NONPARTISAN POLITICAL REDISTRICTING BY COMPUTER*

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OR volunteers developed a compactness measure and a 'warehouse-location' heuristic to draw nonpartisan, Constitutional political districts. The heuristic maps compact and contiguous districts of equal population. The minimization criterion and compactness measure is population moment of inertia—the summed squared distances from each person to his district's center. The districting method is particularly useful when legislative impasse or indifference forces courts to intervene. Federal Courts have received a computer plan for possible use in Delaware and have asked for computer districts in Connecticut.

IN 1962 the Supreme Court[1] said federal courts may review the constitutionality of state legislative apportionments. Citizen interest in reapportioning Delaware followed. To help parties in the Delaware reapportionment suit,[2] the Committee of 39, a nonpartisan Wilmington group, gathered statistics and apportionment information on other states.

The Committee expected the courts would have difficulty implementing reapportionment and redistricting. A rapid and nonpartisan method was clearly needed to develop districting plans to meet prespecified criteria.

A volunteer team of five OR analysts and engineers has developed an appropriate method based on the familiar criteria: equal population, contiguity, and compactness.

CRITERIA

POPULATION equality—"one man–one vote"—was ruled basic to constitutional districting.[3] But many different plans can satisfy this criterion so the Supreme Court told lower courts to develop additional criteria.

Contiguity requires that each district be a single land parcel.

Compactness, although required by many state constitutions and originally by U. S. law for Congress, is not operationally defined, let alone measured. 'Compact' generally means consolidated rather than spread

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out, e.g., square rather than long. No geometric measures of compactness have been widely accepted.[4, 5]

A numerical measure was needed to permit manipulating the data. We proposed to consolidate by population and not just by geography. Our measure is the sum of squared distances from each person to his district's center.[6] This is the moment of inertia of the district's population about the district population center summed over all districts.

Others have suggested or quietly used more debatable criteria:

- Homogeneity (or heterogeneity) of district population.
- Political or historical boundaries.
- Keeping current legislative districts and incumbents.
- Political gain (gerrymandering).

While any of these could be included in a model, we have used only population equality, compactness, and contiguity.

WAREHOUSE-LOCATION HEURISTIC

THE DISTRICTING problem is analogous to the 'warehouse-location' problem.[7-12] We must locate a specified number of warehouses (district centers) and assign customers (population units) to each warehouse. Total cost of assignment (sum of squared distances between each person and his district center) is to be minimized. Warehouse capacities (district populations) must be nearly equal. The latter restriction and the known number of districts are specific to the districting problem but not to the warehouse-location problem.

An integer programming formulation is given in Appendix I. The number of integral variables is equal to the square of the number of population units to be assigned to districts—650 in Delaware. No algorithm currently exists to solve this formulation even for so small an area.[13]

Other warehouse-location techniques were unsatisfactory; either warehouse capacities were unrestrained, or codes were unavailable or too small. We resorted to an approach built around existing transportation codes. While not the ultimate in districting programs, it worked.

Our population units are U. S. Census enumeration districts (ED). This unit has natural boundaries, e.g., rivers, highways, or railroads, desirable for legislative districts (LD). Population equality in Delaware LD's (12,000 people) was possible even though each ED (averaging 1,000 persons) must be entirely within one LD. For many states LD's will be large enough that census tracts (averaging 4,000 people) or even counties can be used as population units.

Essentially the heuristic (Fig. 1) is to:

1. Guess district centers.
2. Use a transportation algorithm to assign population equally to these cen-

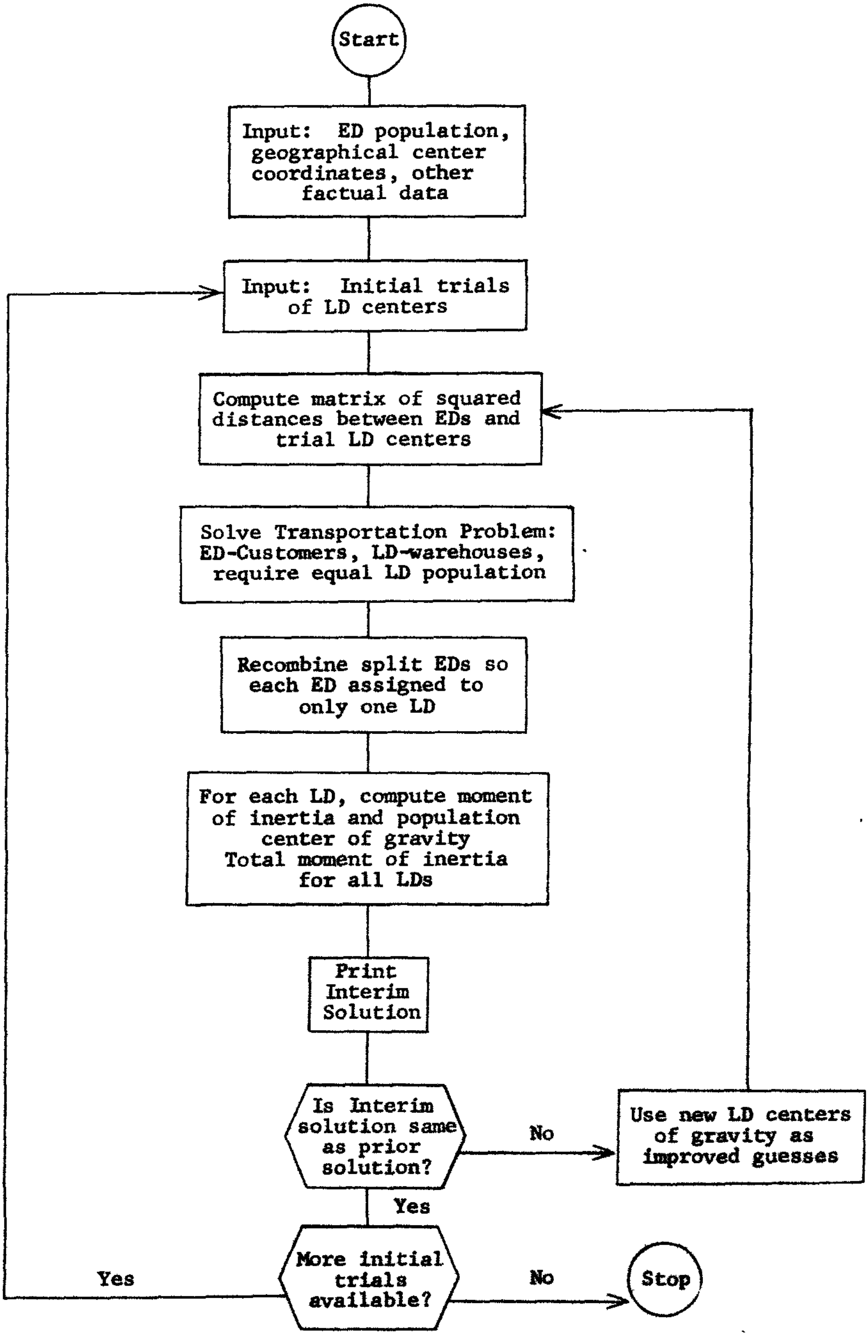
Start
Input: ED population, geographical center coordinates, other factual data
Input: Initial trials of LD centers
Compute matrix of squared distances between EDs and trial LD centers
Solve Transportation Problem: ED-Customers, LD-warehouses, require equal LD population
Recombine split EDs so each ED assigned to only one LD
For each LD, compute moment of inertia and population center of gravity Total moment of inertia for all LDs
Print Interim Solution
Is Interim solution same as prior solution?
No
Use new LD centers of gravity as improved guesses
Yes
More initial trials available?
Yes
No
Stop

Figure 1

ters at minimum 'cost' (i.e., minimum sum of squared distances each person is from his district center).

3. Adjust assignment so each ED is entirely within one LD.
4. Compute centroids and use as improved district centers.
5. Repeat from step 2 until solution converges.
6. Try more initial guesses.

While we have no convergence guarantee, so far all sets of guessed centers have converged to local minima in less than ten 'transportation' solutions. Because step 3 may increase the 'cost' found in step 2, the convergence is not always monotonic.

To pick the best of the local minima, we:

1. Reject noncontiguous solutions.
2. If it exists, select a solution dominant in compactness and population equality.
3. Otherwise, choose the most compact solution within a given population deviation limit.

We could weight compactness and population deviation to form a single criterion. More experience is needed, however, to understand the implied tradeoffs.

APPLICATIONS

New Castle County Council

Early in 1964, Governor Elbert Carvel's Committee to Study Reorganization of the Government of New Castle County asked us to prepare councilmanic districts for that county. We used the computer heuristic to develop compact and contiguous districts whose populations differed from the average by no more than 12 per cent. The high deviation resulted from using large population units (census tracts with population averaging 4,000 people) and the restriction that Wilmington boundaries must be preserved. The efficiency of computer districting let us district for council sizes from 3 to 9.

The Governor's Committee approved our seven-district solution and recommended it to the legislature.[14] In May 1965, however, the legislature—80 per cent Democrats—rejected the computer plan and substituted one of their own.

Delaware Legislature

In June 1964 the Supreme Court declared existing Delaware legislative districts unconstitutional.[15] With the Committee of 39 we used the technique to prepare Constitutional districts for the entire State ignoring county and city lines.

Meanwhile, Democrats in the State Legislature passed new districting legislation. This was also challenged in Federal Court as a 'gerrymander' still not achieving population equality. In July the Committee of 39, as "friend of the court," testified for two days on our computer plan[16] backed by plaintiffs. The court was extremely interested in a nonpartisan districting alternative by computer, but would have preferred a unique solution.

A comparison between the Committee of 39 computer plan and the new districting law (SB336) is given in Table I. The computer plan was

TABLE I

COMPARISON OF DISTRICTING PLANS IN DELAWARE

	Committee of 39 computer plan	Senate bill 366 passed July 1964
Senate		
Number of seats	17	18
Maximum district population	27,197	>27,072[a]
Minimum district population	25,340	20,743
Compactness (10^6 people—miles^2)	8.7	12.2
House		
Number of seats	35	35
Maximum district population	13,371	15,500[b]
Minimum district population	12,102	<11,249[a]
Compactness (10^6 people—miles^2)	3.3	4.0

[a] Legislative district boundaries split smallest census population units; these upper or lower bounds are for extreme districts based on minimum or maximum population from intact census units.

[b] Estimated.

substantially more compact and had no district deviating more than 5 per cent from the average population. Maximum deviation in SB336 was at least 22 per cent.

The Federal Court ruled that the November 3 election proceed under SB336. They felt the new legislation improved the Delaware situation sufficiently not to warrant delay. They will rule on its constitutionality at a later date, however.

Connecticut Legislature

Recognizing the merits of computer districting, a Special Federal Court appointed the Director of the Yale Computing Center as Master to prepare Constitutional legislative districts, should the Connecticut Legislature balk at the task for a third time.[17] Yale was prepared to use our heuristic for this assignment. The computer redistricting threat probably influenced the Legislature to pass Constitutional redistricting ten days before the Court's January 1965 deadline.

FUTURE NEEDS AND PLANS

THE CURRENT version of the computer program is in FORTRAN IV, on the IBM 7040. On smaller problems, those involving 10 or less voting districts and 170 or less population units (ED's), the transportation solution step took about 50 per cent of the total problem solution time. This percentage increased rapidly as the problem size increased. On a 35×299 problem the transportation solution step took 96 per cent of the total problem running time.

Some typical times required by the transportation algorithm to reach an optimal solution using an IBM 7040 computer are shown below.

Problem Size	Solution Time (Min)
5×105	0.27
5×105	0.40
11×98	0.67
8×167	0.55
8×167	1.37
10×170	1.06
10×170	3.92
7×299	2.80
7×299	3.46
17×299	15.80
35×299	29.8
35×299	48.5

We are striving to:

- Increase the maximum problem size.
- Speed up running time.
- Incorporate additional criteria.
- Document the program.
- Make it available to others.

Future efforts will be supported by a Ford Foundation Grant through the National Municipal League.

Other approaches to computer redistricting are summarized in Appendix II.

We urge other analysts to:

- Volunteer your skills to civic groups wrestling with redistricting in your local area.

- Develop an optimizing algorithm.
- Incorporate in this or your own formulation restraints that may seem relevant in special cases.

APPENDIX I

Integer Programming Formulation

The required number of legislative districts, k, will be much smaller than the total number of population units, n, to be assigned. With little loss in compactness measurement we can assume only population unit centers will be district centers. Let

P_j = population of the jth population unit, $(j=1, 2, \cdots, n)$,

d_{ij} = distance between centers of population units i and j, $(i,j=1, 2, \cdots, n)$,

$$x_{ij}=\begin{cases}1 \text{ if the } j\text{th population unit is assigned to the } i\text{th center,}\\ 0 \text{ otherwise,}\end{cases}$$

a = minimum allowable district population, as a per cent of the average district population,

b = maximum allowable district population, as a per cent of the average district population.

We wish to determine the n^2 values of x_{ij} to minimize

$$\text{Moment of Inertia}=\sum_{i=1}^{i=n}\sum_{j=1}^{j=n} d_{ij}^2\, P_j\, x_{ij},$$

subject to the $3n+1$ restraints

$$\sum_{i=1}^{i=n} x_{ij}=1, \qquad (j=1, 2, \cdots, n)$$

$$\sum_{i=1}^{i=n} x_{ii}=k,$$

$$\sum_{j=1}^{j=n} P_j\, x_{ij} \geqq (a/100)\left(\sum_{j=1}^{j=n} P_j/k\right) x_{ii}, \qquad (i=1, 2, \cdots, n)$$

$$\sum_{j=1}^{j=n} P_j\, x_{ij} \leqq (b/100)\left(\sum_{j=1}^{j=n} P_j/k\right) x_{ii}. \qquad (i=1, 2, \cdots, n)$$

The resulting solution must be checked for contiguity.

APPENDIX II

Recent Approaches of Others

Other computer redistricting techniques are in various stages of development. One creates wedge-shaped districts about a circular district centered on the population centroid.[18] Another is proprietary and arbitrarily develops rectangular districts by successively dividing the state in halves.[19] Both programs are nonpartisan and fast, but have no compactness requirement.

A third nonpartisan program[20] attempts to improve old districts by moving each exterior population unit into an adjacent district or swapping it with one in another district. No trades of two for one or higher order are tried. The criterion is a weighted ratio of population equality and geographic compactness measured by the moment of inertia of the area. The program is said to be fast but has not run on problems larger than 12 districts by 101 population units.

A fourth program differs from the third by including partisan considerations.[21] The program can favor either political party or provide a solution with a prespecified number of Republican or Democratic districts. The author feels his approach will speed up redistricting bargaining between representatives of opposite political parties.

No redistricting plans developed by the above approaches have been implemented.

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