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OrganJet: Overcoming Geographical Disparities in Access to Deceased Donor Kidneys in the United States

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Abstract. There are over 90,000 patients in the United States waiting for a kidney transplant. Under the current allocation policy, the vast majority of deceased organs are allocated locally. This causes significant disparities in waiting times and access to transplant across different geographical areas. To ameliorate this inequity, we propose an operational solution that offers affordable jet services (OrganJet) to patients on the transplant waiting list, allowing them to list in multiple different, and possibly very distant, donation service areas (DSAs) of their choosing. First, using a fluid approximation, we formulate the patients' problem of choosing a location to multiple list as a selfish routing game in which each patient tries to minimize his "congestion cost," i.e., maximize his life expectancy. Through a combination of numerical, simulation, and analytical results, we show that multiple listing can lead to a significant improvement in geographic equity. In the special case when sufficiently many patients can multiple list, the geographic inequity disappears. Moreover, the supply of deceased donor organs increases under multiple listing, leading to more transplants and saved lives. We also consider a diffusion approximation and study the resulting multiple-listing game. The equilibrium outcome under the diffusion approximation is a second-order perturbation of that under the selfish routing formulation. In particular, the geographic equity metric, waiting times, and probabilities of receiving a transplant at various DSAs in equilibrium are second-order perturbations of those predicted by the selfish routing equilibrium. Hence, the analysis under the diffusion approximation also supports the finding that multiple listing leads to an improvement in geographic equity. In addition, restricting attention to the special case of sufficiently many patients multiple listing leads to an explicit characterization of the equilibria, which in turn yields additional structural insights. Last, we undertake a simulation study that supports aforementioned findings.

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1. Introduction

Steve Jobs received a liver transplant in Tennessee in March 2009, although he was first put on the transplant waiting list in California, where he lived. Because of the long waiting time there, he would never have gotten a transplant in time, and it was not possible (even for Mr. Jobs) to jump the queue. Isaacson (2011) quotes Laurene Powell, Mr. Jobs' wife: "You can do the math, which I did, and it would have been way past June before he got a liver in California, and the doctors felt that his liver would give out in about April" (p. 483).

Isaacson (2011) writes:

> So she started asking questions and discovered that it was permissible to be on the list in two different states at the same time.... There were two major requirements: The potential recipient had to be able to get to the chosen hospital within eight hours, which Jobs could do thanks to his plane and the doctors from that hospital had to evaluate the patient in person before adding him or her to the list.... James Eason [the head of transplant at the Methodist University Hospital in Memphis]...had no problem allowing people from elsewhere to multiple-list in Memphis....
>
> The awful reality was that upcoming events like St. Patrick's Day and March Madness offered a greater likelihood of getting a donor because the drinking causes a spike in car accidents.... Indeed, on the weekend of March 21, 2009 a young man in his mid-twenties was killed in a car crash, and his organs were made available....
>
> Jobs and his wife flew to Memphis, where they landed just before 4 A.M. (p. 483)

The story of how Steve Jobs received a transplant elucidates the geographical disparity in the organ allocation system. This paper focuses on this very disparity for the kidney transplants, which constitute the great majority of organ transplants in the United States. However, the insights gleaned for the kidney allocation system apply to other organ transplants as well.

Kidney transplantation is both a life-prolonging and life-enhancing treatment compared to dialysis. Unfortunately, there is a large imbalance between the supply of and the demand for deceased donor kidneys. There are over 90,000 patients awaiting a kidney transplant in the United States. The majority of transplanted kidneys are deceased donor kidneys (Organ Procurement Transplantation Network 2016b). Oversight of deceased donor kidney allocation is done by the United Network for Organ Sharing (UNOS). Under the current policy, the vast majority (more than 70%; Davis 2011) of deceased donor kidneys are transplanted locally. Therefore, the differences in supply and demand characteristics of the various geographical regions lead to a significant disparity and variability in waiting times and access to transplant across the United States.

The U.S. Department of Health and Human Services (HHS) stated its beliefs about the current and future national organ transplantation operations in a 1998 final ruling (see Department of Health and Human Services 1998, p. 16298): "In principle, and to the extent technically and practically achievable, any citizen or resident of the United States in need of a transplant should be considered as a potential recipient of each retrieved organ on a basis equal to that of a patient who lives in the area where the organs or tissues are retrieved. Organs and tissues ought to be distributed on the basis of objective priority criteria, and not on the basis of accidents of geography."

Despite the official view of the U.S. Department of Health and Human Services, there are significant geographic disparities in access to deceased organs in practice, as illustrated by the story of Steve Jobs. The HHS rule is a federal mandate and not merely a view point. Strictly speaking, (geographical disparities arising from) the current allocation system is against the law. These discrepancies are rooted largely in the evolution of the current allocation system starting in the 1980s; see Section 2 for details.

Ideally, the geographic disparity should be addressed by a policy change that facilitates complete sharing of organs. However, despite the significant advancements in transportation, communication, and medicine in the last few decades, there remain substantial obstacles to the successful reform of organ allocation policy to eliminate the geographical disparity. The Organ Procurement and Transplantation Network (OPTN) Kidney Transplantation Committee spent nearly 10 years finalizing the changes to the allocation policy recently (see Organ Procurement and Transplantation Network 2016a). However, the tiered geographic allocation policy is left untouched, and the committee has stated that substantial geographic disparities will continue to exist under the newly revised policy.

For the case of liver allocation, Washburn et al. (2011), coauthored by the former UNOS president John Roberts, states that

> As long as there is a disparity between the supply of cadaveric livers and the demand, the distribution system will not please all stakeholder groups. In general, any system that redistributes organs from areas of low need to areas of high need will be accepted by the high-need areas and rejected by the low-need areas.... The clear endorsement of small, incremental changes as the best path forward has provided guidance for future policy recommendations. (p. 1011)

The difficulty highlighted by Washburn et al. (2011) applies to the allocation of cadaveric kidneys as well. Therefore, we seek an operational solution and study its ramifications. As mentioned in the Steve Jobs' story, patients have the right to be listed at multiple centers of their choosing under the current policy.[1] Ardekani and Orlowski (2010) writes: "The UNOS committee acknowledged that the multiple listing policy could address geographic disparities" (p. 720). (Many patient-centric sites provide information about how to multiple list; see txmultilisting.com for example.)[2] The authors also note that only 5.1% of end stage renal disease (ESRD) patients were multiple listed as of January 31, 2009.

Consistent with the view of Washburn et al. (2011), we propose an operational solution to alleviate the geographic disparity in the allocation of cadaveric kidneys. Our approach is incremental and works within the system. To be specific, we propose using jets (OrganJet) for patients on the transplant waiting list, which supports multiple listing of patients of transplant centers in different, and possibly very distant, donation service areas (DSAs) of their choosing. This is feasible in the United States because of the vast number of available choices in buying fractional jet services. The feasibility of aircraft logistics has already been verified as Angel Networks (using prop jets) already provide multiple listing capability (within 350–400 miles); see, for example, the Air Care Alliance (http://www.aircareall.org).

Interestingly, there have been recent discussions in the liver transplant community on reducing inequity. A concept document (not a policy proposal) has circulated outlining a potential redistricting of the liver allocation system to seek public input (see Organ Procurement and Transplantation Network 2014; also see Neidich et al. 2013). Perhaps as expected, this led to a strong backlash immediately (see, e.g., Bruce 2014, Christensen 2014). This effort is currently in a "pause" stage, requiring further analysis for robustness, and to analyze newer, less disruptive proposals.[3] Based on our discussions with the transplant surgeons, policy makers, and other stakeholders, a potential policy change, should it occur, will take several years. Unfortunately, such a change may not happen. Moreover, as mentioned above, the recent change to the kidney allocation policy did not address the geographic inequity. Indeed, the committee has stated that addressing that issue was not a primary goal of theirs (Organ Procurement and Transplantation Network 2016a), which highlights the importance of incremental changes such as the one studied in this paper to improve geographic equity.

The story of how Steve Jobs received a transplant in Tennessee highlights the potential benefit when only one person multiple lists. In contrast, we view multiple listing as an instrument to address geographic inequity without requiring a policy change. Therefore, we focus on the case when many patients multiple list and analyze the resulting game. For simplicity, we restrict attention to the case of double listing; that is, each multiple-listing patient can list at one other DSA.[4] First, we consider a deterministic model that formulates the patients' multiple-listing decisions as a selfish routing problem and characterize the equilibria as a solution to a convex optimization problem. To assess the geographic equity in allocation of deceased donor kidneys, we develop the metric geographic coefficient of variation (GCV) and consider how it changes under multiple listing. Through a combination of analytical, numerical, and simulation results, we show that geographic equity improves significantly under multiple listing. We also consider a special case of the formulation in which sufficiently many patients can multiple list and can do so at any DSA. In this case, the geographic inequity disappears. We also show in this case that the supply of deceased donor organs increases, leading to more transplants and saved lives.

Next, we consider the multiple-listing game under uncertainty using a diffusion approximation. Section 4 shows that the equilibrium geographic coefficient of variation under the diffusion approximation is a second-order perturbation of that in the selfish routing formulation. Similarly, the equilibrium waiting time at a DSA equals to that predicted by the selfish routing formulation plus a small (i.e., second-order) Gaussian random variable. In this sense, the equilibrium outcome under the diffusion approximation is a second-order perturbation of that under the selfish routing formulation. These results are proved without any assumptions on the multiple-listing radius or, hence, the network structure or the number of patients who can multiple list. We also consider the special case in which there are no geographical constraints to multiple listing and sufficiently many people can multiple list. With these additional assumptions, we characterize the equilibria explicitly and show that the earlier results continue to hold. In particular, the improvement in the geographic equity is even stronger, which is consistent with the findings in the context of the selfish routing formulation.

Last, we undertake a simulation study that relaxes several simplifying assumptions made in our model. The results of the simulation study corroborate the earlier finding that multiple listing leads to a more equitable outcome geographically, illustrating the value of OrganJet.

The rest of this paper is structured as follows. Section 2 reviews the related literature. Section 3 introduces and studies the selfish routing formulation of the patients' multiple-listing decisions. Section 4 introduces uncertainty in the model and uses a diffusion approximation to study the patients' multiple-listing decision. Section 5 performs a simulation study. Section 6 concludes. An online supplement includes Appendices A–H.

2. Current Status and Literature Review

Recognizing the national shortage of donated organs, the U.S. Congress passed the National Organ Transplant Act in 1984, which led to the creation of the Organ Procurement and Transplantation Network to address the issue of how to allocate deceased organs nationally. The legislation views the donated organs as national resources and calls for their fair and equitable distribution nationally (not just within the local area of procurement). Since this legislation in 1984, the UNOS has managed the allocation of deceased organs in the United States. As part of this effort, the UNOS divided the country into 11 regions, which are further divided into 58 donation service areas. Associated with each DSA is an organ procurement organization managing all local procurement and allocation procedures.

The revised kidney allocation policy of the UNOS is a point system that prioritizes the potential transplant candidates based on the points they receive on the following dimensions: (i) waiting time;[5] (ii) antibodies (i.e., whether the patient is sensitized or not[6]); (iii) antigen matches (which reflects the quality of the tissue match between the donor and the recipient); (iv) whether the recipient and the organ are among the top 20% in terms of their predicted life expectancy and the quality scores,[7] respectively; and (v) whether the patient is a child. Interestingly, Su and Zenios (2004) note (for the previous kidney allocation policy) that "The continued shortage of organs and the associated explosion in waiting times has contributed to a convergence of this point system to a system that resembles first-come-first-served (FCFS)" (p. 284).

This point system is crucially embedded in the aforementioned geographical structure: When a kidney becomes available, it is first offered to the patients listed in the same DSA (with a few minor exceptions). If no patient within the DSA accepts the offer, it is then offered to the patients in the same region. Finally, if no prospective recipient in the region takes the offer, then the kidney is offered nationally. During each of these steps, patients are ranked with respect to the point system described immediately above. This tiered geographical structure and the limited cold-ischemia time make it difficult for organs to be shared across different geographical regions and nationally; see Organ Procurement and Transplantation Network (2011) for further details of the deceased kidney allocation policy.

Organ transplant operations have received significant research attention in recent years. Several researchers study the patient's problem of accepting/rejecting an organ offer while waiting for a transplant; see David and Yechiali (1995), Ahn and Hornberger (1996), Hornberger and Ahn (1997), Howard (2002), Alagoz et al. (2004, 2007a, b), and Sandıkçi et al. (2008, 2013). Several others use simulation models to study the results of possible changes to the organ allocation policy or alternative policies; see CONSAD Research Corporation (1995), Pritsker et al. (1995), Zenios et al. (1999), Taranto et al. (2000), Kreke et al. (2002), and Shechter et al. (2005). Recent work on modeling delays in data centers with redundant requests has features reminiscent of our setting; see, for example, Shah et al. (2014), who focus on deriving performance bounds. Although our setting is related to the replication-based scheme with redundant requests studied in Shah et al. (2014), it has an important difference: the routing structure is endogenous in our setting, whereas it is exogenous in that setting. Our setting also has problem-driven differences such as abandonments (i.e., patient deaths) and the overloaded nature of the queues. Another related queueing-theoretic paper is Guo and Hassin (2015), where customers place duplicate orders from two parallel queues. The authors characterize the equilibrium for symmetric servers with the same service rate. Guo and Hassin (2015) do not consider abandonments. Unfortunately, their analysis does not extend to the case with more than two queues[8] or abandonments (Guo and Ata 2012). Indeed, the analysis of the multiple-listing game is notoriously difficult in general.

In designing optimal allocation policies, researchers seek to match organs and patients optimally to maximize the social welfare; see Righter (1989), David (1995), and David and Yechiali (1990, 1995). Zenios et al. (2000) explore the efficiency–equity trade-off and use various approximations to develop a dynamic index policy for deceased kidney allocation, which is effective and easy to implement. Akan et al. (2012) explore the trade-off between medical urgency and efficiency in designing the liver allocation system, and propose a policy that ranks patients based on their marginal benefit from transplant, i.e., the difference in expected benefit with versus without the transplant.

Su and Zenios (2004, 2005, 2006) study the impact of patient choice on the kidney allocation system. Su and Zenios (2005) consider a stochastic assignment model and prove asymptotic optimality of a simple partition policy. Su and Zenios (2006) use mechanism design to propose a kidney allocation system in which patients reveal which types of kidneys they would be willing to accept upon joining the transplant waiting list. Finally, Su and Zenios (2004) explore the role of queueing service discipline as an instrument for maximizing social welfare when patients choose whether or not to accept a kidney offer. Bertsimas et al. (2013) design a scalable, data-driven policy that incorporates fairness constraints and offers significant efficiency improvements. Ata et al. (2014) provide an analysis of scoring-based allocation policies taking into account recipient's forward-looking behavior. Roth et al. (2005, 2007) and Zenios (2002) explore paired kidney exchange; also see Ashlagi and Roth (2012).

Important antecedents of this paper are Kaplan (1984, 1985, 1986, 1988) and Caldentey et al. (2009), which use (overloaded) queueing models to study tenant assignment policies in public housing. Talreja and Whitt (2008) study a queueing system arising from the same applications. Our diffusion approximation crucially uses the results of Jennings and Reed (2012).

The literature on addressing geographical disparities is thin. Stahl et al. (2005) and Kong et al. (2010) investigate potential changes to the geographical structure of UNOS regions to improve geographic equity in liver allocation. More recently, Davis (2011) considered addressing geographic inequities in kidney transplantation and proposes probabilistic sharing of available organs in neighboring DSAs. Based on UNOS data from 2000 until 2009, Davis (2011) observes that the overall median waiting time to transplant varies between 0.93 years and 4.14 years depending on the DSA of listing. Note, however, that these statistics reflect the aggregate situation over a decade, which underestimates the current geographic discrepancy. Unfortunately, the number of patients in need of a kidney transplant has increased significantly in the last decade, leading to significantly longer waiting times at many DSAs and an even worse discrepancy in recent years than those reported in Davis (2011).

Although the OPTN Kidney Transplantation Committee spent a decade working on revising the kidney allocation policy, no changes are made to the current geographical structure in the revised kidney allocation policy. Indeed, the committee states addressing that issue was not a primary goal of theirs, and that substantial geographic disparities will continue to exist (see Organ Procurement and Transplantation Network 2016a). Indeed, there seem to be substantial obstacles to the successful reform of the current policy to address the geographic inequity in the United States, as discussed in the introduction.

Fortunately, multiple listing may help alleviate this geographic disparity. The UNOS acknowledges that multiple listing may address the geographical disparity; see Ardekani and Orlowski (2010). Given the significant barriers, which are practically insurmountable, to satisfactorily changing the current policy to address the geographic inequity, we propose an operational solution, which works within the existing system, to alleviate the geographic discrepancy without requiring any policy change. Instead, it simply facilitates multiple listing for the patients who wish to do so.

Last, Yeh et al. (2011) document the geographic inequity in the liver allocation system; also see Washburn et al. (2011), Kohn et al. (2014), and Vagefi et al. (2014). The geographic disparity in that context is also driven by the UNOS' geographically tiered allocation rule as in the kidney allocation system. Hence, our results shed light on that setting as well.

3. A Selfish Routing Formulation of Patients' Multiple-Listing Decisions via a Fluid Model

This section models patients' multiple-listing decisions using a fluid model. For a multiple-listing patient, the key decision is which DSA to list at because the current allocation policy is blind to the specific transplant centers at which patients are listed within each DSA. There are K DSAs in our model. (Currently, $K = 58$ in the United States.) Because of the deterministic nature of the fluid model, a multiple-listing patient can be viewed as essentially "moving" to the DSA where he multiple listed. This follows because in the fluid model a patient either receives a transplant in the DSA that has the shortest waiting time (among the ones he multiple listed) or dies while waiting. Therefore, pretending that the patient moves to that DSA produces identical results in the fluid model.

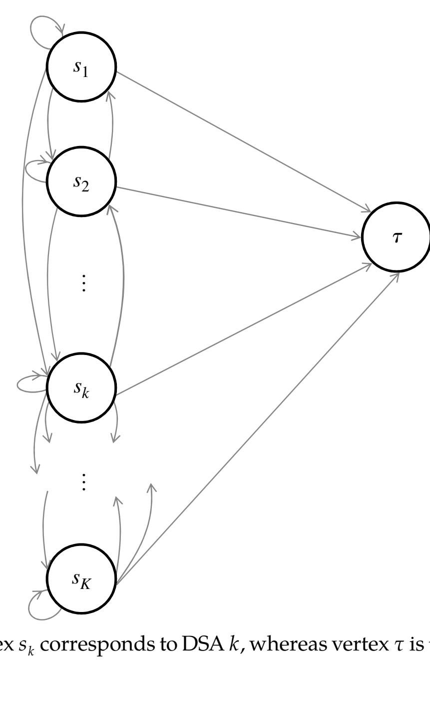
We restrict attention to adult patients with no living donors,[9] who arrive at rate λ_k to DSA k for $k = 1, \ldots, K$. The workings of the organ allocation system are complex, which makes its exact representation by a tractable model infeasible. (Indeed, the UNOS resorts to simulation studies to understand the impact of potential changes to the organ allocation system.) Therefore, to focus on the most important issue of interest to us, i.e., the geographical disparity, while keeping the model tractable, we make several simplifying assumptions. First, we ignore the antigen and sensitization status of patients. The only other key characteristic is the blood type.[10] Port et al. (1991) observe that 93% of all transplants are made between identical blood types. Hence, we make the simplifying assumption that patients with different blood types do not share organs.[11] This allows decomposing the problem across blood types and studying each blood type in isolation. Last, we assume that patients accept all organ offers.[12]

In DSA k, only a fraction π_k of patients can multiple list. In other words, the fraction $(1 - \pi_k)$ of DSA k patients has to wait for a transplant locally. Recall that we only consider double listing;[13] i.e., each multiple-listing patient chooses one additional location in addition to his local DSA. In doing so, he wishes to maximize his life expectancy.

The geography may limit a patient's choice of DSAs to multiple list, because he may not be able to travel quickly beyond a certain travel radius d, e.g., $d = 1{,}200$ miles. For $k = 1, \ldots, K$, let $A(k)$ denote the set of DSAs j whose distance to DSA k is less than or equal to d miles. Then $A(k)$ denotes the set of DSAs to which patients in DSA k can multiple list. Note that $A(k)$ is also the set of DSAs whose patients can multiple list at DSA k.

Restricting attention to those patients who can multiple list, we model their decision through a selfish routing game; see Roughgarden (2007), and also see Parlakturk and Kumar (2004) for a related study. The selfish routing game occurs on a network consisting of a directed graph (V, E), depicted in Figure 1, and a set of source-sink pairs referred to as commodities. The set V of vertices is given by $V = \{\tau, s_k \colon k = 1, \ldots, K\}$, where s_k denotes DSA k, and τ is the terminal node. The set E of edges is given by $E = \{(s_k, s_j), (s_k, \tau) \colon k = 1, \ldots, K,\ j \in A(k)\}$, where the edge (s_k, s_j) is a transport edge[14] corresponding to those patients who live

Figure 1. The Directed Graph Underlying Patients' Multiple-Listing Game



Note. Vertex s_k corresponds to DSA k, whereas vertex τ is the terminal node.

in DSA k but multiple list at DSA $j \in A(k)$. Similarly, the edge (s_k, τ) is a transplant edge that corresponds to patients receiving a transplant (or dying while waiting for it) in DSA k; these patients can be either local or multiple listed from other DSAs. In particular, DSA k patients, who cannot multiple list, go through the edge (s_k, τ) to reach the terminal node τ, which corresponds to leaving the transplant wait list either by receiving a transplant or by dying.

Another primitive of the selfish routing game is the set of source-sink pairs $s_k - \tau$, or commodities. To be specific, we wish to send a flow of $\pi_k \lambda_k$ patients from vertex s_k to vertex τ for $k = 1, \ldots, K$. Each patient is identified with one commodity. Patients originating from source s_k (DSA k) can either wait for a transplant at their local DSA or multiple list at a different DSA. Let $\mathcal{P}_k$ denote the $s_k - \tau$ paths of the network, that is, $\mathcal{P}_k = \{(s_k, s_j, \tau): j \in A(k)\}$, and define $\mathcal{P} = \bigcup_k \mathcal{P}_k$.

The routes chosen by patients are described using a flow, which is simply a nonnegative vector $\mathbf{x}$ indexed by the set $\mathcal{P}$ of source-sink paths. For a flow $\mathbf{x}$ and a path $P \in \mathcal{P}_k$, we interpret $\mathbf{x}_P$ as the amount of commodity k that chooses the path P to travel from s_k to τ. A flow $\mathbf{x}$ is feasible if it routes all patients: for each k,

$$\sum_{P \in \mathcal{P}_k} \mathbf{x}_P = \pi_k \lambda_k. \tag{1}$$

For notational simplicity, let x_{kj} denote the amount of flow going through the path (s_k, s_j, τ). Then constraint (1) can be rewritten as $\sum_{j \in A(k)} x_{kj} = \pi_k \lambda_k$ for $k = 1, \ldots, K$. Note that x_{kk} corresponds to those patients who can multiple list but chose not to; so the local arrival rate to DSA k is $x_{kk} + (1 - \pi_k)\lambda_k$.

Finally, each edge e of the network has a cost function $c_e(\cdot)$, which captures the "congestion cost" $c_e(x)$ on that edge as a function of the total flow x going through the edge e. The cost of multiple listing is taken as zero,[15] i.e., $c_{s_k, s_j}(x) = 0$ for the transport edges (s_k, s_j) for all $k, j \in A(k)$. For the transplant edges (s_k, τ), the cost of the edge corresponds to (the negative of) the expected total life years by listing at DSA k, which includes life-years while waiting for a transplant as well as the posttransplant life expectancy (if the patient receives a transplant).

Arikan et al. (2012) observe that the organ procurement rate, i.e., the fraction of deceased organs procured for transplant, increases with the congestion at the DSA. Motivated by this, we allow the organ procurement rate to depend on the number of patients listed at the DSA; that is, we let $\mu_k(x)$ denote the organ procurement rate at DSA k as a function of the patient arrival rate x to the DSA.[16] To be more specific, we assume

$$\mu_k(x) = m_k f\left(\frac{x}{m_k}\right), \quad k = 1, \ldots, K, \tag{2}$$

where m_k is the maximum rate of organs that can be procured, and $f(\cdot) \in [0, 1]$ is the fraction of organs procured. In what follows, we consider two cases: In the first case, f is concave increasing with $f(0) = 0$. In the second case, $f \equiv 1$, i.e., the organ procurement rate does not change with the congestion at the DSA. Then the cost of the transplant edge (s_k, τ) as a function of the total patient flow x on the edge is given by

$$c_{s_k, \tau}(x) = -[\bar{W}_k(x + (1 - \pi_k)\lambda_k) + \phi_k(x + (1 - \pi_k)\lambda_k)L], \quad k = 1, \ldots, K, \tag{3}$$

where $L > 1/\gamma$ is the posttransplant life expectancy, e.g., 15 years, γ is the death rate, and $1/\gamma$ is the mean time to death. As characterized in Proposition 14 (see Online Appendix A), $\bar{W}_k(x)$ and $\phi_k(x)$ are the expected time spent on the waiting list and the probability of receiving a transplant, respectively, at DSA k as functions of total patient flow x using that edge.

Combining (2) and (3) with Proposition 14 (see Online Appendix A) gives

$$c_{s_k, \tau}(x) = -\left[\frac{1}{\gamma} + \left(L - \frac{1}{\gamma}\right) \frac{\mu_k(x + (1 - \pi_k)\lambda_k)}{x + (1 - \pi_k)\lambda_k}\right], \quad k = 1, \ldots, K. \tag{4}$$

Note that $c_{s_k, \tau}(\cdot)$ is a convex function for all k.

The selfish routing (or multiple listing) game comprises of the triple of the graph (V, E), the commodities, and the costs of the edges, in which each patient tries to minimize his cost. The cost of a patient choosing a path P with respect to a flow x is the sum of costs of the constituent edges, $c_P(x) = \sum_{e \in P} c_e(x^e)$, where, for $k = 1, \ldots, K$ and $j \in A(k)$,

$$x^{s_k, s_j} = x_{kj} \quad \text{and} \quad x^{s_k, \tau} = \sum_{i \in A(k)} x_{ik}$$

denote the patient flows on edges (s_k, s_j) and (s_k, τ), respectively.

We refer the reader to Roughgarden (2002) for a formal definition of an equilibrium of the routing game. The selfish routing game is a special case of the potential games, and its equilibria can be characterized by the potential function method. More specifically, the equilibria of the selfish routing game are precisely the outcome that optimizes the potential function, which is given by

$$\Phi(x) = \sum_{e \in E} \int_0^{x^e} c_e(x)\, dx.$$

Consequently, we characterize the equilibrium flows as the minimizers of the potential function $\Phi(\cdot)$. In particular, defining

$$g_e(x) = \int_0^x c_e(u)\, du, \tag{5}$$

the equilibrium flows of the patients is given by the following mathematical program:[17]

$$\text{minimize} \quad \sum_{k=1}^{K} g_{s_k,\tau}\left(\sum_{j\in A(k)} x_{jk}\right) \tag{6}$$

$$\text{subject to} \quad \sum_{j\in A(k)} x_{kj} = \pi_k \lambda_k, \quad k=1,\ldots,K, \tag{7}$$

$$x_{kj} \geq 0, \quad k=1,\ldots,K,\ j\in A(k). \tag{8}$$

Using the special structure of our problem, we can simplify this formulation further. To that end, for each k, define

$$H_k(x) = \int_0^x \frac{\mu_k(u+(1-\pi_k)\lambda_k)}{u+(1-\pi_k)\lambda_k}\,du, \tag{9}$$

which is a concave decreasing function. Then consider the following mathematical program: Choose x_{kj} so as to

$$\text{maximize} \quad \sum_{k=1}^{K} H_k\left(\sum_{j\in A(k)} x_{jk}\right) \tag{10}$$

$$\text{subject to} \quad (7)\text{–}(8). \tag{11}$$

The following proposition characterizes the equilibria of the selfish routing game.

Proposition 1. *The equilibrium flows of the patients' multiple-listing game are given by the maximizers of the mathematical program* (10)–(11).

Although our formulation allows a general $f(\cdot)$, we restrict attention to the case $f\equiv 1$ for simplicity throughout except in Section 3.1, where we assume that $f(\cdot)$ is strictly concave and increasing and show that multiple listing leads to an increase in the supply of organs in that case.

Intuitively, we expect that the waiting times and access to transplant at the various DSAs become more uniform, and hence more equitable, under multiple listing. Propositions 2 and 3 support this intuition. The following notation is needed to state the results. For $k=1,\ldots,K$, let w_k and ϕ_k denote the time to transplant (conditional on receiving a transplant) and the probability of receiving a transplant, respectively, at DSA k. Also, let $\tilde{w}_k$ and $\tilde{\phi}_k$ denote the corresponding quantities after multiple listing. Similarly, let $\tilde{\lambda}_k$ denote the total effective arrival rate to DSA k after multiple listing. Last, define the vectors $w=(w_k)$, $\phi=(\phi_k)$, $\lambda=(\lambda_k)$, $\tilde{w}=(\tilde{w}_k)$, and $\tilde{\lambda}=(\tilde{\lambda}_k)$.

Consider the directed graph $G(x)$ associated with a feasible flow x of the selfish routing game. The nodes of $G(x)$ are the DSAs $1,\ldots,K$ and there is a directed edge from node k to node j ($\neq k$) if $x_{kj}>0$. Online Appendix D provides graph-theoretic results (and their proofs) on $G(x)$ that facilitate the proofs of the following result and Theorem 1.

The following proposition shows that the ranges of the waiting times and access-to-transplant probabilities across various DSAs decrease under multiple listing.

Proposition 2. *If x denotes the multiple-listing rates for an equilibrium of the selfish routing problem such that $G(x)$ has no directed cycles, then the ranges of time to transplant and access to transplant across various DSAs decreases under that equilibrium, i.e.,*

$$\max_{i,j}(\tilde{w}_i-\tilde{w}_j) \leq \max_{i,j}(w_i-w_j) \quad \text{and} \\ \max_{i,j}(\tilde{\phi}_i-\tilde{\phi}_j) \leq \max_{i,j}(\phi_i-\phi_j). \tag{12}$$

The next proposition provides further evidence of the equity improvements due to multiple listing.

Proposition 3. *If $A(k)=\{1,\ldots,K\}$ and $\mu_k=m_k$ for all k and $\lambda_i/\mu_i \geq \lambda_j/\mu_j$ whenever $\lambda_i\geq\lambda_j$ for all $i,j=1,\ldots,K$, then the effective arrival rate vector $\tilde{\lambda}$ after multiple listing is majorized by λ, i.e., $\tilde{\lambda}\prec\lambda$. Moreover, $\tilde{w}$ and $\tilde{\phi}$ are weakly majorized by w and ϕ, respectively, i.e., $\tilde{w}\prec^{w} w$ and $\tilde{\phi}\prec_{w}\phi$.*

The definition of majorization goes back to Lorenz (1905), who studied wealth inequality and introduced what is known as the Lorenz curve, which leads to an equivalent characterization of majorization. The basic idea is that when a wealth distribution vector majorizes another, then it is considered less equitable than the other. In this sense, Proposition 3 shows that the outcomes under multiple listing are more equitable.

Of course, Proposition 3 hinges on the assumption that $A(k)=\{1,\ldots,K\}$. However, our numerical experiments (see Section 5) show that even when it is violated, multiple listing leads to an improvement in equity. In particular, we see that the results are insensitive to multiple listing radius provided it is larger than 1,000 miles.[18] The other assumption of Proposition 3 corresponds to assuming that larger DSAs are more congested, which tends to hold in practice especially for the largest DSAs. Last, to further explore the equity benefits of multiple listing, Section 3.1 considers the special case where sufficiently many patients can multiple list and shows that multiple listing leads to uniform waits at the various DSAs and, hence, to an equitable outcome. This supports the findings of Propositions 2 and 3.

Quantifying the change in equity under multiple listing in general is challenging. Nonetheless, our numerical analysis in Section 5 shows a substantial improvement in geographic equity under multiple listing. To facilitate that analysis, we develop a metric of geographical coefficient of variation across DSAs, which gives an indication of how uniform (or variable) waiting times (and the probabilities of receiving a transplant) across different DSAs are. The definition of the geographical coefficient of variation requires the following notation: Let $\bar{w}$ and $\bar{\phi}$ denote the average time to transplant and the average probability of receiving a transplant, respectively; that is,

$$\bar{w} = \sum_{k=1}^{K} \frac{\mu_k}{M} w_k \quad \text{and} \quad \bar{\phi} = \sum_{k=1}^{K} \frac{\lambda_k}{\Lambda} \phi_k,$$

where $\Lambda = \sum_{k=1}^{K} \lambda_k$ and $M = \sum_{k=1}^{K} \mu_k$ are the total patient and organ arrival rates nationally, respectively, and $\mu_k = m_k f(\lambda_k/m_k)$. Note that $\bar{\phi} = M/\Lambda$. Then the geographical coefficient of variation for waiting times, denoted by GCV_w, is defined as follows:

$$GCV_w = \sqrt{\sum_k \mu_k^2 (w_k - \bar{w})^2 \Big/ \sum_k \mu_k^2} \Bigg/ \bar{w}.$$

Similarly, define the geographical coefficient of variation for access to transplant GCV_ϕ by

$$GCV_\phi = \sqrt{\sum_k \lambda_k^2 (\phi_k - \bar{\phi})^2 \Big/ \sum_k \lambda_k^2} \Bigg/ \bar{\phi}.$$

Note that GCV_w and GCV_ϕ after multiple listing can be defined similarly by replacing λ_k and μ_k with those after multiple listing.

The next section considers a special case with no geographical constraints to multiple listing and where sufficiently many patients can multiple list to achieve a geographically equitable outcome.

3.1. The Special Case of Sufficiently Many Patients Multiple Listing to Achieve a Global FCFS System

This section considers a relaxed version of the formulation (10)–(11), which allows patients to multiple list at any other DSA, i.e., $A(k) = \{1, \ldots, K\}$ for all k. The following proposition characterizes the equilibrium multiple-listing rates in this case explicitly and shows that both the supply of organs increases under multiple listing and the equilibrium outcome is equitable.

Proposition 4. *Suppose that $A(k) = \{1, \ldots, K\}$ and that*

$$(1 - \pi_k)\lambda_k < \Lambda \frac{m_k}{m} \quad \text{for } k = 1, \ldots, K, \tag{13}$$

where $m = \sum_{k=1}^{K} m_k$. Then $\{x_{kj}\colon j, k = 1, \ldots, K\}$ constitute an equilibrium if and only if they satisfy (7)–(8) and

$$\sum_{j=1}^{K} x_{jk} = \Lambda \frac{m_k}{m} - (1 - \pi_k)\lambda_k, \quad k = 1, \ldots, K. \tag{14}$$

In each equilibrium, the following hold for $k = 1, \ldots, K$:

$$w_k = \frac{1}{\gamma} \ln\left(\frac{\Lambda}{\tilde{M}}\right), \quad \phi_k = \frac{\tilde{M}}{\Lambda}, \quad \text{and} \quad \bar{W}_k = \frac{1}{\gamma}\left(1 - \frac{\tilde{M}}{\Lambda}\right), \tag{15}$$

where $\tilde{M} = m f(\Lambda/m)$ and $m = \sum_{k=1}^{K} m_k$. Therefore, we have that $GCV_w = 0$ and $GCV_\phi = 0$. Moreover, under multiple listing, the nationwide organ supply, i.e., total number of deceased donor organs procured, increases.

There are many equilibria that satisfy (7) and (8) and (14); see Online Appendix F for one in which the multiple-listing rates are minimal. However, all equilibria are payoff equivalent and lead to same aggregate system characteristics;[19] see (15).

Proposition 4 shows that when sufficiently many people can multiple list (and when there are no geographical constraints), then an equitable distribution of deceased donor kidneys is achieved. Although Proposition 4 makes strong assumptions, our numerical study (using the UNOS deceased donor kidney transplant data and without making those assumptions) yields the same insight that multiple listing can alleviate geographic inequity significantly; see Section 5.

The fluid model does not fully capture the multiple-listing phenomenon because of its deterministic nature. Therefore, in the next section, we use the diffusion approximation laid out in Online Appendix A for the virtual waiting time (see Equation (43)), which helps in modeling the multiple-listing phenomenon more accurately. Nonetheless, the fluid model captures the first-order (aggregate) effects of multiple listing and provides a useful building block for the analysis of Section 4.

4. A Diffusion Model Analysis of Multiple Listing

This section studies the multiple-listing game using a diffusion approximation for the congestion at the various DSAs. Roughly speaking, we construct an equilibrium that can be viewed as a second-order perturbation of the selfish routing equilibrium of Section 3. More specifically, patients' strategies (defined in Section 4.1) yield multiple-listing rates that are second-order perturbations of those in the selfish routing equilibrium. In other words, the first-order (or the fluid-scale) transplant rates under the patients' strategies coincide with the multiple-listing rates of the selfish routing game. This result is proved in Section 4.2 (see Theorem 1). Building on this result, Section 4.3 develops a diffusion approximation for the waiting times at the various DSAs under the strategies of the form described in Section 4.1. This approximation leads to a natural approximation of the patients' utility under multiple listing as well. Section 4.4 establishes the existence of an equilibrium and proves a useful result that characterizes the various metrics in equilibrium in relation to their analogs in the selfish routing equilibrium (see Section 3). In particular, Proposition 7 shows that the equilibrium geographic coefficient of variation under the diffusion approximation is a second-order perturbation of that in the selfish routing formulation. Similarly, the equilibrium mean time to transplant and the probability of transplant at the various DSAs are second-order perturbations of those in the selfish routing formulation.

Last, Section 4.5 considers the special case where there are no geographical constraints, i.e., $A(k) = \{1, \ldots, K\}$ for all k, and that sufficiently many patients can multiple list. The additional structure imposed on the formulation allows a sharper characterization of the equilibrium. The findings in this section are consistent with those in Section 4.4, but the equilibrium is characterized explicitly, which yields additional insights.

4.1. Patients' Strategy

Defining $\tilde{\mathcal{P}}_k$ as the set of probability distributions on $A(k)$, i.e.,

$$\tilde{\mathcal{P}}_k = \left\{p_k\colon A(k) \to [0,1] \text{ such that } \sum_{j\in A(k)} p_k(j) = 1\right\},$$

and letting $\tilde{\mathcal{P}} = \prod_{k=1}^{K} \tilde{\mathcal{P}}_k$, the patients' strategy is denoted by $p \in \tilde{\mathcal{P}}$. We refer to such an equilibrium as a symmetric mixed strategy equilibrium. For notational convenience, we denote $p \in \tilde{\mathcal{P}}$ by $p = (p_1, \ldots, p_k, \ldots, p_K)$, where $p_k \in \tilde{\mathcal{P}}_k$ is the strategy of all patients living in DSA k.

In deriving the diffusion approximation, we consider an asymptotic regime where arrival rates of patients and organs grow proportionally large where time-to-death distributions remain fixed; that is, we consider a sequence of systems[20] indexed by n, where $\lambda_k^n = n\lambda_k$ and $\mu_k^n = n\mu_k$ for $k = 1, \ldots, K$ and $n \geq 1$. We denote the patients' strategy in the nth system by p^n. Given $p^n \in \tilde{\mathcal{P}}$, define

$$x_{kj}^n(p^n) = \pi_k \lambda_k n p_k^n(j) \quad \text{for } j \in A(k)$$

as the rate of DSA k patients multiple listing at DSA j. Note that the number of DSA k patients not multiple listing (and hence waiting for a transplant locally) is equal to

$$(1-\pi_k)\lambda_k n + x_{kk}^n(p^n) = [(1-\pi_k) + \pi_k p_k^n(k)]\lambda_k n.$$

Loosely speaking, we restrict attention to equilibria that can be viewed as a second-order perturbation of the selfish routing equilibrium. Indeed, we construct an equilibrium where the first-order multiple-listing rates coincide with (the appropriately scaled versions of) those in the selfish routing equilibrium. To this end, we adopt a diffusion approximation and consider strategies of the form[21]

$$p^n = \bar{p} + \frac{1}{\sqrt{n}} q^n, \tag{16}$$

where $\bar{p} \in \tilde{\mathcal{P}}$ and $\sum_{j\in A(k)} q_k^n(j) = 0$ for all k. We also require that $|q^n| \leq c^n < \infty$ and that $p^n \in \tilde{\mathcal{P}}$ for all n. In Equation (16), the term $\bar{p}$ captures the first-order effects, whereas q^n captures the second-order effects, which can be attributed to the (second-order) stochasticity in waiting times; see Online Appendix A.

4.2. Patients' Payoffs and Fluid Model Routing Equations and Their Solutions

To formulate the patients' payoff under a given strategy profile $p^n = \bar{p} + (1/\sqrt{n})q^n$, we next characterize the congestion in each DSA under that strategy profile. To this end, note that

$$x_{kj}^n(p^n) = n\left[\bar{x}_{kj}(\bar{p}) + \frac{1}{\sqrt{n}}\tilde{x}_{kj}(q^n)\right], \tag{17}$$

where

$$\bar{x}_{kj}(\bar{p}) = \pi_k\lambda_k\bar{p}_k(j) \quad \text{and} \quad \tilde{x}_{kj}(q^n) = \pi_k\lambda_k q_k^n(j). \tag{18}$$

In other words, for DSA k, the strategy profile p^n results in the first-order multiple-listing rates

$$\bar{x}_{kj}(\bar{p}) = \pi_k\lambda_k\bar{p}_k(j), \quad j \in A(k),\ j \neq k. \tag{19}$$

Also, the (first-order) rate of DSA k patients who do not multiple list (and hence wait for a transplant locally) is equal to $(1-\pi_k)\lambda_k + \bar{x}_{kk}(\bar{p}) = [(1-\pi_k) + \pi_k\bar{p}_k(k)]\lambda_k$.

Given these first-order rates, we next focus on the resulting transplant rates[22] in the fluid model of the corresponding queueing network, where each node is a multiclass, FCFS queue and corresponds to a DSA. The first-order rates $\bar{x}_{kj}$ define the topology of the queueing network, which we elaborate on next. To understand the structure of the resulting queueing network, it is helpful to imagine a separate buffer for each flow $\bar{x}_{kj}$ for $j \in A(k)$ and a separate buffer for the patients of DSA k who do not have the option to multiple list.

Figure 2(a) shows DSA k patients who multiple list in DSA $j \neq k$, $j \in A(k)$. Note that y_{kj} denotes the transplant (or service) rate they receive from DSA j (or server j), whereas z_{kj} denotes the transplant (service) rate they receive from DSA k. Figure 2(b) shows DSA k patients who choose not to multiple list. The transplant (service) rate they receive is denoted by z_{kk}. Figure 2(c) shows DSA k patients who do not have the option to multiple list; ν_k denotes the transplant (service) rate they receive. We also let $\bar{w}_k$ denote the time to transplant at DSA k conditional on receiving a transplant (in DSA k). Finally, let $\bar{\phi}_k = e^{-\gamma\bar{w}_k}$, which can be interpreted as the effective transplant probability[23] at DSA k.

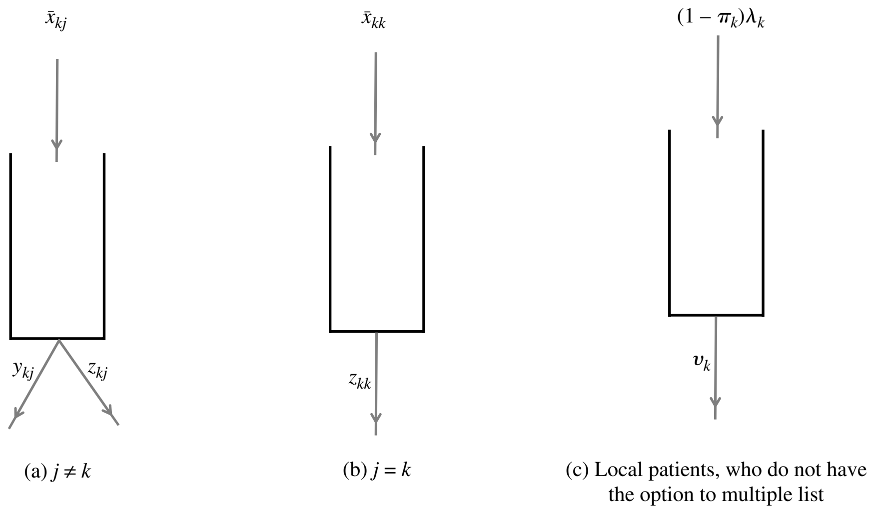
Given the routing rates $\bar{x} = (\bar{x}_{kj})$ and the resulting queueing network described in Figure 1, the transplant (or service) rates $\bar{y} = (\bar{y}_{kj})$, $\bar{z} = (\bar{z}_{kj})$, and $\nu = (\nu_k)$, the waiting times $\bar{w} = (\bar{w}_k)$, and the transplant probabilities $\bar{\phi} = (\bar{\phi}_k)$ must satisfy the following fluid model routing equations (Equations (20)–(30)).

4.2.1. Fluid Model Routing Equations. For all $k = 1, \ldots, K$ and $j \in A(k)$ with $j \neq k$ and $\bar{x}_{kj} > 0$, Equations (20)–(25) must hold:

$$\text{if } \bar{w}_k < \bar{w}_j, \text{ then } \bar{y}_{kj} = 0; \tag{20}$$

$$\text{if } \bar{y}_{kj} = 0, \text{ then } \bar{w}_k \leq \bar{w}_j; \tag{21}$$

Figure 2. Transplant Rates for DSA k Patients



Note. Panel (a) shows the rates for DSA k patients multiple listing to DSA j, panel (b) shows the rates for DSA k patients choosing not to multiple list, and panel (c) shows DSA k patients who do not have the option to multiple list.

$$\text{if } \bar{w}_j < \bar{w}_k, \text{ then } \bar{z}_{kj} = 0; \tag{22}$$

$$\text{if } \bar{z}_{kj} = 0, \text{ then } \bar{w}_k \geq \bar{w}_j; \tag{23}$$

$$\bar{w}_k = \frac{1}{\gamma} \ln\left(\frac{1}{\bar{\phi}_k}\right); \tag{24}$$

$$\frac{\bar{y}_{kj} + \bar{z}_{kj}}{\bar{x}_{kj}} = \max\{\bar{\phi}_j, \bar{\phi}_k\}. \tag{25}$$

For DSA k patients, who do not multiple list, we have

$$\frac{\nu_k}{\lambda_k(1-\pi_k)} = \bar{\phi}_k \quad \text{for all } k, \tag{26}$$

$$\frac{\bar{z}_{kk}}{\bar{x}_{kk}} = \bar{\phi}_k \quad \text{provided } \bar{x}_{kk} > 0, \text{ for all } k. \tag{27}$$

The access to transplant, i.e., the probability of receiving a transplant, at DSA k, $\bar{\phi}_k$ must satisfy

$$\bar{\phi}_k = \frac{\mu_k + \sum_{j \in A(k), j\neq k} \bar{z}_{jk} \mathbb{1}_{\{\bar{\phi}_j \leq \bar{\phi}_k\}} - \sum_{j\in A(k), j \neq k} \bar{z}_{kj}}{\lambda_k + \sum_{j \in A(k), j\neq k} \bar{x}_{jk} \mathbb{1}_{\{\bar{\phi}_j \leq \bar{\phi}_k\}} - \sum_{j\in A(k), j \neq k} \bar{x}_{kj}}. \tag{28}$$

Also, the following flow conservation equation must hold for each DSA:

$$\nu_k + \sum_{j\in A(k), j\neq k} \bar{y}_{jk} + \sum_{j \in A(k)} \bar{z}_{kj} = \mu_k \quad \text{for all } k. \tag{29}$$

Last, we have the nonnegativity constraints:

$$\bar{y}_{kj} \geq 0, \quad \bar{z}_{kj} \geq 0, \quad \bar{w}_k > 0, \quad \bar{\phi}_k \in (0,1). \tag{30}$$

Equations (20)–(30) follow because the service discipline at each DSA is FCFS. To elaborate further, consider Equation (20): if $\bar{w}_k < \bar{w}_j$, then, since the time to transplant is shorter at DSA k, no DSA k patient who multiple lists to DSA j will receive a transplant at DSA j. Thus, we conclude $\bar{y}_{kj} = 0$. Similarly, if no DSA k patient (multiple listing to DSA j) receives a transplant from DSA j, then it must be that $\bar{w}_k \leq \bar{w}_j$. Otherwise, if $\bar{w}_j < \bar{w}_k$, then we would have $\bar{y}_{kj} > 0$, since time to transplant $\bar{w}_j$ at DSA j is shorter than that in DSA k. In the same vein, if $\bar{w}_j < \bar{w}_k$,then no DSA k patient who multiple lists to DSA j receives a transplant at DSA k in the fluid model because the time to transplant is shorter at DSA j. Thus, $\bar{z}_{kj} = 0$. Equation (23) follows similarly. Equation (24) follows from Proposition 12 (in Online Appendix A), where the traffic intensity is given by $1/\bar{\phi}_k$.

Note that DSA k patients who multiple list to DSA j receive a total transplant rate of $\bar{y}_{kj} + \bar{z}_{kj}$. Thus, their probability of receiving a transplant equals $(\bar{y}_{kj} + \bar{z}_{kj})/\bar{x}_{kj}$. At the same time, since both nodes (DSA k and DSA j) operate under FCFS, this should equal the maximum of the transplant probabilities at the two DSAs. Therefore, Equation (25) must hold. Equations (26) and (27) follow similarly.

Equation (28) looks at the transplant probability at DSA k, but it excludes the rate of DSA k patients who multiple list elsewhere, i.e., $\sum_{j\in A(k), j\neq k} \bar{x}_{kj}$. In doing so it appropriately subtracts the transplant rate these patients receive from DSA k, i.e., $\sum_{j\in A(k), j\neq k} \bar{z}_{kj}$. Note that if $\bar{z}_{kj} = 0$, then it means that $\bar{y}_{kj} > 0$ and they should be accounted for in calculating $\bar{\phi}_j$ since their transplant probability will be $\bar{\phi}_j > \bar{\phi}_k$. On the other hand, if $\bar{z}_{kj} > 0$, then it must be that $\bar{z}_{kj}/\bar{x}_{kj} = \bar{\phi}_k$ (unless[24] $\bar{\phi}_j = \bar{\phi}_k$) by (25). So excluding them in Equation (28) causes no harm. At the same time, Equation (28) also accounts for patients multiple listing to DSA k from other DSAs. Note that these patients should be accounted for only if they receive a transplant at DSA k, which may happen only if $\bar{\phi}_j \leq \bar{\phi}_k$ for a DSA j patient multiple listing to DSA k.

Remark 1 (Fluid Model Routing Equations Under Global FCFS). Equations (20)–(30) simplify significantly under global FCFS, i.e., when $\bar{w}_k = \bar{w}_j$ for all i, j. Letting $\bar{\phi}_k = M/\Lambda$ for all k yields the new set of equations.

The next major step in the analysis is to show that Equations (20)–(30) pin down the first-order transplant (i.e., service) rates, waiting times, and transplant probabilities under multiple listing for the queueing network described above.

Theorem 1 ties in the queueing network formulated in this section to the selfish routing formulation of §3 and helps characterize the first-order behavior of the organ allocation system under multiple listing.

Theorem 1. *Let x be an equilibrium of the selfish routing problem such that $G(x)$ contains no directed cycles. Then the fluid model routing Equations* (20)–(30) *corresponding to flow x (i.e., putting x in place of $\bar{x}$ in* (20)–(30)) *have the unique solution given by*

$$\bar{\phi}_k = \phi_k \quad \text{for } k = 1, \ldots, K,$$

and for $k = 1, \ldots, K$, $j \in A(k)$ with $j \neq k$, we have that $\bar{z}_{kj} = 0$,

$$\bar{y}_{kj} = \begin{cases} \phi_j x_{kj} & \text{if } x_{kj} > 0, \\ 0 & \text{otherwise,} \end{cases}$$

$\bar{z}_{kk} = \phi_k x_{kk}$, and $\nu_k = \phi_k(1-\pi_k)\lambda_k$, where

$$\phi_j = \frac{\mu_j}{\lambda_j + \sum_{l \in A(j), l \neq j} x_{lj} - \sum_{l \in A(j), l \neq j} x_{jl}}. \tag{31}$$

Remark 2. Theorem 1 pins down the first-order rates given the multiple listing rates x, which in turn yields the first-order component $\bar{p}$ of the strategy profile p^n; see (17)–(19).

The next subsection refines this characterization by providing a second-order approximation to patients' utility under multiple listing.

4.3. A Diffusion Approximation to Waiting Times and Patients' Utility Under Multiple Listing

Using the approximation based on the limit theorem of Jennings and Reed (2012), we model the steady-state waiting time W_j^n at DSA j by a normal random variable:

$$W_j^n \stackrel{D}{=} N(w_j^n, (\sigma_j^n)^2), \tag{32}$$

where[25]

$$w_j^n = \frac{1}{\gamma} \ln\left(\frac{\Lambda_j^n}{\mu_j}\right) \quad \text{and} \quad (\sigma_j^n)^2 = \frac{1}{n\gamma\mu_j}, \quad \text{and}$$

$$\Lambda_j^n = \lambda_j + \sum_{k \in A(j), k \neq j} \bar{x}_{kj}(\bar{p}) + \frac{1}{\sqrt{n}} \sum_{k \in A(j), k \neq j} \tilde{x}_{kj}(q^n) - \sum_{l \in A(j), l \neq j} \bar{x}_{jl}(\bar{p}) - \frac{1}{\sqrt{n}} \sum_{l \in A(j), l \neq j} \tilde{x}_{jl}(q^n).$$

Rearranging terms, we write

$$\Lambda_j^n = \bar{\Lambda}_j + \frac{1}{\sqrt{n}} \theta_j, \tag{33}$$

where

$$\bar{\Lambda}_j = \lambda_j + \sum_{k \in A(j), k \neq j} \bar{x}_{kj}(\bar{p}) - \sum_{l \in A(j), l \neq j} \bar{x}_{jl}(\bar{p}), \tag{34}$$

$$\theta_j = \sum_{k \in A(j), k \neq j} \tilde{x}_{kj}(q^n) - \sum_{l \in A(j), l \neq j} \tilde{x}_{jl}(q^n). \tag{35}$$

Also note that $\bar{\Lambda}_j/\mu_j = 1/\bar{\phi}_j$. Then

$$w_j^n \simeq \bar{w}_j + \frac{1}{\gamma\sqrt{n}} \bar{\phi}_j \frac{\theta_j}{\mu_j} = \bar{w}_j + \frac{1}{\gamma\sqrt{n}} \frac{\theta_j}{\bar{\Lambda}_j},$$

where $\bar{w}_j = (1/\gamma)\ln(\bar{\Lambda}_j/\mu_j)$.

Given the virtual waiting times W_j^n $(j = 1, \ldots, K)$ at various DSAs, consider a patient living in DSA k who multiple lists at DSA j. His virtual waiting time to receive a transplant (conditional on surviving on dialysis until then), denoted by W_{kj}^n, is given by

$$W_{kj}^n = \min\{W_k^n, W_j^n\}.$$

Then the patient's waiting time (more precisely, the time spent on the waiting list) is $\min\{X, W_{kj}^n\}$, where X is the time to death on dialysis without a transplant and is an exponential random variable with mean $1/\gamma$. As done in §3, we assume that each multiple-listing patient in DSA k wishes to maximize his life expectancy. The life expectancy of a DSA k patient multiple listing to DSA j is given by the following:

$$\mathbb{E}[\min\{X, W_{kj}\}] + L\mathbb{P}(W_{kj} < X),$$

where L is the posttransplant life expectancy, and $L > 1/\gamma$.

Proposition 5. *We have that*

$$\mathbb{E}[\min\{X, W_{kj}\}] + L\mathbb{P}(W_{kj} < X) = L - \left(L - \frac{1}{\gamma}\right)\mathbb{P}(W_{kj} > X),$$

where

$$\mathbb{P}(W_{kj} > X) = \int_0^\infty \gamma e^{-\gamma x} \bar{\Phi}\left(\frac{x - w_k^n}{\sigma_k^n}\right) \bar{\Phi}\left(\frac{x - w_j^n}{\sigma_j^n}\right) dx.$$

Since $L > 1/\gamma$, the patients' objective is equivalent to maximizing the probability of receiving a transplant, i.e., $\mathbb{P}(W_{kj} < X)$. Moreover, the patients' strategy can be viewed as choosing q^n since p^n is of the form (16). Then, fixing everyone else's strategy q^n, the utility of a patient living in DSA k taking action j, denoted by $u_k(j; q^n)$, is given by

$$u_k(j; q^n) = 1 - \mathbb{P}(W_{kj} > X) = 1 - \int_0^\infty \gamma e^{-\gamma x} \bar{\Phi}\left(\frac{x - w_k^n}{\sigma_k^n}\right) \bar{\Phi}\left(\frac{x - w_j^n}{\sigma_j^n}\right) dx.$$

4.4. Existence and Characterization of Equilibria

The following proposition establishes existence of an equilibrium.[26]

Proposition 6. *There exists a symmetric mixed-strategy equilibrium of the patients' multiple-listing game.*

Next, we turn to characterizing the properties of the equilibria. In Online Appendix B, using a result from Cramér (1946), we derive a tractable approximation to patients' utility $u_k(j;q^n)$ for all k. To state the approximation, consider DSAs j,k and recall that the expected virtual waiting time w_i^n (for $i=j,k$) and the standard deviation σ_i^n (for $i=j,k$) are given by

$$w_i^n=\bar{w}_i+\frac{1}{\gamma\sqrt{n}}\bar{\phi}_i\frac{\theta_i}{\mu_i}\quad\text{and}\quad(\sigma_i^n)^2=\frac{1}{\gamma n\mu_i},\qquad i=j,k.$$

There are two cases to consider for a patient listed at both DSAs j and k: case (i) is $\bar{w}_j<\bar{w}_k$; case (ii) is $\bar{w}_j=\bar{w}_k$. In Online Appendix B, we show that

$$\mathbb{P}(\min\{W_j,W_k\}>X)=\begin{cases}\mathbb{P}(W_j>X)+o(1/\sqrt{n}) & \text{in case (i)},\\ \mathbb{P}(W_j>X)+O(1/\sqrt{n}) & \text{in case (ii)}.\end{cases}$$

In other words, the improvement in the patient's utility from listing at both DSAs (as opposed to listing only at DSA j) is negligible in the diffusion scale, i.e., $o(1/\sqrt{n})$, in case (i), whereas in case (ii) it is significant, i.e., $O(1/\sqrt{n})$.

Therefore, in what follows, we adopt the following second-order approximation: In case (i),

$$\mathbb{P}(W_{jk}>X)\simeq\mathbb{P}(W_j>X),\tag{36}$$

and in case (ii),

$$\log\left(\sqrt{n}\left[1-\frac{1}{\phi}\mathbb{P}(X>W_{kj})\right]+\gamma\Delta\right)\simeq C_1+C_2\left(\sum_{l=j,k}\Delta\sqrt{\gamma\mu_l}+\frac{\phi_l\theta_l}{\sqrt{\gamma\mu_l}}\right),\tag{37}$$

where Δ, C_1, and C_2 are positive constants specified in Online Appendix B, and $\phi=e^{-\gamma\bar{w}_k}$.

Using the second-order approximations in (36) and (37), the following proposition characterizes an equilibrium that is a (second-order) perturbation of the selfish routing equilibrium solved for in Section 3. This result is proved for general $A(k)$ and $\pi_k, k=1,\ldots,K$. To state the result, we attach the superscript of $d(f)$ to represent quantities under the diffusion (fluid) approximation.[27]

Proposition 7. *There exists an equilibrium under the diffusion approximation such that the geographic coefficients of variation $GCV_\phi^{n,d}$ and $GCV_w^{n,d}$ satisfy the following:*

$$GCV_i^{n,d}=GCV_i^{n,f}+O(1/\sqrt{n})\quad\text{for } i=\phi,w.$$

Moreover, for $k=1,\ldots,K$, the following hold:

$$w_k^{n,d}=w_k^f+O(1/\sqrt{n}),$$
$$\phi_k^{n,d}=\phi_k^f+O(1/\sqrt{n}).$$

4.5. The Case of Widespread Multiple Listing and No Travel-Radius Restrictions

In this section, we revisit the special case studied in Section 3.1. In particular, we assume that (13) holds, i.e., π_k's are sufficiently large and $A(k)=\{1,\ldots,K\}$ for all k. As shown in Proposition 4, we have that

$$w_k=\frac{1}{\gamma}\ln\left(\frac{\Lambda}{\tilde{M}}\right)=w^*\quad\text{and}\quad\phi_k=\frac{\tilde{M}}{\Lambda}=\phi^*\quad\text{for all } k.\tag{38}$$

We further assume[28] that the equilibrium flows of the selfish routing problem satisfy the following:

$$\sum_{j\neq k}x_{kj}<\pi_k\quad\text{for all } k.\tag{39}$$

Under these additional assumptions and approximations in (36) and (37), we characterize the equilibria under the diffusion approximation further. To that end, note that

$$w_l^n=w^*+\frac{1}{\gamma\sqrt{n}}\phi^*\frac{\theta_l}{\mu_l}\quad\text{and}\quad\sigma_l^n=\frac{1}{\sqrt{n\gamma\mu_l}},\qquad l=1,\ldots,K,\tag{40}$$

where θ_l is given by (35). Also note that $\sum_{l=1}^K\theta_l=0$.

Proposition 8. *All equilibria q^n are payoff equivalent. Moreover, every equilibrium leads to the same set of virtual waiting time distributions at the various DSAs characterized by (32) and (40), where*

$$\theta_l^*=\Delta\gamma\Lambda\left(\frac{\sqrt{\mu_l}}{\sum_{j=1}^K\sqrt{\mu_j}}-\frac{\mu_l}{\sum_{j=1}^K\mu_j}\right)\quad\text{for } l=1,\ldots,K.\tag{41}$$

Without loss of generality, we relabel the DSAs such that

$$\mu_1>\cdots>\mu_K;\tag{42}$$

i.e., DSAs with larger service rates have lower indices. Since the variance σ_l^2 of the virtual waiting time at DSA l is inversely proportional to the service rate μ_l (see Equation (40)), the virtual waiting times at the larger DSAs have smaller variances: $\sigma_1<\cdots<\sigma_K$. The following proposition helps characterize the equilibrium further.

Proposition 9. *We have that $\theta_1^*<0<\theta_K^*$. Moreover, there exists $l^*\geq 2$ such that*

$$\theta_1^*<\theta_2^*<\cdots<\theta_{l^*-1}^*<0\leq\theta_{l^*}^*\quad\text{and}\quad\theta_j^*>0\quad\text{for all } j>l^*.$$

Recall that the nominal offered load is $\Lambda/\tilde{M}$ for every DSA in the fluid-scale under the current assumptions. Proposition 9 shows that the offered load of the larger DSAs will be perturbed downward from this nominal value, whereas it will be perturbed upward for the smaller DSAs. This result may seem counterintuitive at first. The intuition will be provided after the next proposition, which is immediate from (38), (42), and Proposition 8.

Proposition 10. *Expected virtual waiting times are decreasing with the size of the DSA and, hence, increasing with the variance of waiting times; that is,* $w_1^n < w_2^n < \cdots < w_K^n$.

To see the intuition behind this result, note that all DSAs offer the same virtual waiting time w^* at the fluid scale. Consider a patient choosing between two DSAs (in addition to his own DSA) that have the same waiting time w^*. Then since he wishes to minimize the minimum of the waiting times at the chosen DSA and his own, this "extreme seeking" behavior leads him to pick the DSA with the larger variance. Since the smaller DSAs have larger variances, they are more appealing (at equal expected waiting time). Thus, in equilibrium we expect the larger DSAs to have slightly shorter waits to offset the fact that their variance is smaller. Although we relied on the approximation (37) to arrive at this characterization, we arrive at the same insight in Online Appendix E using a result of Clark (1961) and verify that it is robust to the particular approximation we used here.

We close this section by a characterization of the geographical equity in equilibrium.

Proposition 11. *The geographical coefficients of variation in equilibrium are as follows:*

$$GCV_w^d = \frac{1}{\sqrt{n}}\frac{\Delta}{w^*}\sqrt{1+\left(\frac{M}{\hat{M}}\right)^2\frac{M}{\sum_k \mu_k^2}-2\left(\frac{M}{\hat{M}}\right)\frac{\sum_k \mu_k\sqrt{\mu_k}}{\sum_k \mu_k^2}},$$

$$GCV_\phi^d = \frac{1}{\sqrt{n}}\sqrt{\frac{\sum_k(\mu_k(\Lambda/M)+(1/\sqrt{n})\theta_k^*)^2(\theta_k^*/\mu_k)^2}{\sum_k(\mu_k(\Lambda/M)+(1/\sqrt{n})\theta_k^*)^2}},$$

where $\hat{M}=\sum_{k=1}^K\sqrt{\mu_k}$ *and* θ_k^* *is as given in Proposition 8.*

5. Simulation Study

This section undertakes a simulation study to assess the potential impact of multiple listing on geographic equity. It explores the value of OrganJet and how it depends on the problem parameters $\pi = (\pi_k)$ (the fraction of patients who can multiple list) and d (geographic coverage). Our primary focus is on blood types B and O, for which the organ shortage is most severe.

The analytical model advanced in this paper makes several simplifying assumptions for tractability. From the perspective of geographic equity, the focus of our paper, the single most important simplification is that it ignores the geographical sharing of organs under the current UNOS policy. (Recall that about 30% of organs are eventually offered to patients outside the DSAs in which they are procured;[29] see Davis 2011). The simulation study accurately captures the geographic sharing of organs under the current UNOS policy. We also incorporate the patients' accept/reject decisions via a simple model, which is calibrated using UNOS offer data.

We use public UNOS data to estimate the patient and organ arrival rates to various DSAs during 2009–2013 (http://optn.transplant.hrsa.gov/data/), and private UNOS organ offer and patient data to estimate patients' organ acceptance probabilities and the death rate γ while waiting for a transplant ($\gamma = 0.17$); see Online Appendix H for details.

We assume that each DSA k patient accepts an organ offer with probability α_k ($k = 1, \ldots, K$). If an organ is offered to N patients, then the probability that it will be accepted is $1-(1-\alpha_k)^N$. Setting N to the average number of organ offers in our data set and $1-(1-\alpha_k)^N$ to the percentage of organs accepted in DSA k, we estimate α_k, which is then used to simulate the accept/reject decisions of DSA k patients.[30]

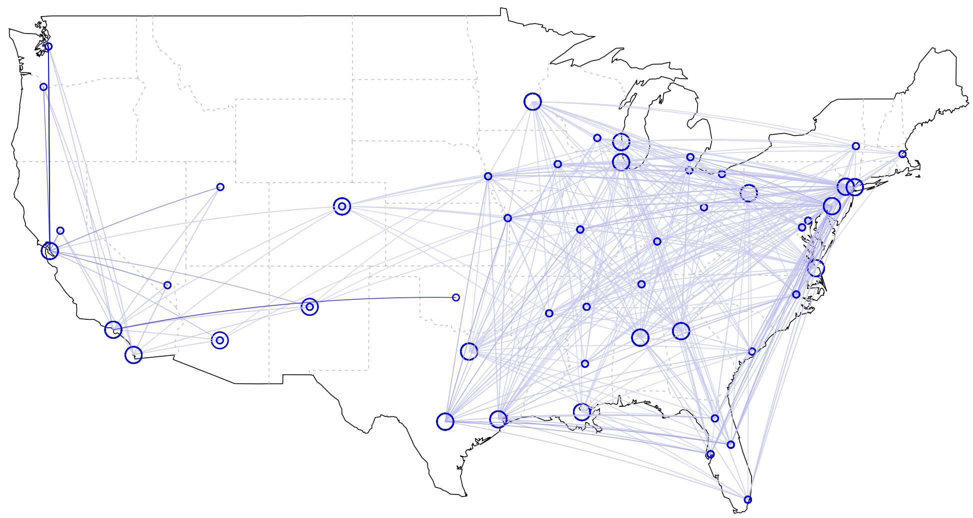
The analysis is done for each blood type separately. To validate the model, we run the simulation with no multiple listing and compare the resulting median waiting times to those provided by the UNOS (see Table 1).[31]

As a preliminary to simulating the system under multiple listing, we derive the equilibrium multiple-listing rates using the patient and organ arrival rates as well as the death rate. This is done for a combination of parameters $\pi \in \{0.05, 0.10, 0.15, 0.20, 0.25, 0.50\}$ and $d \in \{700, 1{,}000, 1{,}200, 1{,}400, 1{,}600, 1{,}800, 2{,}000\}$. Recall that patients can multiple list to DSAs that are within a radius of d miles from their own DSA, and a multiple-listing patient gets transplanted in the DSA that offers him a desirable organ first. Our analysis reveals that the multiple-listing equilibrium is robust to changes in d for $d \geq 700$. The equilibrium outcomes (e.g., arrival rates, waiting times, access probabilities, and GCV after multiple listing) are virtually identical for $d \geq 1{,}000$ miles and change only slightly for $d \in [700, 1{,}000]$. In contrast, the equilibrium outcomes are sensitive to π.

Table 1. Median Waiting Times Resulting from Simulation and Those Provided by the UNOS for Blood Types B and O, Respectively

	Year listed	Simulation	UNOS data
B	2001–2002	1,712	2,030
	2003–2004	1,806	1,935
O	2001–2002	1,707	1,832
	2003–2004	1,733	1,851

Figure 3. (Color online) Equilibrium Multiple-Listing Rates for Blood Type O on a U.S. Map for the Case of $\pi = 0.25$ and $d = 1{,}200$



Note. Each line corresponds to a particular flow, and for each flow, the large circle indicates the fly-out DSA and the small circle corresponds to the associated fly-in DSA.

Figure 3 (and Figure 8 in Online Appendix G) displays multiple-listing rates for blood type O (and B) for the case of $\pi = 0.25$ and $d = 1{,}200$.[32]

Waiting times of all DSAs for blood type O (and B), before and after multiple listing, are shown in Figure 4 (and Figure 9 in Online Appendix G). Table 2 shows

Figure 4. (Color online) Waiting Times at 58 DSAs (Based on Patient and Organ Arrival Data from 2009–2013 Provided by the UNOS) for Blood Type O Before and After Multiple Listing, for $d = 1{,}200$ and $\pi = 0.25$

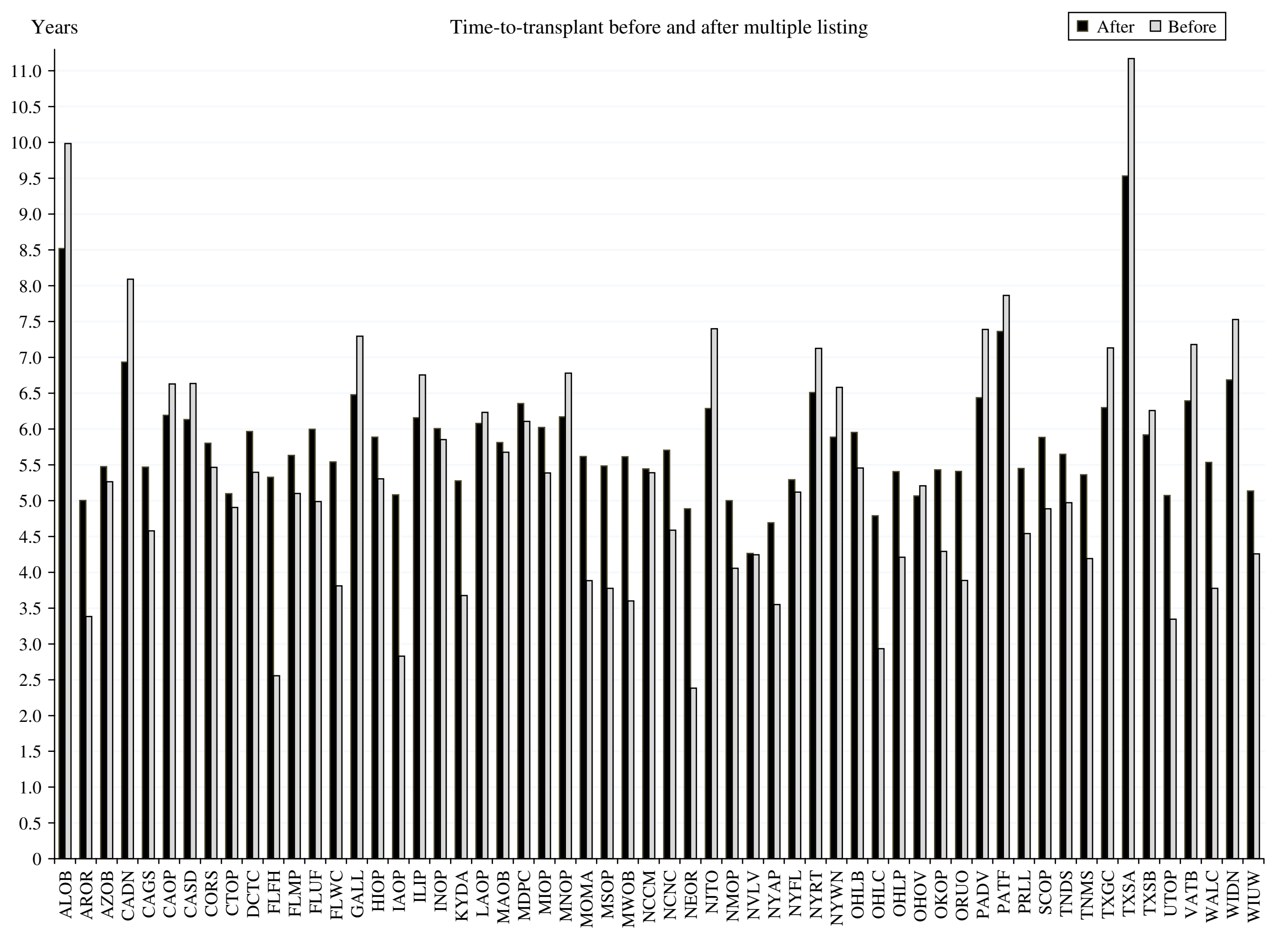


Table 2. Changes in Average Waiting Times of Blood Type O Patients (in Years) After Multiple Listing at the Most and the Least Congested Five DSAs

	DSA	Before	After
Most congested	New Providence, NJ	7.40	6.29
	San Antonio, TX	11.17	9.53
	Birmingham, AL	9.99	8.52
	Oakland, CA	8.09	6.93
	Philadelphia, PA	7.39	6.44
Least congested	Winter Park, FL	2.56	5.33
	Omaha, NE	2.38	4.89
	North Liberty, IA	2.83	5.08
	Maumee, OH	2.93	4.79
	Westwood, KS	3.60	5.61

the changes in average waiting time for the most and least congested five DSAs. Table 3 shows how GCV_w and GCV_ϕ change[33] with π and d for blood types B and O.

We observe that multiple listing improves geographic equity. To be specific, both GCV_w and GCV_ϕ decrease as π increases. Moreover, the geographic equity is not sensitive to the multiple-listing radius provided $d \geq 700$ miles. In particular, this suggests that the assumption of $A(k) = \{1, \ldots, K\}$ for all k, which was made for tractability in Proposition 3 and Sections 3.1 and 4.5, is not too critical from an equity perspective.

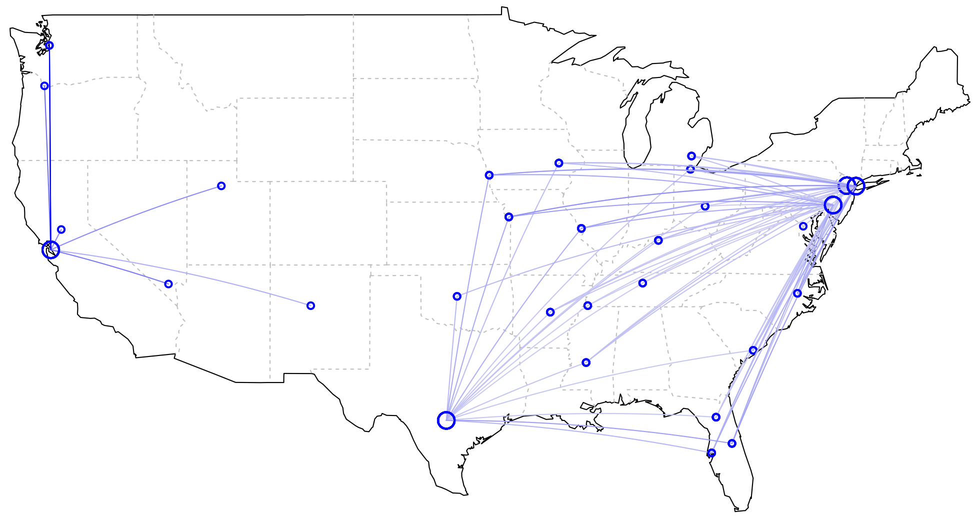
Interestingly, even if everyone has the ability to multiple list, only 12% of the patients choose to multiple list ex post, regardless of the blood type. However, the resulting network of flows is complex, as can be seen from Figure 3 (and Figure 8 in Online Appendix G). This may make it operationally challenging to implement the solution. Nonetheless, as can be seen from Figures 3 and 8 (in Online Appendix G), much of the traffic is sourced from a small number of DSAs. Therefore, focusing on these sources (or fly-out DSAs) leads to a solution that is much easier to implement and yet captures much of the value OrganJet offers; namely, one can consider the case where OrganJet offers its services only in certain DSAs.

First, we consider the fly-out DSAs: Oakland, California; New York, New York; New Providence, New Jersey; and Philadelphia, Pennsylvania. Then we include Waltham, Massachusetts, as well. In the former case, the number of people multiple listing is 35% (38%) of what it would have been for blood type O (B) if OrganJet was available everywhere. Including Waltham, Massachusetts, as a fly-out DSA increases this number to 37% (38%) for blood type O (B). Finally, we also include San Antonio, Texas, which brings the number of people multiple listing to 44% (42%) of what it would have been for blood type O (B) if OrganJet was available everywhere. Figure 5 displays the resulting flows for blood type O when OrganJet offers its services only in the following DSAs: Oakland, New York, New Providence, Philadelphia, Waltham, and San Antonio. Figures 10–14 in Online Appendix G display the resulting flows for other cases discussed.

Table 3. How GCV_w and GCV_ϕ Change with π and d for Blood Types B and O, Respectively

		d	$\pi = 0.05$	$\pi = 0.10$	$\pi = 0.15$	$\pi = 0.20$	$\pi = 0.25$	$\pi = 0.50$
B	GCV_w	700	0.2158	0.1857	0.1627	0.1460	0.1353	0.1258
		1,000	0.2125	0.1828	0.1573	0.1467	0.1369	0.1235
		1,200	0.2153	0.1835	0.1609	0.1436	0.1368	0.1222
		1,400	0.2146	0.1817	0.1622	0.1446	0.1345	0.1272
		1,600	0.2123	0.1806	0.1611	0.1464	0.1359	0.1233
		2,000	0.2179	0.1787	0.1592	0.1441	0.1385	0.1258
B	GCV_ϕ	700	0.2059	0.1686	0.1365	0.1160	0.0884	0.0525
		1,000	0.2047	0.1710	0.1337	0.1087	0.0857	0.0576
		1,200	0.2048	0.1657	0.1386	0.1084	0.0886	0.0550
		1,400	0.2040	0.1640	0.1325	0.1163	0.0872	0.0533
		1,600	0.2088	0.1659	0.1328	0.1068	0.0938	0.0556
		2,000	0.2079	0.1665	0.1401	0.1097	0.0873	0.0519
O	GCV_w	700	0.1998	0.1553	0.1308	0.1102	0.1025	0.0781
		1,000	0.1995	0.1582	0.1315	0.1096	0.1008	0.0741
		1,200	0.1994	0.1573	0.1321	0.1095	0.1020	0.0745
		1,400	0.1993	0.1558	0.1315	0.1101	0.1007	0.0757
		1,600	0.1997	0.1575	0.1314	0.1088	0.1004	0.0750
		2,000	0.1983	0.1580	0.1310	0.1091	0.1007	0.0730
O	GCV_ϕ	700	0.2185	0.1819	0.1528	0.1294	0.1096	0.0528
		1,000	0.2187	0.1827	0.1538	0.1303	0.1099	0.0522
		1,200	0.2185	0.1834	0.1539	0.1299	0.1106	0.0493
		1,400	0.2191	0.1815	0.1539	0.1310	0.1116	0.0504
		1,600	0.2183	0.1828	0.1534	0.1299	0.1103	0.0515
		2,000	0.2189	0.1834	0.1539	0.1297	0.1108	0.0511

Figure 5. (Color online) Equilibrium Multiple-Listing Rates for the Partial Solution for Blood Type O Patients ($\pi = 0.25$ and $d = 1{,}200$)



Note. In this illustrative scenario, OrganJet offers its services only in certain DSAs: Oakland, California; New York, New York; New Providence, New Jersey; Philadelphia, Pennsylvania; Waltham, Massachusetts; and San Antonio, Texas.

6. Concluding Remarks

Our goal was to understand the impact of the multiple listing of ESRD patients when a timely jet option is available on several important metrics, including the waiting times, the fraction of patients who would multiple list, the flow of such patients, the number of organs harvested, and the mortality rate. Unfortunately, a policy change to address the geographic disparity does not seem feasible in the near future[34] (see Washburn et al. 2011). Our proposal to improve geographic equity in the United States is in line with the goals of the UNOS. The key innovation is to dramatically increase the range of options where a patient can multiple list by providing jet access, now possible in the United States (and parts of Europe) because of wide availability of fractional use of midrange business jets.

Our analysis shows that multiple listing benefits not only those who multiple list, but all patients in the congested areas. Despite this, it is plausible that wealthier patients may have improved access to transplants. However, Yeh et al. (2011) note that there is greater representation of minorities (especially African American and Asian American patients) in the congested areas, who will benefit as more patients from their DSAs multiple list elsewhere. Therefore, it is not a priori clear what the net impact of these two forces will be, and it constitutes an interesting future research topic.

The stylized model in this paper is just the first step in understanding the important issue of geographic disparity and possible solutions. There are several interesting future topics of research, such as incorporating in the model the possibility of transplants across compatible blood types, allowing nonstationarities in the arrivals of organs and patients, and general queueing service discipline capturing further details of the organ allocation policy.

In parallel to this analysis, we are in discussions with fractional jet companies, the UNOS, Medicare policy makers, insurance companies, the transplant centers in key fly-out and fly-in OPOs, and angel networks (who can provide certain jet hours for free). This analysis has given us the design parameters for the creation of an operating system.

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Endnotes

[1] There is one exception to this: No patient can list at a transplant center in New York State if he is already listed at another transplant center. However, people from New York can multiple list elsewhere after they have first listed in New York.

[2] Although the current practice of multiple listing may alleviate geographical disparities in principle, its impact is limited because of the limited time a kidney can spend outside the human body, i.e., the cold ischemia time. The cold ischemia time severely restricts a

patient's ability to multiple list at transplant centers of his choosing because he will have limited time to travel when he accepts a cadaveric organ offer. Indeed, Merion et al. (2004) studies the multiple-listing patients' choices of the second transplant center to list and notes that multiple-listing patients are typically willing to make a two to three hour highway trip, which, of course, limits their options to those transplant centers within a radius of about 200 miles.

[3] See https://www.unos.org/liver-public-forum-final-agenda-and-presentations-available/ (accessed June 19, 2015).

[4] Currently, only a small percentage of multiple-listing patients lists to more than one DSA in addition to their own DSA.

[5] The waiting time of a potential recipient is measured from the start of dialysis under the new allocation policy.

[6] A sensitized patient has exceptionally high antibody levels that react to foreign tissues.

[7] This is introduced in the revised allocation policy. However, because every DSA is overloaded, we expect that the top 20% of the organs will be allocated locally, and hence this change to the policy will have no significant impact on the existing geographic inequity as also predicted by the Kidney Allocation Committee.

[8] Kidney allocation system in the United States can be viewed as having 1 queue per DSA, resulting in 58 queues.

[9] We assume that patients with living donors do not wait in the system; i.e., they arrive with living donors and receive a transplant right away. Although the assumption is not completely realistic, there is some empirical evidence supporting it, namely, the median waiting times for transplants with living donors are much less than those with deceased donors. (The median wait times for living donors vary from five to nine months across different regions, whereas the wait times for deceased donors are several years.) In other words, the patients with living donors are deleted from our model. Moreover, a simulation study (available from the authors) incorporates the patients with living donors and shows that the results are insensitive to this. To repeat, this is because the time to receive a transplant from a living donor (conditional on receiving one) is significantly shorter than that from a deceased donor. Similarly, the pediatric patients constitute only a small fraction of patients, and they wait only a few months, in stark contrast to adult patients.

[10] Although one can further refine the patient types along other attributes, this would offer no additional insights given that the current allocation policy is blind to such attributes.

[11] This is largely driven by the deceased donor kidney allocation policy.

[12] To study the impact of multiple listing on organ wastage, we undertake a simulation study in Section 5 that takes into account patients' accept/reject decisions.

[13] An extensive simulation study (available from the authors) shows that the incremental improvement in geographic equity under triple listing beyond that under double listing is not significant.

[14] The edge (s_k, s_k) corresponds to patients in DSA k who can multiple list but choose not to and wait for a transplant locally.

[15] Recent work by Lee et al. (2009) estimates the value of life at \$129,000 per year, which suggests that the dollar value of receiving a transplant is several millions of dollars. Multiple listing can improve the probability of receiving a transplant significantly, and hence is valued similarly. In contrast, we estimate that the financial cost of multiple listing is around (or less than) ten thousand dollars and is negligible when compared with its potential benefit.

[16] This rate includes local patients as well as those patients multiple listing from other DSAs.

[17] To see this, observe that (6)–(8) is equivalent to the selfish routing formulation (NLP2) given on page 29 of Roughgarden (2002).

[18] As our base case of analysis, we take $d = 1{,}200$ miles, which corresponds to three hours of flight time with a midrange jet at the speed of 400 miles per hour.

[19] Under conditions of Proposition 4, multiple listing enables the pooling of the distributed resources in a strong sense, which resembles the complete resource pooling phenomenon observed in queueing networks; see, for example, Harrison and Lopez (1999) and Ata and Kumar (2005). Indeed, every patient experiences the same time to transplant (conditional on receiving a transplant) and probability of receiving a transplant regardless of the DSA he lives in and whether or not he multiple lists. The equal waiting times at different DSAs are closely related to the notion of the global FCFS phenomenon; see Talreja and Whitt (2008). The authors consider a multiclass, parallel-server, overloaded queueing system with abandonments. They note that global FCFS holds in the fluid model if when one atom of fluid arrives to the system before a second atom of fluid, it starts service before the second atom of fluid. They show that global FCFS is equivalent to having identical waiting times for different queues. Interestingly, Talreja and Whitt (2008) show that in a parallel-server fluid model with an arbitrary exogenous routing structure, global FCFS may not be achieved. In contrast, the routing structure is endogenous in our setting, and global FCFS is always achieved, as shown in Proposition 4 (provided that there are sufficiently many patients who can multiple list). Intuitively, whenever $w_i < w_j$ for DSAs i and j, patients will gravitate toward DSA i, and away from DSA j to achieve $w_i = w_j$. Thus, it is natural to have global FCFS and equal waiting times. Also note that our setting has another important difference from Talreja and Whitt (2008): they consider a many-server asymptotic regime, whereas we focus on a single-server asymptotic regime as in Jennings and Reed (2012).

[20] A superscript of n will be attached to the quantities of interest corresponding to the nth system.

[21] Candogan et al. (2011, 2013) study finite-player potential games and the games that are close to those, referred to as near-potential games, and establish that the (approximate) equilibria of the two are close. Although our setting is different from theirs, their findings and our choice of strategy space (see (16)) can be traced back to the same intuition: A small perturbation of the problem primitives should lead to a small perturbation of the outcomes. Loosely speaking, once we restrict attention to strategies of the form (16), the resulting game studied in this section can be viewed as a near-potential game (by virtue of Theorem 1), albeit with a continuum of agents. Thus, based on the intuition from the work of Candogan et al. (2011, 2013), it is natural to expect that the equilibrium outcomes are close to those of the selfish routing game studied in Section 3. This intuition is consistent with Proposition 7. However, without Theorem 1 and (16), patients' utilities are not necessarily close to those in the selfish routing game of Section 3. Hence, the game studied in this section is not a near-potential game a priori. Therefore, the aforementioned intuition hinges critically on (16) and Theorem 1.

[22] Or, using the terminology that is standard in queueing theory, those are the service rates received from various DSAs, which correspond to the servers.

[23] Recall from Propositions 12 and 14 of Online Appendix A that $\bar{w}_k = (1/\gamma)\ln(\lambda_k/\mu_k)$ and $\bar{\phi}_k = \mu_k/\lambda_k$, which yield $\bar{\phi}_k = e^{-\gamma\bar{w}_k}$.

[24] If $\bar{\phi}_j = \bar{\phi}_k$, then the same reasoning applies.

[25] Note that $\bar{x}_{kj}$ corresponds to the (appropriately scaled) equilibrium flows of the selfish routing problem.

[26] We refer the reader to Mas-Colell (1984) (and Schmeidler 1973) for a mathematically precise definition of Nash equilibrium for games with a continuum of agents.

[27] The quantities under the fluid approximation refer to those derived in the selfish routing equilibrium; see Section 3.

[28] Note that if (39) does not hold, then increasing all π_k's by a small $\epsilon > 0$ ensures (39).

[29] Presumably, these are mostly lower-quality organs for which the rejection rates, and hence, the discard rates, are significantly higher.

[30] When a DSA k patient multiple lists to DSA j, he is assumed to have the same probability, namely, α_k, of accepting an organ offer in both locations. A sensitivity analysis (available from the authors) varies this probability for DSA j between α_k and α_j and shows that the results are robust to this assumption.

[31] The median waiting times are listed in this table since average waiting times are not provided by the UNOS. However, the simulation model also outputs the average waiting times, which are 1,804 and 1,826 for blood type B, respectively, for years 2001–2002 and 2003–2004. For blood type O, average waiting times are 1,737 and 1,740, respectively, for years 2001–2002 and 2003–2004.

[32] Note that in Figure 3, some locations are both fly-in and fly-out DSAs, e.g., Denver, Colorado, and Phoenix, Arizona. These DSAs are net fly-in DSAs, and they help connect the West to the East, which results in more balanced waiting times and access to transplants across the country.

[33] For brevity, we report only the outcomes of the simulation study because we believe that they are more accurate representations of reality. Nonetheless, we also observed that the analytical approximations (based on the fluid model) are close to (and, hence, consistent with) the simulation results.

[34] Otherwise, flying the organs could have been easier to implement.

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