

# Overview on kidney exchange programs

Ana Viana et al

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- Kidney failure: some figures.
- The past: deceased and living donor transplants.
- The present: Kidney exchange programmes.
  - Current state of practice.
  - Current state of research.
- The future: multi-country kidney exchange programmes.

- 10% of the Portuguese population suffers from chronic kidney disease.
- Over 12 000 people with end stage kidney disease (ESKD); 24 000 people expected in 2025<sup>1</sup>.
- It is the 9<sup>th</sup> leading cause of death each year in the United States (more than breast or prostate cancer).
- Has high economic impact on national health services (NHS)
  - in the UK, the cost of treating ESKD in 2010 was estimated to be 1–2% of the total NHS budget although ESKD patients comprise of only 0.05% of the total population.

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# The past

Two treatment options:

- Dialysis
- Transplantation
  - Deceased donors
    - Portugal listed as 2<sup>nd</sup> country in the world in number of transplants.
  - Living donors (spouse, sibling, ...)
    - In 2010 there were 51 living donor transplants out of a total of 573 in Portugal;
    - Several potential transplants were not performed due to incompatibility between patient and donor.



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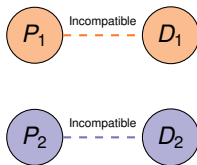
- Blood incompatibility:

Donor	Patient			
	O	A	B	AB
O	V	V	V	V
A	X	V	X	V
B	X	X	V	V
AB	X	X	X	V

- Immunological incompatibility.

# Living donor kidney transplants

The past



Transplants could not be done.

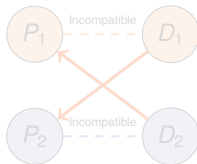
# The present

## Kidney exchange programmes

## Current state of practice

# Kidney exchange programmes

Many countries set new policies that allow exchange of organs between incompatible pairs - **Kidney exchange programmes**.

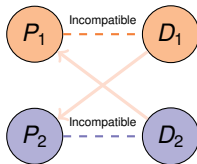


2-way kidney exchange

Two transplants are now possible.

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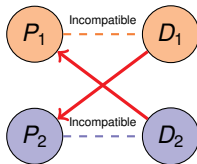
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# Kidney exchange programmes

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2-way kidney exchange

Two transplants are now possible.

# Kidney exchange programmes

**Objective:** Maximise number of transplants AND reduce illegal donation.

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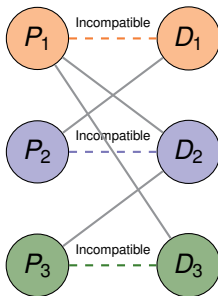
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# Kidney exchange programmes

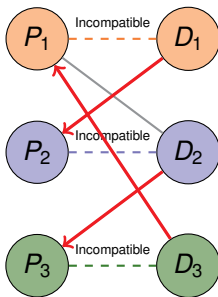
3-way exchange

The idea can be extended to more pairs.



# Kidney exchange programmes

3-way exchange

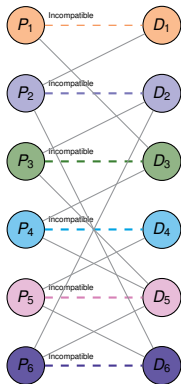


3-way kidney exchange

Three transplants are possible if we allow at most three pairs in an exchange.

# Kidney exchange programmes

$\infty$ -way exchange

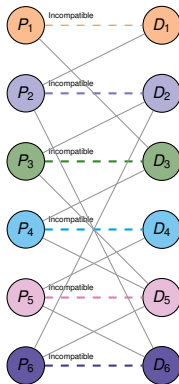


$\infty$ -way kidney exchange

How many transplants?

# Kidney exchange programmes

$\infty$ -way exchange



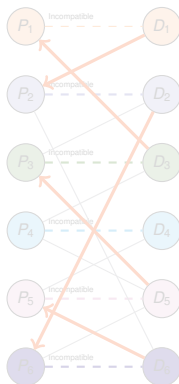
$\infty$ -way kidney exchange

How many transplants?

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$\infty$ -way exchange

At most **5** transplants:  $1 \rightarrow 2 \rightarrow 6 \rightarrow 5 \rightarrow 3 \rightarrow 1$

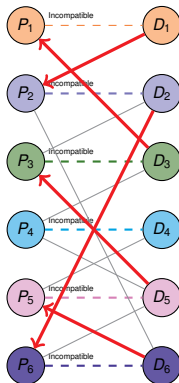


$\infty$ -way kidney exchange

# Kidney exchange programmes

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# Kidney exchange programmes

A need for bounded cycle size

But...

– because all transplants in a cycle must be done simultaneously,  
in practice the maximum number of pairs involved in an exchange  
must be bounded.

- Logistic/personnel issues constrain the number of such simultaneous operations.

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A need for bounded cycle size

Besides...

- final compatibility tests may detect **new incompatibilities**
- if pairs X and Y in a cycle are found to be incompatible, all transplants in the cycle involving X and Y have to be cancelled:  
the bigger the cycle the more pairs are affected.

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# Kidney exchange programmes

Problem extensions

- **Altruistic** donors
- Multiple donors associated to one patient
- Compatible pairs

It is now time for...

Optimisation

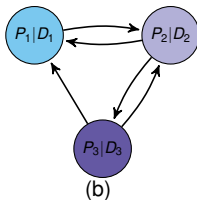
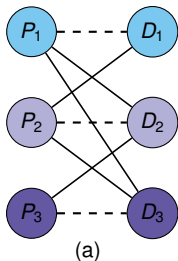


## Optimisation problem at hand

Given a pool of  $N$  incompatible Patient-Donor pairs, find the maximum number of kidney exchanges (transplants) that involve cycles of size at most  $K$ .

# Kidney exchange problem

**Pre-processing:** – transform the bipartite graph of compatibilities into a directed graph in which vertices represent incompatible patient-donor pairs and arcs between vertices represent compatibilities.



Bipartite versus Directed graph

A cycle with  $k$  nodes in the directed graph corresponds to a  $k$ -exchange.

Let  $G(V, A)$  be a directed graph with:

- $V$  – the set of vertices consisting of all incompatible patient-donor pairs;
- $A$  – the set of arcs for designating compatibilities between the vertices.

Two vertices  $i, j \in V$  are connected by arc  $(i, j)$  if the patient in pair  $j$  is compatible with the donor in pair  $i$ <sup>2</sup>.

**Definition:** The **Kidney Exchange Problem** can be defined as follows:

*Find a maximum weight packing of vertex-disjoint cycles having length at most  $k$ .*

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For:

- $k = 2$  – the problem reduces to finding a maximum matching which can be solved efficiently (Edmonds 1965);
- $k = \infty$  – the problem can be formulated as an assignment problem and solved efficiently by the Hungarian algorithm;
- $k \geq 3$  – *NP*-hard.

# Integer Programming Formulations

# Integer Programming Formulations

- Two Integer Programming models have been presented in (Abraham et al)<sup>3</sup>
  - Edge formulation;
  - Cycle formulation.
- None of the above mentioned formulations is compact: the number of constraints or variables grows exponentially with  $k$  or  $N$ .
- **New contribution:** In (Constantino et al)<sup>4</sup> we propose an Integer Programming formulation whose number of variables and constraints does not depend on  $k$  and grows polynomially with  $N$ .

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For each cycle  $c$  of length less or equal to  $k$  in the graph:

$$z_c = \begin{cases} 1 & \text{if cycle } c \text{ is selected for the exchange,} \\ 0 & \text{otherwise.} \end{cases}$$

$$\text{Maximize} \quad \sum_{c \in \mathcal{C}(k)} w_c z_c \quad (1a)$$

$$\text{Subject to:} \quad \sum_{c: i \in c} z_c \leq 1 \quad \forall i \in V \quad (1b)$$

$$z_c \in \{0, 1\} \quad \forall c \in \mathcal{C}(k). \quad (1c)$$

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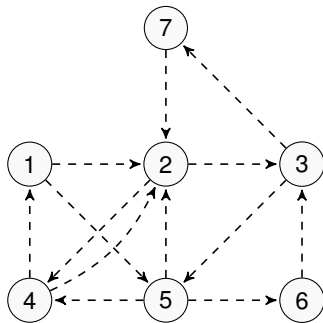
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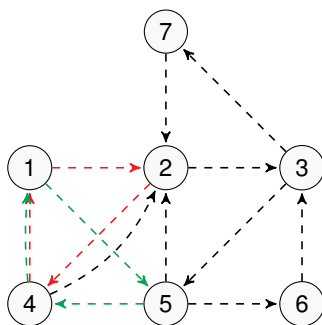


Base graph



# Kidney exchange problem

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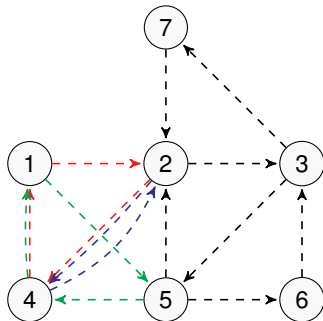


Base graph

$C_1, C_2$

# Kidney exchange problem

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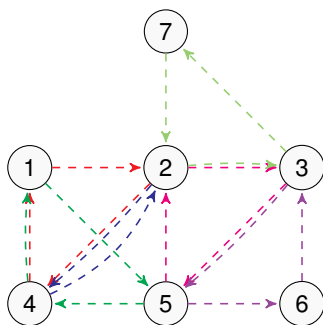


Base graph

$C_1, C_2, C_3$

# Kidney exchange problem

Example

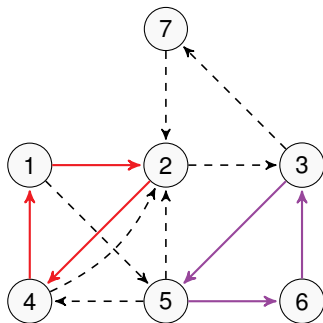


Base graph

$C_1, C_2, C_3, C_4, C_5, C_6$

# Kidney exchange problem

Example



(One) optimal solution



- Programmes are now set in several countries, e.g.:
  - Portugal\*, South Korea, USA\*, Switzerland, Turkey, Romania, Netherlands, UK\*, Canada, Australia, New Zealand, Spain.

\* – These countries use Integer Programming models to solve the underlying optimisation problem.

Optimisation tools developed under **project KEP<sup>5</sup>** are used by the **National Authority for Transplantation**.

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<sup>5</sup>KEP - New models for enhancing the kidney transplantation process.

## Primeiro duplo transplante renal cruzado em dadores vivos

Paula Costa/Paulo José Oliveira - RTP

16 Abr. 2013, 23:00 / atualizado em 19 Abr. 2013, 11:15 | País



## Seis cirurgias num só dia permitem primeiro transplante renal triplo em Portugal

Paula Rebelo, Rui César, Marcelo Sá Carvalho - RTP

19 Set. 2015, 14:02 / atualizado em 19 Set. 2015, 15:14 | País

A cirurgia, à  
Coimbra, em  
horas.



# Kidney exchange programmes

Web platform used by IPST

[KEP](#) [Dadores](#) [Pacientes](#) [Inserir](#) [Checklist](#) [Compatibilidade](#) [Parâmetros](#) [Resultados](#) [logout](#)

Ciclo máximo:

Caminho máximo:

Número de soluções:

[submeter](#)



**COMPETE**  
PROGRAMA OPERACIONAL FACTORES DE COMPETITIVIDADE



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**FCT**  
Fundação para a Ciência e a Tecnologia  
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INESC TEC 2014

# The present

Current state of research

The models that are currently used in practice consider that **data is certain** which is **not true**:

- Incompatibilities may be detected between pair matching and actual transplantation (**arc failure**).
- Pairs may dropout of the program (**node failure**).
- Patients and/or donors may be physically unfit when the operation is scheduled (**node failure**).
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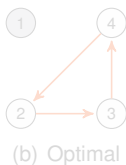
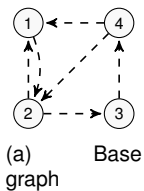
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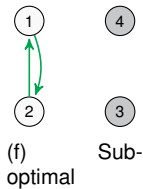
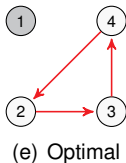
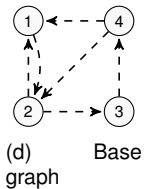
# Kidney exchange programmes

Arc/node failure: an example



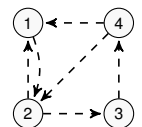
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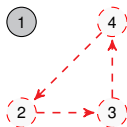


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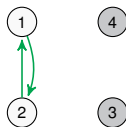
Arc/node failure: an example



(g) Base graph



(h) Optimal



(i) Sub-optimal



## How have we handled data uncertainty so far?

- Assigning probabilities to node and arc failure
  - Maximise expected number of transplants.
- Robust optimisation
  - Maximise the number of pairs selected in both the initial and the final solution, given a specific scenario.

## Plus:

- Recourse policies.
  - Reconstruction policies that can be implemented in a solution if one/some of its nodes and/or arcs fail.

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  - Maximise the expected number of transplants.
  - Robust optimisation: maximise the number of pairs selected in both the initial and the final solution, in the worst case.
- Backarcs recourse
- Subset-recourse (S.O.)
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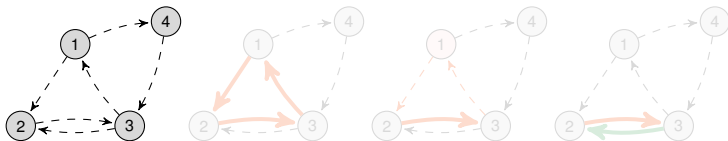


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# Kidney exchange programmes

Examples of recourse policies

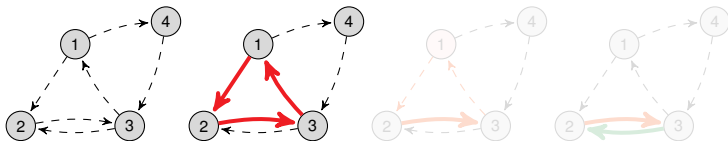
- Backarcs recourse



# Kidney exchange programmes

Examples of recourse policies

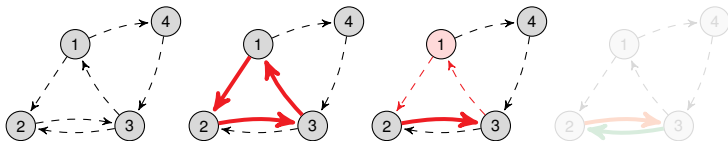
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# Kidney exchange programmes

Examples of recourse policies

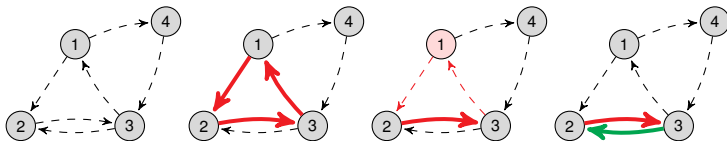
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# Kidney exchange programmes

Examples of recourse policies

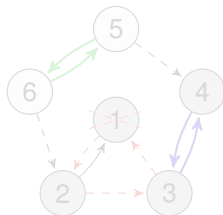
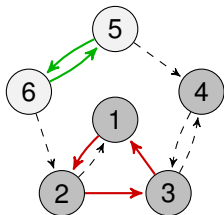
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# Kidney exchange programmes

Examples of recourse policies

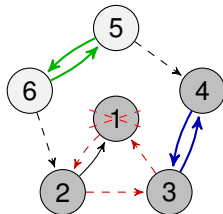
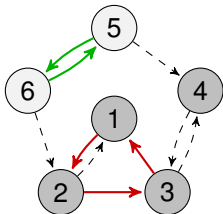
- Subset recourse
  - considers the possibility of involving in the rearrangement vertices not enclosed in the cycle.



# Kidney exchange programmes

Examples of recourse policies

- Subset recourse
  - considers the possibility of involving in the rearrangement vertices not enclosed in the cycle.



Obtaining optimal results for these problems within reasonable computational time requests for adequate formulations and algorithms.

- Klimentova, Pedroso, Viana. “Maximising expectation of the number of transplants in kidney exchange programmes”. Accepted for publication at Computers & OR.
- Glorie et al. “Robust Models for the Kidney Exchange Problem.” Submitted to Operations Research.



# The future

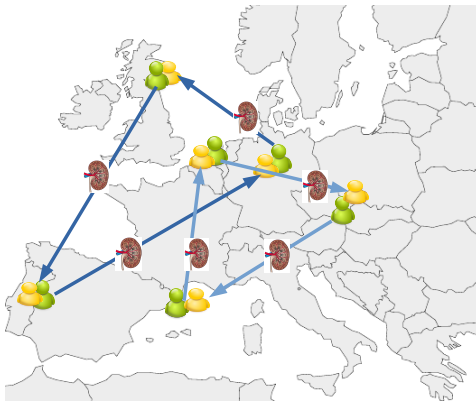
Multi-country kidney exchange programmes

**Project mKEP** – Models and optimisation algorithms for multicountry kidney exchange programs.

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# Multi-country KEP



**One concern:** give countries the incentive to participate fully, in order to achieve the gains that kidney exchange on a large scale makes possible.

$$\begin{aligned} \text{maximize } Z^* &= \sum_{c \in \mathcal{C}} w_c x_c \\ \text{subject to } \sum_{c: i \in c} x_c &\leq 1, & \forall i \in \mathcal{V} \\ \sum_{c \in \mathcal{C}} w_{cp} x_c &\geq D_p, & \forall p \in \mathcal{P} \\ x_c &\in \{0, 1\}, & \forall c \in \mathcal{C} \end{aligned}$$

Individual rationality (IR) constraints

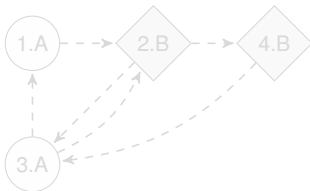
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**Individual rationality (IR) constraints**

**Our vision:** IR constraints attack part of the problem but we think there is more...

- New challenges:

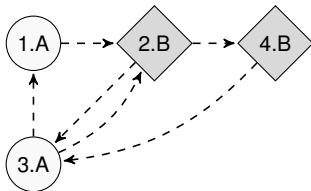


- Country A cannot do any transplants without the help of country B.
- The same is valid for country B.

But...

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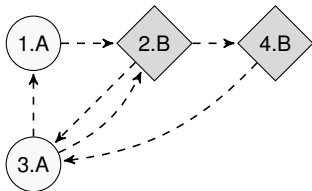
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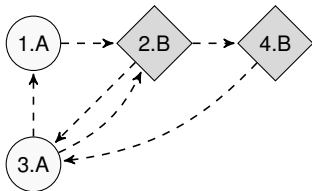


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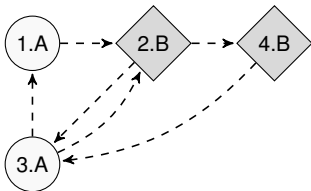


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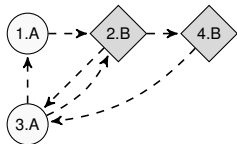
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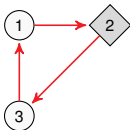


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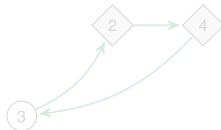
But...



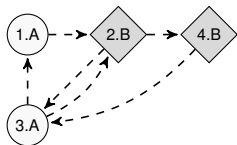
(j)



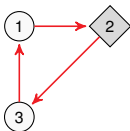
(k) Country A would be happy with this solution...



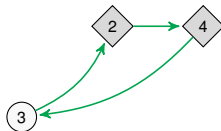
(l) ... while country B would be happy with this one.



(m)



(n) Country A would be happy with this solution...



(o) ... while country B would be happy with this one.

- Current and future research on mKEP will focus on developing models and algorithms for a **fair and equitable share of resources** among the multiple agents (countries, hospitals, etc) participating in the shared pool.
- Approaches followed so far:
  - Game theory: 2-KEG
    - Carvalho et al. "Two-Player Kidney Exchange Game". Submitted to Math. Programming.
  - Integer Programming models: equitable share of resources (kidneys) in the long-term for all countries involved.

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## Final remarks

# The team



# Overview on kidney exchange programs

Ana Viana et al

`ana.viana@inesctec.pt`

11 March 2016