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Solving a home-care districting problem in an urban setting

M Blais[1], SD Lapierre[2,3] and G Laporte[3,4,*]

[1]*Urgences-Santé, 3232, rue Bélanger, Montreal, Canada H1Y 3H5;* [2]*École Polytechnique, C.P. 6079, Succursale Centre-Ville, Montreal, Canada H3C 3A7;* [3]*Centre de recherche sur les transports, (CRT) Universite de Montréal, C.P. 6128, Succursale Centre-Ville, Montreal, Canada H3C 3J7; and* [4]*Canada Research Chair in Distribution Management, HEC Montréal, 3000 chemin de la Côte-Sainte-Catherine, Montreal, Canada H3T 2A7*

This article describes a districting study undertaken for the Côte-des-Neiges local community health clinic in Montreal. A territory must be partitioned into six districts by suitably grouping territorial basic units. Five districting criteria must be respected: indivisibility of basic units, respect for borough boundaries, connectivity, visiting personnel mobility, and workload equilibrium. The last two criteria are combined into a single objective function and the problem is solved by means of a tabu search technique that iteratively moves a basic unit to an adjacent district or swaps two basic units between adjacent districts. The problem was solved and the clinic management confirmed its satisfaction after a 2 year implementation period.

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Introduction

In Canada, as in many Western countries, there has been an increase in recent years in the provision of home-care services. This is due in part to the ageing of the population, and also to the need to reduce or at least contain health expenditures. In the province of Quebec, home-care services fall under the jurisdiction of local community health clinics known as CLSC (for *Centre local de santé communautaire*). In particular, each clinic is responsible for the logistics of home-care visits by health-care personnel including nurses, nurse assistants, ergotherapists, physiotherapists, dietetists, family helpers, social workers, physicians, etc. Contrary to hospitals that are open to the population at large, public health clinics are responsible for the well-being of the population in a given *territory*. When they exceed a certain size, these territories are often partitioned into *districts*, each falling under the responsibility of a different multi-disciplinary team.

This study stems from a practical case encountered in the Côte-des-Neiges CLSC in Montreal that caters to 125 000 inhabitants, among which 5200 are regular service users. With recent changes in Quebec health-care policies, there has been a shift from institutional to community services, thereby increasing the workload of the CLSC. At the time of the study there were four districts, each under the supervision of a different team. The use of districts allows a decentralization of service dispatching and, therefore, facilitates the management of home-care services. When a person calls the CLSC to request home-care services, the central home-care service identifies the district associated with the address and a nurse attached to that district is sent. The workload of nurses is about the same in each district. The number of nurses in each district is relatively small, which facilitates management and scheduling.

Nursing teams in each of the four CLSC districts had grown to a point where management had become difficult and the workload between the different districts had become uneven. To alleviate this problem, the home-care services managers decided to partition the territory into six districts instead of four. (They first considered the possibility of having five districts, but the staff size of each district would still have been too large.) Owing to the administrative complexity it generates, redistricting is not carried out lightheartedly. The costs involved are significant and the effects on performance, staff morale and human relations are also significant. The last districting plan had been in effect for seven years and the next one should last at least as long.

In the following sections, we will describe how we have modelled and solved this problem to the satisfaction of the Côte-des-Neiges CLSC. We will first describe the criteria used in our model, the model itself, a tabu search algorithm and results. A conclusion follows.

Correspondence: G Laporte, Centre de Recherche sur les Transports, (CRT) Universite de Montreal, C.P. 6128, Succursale Centre-Ville, Montreal, Canada H3C 3J7.
E-mail: gilbert@crt.umontreal.ca

Districting criteria

There are two major districting issues associated with the provision of health-care services. The first consists of optimally locating health-care facilities and then drawing their associated territory. This can be solved by means of a location-allocation algorithm. An example of this type of problem is presented in Lapierre *et al*.[1] The second problem consists of partitioning each territory into suitable districts. The home-care districting problem belongs to this category. It consists of constructing m districts made up of several territorial *basic units* in order to optimize a multi-criteria objective. This problem bears similarities with the classical *political districting problem*, but is fundamentally different in the sense that its solution has organizational and human impacts that are felt day after day by the clinic's employees and the client population. The problem shares some features with the *police district design problem* recently studied by D'Amico *et al*,[2] where human factors are also important. This feature is also present, albeit to a lesser extent, in the drawing of sales districts.[3,4] We now review the five main criteria used in home-care districting.

Indivisibility of basic units

As is common in the districting literature (see, eg, Bozkaya *et al*,[5] Mehrotra *et al*,[6] Grilli di Cortona *et al*,[7] Minciardi *et al*[8]), districts are constituted from the grouping of indivisible basic units that must be sufficiently small to allow enough flexibility in the solution design, but not too small in order to keep the problem size manageable. It is also convenient to choose basic units for which sufficient geographic, demographic and socio-economic data are available. The census tracts or the enumeration area used by Statistics Canada fit these requirements. There are 32 census tracts containing 193 enumeration areas in the Côte-des-Neiges territory.

Respect of borough boundaries

The Côte-des-Neiges territory spans three of the City of Montreal boroughs (see Figure 1): Ville Mont-Royal, Outremont and part of Côte-des-Neiges/Notre-Dame-de-Grâce. Both Ville Mont-Royal and Outremont are rather wealthy areas, whereas the Côte-des-Neiges area is poorer. One of the constraints imposed on the districting plans keep the Ville Mont-Royal and Outremont districts intact since these two boroughs manage their own community services.

Connectivity

As is common in political districting, each district should be connected and should contain no enclaves.

Mobility

Most visiting staff uses public transit to move from one location to another. It is therefore important to design districts within which travel by bus is easy. This criterion is absent, for example, from other districting problems encountered in politics or sales force planning. Also, in the interest of mobility, travel should not be constrained by major barriers such as railway lines, motorways, etc.

Workload equilibrium

One important consideration to take into account when designing health-care districts is ensuring that the total workload (expressed in hours per year) of each district is roughly the same. This criterion is also present in a number of districting problems.[9] In our problem, workload is measured not only as the time spent with clients but also by the time spent travelling between visit locations.

It is worth noting that, when the study was initiated, the four districts currently used by the Côte-des-Neiges CLSC did not fully satisfy these criteria. In particular, two districts were not connected and one contained an area isolated by a railway line (see Figures 1 and 2). In addition, travel by public transit was not ideal as far as directeness of travel was concerned. Workload equilibrium was quite satisfactory for nurses, but not so for nurse assistants. However, from what we were told, it is more important to first obtain a good equilibrium for nurses and then appropriately allocate nurse assistants to meet the demand.

Model

There are several ways to model and to solve districting problems. The choice of a model is of course linked to that of a solution technique. A number of mathematical

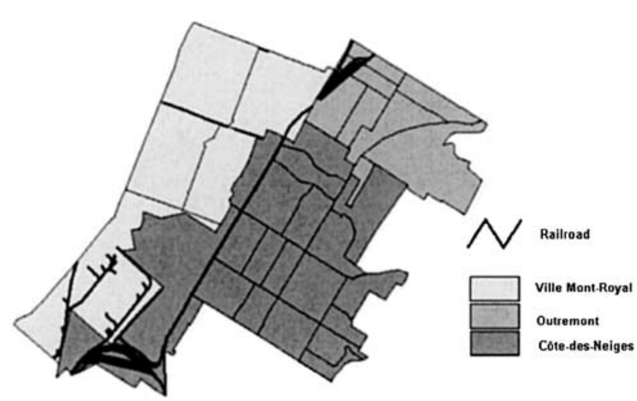

Figure 1 The Côte-des-Neiges CLSC territory.

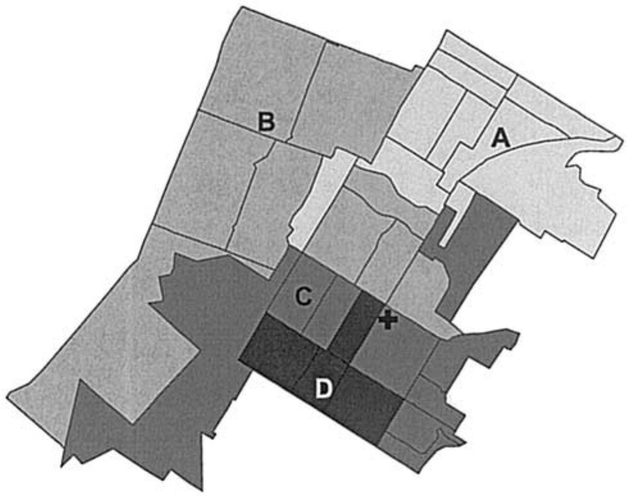

Figure 2 The four initial Côte-des-Neiges CLSC districts.

programming formulations exist, based on location-allocation (eg, Hess *et al*[10]) and set covering models (eg, Garfinkel and Nemhauser,[11] Mehrotra *et al*[6]). Location-allocation models do not adequately account for the numerous side constraints of our problem. Set covering models are more flexible. These are usually tackled by means of column generation methods that can, however, be difficult to implement and can be time consuming as witnessed by Mehrotra *et al*,[6] who had to resort to a truncated optimization procedure. The most efficient solution techniques are clustering heuristics that seek combinations of basic units into feasible district while minimizing a multi-criteria objective function. An early local search algorithm based on this scheme is that of Bourjolly *et al.*[12] Simulated annealing algorithms have later been proposed by Browdy[13] and Macmillan and Pierce.[14] More recently, Bozkaya *et al*[5] have developed an efficient tabu search algorithm for the political districting problem.

Our problem can be modelled by a multi-criteria approach. We will now explain how we have modelled the five criteria described earlier. As is common in multi-criteria optimization, some criteria are treated as hard constraints, while others are incorporated in the objective function (see, eg, Eiselt and Laporte[15]). After discussion with the CLSC management, we have opted to treat the first three requirements as hard constraints and to include the remaining two in a weighted objective of the form

$$f(s) = \alpha f_1(s) + (1-\alpha) f_2(s) \tag{1}$$

where $f_1(s)$ and $f_2(s)$ measure the mobility level and the workload equilibrium of solution s, respectively, and α is a user-controlled parameter in the range [0, 1]. The mobility level associated with solution s is measured as

$$f_1(s) = \sum_{k=1}^{m} \left(\sum_{\substack{i,j \in D_k \\ i<j}} v_i v_j d_{ij} \right) \Bigg/ \left[(n_k(n_k-1)/2) \left(\sum_{i \in D_k} v_i \right)^2 \right] \tag{2}$$

where m is the number of districts, i and j are basic units of district D_k, d_{ij} is the distance between the centres of basic units i and j, n_k is the number of basic units of D_k, and v_i is the yearly number of visits made in unit i. For each k, the numerator of this expression computes the total distance travelled within district D_k each year, while the denominator is used for scaling. The factor $n_k(n_k-1)/2$ corresponds to the number of i, j pairs in the numerator, while the last factor removes the measurement unit effect. The distance d_{ij} is computed as the shortest travel time between i and j, using public transit and walking.

A low value of $f_1(s)$ will correspond to a situation where ease of travel within each district is high. Note, however, that this function does not accurately measure the distance effectively travelled, since it adds up a term for each i, j pair whereas in practice clients are visited by means of *tours*. However, our choice is perfectly appropriate at a tactical planning level where an approximate figure is sufficient. The function $f_1(s)$ also helps achieve compact districts. Our measure differs from those commonly used in political districting (see, eg, Niemi *et al*[16] and Young[17]), since our aim is not to obtain compact geometrical shapes but districts that favour mobility. In practice, we found the use of (1) to define $f_1(s)$ to be perfectly adequate. It tends to favour the creation of a few high-density and a few low-density districts as opposed to several medium-density districts. This behaviour is consistent with the quadratic form of $f_1(s)$.

To compute the workload equilibrium $f_2(s)$, we use a piecewise linear function by which the workload of a district is only penalized if it lies outside the interval $[(1-\beta)\overline{W}, (1+\beta)\overline{W}]$ around the average district workload $\overline{W}$ in the territory, where $0 \leqslant \beta \leqslant 1$. More precisely, the workload equilibrium function is defined as

$$f_2(s) = \left(\sum_{k=1}^{m} \max\{W_k - (1+\beta)\overline{W}, (1-\beta)\overline{W} - W_k, 0\} \right) \Big/ \overline{W} \tag{3}$$

where W_k is the workload in district k.

The workload W_k is itself the sum of V_k and T_k representing, respectively, the total visit time and the total travel time in district k within a given period. The values W_k, V_k and T_k are of course dependent on the districting solution, but even the total $\sum_{k=1}^{m} T_k$ and the total visit time $\sum_{k=1}^{m} V_k$ are solution dependent. The first case is relatively easy to understand since total travel time can be reduced through better clustering of clients and better planning of visiting personnel schedules. The total visit time is also partly solution dependent since less time spent on travelling allows for more visiting time.

We have used historical data to approximate $\sum_{k=1}^{m} V_k$ and we have also made use of the historical ratio $\lambda = \sum_{k=1}^{m} T_k / \sum_{k=1}^{m} W_k$, equal to 18%. (In our final solution, this ratio was reduced to approximately 16%.) In practice, it is more convenient to work with the identity $\sum_{k=1}^{m} T_k = \lambda \sum_{k=1}^{m} V_k/(1-\lambda)$. Thus, the value of T_k associated with solution s is given by

$$T_k = \left(\frac{\lambda \Sigma_{k=1}^{m} V_k}{1-\lambda}\right)\left(\frac{\Sigma_{\substack{i,j \in D_k \\ i<j}} v_i v_j d_{ij}}{(n_k(n_k-1)/2)(\Sigma_{i \in D_k} v_i)^2} \frac{1}{f_1(s)}\right) \quad (4)$$

where the first factor estimates the total travel time in the entire territory and the second factor is the fraction of the total travel time spent in district k over the total travel time spent in the territory.

Algorithm

We have chosen to solve the health-care districting problem with the tabu search heuristic developed by Bozkaya *et al*[5] for political districting. Since this algorithm has recently been described at length and we only worked with a different objective function, we will sketch it here and refer the reader to the original reference for further details.

The heuristic works on solutions consisting of m districts made up of territorial basic units corresponding to census tracts. Starting from an initial solution, the algorithm iteratively proceeds from one to another solution in its neighbourhood by performing two types of movement. It either moves a basic unit from its current district to an adjacent district, or swaps two basic units at the frontier of two different contiguous districts.

The initial solution can be any hand-made solution, or a solution iteratively constructed by using seed basic units. Using these seeds, districts are gradually constructed by adjoining at each step a basic unit adjacent to district k having the least workload. More precisely, let $S(k)$ be the set of basic units in district k, and $S'(k)$ the set of unassigned basic units adjacent to district k. Then basic unit i^* is included in district k^*, where k^* and i^* yield

$$\min_{k} \min_{i \in S'(k)} \{g(i,k)\} \quad (5)$$

and

$$g(i,k) = \sum_{h,j \in S(k) \cup \{i\}} v_h v_j d_{h_j} \quad (6)$$

At a general iteration, the algorithm enumerates all feasible neighbours and moves to the best one, even if this causes the objective function f to deteriorate. To prevent cycling, moving a basic unit back to its original district is forbidden, or tabu, for θ iterations, where θ is randomly selected in the interval [1, 10] according to a discrete uniform distribution. The tabu status of a move is revoked whenever implementing it yields a new overall best solution. This rule is often refereed to as *aspiration criterion*.[18] The algorithm operates in two phases. In the first phase, single basic unit moves are applied until the objective f does not improve from $[100\sqrt{n}\,]$ consecutive iterations, where n is the number of basic units. In the second phase, basic units are exchanged between two adjacent districts until no improvement has been observed for $[10\sqrt{n}]$ consecutive iterations. As is done in a number of tabu search implementations (see, eg, Gendreau *et al*[19]), a continuous diversification process is implemented by assigning a penalty to solutions corresponding to frequently moved basic units.

We now illustrate the algorithm in a simple example containing 10 basic units (Figure 3) to be assigned to two districts. The workload v_i of each unit i is provided in Table 1, while the travel time matrix (d_{ij}) is given in Table 2. An initial solution is constructed by taking units 3 and 9 as seeds. Adjacent units are gradually combined with these seeds in order to minimize the workload difference between the two (partially constructed) districts, as shown in Table 3. Thus, in iteration 2, basic unit 2 is added to district 1 since this operation yields the minimum value of $g(i, k)$. At the end of iteration 10, district 1 contains units 3, 2, 1, 4, 7 and 6 while district 2 contains units 9, 10, 5, 8, yielding $f(s) = 1.193$ for $\alpha = 0.9$, $\beta = 0.25$, $\lambda = 0.18$. A search procedure is then applied to the current solution s by either transferring a basic unit to an adjacent districtor by swapping two units between adjacent districts in order to improve the value of $f(s)$. Thus, moving basic unit 4 to district 2 yields $f(s) = 0.814$, while swapping basic units 4 and 8 between districts 1 and 2 yields $f(s) = 0.815$. In this case, the first move would be preferred. This process is repeated (subject to the tabu mechanism) until no improvement has been observed for $[100\sqrt{10}] = 316$ consecutive iterations. The

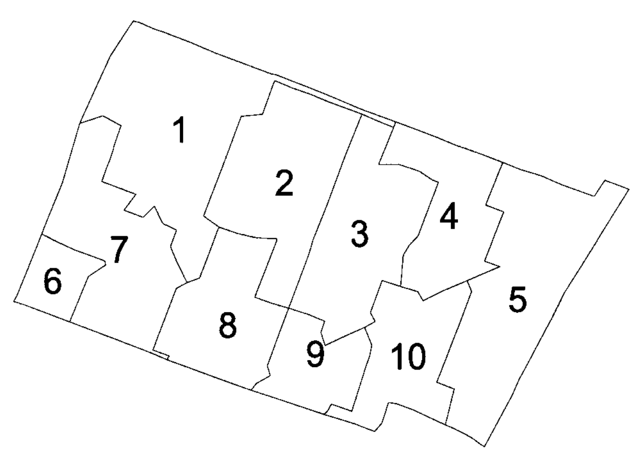

Figure 3 Territory partitioned into 10 basic units.

optimal solution s is made up of district 1 with units 1, 2, 6 and 7, and district 2 with units 3, 4, 5, 8, 9 and 10, yielding $f(s)=0.403$.

Table 1 Workload v_i in each basic unit i

Basic unit i	*Workload* v_i
1	79
2	50
3	35
4	113
5	82
6	229
7	357
8	154
9	43
10	74

Computational results

The algorithm just described was coded in C^{++} and was executed on a Sun Enterprise 10 000 (400 MHz). The algorithm was tested on several artificial instances and four CLSC territories on the Island of Montreal: Côte-des-Neiges, Petite-Patrie, Rosemont and Villeray (Figure 4). In all cases, good-quality solutions were produced within less than 300 CPU seconds. The algorithm was run using enumeration areas as basic units. Since the solution contained several small protrusions, a coarser solution made up of census tracts was produced. We present the results obtained for the Côte-des-Neiges territory and a comparison with the districting solution produced manually by the CLSC staff and two of the authors (Figure 5). It is difficult to assess the similarities and differences between these two solutions by looking at the figures alone. However, someone

Table 2 Travel time d_{ij} between basic units i and j

	j									
i/j	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
1	0.0	7.4	13.9	17.8	25.2	12.1	7.5	13.6	21.5	21.9
2	7.4	0.0	7.6	13.8	21.2	19.5	14.9	7.7	14.1	15.5
3	13.9	7.6	0.0	6.2	13.6	25.9	21.3	12.0	8.1	7.9
4	17.8	13.8	6.2	0.0	7.4	29.9	25.3	18.2	14.3	8.2
5	25.2	21.2	13.6	7.4	0.0	37.3	32.8	25.7	17.6	9.5
6	12.1	19.5	25.9	29.9	37.3	0.0	4.5	13.8	23.1	31.2
7	7.5	14.9	21.3	25.3	32.8	4.5	0.0	9.3	18.5	26.6
8	13.6	7.7	12.0	18.2	25.7	13.8	9.3	0.0	9.2	17.4
9	21.5	14.1	8.1	14.3	17.6	23.1	18.5	9.2	0.0	8.1
10	21.9	15.5	7.9	8.2	9.5	31.2	26.6	17.4	8.1	0.0

Table 3 Generation of the initial solution

Iteration	*Basic units in district k = 1*	$i \in S'(1)$	*g (i, 1)*	*Basic units in district k = 2*	$i \in S'(2)$	*g (i, 2)*
1	3	—	—	9	—	—
2	3	2	13 213	9	8	61 121
		4	24 561		10	25 838
		10	20 513			
3	3, 2	1	67 735	9	8	61 121
		4	102 305		10	25 838
		8	124 417			
		10	77 752			
4	3, 2	1	67 735	9, 10	4	138 192
		4	102 305		5	119 870
		8	124 417		8	258 842
5	3, 2, 1	4	261 206	9, 10	4	138 192
		7	745 669		5	119 870
		8	290 361		8	258 842
6	3, 2, 1	4	261 206	9, 10, 5	4	206 853
		7	745 669		8	582 876
		8	290 361			
7	3, 2, 1, 4	7	1 767 909	9, 10, 5	8	582 876
		8	607 947			
8	3, 2, 1, 4	7	1 767 909	9, 10, 5, 8	7	2 457 547
9	3, 2, 1, 4, 7	6	1 794 634	9, 10, 5, 8	—	—
10	3, 2, 1, 4, 7, 6	—	—	9, 10, 5, 8	—	—

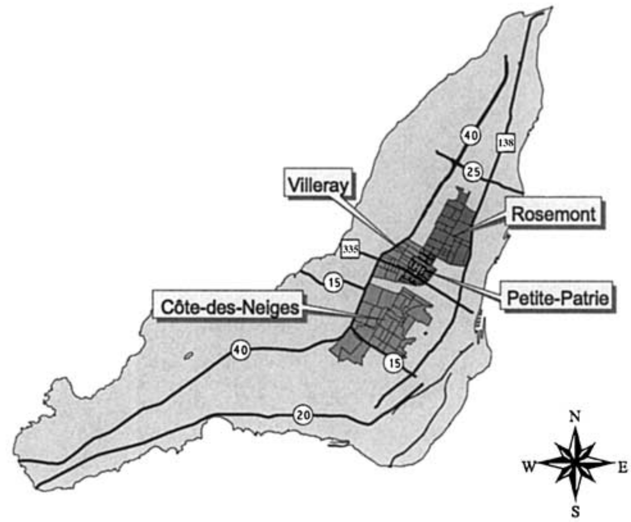

Figure 4 Geographic locations of the other three test problems.

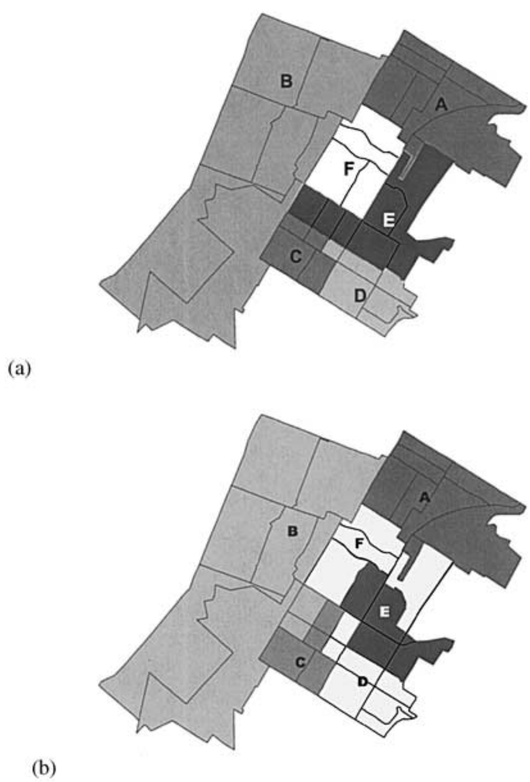

Figure 5 A comparison of two districting plans for the Côte-des-Neiges CLSC: (a) Manual solution produced by the CLSC staff and two of the authors, (b) Automatic solution produced with the tabu search algorithm.

acquainted with the Montreal bus network would see that travel by bus in districts C and E should be easier in the automatically generated solution than in the manual solution. To really appreciate the results, one should look instead at the workload distributions in each solution

Table 4 Annual number of home visits

District	*Manual solution*	*Automatic solution*
A	5014	5130
B	3739	4991
C	5652	5256
D	4538	5228
E	6206	5280
F	6059	5323
Mean	5201.3	5201.3
Standard deviation	871.9	111.1

(Table 4). This distribution is much more uniform in the automatic solution in which the standard deviation is 83.3% less than that of the manual solution. Mobility, as measured by Equation (2), is also better in the automatic solution, reducing travelling time from 18 to 16% of the daily workload. This confirms that an automatic districting process can generate solutions at least as good as those produced by a team of experts.

The proposed automatic solution was adopted and implemented in September 2000. The transition to the new districting plan went smoothly and the CLSC management confirmed its satisfaction 2 years after the implementation.

Conclusion

We have modelled and solved a practical districting problem arising in the management of public health-care clinics in Quebec. While the problem bears similarities with the classical political problem, it is also different from it. In particular, districts must be designed so as to ease travel by public transit. Also, the workload between districts must be well balanced. The districting plan developed for the Côte-des-Neiges CLSC in Montreal was adopted by management. At 2 years after the solution was implemented, the level of satisfaction remains high.

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References

1 Lapierre SD, Myrick JA and Russell G (1999). The public health care planning problem: a case study using geographic information systems. *J Med Systems* **23**: 401–417.

2 D'Amico SJ, Wang S-J, Batta R and Rump CM (2002). A simulated annealing approach to police district design. *Comput Oper Res* **29**: 667–684.
3 Skiera B and Albers S (1998). Costa: contribution optimizing sales territory alignment. *Market Sci* **17**: 196–213.
4 Drexl A and Haase K (1999). Fast approximation methods for sales force deployment. *Mngt Sci* **45**: 1307–1323.
5 Bozkaya B, Erkut E and Laporte G (2003). A tabu search heuristic and adaptive memory procedure for political districting. *Eur J Oper Res* **144**: 12–26.
6 Mehrotra A, Johnson EL and Nemhauser GL (1998). An optimization based heuristic for political districting. *Mngt Sci* **44**: 1100–1114.
7 Grilli di Cortona P, Manzi C, Pennisi A, Ricca F and Simeone B (1999). *Evaluation and Optimization of Electoral Systems*. SIAM Monographs on Discrete Mathematics and Applications: Philadelphia.
8 Minciardi R, Puliafito PP and Zoppoli R (1981). A districting procedure for social organization. *Eur J Oper Res* **8**: 47–57.
9 Muyldermans L, Cattrysse D, Van Oudheusden D and Lotan T (2002). Districting for salt spreading operations. *Eur J Oper Res* **139**: 521–532.
10 Hess SW, Weaver JB, Siegfeldt JN, Whelan JN and Zhitlan PA (1965). Nonpartisan political districting by computer. *Oper Res* **13**: 998–1006.
11 Garfinkel RS and Nemhauser GL (1970). Optimal political districting by implicit enumeration techniques. *Mngt Sci* **16**: 495–508.
12 Bourjolly J-M, Laporte G and Rousseau J-M (1981). Découpage électoral automatisé: application à l'Île de Montréal. *INFOR* **19**: 113–124.
13 Browdy MH (1990). Simulated annealing: an improved computer model for political districting. *Yale Law Policy Rev* **8**: 163–179.
14 Macmillan W and Pierce T (1994). Optimization modelling in a GIS framework: the problem of political districting. In: Fotheringham S and Rogerson P. *Spatial Analysis and GIS*. Taylor and Francis: London.
15 Eiselt HA and Laporte G (1987). Combinatorial optimization problems with soft and hard requirements. *J Oper Res Soc* **38**: 785–795.
16 Niemi RG, Grofman B, Carlucci C and Hofeller T (1990). Measuring compactness and the role of a compactness standard in a test for partisan and racial gerrymandering. *J Polit* **52**: 1155–1181.
17 Young HP (1988). Measuring the compactness of legislative districts. *Legislative Stud Q* **13**: 105–115.
18 Glover F and Laguna M (1997). *Tabu Search*. Kluwer: Boston.
19 Gendreau M, Hertz A and Laporte G (1994). A tabu search heuristic for the vehicle routing problem. *Mngt Sci* **40**: 1276–1290.

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