indegree and outdegree constraints, and impose that exactly one arc enters and leaves each node. Analogously, (3) and (4) impose the degree requirements for the depot; (5) and (6) are the alternative family of constraints equivalent to the capacity–cut constraints and (7) to (9) are the new constraints wich model the flexibility in the delivery date.

3   The solution approach

Experimental analyses were performed to validate the model. Several instances were randomly generated and Cplex v.9.0 was used to solve them for different values of F and n.

We observed that as n increased the solver becomes very time consuming, especially when $F = 2$ or 3. When $F = 4$ or 5 the model identifies that the problem is a VRP without additional constraints and it is solved faster.

The proposed model is obviously a generalization of CVRP, so we designed a solution approach based on a GRASP metaheuristic which is able to find near optimal solutions in reasonable computational time. GRASP [2] is an iterative procedure. Each iteration consists basically of two phases: construction and local search. The construction phase tries to build a feasible solution step by step, and its neighborhood is explored by a local search.

The construction phase is characterized by an adaptive greedy function, which estimates the gain of including a candidate element in a solution being created, and by a randomized selection from a restricted candidate list of elements with the best values of the greedy function.

The proposed method can be sketched as follows.

- **Constructive phase**

    - Built k clusters of nodes as follows:
        * Approach the problem by a GAP (as suggested by Fisher and Jaikumar [3]).
        * Solve this GAP using a modification of Martello and Toth algorithm [5].
    - For each cluster the route is designed using the GENI heuristic.

- **Local Search** Performs exchanges within routes (Or interchanges [6]) and/or between routes (Cross–interchanges, Taillard *et. al.* [7]).

We describe the constructive phase a detail.

Constructive phase

The constructive phase consists in designing k clusters, one for each vehicle, and solve the Traveling Salesman Problem for the nodes belonging to each cluster. The clustering phase follows the Fisher and Jaikumar idea of approaching the problem by a Generalized Assignment Problem (GAP) [3]. As we mentioned before, to solve the GAP we use a modified version of the algorithm of Martello and Toth [5].

Let a_{il} the cost to assign the node i to the route l, obtained by the Fisher and Jaikumar approach, the proposed modified algorithm is as follows.

$S = \emptyset$, $P = \{1, 2, \ldots, n\}$
Repeat
      Compute Δ_i, $\forall i \in P \backslash S$
      Compute $\Delta_{\min}$, $\Delta_{\max}$
      $L = \{i \in P \backslash S \ / \Delta_i \geq \alpha \Delta_{\max} - (1 - \alpha) \Delta_{\min}\}$
      Choose $i^* \in L$ randomly
      $S = S \cup \{i^*\}$ and assign i^* to the best route
Until $|S| = n$

    Where

$$\Delta_i = a_{il_{\min 1(i)}} - a_{il_{\min 2(i)}}$$

$$
\begin{aligned}
a_{il_{\min 1(i)}} &= \min\{a_{il} \text{ such that the assignment } i \text{ to } l \text{ is feasible} \\
&\qquad \text{for } l = 1, 2, \ldots, k\} \\
a_{il_{\min 2(i)}} &= \min\{a_{il}, l \neq l_{\min 1(i)}, \text{ such that the assignment } i \text{ to } l \text{ is feasible} \\
&\qquad \text{for } l = 1, 2, \ldots, k\}
\end{aligned}
$$

To insert a vertex into a route we use the Generalized Insertion heuristic, GENI [4].

4 Some Computational Results

We show some computational results, obtained on random instances. Nodes are randomly generated in the plane in the region $[-10, 10] \times [-10, 10]$ and the matrix cost is the rounded euclidean distance, the demands were generated between 1 to 25 to each node and the capacity of the vehicles was set as 100 and 250. The time is reported in seconds. We solved exactly with CPLEX v.9.0 in a Sun Fire V440 Ultra Sparc III, 1062 GHz and the GRASP was run in a PC, 1GHz, the stop criterium is 10 iterations without improvement.

Table 1: Computational Results

				CPLEX		GRASP	
n	k	Q	F	Cost	Time	Cost	Time
	3	100	1	157	1.84	157	0.201
10	2	100	2	136	4.61	136	0.130
	2	100	3	147	37.58	147	0.621
	2	100	4	132	53.00	132	3.245
	3	250	1	157	2.97	157	0.23
10	2	250	2	119	8.35	119	0.15
	2	250	3	116	46.35	116	0.72
	1	250	4	78	20.90	78	1.23

As we can see our GRASP is able to find optimal solutions in very short computational time for this small instances. For larger instances ($n = 15, 18$), CPLEX is not able to finish in a moderate time, even using some hours, and the upper bound obtained is worst than solutions obtained by our GRASP that employed only few seconds. These tests were performed to validate that the commercial software has troubles to solve even small instances of the problem while the GRASP only needs a few seconds to get the optimal solution. Tests using instances with more nodes are in progress.

Acknowledgment

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