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A new formulation for the conditional p-median and p-center problems

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Abstract

In this paper we discuss the conditional p-median and p-center problems on a network. Demand nodes are served by the closest facility whether existing or new. The formulation presented in this paper provided better results than those obtained by the best known formulation.

Keywords: p-median; p-center; Conditional; Location; Network

1. Introduction

The conditional location problem is to locate p new facilities to serve a set of demand points given that q facilities are already located. When $q = 0$, the problem is unconditional. In the conditional p-median and p-center problems, once the new p locations are determined, a demand can be served either by one of the existing or by one of the new facilities whichever is the closest facility to the demand. Note that the conditional p-max-cover problem is trivial because all nodes covered by existing facilities are removed from the problem.

The p-median and p-center problems are probably the most commonly used models in location analysis [5]. Applications to the conditional p-median or p-center problems are very common. Every application to the p-median or p-center problems turns to the conditional model when there are existing facilities in the area. For example, if one wishes to locate p warehouses in an area, it is an unconditional p-median problem. However, when q warehouses already exist in the area and we need to add p new warehouses, it is the conditional p-median problem.

References [8,9] are the earliest references on conditional location problems. The problem studied in both [8] and [9] is the conditional 1-center problem (i.e. $p = 1$, and $q \geq 1$). The conditional location problem with $p \geq 1$ new facilities on the plane was considered in [3,4]: for the p-median and p-center problems in [3] and for the p-center problem in [4]. The method proposed in [6,7] is applicable to both planar and network topologies. Drezner [6] suggests to solve the conditional p-center problem by an algorithm that requires solving $O(\log n)$ unconditional p-center problems (n is the number of demand nodes). A heuristic that requires solving several unconditional p-median problems is discussed in [7] for the conditional p-median problem. Berman and Simchi-Levi [2] suggest to solve the conditional p-median and p-center problems on a network by an algorithm that requires one-time solution of an unconditional $(p + 1)$-median or $(p + 1)$-center problem (we briefly describe the algorithm in the next section).

In this paper, we discuss a very simple algorithm that solves both the conditional p-median and p-center problems on a network. The algorithm requires one-time solution of an unconditional p-median or p-center problem using an appropriate shortest distance matrix. As can be seen from our computational experience in the last section, conditional p-median problems by CPLEX provided considerable better results than those obtained by the formulation proposed in [2].

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2. Analysis

Consider a network $G = (N, L)$ where N is the set of nodes, $|N| = n$ and L is the set of links. Let w_i be a non-negative number that represents the demand weight at node $i \in N$. Let d_{xy} be the shortest distance between any $x, y \in G$.

Suppose that there is a set Q ($|Q| = q$) of existing facilities. Let $Y = (Y_1, \ldots, Y_q)$ and $X = (X_1, X_2, \ldots, X_p)$ be vectors of size q and p respectively, where Y_i is the location of existing facility i and X_i is the location of new facility i. Without any loss of generality we do not need to assume that $Y_i \in N$. The conditional p-median location problem is to minimize

$$f(x) = \sum_{i=1}^{n} w_i \min\{d(X, i), d(Y, i)\} \tag{1}$$

and the conditional p-center problem is to minimize

$$g(x) = \max_{i=1,\ldots,n} \min\{d(X, i), d(Y, i)\} \tag{2}$$

where $d(X, i)$ (or $(d(Y, i))$) is the shortest distance from the closest facility in X (or Y) to node i.

It is known that for the unconditional p-median problem the dominant set of locations is N and for the p-center problem the dominant set of locations is the set of local centers [8, 10]. Let D be a distance matrix with rows corresponding to demands and columns corresponding to potential locations. For the unconditional p-median location problem the columns of D correspond to the set of nodes and for the p-center problem the columns of D correspond to the set of local centers C.

We first briefly describe the algorithm suggested in Berman and Simchi-Levi [2]. The idea is to create a new potential location representing all existing facilities. If a demand point is utilizing the services of an existing facility, it will use the services of the closest existing facility. Therefore, the distance between a demand point and the new location is the minimum distance to all existing facilities. To force the creation of a facility at the new location, a new demand point is created with a distance of zero to the new potential location and a large distance to all other potential locations. The new distance matrix $\hat{\hat{D}}$ is constructed by adding a new location a_0 (a new column) to D that represents the Q existing locations and a new demand point v_0 with an arbitrary positive weight. For each regular demand point (node) i, $d(i, a_0) = \min_{k\in Q}\{d_{ik}\}$ and $d(v_0, a_0) = 0$. For each regular potential location (node) j, $d(v_0, j) = M$ (M is a large number). Again the nodes in Q and in potential locations Q are removed. The program is solved by solving the unconditional $p + 1$ median (or center) problem using $\hat{\hat{D}}$.

We suggest to solve both conditional problems by defining a modified shortest distance matrix $\hat{D}$:

$$\hat{D}_{ij} = \min\left\{d_{ij}, \min_{k\in Q}\{d_{ik}\}\right\} \quad \forall i \in N,\ j \in N(\text{median}),\ j \in C(\text{center})$$

Note that $\hat{D}$ is not symmetric even when D is symmetric.

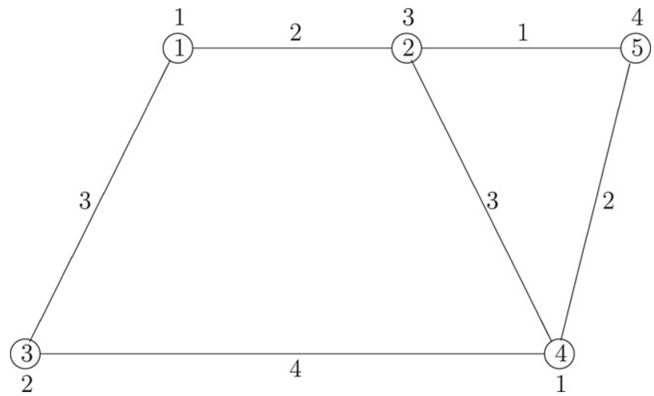

Fig. 1. A 5-node network.

Table 1

The shortest distance matrix $\hat{D}$

Demand nodes	Potential location		
	1	4	5
1	0	2	2
4	3	0	2
5	1	1	0

The unconditional p-median or p-center problem using the appropriate $\hat{D}$ solves the conditional p-median or p-center problem. This is so since if the shortest distance from node i to the new p facilities is larger than $\min_{k\in Q}\{d_{ik}\}$, then the shortest distance to the existing q facilities is utilized. Notice that the size of $\hat{D}$ is $n \times n$ and $n \times |C|$ for the conditional p-median and p-center, respectively. The appropriate shortest distance used in [2] is $(n + 1) \times n$ and $(n + 1) \times |C|$ for the conditional p-median and p-center, respectively. If $Y_i \in N \quad i = 1, \ldots, q$, then the size of the distance matrix is $(n - q) \times (n - q)$ and $(n - q) \times (|C| - q)$ for the conditional p-median and p-center problems, respectively. It is $(n + 1 - q) \times (n - q)$ and $(n+1-q) \times (|C|-q)$ for the p-median and p-center problem, respectively in [2]. In our proposed approach the nodes in Q and potential locations in Q are removed.

Our approach is simpler than the approach by Berman and Simchi-Levi [2]. Rather than creating a new location for an artificial facility and force the algorithm to locate a new facility there by creating an artificial demand point, we just modify the distance matrix.

3. Example

To demonstrate the algorithm we consider the 5-node network depicted in Fig. 1 where the numbers next to the links are lengths and the numbers next to the nodes are weights.

We solve the 1-median problem. Suppose that $Q = \{2, 3\}$ and $p = 1$. It is easy to verify that $\hat{D}$ and $\hat{\hat{D}}$ are the distance matrix shown in Tables 1 and 2 (nodes 2 and 3 are removed). The optimal new location using $\hat{D}$ is node 5 with an objective function value of $(2)(1) + (2)(1) + (0)(4) = 4$ (the optimal 2-median problem using $\hat{\hat{D}}$ is $\{5, a_0\}$).

Table 2
The shortest distance matrix $\hat{\hat{D}}$

Demand nodes	Potential location			
	1	4	5	a_0
1	0	5	3	2
4	5	0	2	3
5	3	2	0	1
v_0	M	M	M	0

Table 3
Results by CPLEX

n	p	Opt.	[2][a]	New[a]
100	5	5 819	1.12	1.64
100	10	4 093	1.20	1.73
100	10	4 250	1.51	2.31
100	20	3 034	0.87	1.28
100	33	1 355	0.90	1.26
200	5	7 824	71.18	62.96
200	10	5 631	5.42	5.31
200	20	4 445	4.50	4.46
200	40	2 734	4.26	4.25
200	67	1 255	4.02	3.91
300	5	7 696	51.05	35.05
300	10	6 634	82.70	90.71
300	30	4 374	12.75	12.35
300	60	2 968	14.61	15.24
300	100	1 729	11.84	11.86
400	5	8 162	1221.44	1094.01
400	10	6 999	742.42	577.35
400	40	4 809	41.44	42.31
400	80	2 845	27.22	28.13
400	133	1 789	26.74	27.39
500	5	9 138	60.06	57.56
500	10	8 579	1113.70	645.17
500	50	4 619	58.50	57.50
500	100	2 961	50.70	52.44
500	167	1 828	48.95	48.05
600	5	9 917	5458.86	3674.21
600	10	*	*	*
600	60	4 498	102.58	99.22
600	120	3 033	82.49	80.55
600	200	1 989	74.47	77.35
700	5	10 086	*	2449.34
700	10	*	*	*
700	70	4 700	154.18	157.55
700	140	*	*	*
Average[b]			317.72	232.44

* No optimal solution found within 2 h.
[a] Time in seconds.
[b] Average for problems solved by both.

By the proposed method the example problem is converted to an unconditional 1-median problem while by the method in [2] the example problem is converted to an unconditional 2-median problem.

4. Computational experiments

We solved the Beasley (1990) p-median problems [1] by CPLEX using the two formulations. For each problem we assumed that the first ten nodes of the network are existing facilities and p new facilities need to be located. The results are summarized in Table 3. None of the problems with $n \geq 800$ nodes could be solved due to lack of memory. Of the 34 problems with $n \leq 700$ nodes, the formulation in [2] solved 30 problems to optimality and the new approach solved 31 problems. The new approach was about 27% faster. When the run time is short, the new approach may sometimes take a bit longer. However, when run time is long the new approach is significantly faster. The new approach is superior when it counts the most — for larger problems.

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