A Divide-and-Conquer Approach to Commercial Territory Design

M.Angélica Salazar-Aguilar, J. Luis González-Velarde, R. Z. Ríos-Mercado

Interuniversity Research Centre on Enterprise Networks, Logistics and Transportation (CIRRELT), HEC Montréal
Montréal, Canada
angelica.salazar@cirrelt.ca

Center for Quality and Manufacturing, Tecnológico de Monterrey
Monterrey, Mexico
gonzalez.velarde@itesm.mx

Graduate Program in Systems Engineering, Universidad Autónoma de Nuevo León
San Nicolás de los Garza, Mexico
roger@yalma.fime.uanl.mx

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Abstract A new heuristic procedure for a commercial territory design problem is introduced in this work. The proposed procedure, based on the divide-and-conquer paradigm, consists basically of a successive dichotomy process of a given large instance of the problem. During this process, a series of integer quadratic subproblems is solved. Computational results showed that the proposed heuristic is an attractive technique for obtaining locally optimal solutions for large instances which are intractable by using exact optimization methods.

Keywords: Territory design, Heuristic optimization, Integer quadratic programming, Divide and conquer approach

Resumen. En este trabajo se presenta un procedimiento heurístico para el diseño de territorios comerciales. El procedimiento propuesto, basado en el paradigma dividir-y-vencer, consiste básicamente en un proceso de dicotomías sucesivas a partir de una estancia dada. Durante este proceso se resuelven una serie de subproblemas de programación cuadrática entera. Resultados computacionales muestran que la heurística propuesta es una técnica de solución atractiva que permite la obtención de soluciones óptimas locales para instancias grandes del problema, las cuales resultan intratables al intentar resolverlas a través de métodos exactos.

Palabras Clave: Diseño territorial, Optimización heurística, Programación cuadrática entera, Procedimiento dividir y vencer.

1 Introduction

The problem addressed in this work is motivated by a real-world application from a beverage distribution firm in the city of Monterrey, Mexico. The problem consists of finding a partition of the entire set of city blocks (basic units, BUs) into p territories, such that a measure of territory compactness is maximized. Additionally, it is required to find territories that are connected and balanced (similar in size) with respect to the number of customers and product demand. A territory is connected if the set of BUs belonging to it induces a connected subgraph.

This problem can be found in every distribution firm before the routing plan takes place. Having shorter routes in product distribution is a direct consequence of having compact territories in the design stage. In addition, it is well established by the firm that compact territories reduce the number of unsatisfied customers caused by different deals offered to their customers.

The first related work that appears in the literature is that one studied by [22]. In this work, a reactive GRASP procedure is developed in order to minimize a dispersion measure (based on the p-Center Problem objective) subject to multiple balancing constraints (number of customers, product demand, and workload). Caballero-Hernandez et al. [6] studied a related model by considering BU joint-assignment constraints. They develop a GRASP including a pre-processing phase that first satisfies the joint-assignment constraints and then a construction phase based on a territory merging mechanism with relatively good results.

Salazar-Aguilar et al. [23] present an exact optimization framework for solving small- to medium-size instances of the problem. This method is successfully applied to both p-Median and p-Center objective models. In addition, the authors propose new integer quadratic programming models that allowed to efficiently solve larges instances by commercial MINLP solvers such as DI-

COPT and AlphaECP. These reported results motivate the solution procedure proposed in this work.

In this work, we proposed a divide-and-conquer heuristic aiming at solving large instances of the commercial territory design problem based on the p-Median Objective for measuring dispersion. This work can be seen as an extension of the work by Salazar-Aguilar et al. [23] focusing on exact methods for small- and medium-size instances of the problem.

In particular, our proposed heuristic follows a successive dichotomy' idea where at each iteration a given subproblem is partitioned into two smaller subproblems by solving an associated territory design problem with two territories. When a given subproblem is small enough, it is solved exactly by means of an integer quadratic programming model.

The proposed procedure (IQPHTDP) was evaluated over a set of randomly generated instances based on real-world data. Results revealed that IQPHTDP is a very attractive technique that allows to obtain good quality solutions for large instances in reasonable times.

The remainder of this paper is organized as follows. Section 2 is devoted to the description of the problem. Section 3 highlights relevant works on the territory design/districting literature. The proposed procedure is described in Section 4. Computational results are presented in Section 5, followed by conclusions in Section 6.

2 Problem Statement

Territory design or districting consists of dividing a set of basic units (typically city blocks, zip-codes or individual customers) into subsets or groups according to specific planning criteria. These groups are known as territories or districts. Diverse applications from different areas require the creation of territories. For instance, school districts, political districting, and sales territory design (see Kalcsics et al. [15]). There are a few works related to this commercial territory design problem. The first work related to this problem was introduced by Ríos-Mercado and Fernández [22]. Different versions of this problem have been studied by Caballero-Hernández et al. [6] and Salazar-Aguilar et al. [23].

Specifically, the firm wants to partition the basic units (blocks) of the city into a specific number of disjoint territories that are suitable according to their logistic, marketing and planning requirements. The company wishes to create a specific number of territories (p) that are balanced with respect to each of two attributes (number of customers and product demand). Additionally, each territory needs to be connected, so basic units (BUs) in the same territory can reach each other without leaving the territory. Territory compactness is required to guarantee that customers within a territory are relatively close to each other. The problem is modeled by a graph $G=(V,E)$, where V is the set of nodes (city blocks) and E is the set of edges that represents adjacency between blocks. That is, a block or BU j is associated with a node, and an edge connecting nodes i and j exists if BUs i and j are located in adjacent blocks. Multiple attributes such as geographical coordinates (c_x, c_y), number of customers, and product demand are associated to each node $j \in V$. It is required that each node is assigned to only one territory (exclusive assignment). In particular, the firm seeks perfect balance among territories, it means each territory must have around the same number of customers and product demand associated. Let $A=\{1,2\}$ be the set of node activities, where 1 refers to the number of customers and 2 refers to product demand. We define the size of territory V_k with respect to activity a as $w^a(V_k)=\sum_{i\in V_k}(w_i^a)$, where $a \in A$ and w_i^a is the value associated to activity a in the node $i \in V_k$. Hence, the target value is given by $\mu^a = \sum_{j\in V}\frac{w_j^a}{p}$. Another important constraint is that of connectivity, i.e., for each pair of nodes i, j that belong to the same territory, there must exist a path between them such that it is totally contained in the territory. In addition, in each territory the BUs must be relatively close to each other (compactness).

Depending on how the dispersion is measured, different models can be obtained. In this work we consider a dispersion measure based on the p-Median Problem. Full description of this model can be found in [23]. For completeness, we include here the combinatorial formulation of the MPTDP model studied in this work. Let Π be the set of all possible p-partitions of V. For a particular territory B_k, $c(k)$ is a territory center and d_{ij} is the Euclidian distance between nodes i and j, $i,j \in B_k$. A territory center is computed as

$$c(k) = \arg\min_{j\in B_k} \sum_{i\in B_k} d_{ij}$$

$$\text{(MPTDP)} \quad \min_{B \in \Pi} \quad f(B) = \sum_{k=1,\ldots p} \sum_{i \in B_k} d_{ic(k)} \tag{1}$$

Subject to :

$$w^a(B_k) \in [(1-\tau^a)\mu^a, (1+\tau^a)\mu^a], a \in A; k = 1, \ldots, p \tag{2}$$

$$G = (B_k, E(B_k)) \quad \text{is connected} \quad \forall k = 1, \ldots, p \tag{3}$$

In this model, the objective is to find a p-partition of V, such that the dispersion (1) on each territory B_k is minimized. Constraints (2) establish that the territory size (number of customers and product demand) should be in the range allowed by the tolerance parameter τ^a. In addition, each territory should induce a connected subgraph (3). It has been shown that MDTDP is NP-hard [24].

Furthermore, as shown in [23], there are two mathematical programming models for this problem. In our solution procedure, we make use of the quadratic integer programming (IQP) model introduced in [23] since it was shown it allows to optimally solve instances of up to 400-500 BUs. When using the linear model, the size of the instances that could be optimally solved is the range of 250 BUs.

Table 1: Summary of territory design applications, part 1.

Author	**Application**	**Criteria**	**Objective**	**Solution Technique**
Hess and Weaver [13]	Political	B,C,F	Single	Location-allocation
Garfinkel and Nemhauser [10]	Political	B,C,F	Single	Exact procedure
Hess and Samuels [12]	Sales	B,-,F	Single	Location-allocation
Bertolazzi et al. [2]	Services	B,-,F	Single	Exact procedure
Marlin [16]	Services	B,-,F	Single	Location-allocation
Pezzella et al. [19]	Services	B,C,F	Single	Location-allocation
Fleischman and Paraschis [9]	Sales	B,-,F	Single	Location-allocation
Hojati [14]	Political	B,C,F	Single	Location-allocation
Mehrotra [17]	Political	B,C,V	Single	Heuristic based on Branch & Price
Drexl and Haase [8]	Sales	B,C,V	Single	Heuristic
Guo et al. [11]	Political	B,C,F	Bi-objective	MOZART
Muyldermans et al. [18]	Services	B,C,F	Single($\sum$)	Heuristic of two phases
Blais et al. [3]	Services	B,C,F	Single($\sum$)	Tabu search

Table 2: Summary of territory design applications, part 2.

Author	Application	Criteria	Objective	Solution Technique
Bozkaya et al. [5]	Political	B,C,F	Single($\sum$)	Tabu search and adaptive memory
Ricca and Simeone [20]	Political	B,C,F	Single($\sum$)	Old bachelor acceptance
Bong and Wang [4]	Political	B,C,F	Three-objective	Tabu search and scatter search
Bação et al. [1]	Political	B,C,F	Single	Genetic algorithms
Chou et al. [7]	Political	B,C,F	Single($\sum$)	Simulated annealing and genetic algorithms
Tavares and Figueira [26]	Services	B,C,F	Bi-objective	Evolutionary algorithm with local search
Caballero-Hernández et al. [6]	Commercial	B,C,F	Single	GRASP
Segura-Ramiro et al. [25]	Commercial	B,C,F	Single	Location-allocation
Ricca and Simeone [21]	Political	B,C,F	Single($\sum$)	Descent, tabu search, old bachelor acceptance, and simulated annealing
Ríos-Mercado and Fernández [22]	Commercial	B,C,F	Single	Reactive GRASP
Salazar-Aguilar [23]	Commercial	B,C,F	Single	Exact procedure

3 Related Work

Districting problems are similar to clustering problems in the sense that both seek to find suitable partitions of the problem; however, there are fundamental differences that make clustering methods not applicable to districting problems. The presence, for instance, of balancing and connectivity constraints make districting problems unique in this regard. For an extensive survey on clustering methods the reader is referred to the work of Xu and Wunsch [27]. There is also commercial software available such as TerrAlign (*http://www.terralign.com*) and AlignStar (*http://www.alignstar.com/*); however, this software is limited to handling sales force deployment in territory design with different objective and planning requirement measures and therefore cannot be used in our particular districting application.

Tables 1 and 2 contain a summary of the most important work on territory design that have been developed in diverse fields such as political districting, sales districting, and public services. These tables illustrate the main features included on these applications. Planning criteria (third column) as balancing, connectivity, and fixed number of territories are shown as 'B', 'C', and 'F', respectively. In those works where the number of territories is not fixed, the capital letter 'F' is replaced by 'V', and '-' appears in the cases where connectivity is not a constraint. In the fourth column, 'Single($\sum$)' means that two or more criteria were placed together in a weighted sum objective function.

This survey reveals that there are only a few works addressing the commercial territory design problem. Furthermore, among those works, the only studying p-Median based dispersion measures focus on exact methods for small- and medium-size instances. Therefore, the contribution of our work is to present a heuristic for solving large instances of the commercial TDP with p-Median based objective function.

4 Proposed Divide-and-Conquer Procedure

The main idea behind is to decompose the problem (or subproblem) into two smaller subproblems by solving a TDP model with $p = 2$ super-territories. This stems from the fact that solving a TDP with $p = 2$ is considerably easier to solve than solving a TDP with a large value of p. When building this subproblem with $p = 2$ special attention must be paid to how to choose the tolerance parameter for the balancing constraints. As we recall, a feasible design is one which presents imbalances within τ^a percent from the target value μ^a. If this value were to be used in the subproblems, the error would accumulate yielding infeasible designs. This motivates the introduction of a control parameter ρ whose main role is to adjust the tolerance level in the subproblems aiming at yielding feasible designs as output. This parameter is typically fine-tuned empirically. This 2-partition procedure is iteratively performed to create subproblems of smaller size with respect to the number of BUs. When this number of BUs for a given subproblem is smaller than a user-specified threshold *maxN*, the subproblem is no longer 2-partitioned, but solved optimally with an appropriate value of p. As stated before, a reasonable value for *maxN* is 300.

Algorithm 1 IQPHTDP(*I, maxN,* ρ)

Require:
- $I = I(V, p)$:= Instance of TDP, where V is the set of BUs and p is the number of territories
- *maxN* := Threshold on BUs for solving the subproblems
- ρ := Control parameter for adjusting the range of the balance constraints

Ensure: $S = (V_1, \ldots, V_p)$:= A p-partition of V

```
S ← ∅
I_0(V, p) ← I(V, p) {Original instance}
L ← {I_0} {Subproblem list}
while ( L ≠ ∅ ) do
    I_c(V_c, p_c) ← POP(L) {Remove instance from L}
    if ( |V_c| ≤ maxN ) then {Solve the subproblem}
        S_c = (S_1, ..., S_{p_c}) ← SOLVE(V_c, p_c)
        S ← S ∪ S_c {Add partition to solution set}
    else {Partition the subproblem into 2 subproblems}
        S_c = (S_1, S_2) ← SOLVE(V_c, 2)
        p_1 ← ⌈p_c / 2⌉
        p_2 ← p_c − p_1
        L ← L ∪ {I(S_1, p_1), I(S_2, p_2)} {Add the two new subproblems to L}
    end if
end while
return S = (V_1, ..., V_p)
```

Algorithm 1 shows the proposed solution procedure in pseudocode. The algorithm takes as input a problem instance I. Note that when solving a subproblem by means of $SOLVE(V_c, p_c)$, a p_c-partition $S_c = (S_1, \ldots, S_{p_c})$ is sought and the balancing constraints are adjusted as follows:

$$(1-\rho\tau^a)\mu_c^{(a)} \leq \sum_{j\in S_k} w_j^a \leq (1+\rho\tau^a)\mu_c^{(a)},$$

where the target $\mu_c^{(a)}$ is computed as:

$$\mu_c^{(a)} = \frac{1}{p_c}\sum_{i\in V_c} w_i^a.$$

The control parameter ρ should be fine-tuned. Typical values are in the [0.1,0.5] range. It helps to keep balanced partitions as much as possible and it is required because if the initial dichotomy produces a 2-partition with high relative deviation with respect to the average (target value), in the following dichotomy this value carries an aggregated effect that may render some unbalanaced territories at the end.

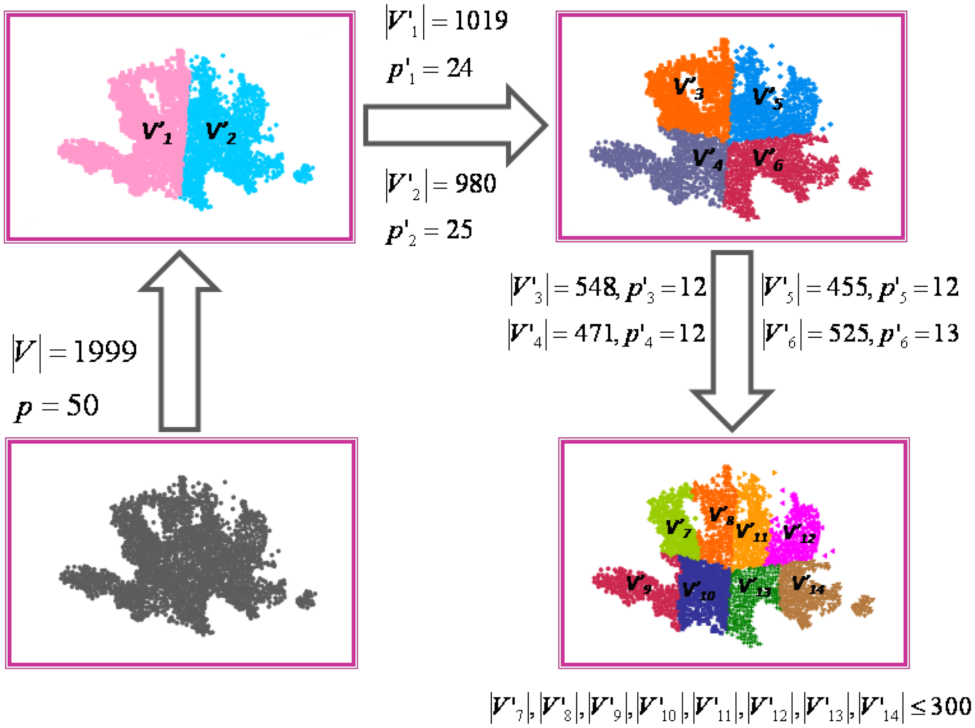

Figure 1: Successive dichotomy process for solving instance I.

Computational complexity: The number of subproblems thar are solved by IQPHTDP for an instance of size (n, p) is bounded by $O((2^{\alpha+1}-1)/(\alpha-1))$ where $\alpha = log_2\lceil n/maxN\rceil$. Now, each subproblem requires solving an IQP which is basically an enumerative procedure such as branch and bound that has an exponential worst-case theoretical bound. However, in practice, relative large instances can be handled. For instance, consider an instance of size (2000, 40). IQPHTDP would solve 1 subproblem of size (2000, 2), two subproblems of size (1000, 2), four subproblems of size (500, 2) and eight subproblems of size (250, 5), that is 15 subproblems. Each of them took from 1 minute up to 30 minutes and the most time consuming were those subpoblems with $p > 2$. We should point out that attempting to solve directly an instance of size (2000, 40) by IQP is useless.

An Illustrative Example

Suppose that IQPHTDP is used for solving an instance I with $(n, p) = (1999{,}50)$ and input parameters $maxN = 300$, and $\rho = 0.8$. Figure 1 shows the dichotomy process. Note that in the first dichotomy each partition V_1' and V_2' contains half the total number of required territories (thus 25 out of 50) and the number of BUs on each of them is greater than $maxN$, thus another dichotomy is needed. Partitions V_1' and V_2' are used to generate two subproblems of TDP ($(G_1' = (V_1', E(V_1'))) \subset G$, and $(G_2' = (V_2', E(V_2'))) \subset G$, respectively) which are solved using $p = 2$. In Figure 1, (V_3', V_4') corresponds to the 2-partition of V_1', and (V_5', V_6') is a 2-partition of V_2'. These partitions V_3', V_4', V_5', and V_6' contain more BUs than the allowed by $maxN$, so the dichotomy process is applied on each of them until the last obtained partitions $V_l' : l = 7, \ldots, 14$ contain less BUs than the limit value (given by $maxN$). The latter are solved using the number of territories contained on each partition. For instance, the subproblem given by V_7' is solved for $p_7' = 6$ and the subproblem given by V_8' is solved for $p_8' = 6$. The upper and lower balancing requirements are taken from the original instance I. Note that the balancing requirements for dichotomy are computed using the control parameter ρ and the number of territories contained on each sub-instance (see Algorithm 1).
The final solution for instance I is computed by putting together all partitions obtained for solving the small subproblems (in the example the small subproblems are those generated by $V_l' : l = 7, \ldots, 14$). Figure 2 shows the final solution

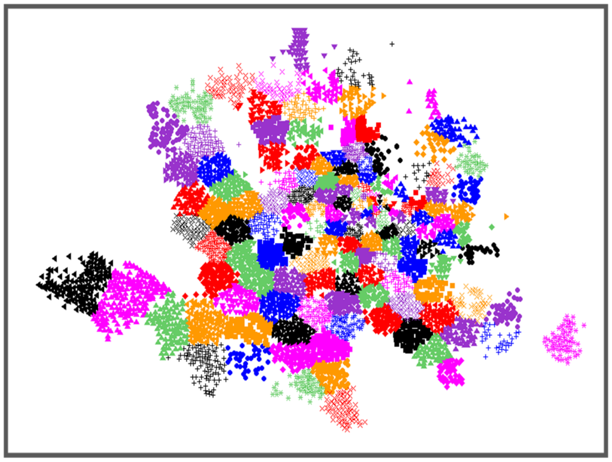

Figure 2: Final solution for instance I (using IQPHTDP).

obtained for instance I by applying IQPHTDP.

Note that some small subproblems may be infeasible with respect to the balancing constraints, so the solution for the original instance will be infeasible. This can be avoided by selecting a suitable value for the ρ parameter. In any other case, a simple local search procedure can be applied to the final solution given by IQPHTDP in order to reach a feasible solution.

5 Experimental Work

The procedure was coded in C++, and compiled with the Sun C++ compiler workshop 8.0 under the Solaris 9 operating system and run on a SunFire V440. Each integer quadratic subproblem is solved by calling GAMS/DICOPT MINLP solver. The data sets were taken from the library developed by [22]. These data set contains randomly generated instances based on real-world data provided by the firm. The number of customers, and product demand are generated from distributions based on historical data. The experimental work was carried out over two instance sets $(n, p) \in \{(1000,50),\ (2000,50)\}$ with $\tau^a = 0.05$. For each of them 10 instances were generated. Different values of ρ were used in order to determine the effect produced by this parameter in the final solution reported by the IQPHTDP procedure.

In Table 3, the first column contains the instance name and each of the following columns show the objective value reported by the IQPHTDP for $\rho \in \{1.0, 0.1, 0.2\}$. Appopiate selection of parameter ρ is very important for the successful of the proposed procedure. If $\rho = 1.0$ it means that the balancing deviation in all IQP subproblems is given by τ^a. This implies that, when the size of a partition is really close to the balancing bounds, subsequent partitions created from this partition may be very unbalanced with respect to the target value in the original instance. Hence, the final solution reported by IQPHTDP is infeasible with respect to the balance constraints in the original problem. In contrast, if the ρ value is very restrictive, some subproblems can not be solved with feasibility (see $\rho = 0.1$) and then, an infeasible solution to the original instance is obtained. When $\rho = 0.2$ was set it allowed to solve more instances than $\rho = 0.1$. Similar behavior was observed for those instances with $(1000, 50)$. However, for these instances, $\rho = 0.1$ was a good choice for getting feasible solutions, see Table 4.

To the best of our knowledge, there is not a heuristic procedure that allows to obtain solutions for the problem addressed in this work. In [22], the authors develop a reactive GRASP for the TDP under a p-Center based objective function. Even though that heuristic was developed for a different problem (i.e., different objective function and three balancing constraints rather than two), we have adapted that procedure for using two balancing constraints, and measure the quality of

Table 3: Best dispersion values (p-Median) for instances from (2000, 50).

Instance	$\rho = 1.0$	$\rho = 0.1$	$\rho = 0.2$
DU2k-1	Infeas	Infeas	54423.02
DU2k-2	Infeas	54337.56	54487.95
DU2k-3	Infeas	Infeas	55111.29
DU2k-4	Infeas	55642.04	54963.38
DU2k-5	Infeas	54616.84	55122.05
DU2k-6	Infeas	54145.92	55070.89
DU2k-7	Infeas	54813.34	Infeas
DU2k-8	Infeas	53048.47	54722.55
DU2k-9	Infeas	54968.87	55402.97
DU2k-10	Infeas	Infeas	55085.06

Table 4: Best dispersion values for instances from (1000,50).

Instance	$\rho = 1$	$\rho = 0.1$
DU1k-1	Infeas	25679.38
DU1k-2	Infeas	26455.53
DU1k-3	Infeas	25965.95
DU1k-4	Infeas	26286.99
DU1k-5	Infeas	26522.25
DU1k-6	Infeas	26180.19
DU1k-7	Infeas	26325.41
DU1k-8	Infeas	27022.62
DU1k-9	Infeas	26347.22
DU1k-10	Infeas	26896.69

the design obtained in terms of the intended TDP with p-Median objective. We called this modified procedure GRASP-RF. We solved the two instance sets using both IQPHTDP and GRASP-RF. We compare the quality of the designs obtained by each method under the TDP with p-Median objective. In addition, we also assess the quality of the solutions found by our method when aimed at solving the other problem, that is, the TDP under p-Center objective, and compare them with the solutions obtained by GRASP-RF. Tables 5 and 6 show a summary of this test for the two different data sets. In these tables, column 1 show the instance name. Columns 2 and 3 show the comparison of the heuristics for the TDP under the p-Median objective function, which is the problem addessed in this work. As can be seen, the solutions obtained by IQPHTDP are best in 19 out of 20 instances. The only instance where IQPHTDP failed was DU2K-07.

Now, columns 4 and 5 in Tables 5 and 6 show the comparison between heuristics for the TDP under the p-Center objective (TDPC). Note that even though GRASP-RF was specifically designed for addressing the TDPC, and therefore obtained in general better solutions for this problem than the ones found by IQPHTDP, our method is still very competitive, helping find some better solutions in some cases. For instance, we observed that for instances from $(1000, 50)$ the GRASP-RF did not report feasible solutions for 2 out of 10 whereas our method did find feasible solutions in all cases. Furthermore, there were 5 out of 10 instances where the solution reported by IQPHTDP was better than the solution obtained by GRASP-RF.

Table 5: Comparison between IQPHTDP and GRASP-RF. Instances from (1000,50).

Instance	*p*-Median		*p*-Center	
	IQPHTDP	GRASP-RF	IQPHTDP	GRASP-RF
DU1K-01	25679.38	31541.49	71.89	74.68
DU1K-02	26455.53	30289.81	82.13	69.38
DU1K-03	25965.95	30350.12	73.56	72.77
DU1K-04	26286.99	31084.62	68.1	69.87
DU1K-05	26522.25	30154.66	72.79	67.54
DU1K-06	26180.19	Infeas	68.47	Infeas
DU1K-07	26325.41	29173.25	64.28	71.04
DU1K-08	27022.61	Infeas	69.78	Infeas
DU1K-09	26347.22	30048.23	70.09	67.07
DU1K-10	26896.69	29369.11	77.31	62.17

Table 6: Comparison between IQPHTDP and GRASP-RF. Instances from (2000,50).

Instance	*p*-Median		*p*-Center	
	IQPHTDP	GRASP-RF	IQPHTDP	GRASP-RF
DU2K-01	54423.02	58909.07	76.69	66.07
DU2K-02	54487.96	61133.65	85.41	63.39
DU2K-03	55111.29	58654.13	75	63.85
DU2K-04	54963.32	58916.57	67.73	62.3
DU2K-05	55122.05	58676.64	67.71	61.15
DU2K-06	55070.89	59558.59	81.36	65.72
DU2K-07	Infeas	62371.46	Infeas	68.38
DU2K-08	54722.55	59908.42	80.83	67.55
DU2K-09	55402.97	58590.57	74.74	66.58
DU2K-10	55085.06	58560.103	77.37	60.55

6 Conclusions

In this paper, the commercial districting problem under a *p*-Median objective for minimizing territory dispersion was addressed. A novel heuristic procedure based on the divide-and-conquer paradigm called IQPHTDP has been proposed. This procedure allows to obtain locally optimal solutions for large instances (1000 and 2000 BUs) in short time. These instances were intractable by using existing exact methods. However, the performance of this procedure depends on the choice of the control parameter ρ. As we showed in the experimental work, the best ρ value was 0.2 for those instances with 2000 BUs and 0.1 for instances with 1000 BUs. Bad values of ρ may yield highly infeasible solutions with respect to the balancing requirements. Therefore, when the final solution is infeasible, the IQPHTDP procedure can be applied by using another ρ value, however, this change does not guarantee that the new solution will be feasible and the time increases for each trial-and-error attempt of the ρ value.

In addition, Empirical evidence showed how the proposed method consistently outperformed the only available existing method from literature, to the best of our knowledge.

A natural extension of this work could be the derivation of a local search procedure to reach feasibility in those cases where IQPHTDP is not able to find feasible solutions.

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