

Brief Announcement: New Mechanisms for Pairwise Kidney Exchange

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Abstract. In this paper, we consider the *pairwise kidney exchange game*. Ashlagi et al. [1] present a 2-approximation randomized truthful mechanism for this problem. We note that the variance of the utility of an agent in this mechanism may be as large as $\Omega(n^2)$, which is not desirable in a real application. Here, we resolve this issue by providing a 2-approximation randomized truthful mechanism in which the variance of the utility of each agent is at most $2 + \epsilon$. Later, we derandomize our mechanism and provide a *deterministic* mechanism such that, if an agent deviates from the mechanism, she does not gain more than $2\lceil \log_2 m \rceil$.

1 Introduction

Kidney transplant is the only treatment for several types of kidney diseases. Since people have two kidneys and can survive with only one kidney, they can potentially donate one of their kidneys. It may be the case that a patient finds a family member or a friend willing to donate her kidney. Nevertheless, at times the kidney's donor is not compatible with the patient. Consider two incompatible patient-donor pairs. If the donor of the first pair is compatible with the patient of the second pair and vice-versa, we can efficiently serve both patients without affecting the donors.

To make the pool of donor-patient pairs larger, hospitals combine their lists of pairs to one big pool, trying to increase the number of treated patients by exchanging pairs from different hospitals. This process is managed by some national supervisor. A centralized mechanism can look at all of the hospitals together and increase the total number of kidney exchanges. The problem is that each hospital is interested in increasing the number of its own served patients. Thus, the hospital may not report some patient-donors pairs, namely, the hospital may report a partial list. This partial list is then matched by the national supervisors. Undisclosed set of pairs are matched by the hospitals locally, without the knowledge of the supervisor. This may have a negative effect on the number of served patients.

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Notations and Definitions. In a kidney exchange game we have a graph G , and each agent owns a disjoint set of vertices of G . A mechanism for this game receives the reported set of vertices from every agent. After the vertices are reported, the mechanism chooses a matching on the induced subgraph of the reported vertices. After this global run, each agent matches her unmatched vertices, including her undisclosed vertices, privately.

In this game, the utility of each agent is the expected number of her matched vertices and the *social welfare* of a mechanism is the size of the output matching. A kidney exchange mechanism is *truthful* if no agent gains more by reporting a partial subset of her vertex. A kidney exchange mechanism F is α -approximation if for every graph G the number of matched vertices in the maximum matching of G is at most α times the expected number of matched vertices by F in G .

Related Work. Ashlagi et al. [1] provide a randomized 2-approximation truthful mechanism for the multi-agent kidney exchange game. Moreover, they show that there is no truthful mechanism with an approximation ratio better than $8/7$. They also introduce a deterministic 2-approximation truthful mechanism for two player kidney exchange game. However, they conjectured that there is no deterministic constant-approximation truthful mechanism for the multi-agent kidney exchange game, even with three agents.

Our Results. In this paper, first, we show that the variance of the utility of an agent in the mechanism proposed by Ashlagi et al. may be as large as $\Omega(n^2)$, where n is the number of vertices. The variance of the utility can be interpreted as the risk of the agent caused by the randomness in the mechanism. Indeed, in a real application agents prefer to take less risk for the same expected utility. In this paper, we provide a tool to lower the variance of the utility of each agent in a kidney exchange mechanism, while keeping the expected utility of each agent the same. We used this tool to provide a 2-approximation randomized truthful mechanism in which the variance of the utility of each agent is at most $2 + \epsilon$.

Theorem 1. *There exists a truthful 2-approximation mechanism for multi-agent kidney exchange such that the variance of the utility of each agent is at most $2 + \epsilon$, where ϵ is an arbitrary small constant.*

Interestingly, we could apply our technique to design a 2-approximation *deterministic* mechanism such that if an agent deviates from the mechanism, she does not gain more than $2\lceil \log_2 m \rceil$. We call such a mechanism *almost truthful*. Indeed, in a practical scenario an almost truthful mechanism is likely to imply a truthful mechanism. To the best of our knowledge this is the first deterministic mechanism for the multi-agent kidney exchange game.

Theorem 2. *There exists an almost truthful deterministic 2-approximation mechanism for multi-agent kidney exchange.*

Reference

1. Ashlagi, I., Fischer, F., Kash, I.A., Procaccia, A.D.: Mix and match: A strategyproof mechanism for multi-hospital kidney exchange. *Games and Economic Behavior* (2013)