A system approach to the optimal health-care districting

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The paper presents a methodology for partitioning a given region in geographical areas in such a way as to insure an optimal allocation of the available health services. Two steps compose the proposed approach. In the first step, via mathematical programming one determines optimal hospital districts by taking into account demand and capacity, measured in number of hospital-beds. In the second step, one determines health-districts by aggregating together hospital districts, by taking into account lower and upper bounds for the population in each district and the existence of districts with a different nature (political, educational, etc.). A case study is presented for the Italian province of Cosenza. Although developed within the context of health-care services, the approach is general enough to be applicable also to the partition in a given region of other social services, i.e. school districts.

1. Introduction

One of the basic problems in the organization of social services is the partition of the territory under study into a series of districts, each of which offers a series of integrated and possibly complete set of services to the population living within its boundary. This kind of problem will be designated as optimal districting. For each district it is necessary to ensure a uniform set of services and a district size which takes advantage of scale economies. At the base of this multi-level system, there are small centres offering only basic services.

They depend on districts offering a set of more complete services. An analytical solution of the optimal districting problem is rather difficult as in setting up districts one has to take into account a series of factors, not always easily measurable and quantifiable.

In this paper, a particular case is taken into consideration: the organization of regional health services into Local Health Departments (LHDs). They represent districts offering a set of integrated and complete health services to some subset of counties, within a region, each of them covering a population ranging between 50000 and 200000, according to geographical and socio-economical characteristics of the territory. The approach proposed in this paper might also be applied to other social resource services (such as schools, etc.) for which an equitable allocation among areas or among different client groups is desired. The main benefit of this approach is that it represents a rational, scientific study of the problem which is relatively simple in concept and allows a number of possible partitions to be deduced, based on different assumptions. There is not just one 'optimal' answer. The job of senior management and planners still remains to make the decision as to which partition appears to be the most desirable and feasible to implement, taking into account the assumption on which each is based and other qualitative constraints.

The problem under study is stated in Section 2. The proposed methodology is discussed in Section 3. In Section 4 and 5 the methodology is applied to the determination of the health districts. Finally, in Section 6 a case study related to the Italian province of Cosenza is considered.

2. Problem statement

The region to be partitioned into districts can be modeled by a graph $G(N,A)$ where the basic units to be combined, the counties, are represented by the set of N nodes. The arc between two nodes represents the shortest road, if it exists, that connects the corresponding counties without going through other counties. The resulting graph is simple, connected and not complete.

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Definition 1. A *district* j is a connected subset of nodes $n_j \subset N$ which contains at least one node representing a county where a hospital is present.

Definition 2. Given a graph $G(N,A)$, its division into districts is called a *district partition.*

The region is composed of n counties, m of which have a hospital, with known capacities b_j $(j=1,2,\dots,m)$.

Definition 3. A *hospital district-partition* is a m-partition of graph $G(N,A)$ in connected subgraphs so that each subset contains one and only one of the nodes which have a hospital.

Definition 4. A hospital district partition is *feasible* if for each district the following condition is verified:

$$\sum_{k \in n_j} a_k \leqslant b_j,$$

where n_j is the subset of counties of the jth hospital district, a_k is a measure of the demand of services of the kth county and b_j is the capacity of services offered by the jth hospital district.

It is now possible to state:

Problem (A): Among all feasible hospital district partitions find the optimal one according to a given performance index.

The Italian health-care law states that a Local Health Department should have a population ranging between 50 000 and 200 000 people, according to the geographical and socio-economical conditions of the region under study, and that:
— every LHD should contain one or more contiguous hospital districts;
— every hospital district should belong to one and just one LHD.

It is now possible to give:

Definition 5. An *LHD partition* is a r-partition $(r \leqslant m)$ of graph $G(N,A)$ in connected subgraphs. This partition is *feasible* if for every LHD the following conditions hold:

$$p_{\min} \leqslant \sum_{k \in n_l} p_k \leqslant p_{\max},$$

where $n_l \subset N$ is a subset of the counties in the lth LHD, $p_{\min}$ and $p_{\max}$ are the lower and upper bounds for the population and p_k is the population of the kth county.

The second problem considered in this paper is:

Problem (B): Among all the feasible partitions in LHDs find the optimal one, according to some geographical and socio-economical characteristics.

3. Methodology

To take into account the quantitative and qualitative factors involved, the proposed methodology combines analitical and heuristic techniques. Particularly, as a first step, via mathematical programming it is possible to determine the hospital districts by assigning the counties to the existing hospitals in the region under consideration. As a second step, by using a heuristic technique, it is possible to find a set of LHDs satisfying some predefined criteria. In this latter step, the qualitative factors (geographical and socio-economical characteristics) have a higher influence. Furthermore one tries to make the LHDs as overlapping as possible to existing administrative or school districts. A basic hypothesis for applying the proposed methodology is that the geographical distribution of hospitals in the region under study is fairly uniform. A flow chart of the proposed methodology is shown in Fig. 1. The main steps to be performed are the following ones:

(a) *Analysis of the demand for health services.* Some methods available to perform this analysis are:

(1) analysis of the data related to the demographical, socio-economical and geographical situation for each county in the region under study;

(2) analysis of the nosological picture of the population, estimated by compulsory declarations and certifications, the clinical cards of the hospital and dispensaries, etc.;

(3) surveys directed to study special problems, performed on samples of the population, special categories, special areas, etc.;

(4) interviews with experts.

In this study, the first method has been used to measure the demand, while the last two should be used to verify the results and the measures of socio-health demand.

In the presence of an efficient health information system, it would be possible to estimate,

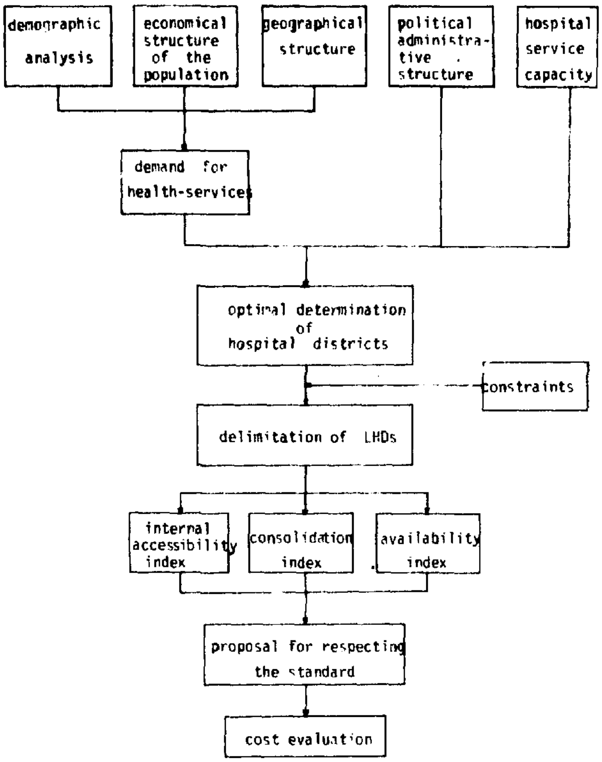

Fig. 1. Diagrammatical representation of the districting process.

through statistical techniques, the correlation between the indices of the nosological picture of the population, and the indices relative to the demographic, socio-economical and geographical situation, disaggregated for each area.

(b) *Analysis of the available hospital services.* The existing, or nearly completed, general hospitals have been taken into account: for each of them, the service was measured by the number of available beds.

(c) *Clustering of the counties for the optimal determination of the hospital districts.* For the districting problem, some authors have proposed mathematical programming models. They may be classified into set partitioning or generalized assignment models [3,7] and location and allocation models [2,6]. In this paper, the proposed mathematical model is based on the transportation algorithm [4]. By solving a suitable linear program, one can obtain:

- the counties assigned to one hospital which form a district (nuclei);
- the counties assigned to one hospital which do not form districts (unconnected zones);
- the counties not uniquely assigned.

For these last two types of counties, one can use an implicit enumeration algorithm which allows one to aggregate the counties to nuclei in such a way as to obtain hospital districts.

In this paper, the following objectives for the optimal districting are considered:

- minimize the average distance of individuals from their nearest centre;
- minimize the 'deviation' between the proposed and existing districting based on other factors.

To consider the former factor, one can take a linear function of the road distance, weighted to take into account the type of road.

A measure of the deviation is given by the weighted distances, modified by a suitable coefficient to take into account the fact that a given county belongs to some existing district.

The sum of the previous factors extended to the entire region gives the value of the final performance function to be minimized.

(d) *Clustering of the hospital districts for the optimal determination of the LHDs.* In this step, by using a heuristic technique and evaluation indices, it is possible to find the feasible partitions in LHDs and the optimal partition.

4. Mathematical model of optimal districtingl

For a given area, let:

- $K=\{1, 2,\ldots, n\}$ the set of counties with a given population p_k, $k \in K$;
- $J \subset K$, $J=\{j : j=1, 2,\ldots, m\}$ the set of counties with a hospital having a given number of beds b_j;
- $T=\{t_{kj}\}$, $(k \in K, j \in J)$ the matrix of the minimal 'weighted distances' computed from the matrix of the 'weighted distances' using the decomposition algorithm proposed by Hu [5];
- x_{kj} the number of persons living in the kth county which uses the hospital in the jth county (if $x_{kj}=p_k$, the kth county is assigned to the hospital in the jth county);
- α the number of people who could be covered by the services associated with one hospital bed (ratio: persons to beds);
- β a number between zero and one which represents the weight given by a planner. It is connected with the importance given to the reduction of the factor distance with respect to the 'deviation' from existing districting;
- γ a number between zero and one, which represents an attraction coefficient given by the planner;

— $T' = \{t'_{kj}\}$ the matrix of the coefficients of the deviations of one solution from existing districting. $t'_{kj} = \gamma \cdot t_{kj}$ if the kth and the jth counties belong to the same existing district (i.e. school district), $t'_{kj} = t_{kj}$ otherwise.

One can now state the problem of optimal districting as:

$$\min z = \sum_{k=1}^{n} \sum_{j=1}^{m} c_{kj} x_{kj}, \quad x_{kj} \geq 0, \tag{1}$$

$$\sum_{j=1}^{m} x_{kj} = p_k, \quad \forall k \in K, \tag{2}$$

$$\sum_{k=1}^{n} x_{kj} \leq \alpha b_j, \quad \forall j \in J, \tag{3}$$

with

$$c_{kj} = (1-\beta) t_{kj} + \beta t'_{kj}, \quad \forall k \in K, \forall j \in J.$$

The stated problem has a solution if:

$$\sum_{k=1}^{n} p_k \leq \alpha \sum_{j=1}^{m} b_j$$

which can be expressed as

$$\alpha \geq \bar{\alpha} = \frac{\sum_{k=1}^{n} p_k}{\sum_{j=1}^{m} b_j}.$$

Since p_k and b_j are not homogeneous quantities, α is a conversion coefficient which allows one to compare the demand and the capacity of the hospital system.

By solving the transportation problem, one can obtain the nuclei, the unconnected zones and the counties not uniquely assigned.

However the number of counties not uniquely assigned to one hospital is less than the number of counties where there is a hospital, since in a transportation problem the number of non-negative variables x_{kj} is less or equal to $m + n - 1$. Nevertheless when α increases, the number of counties not uniquely assigned and the number of unconnected zones tend to decrease.

Since α may be considered as a planning coefficient, one can choose an $\alpha = \alpha^* > \bar{\alpha}$, which is the largest value of α^* for which at least the inhabitants of the jth county make use of the jth hospital ($\alpha^* = \max_{j \in J}(p_k / b_j)$ for $k = j$).

By solving the transportation problem for $\alpha = \alpha^*$, one can have again unconnected zones and counties not uniquely assigned. However, the new assignment problem is much simpler than the initial one. The initial graph G is reduced to a simpler graph G_{α^*}, whose nodes represent the nuclei, the counties not uniquely assigned and the counties which, even if they are associated to one hospital, are not connected with the corresponding nuclei. Also these counties are called 'unassigned'.

For solving the assignment problem, one can use the following algorithm, obtained by modifying the one proposed in [4]:

(1) to each nucleus j associate the residual availability

$$R_j = \alpha^* b_j - \sum_{k \in n_j} p_k$$

where n_j represents the set of counties belonging to the nucleus j;

(2) choose a nucleus as a starting node;

(3) build on the reduced graph all the possible trees of the minimal weighted distances having as a root that node individuated in step 2, and having the population which is not greater than the residual availability of the relative nucleus;

(4) change the starting node and go back to step 2 until all the nuclei are considered. The algorithm can be implemented according to an enumerative technique of the type 'Backtrack Programming' [1].

The proposed algorithm allows one to obtain all the feasible hospital districts (all possible solutions to the afore mentioned problem (A)).

An evaluation measure for choosing the 'optimal' solution, may be based on selecting among the feasible hospital districts the one which minimizes the objective function (1).

The number of steps necessary to solve the new assignment problem decreases when α increases. So it might be interesting to analyze the sensitivity to the value of α, since this parameter represents the basic requirements for planning the hospital services.

Finally, for every fixed β, there exists a value α_{max} such that for $\alpha \geq \alpha_{max}$ the solution of the transportation problem allows one to find directly an 'optimal' hospital districting; α_{max} can be found by sensitivity analysis.

For $\beta = 0$ and $\alpha \geq \alpha_{max}$, the solution of the transportation problem will generate assignments according to the minimal weighted distances.

5. Optimal determination of the LHDs

To find all the feasible LHDs (see Section 2), one has to take into consideration all the possible combinations of hospital districts which represent connected zones of population

$$p_{\min} \leq \sum_{k \in n_l} p_k \leq p_{\max}.$$

The use of heuristic rules can greatly shorten the search by singling out immediately the 'reasonable solutions'.

To do so $\forall h \in J$ let:

$C_h = \{j \in J \mid$ the counties, with a hospital, of the hth and jth hospital districts belonging to the same district already existing (i.e. a school district)$\}$,

$P_h = \{j \in J \mid$ the counties, with a hospital, of the hth and jth hospital districts belonging to the same administrative district (i.e. a mountain community)$\}$,

$G_h = \{j \in J \mid$ the counties, with a hospital, of the hth and jth hospital districts belonging to contiguous hospital districts$\}$.

Let $CP_h = \{(C_h \cup P_h) \cap C_h\}$ represent the set of 'preferable combinations' for the hth hospital district.

Let two hospital districts h and q $(h, q \in J)$ be
— combinable if $CP_h \cap CP_q \neq \varnothing$,
— uncombinable if $CP_h \cap CP_q = \varnothing$.

In formal terms, one can define a symmetrical matrix $S = [s_{hq}]$, $m \times m$, where $s_{hq} = 1$ if $CP_h \cap CP_q \neq \varnothing$, $\forall h, q \in J$, otherwise $s_{hq} = 0$.

This matrix can be used as a neighbourhood matrix of a graph whose nodes represent the hospital districts such that an arc exists between nodes h and q if and only if $s_{hq} = 1$.

One can use an implicit enumeration algorithm to find the admissible health districts by aggregating combinable hospital districts.

The choice of the 'optimal' health districting may be made according to the objectives pursued by the planner (i.e. the maximization of the average of the percentage indexes of consolidation of the LHDs, as shown in Section 7).

6. A case study

Even if the proposed approach is best suited for dealing with an entire region, the following case study refers to a smaller area, the Italian province of Cosenza.

The case however is characteristic of many areas in southern Italy. The province of Cosenza had in 1977 a resident population of 716661 inhabitants distributed in 155 counties, with only 17 hospitals (see Fig. 2).

Most people live in the larger towns: out of 155 counties, 142 have a population less than 10000 inhabitants, 9 between 10000 and 20000 inhabitants, 3 between 20000 and 50000 inhabitants and the main town, Cosenza, has over 100000 inhabitants. Between 1951 and 1977 there has been a migration from the mountain counties to the valleys, with the population increasing in the more populated counties (above 10000 inhabitants) and especially near the major town).

The province has a regional general hospital, and 16 zone general hospitals (of which 8 are about to open).

In the case study, specialized hospitals and private clinics are not considered, even if they might be used by the population in some cases.

The total number of beds, including the new hospitals, is 4191 corresponding to 5.85 beds for 1000 inhabitants.

The minimal distances between counties have been computed from the road distances weighted in such a way as to take into account the road type (highways, turnpikes, state and provincial roads).

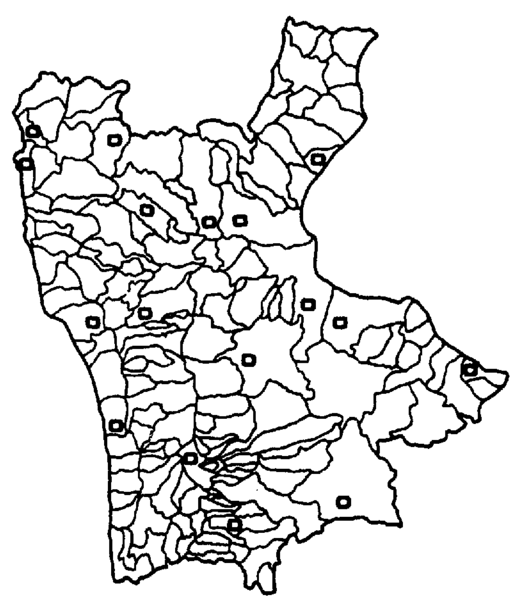

Fig. 2. Province of Cosenza: counties; □ Counties allocation for hospitals.

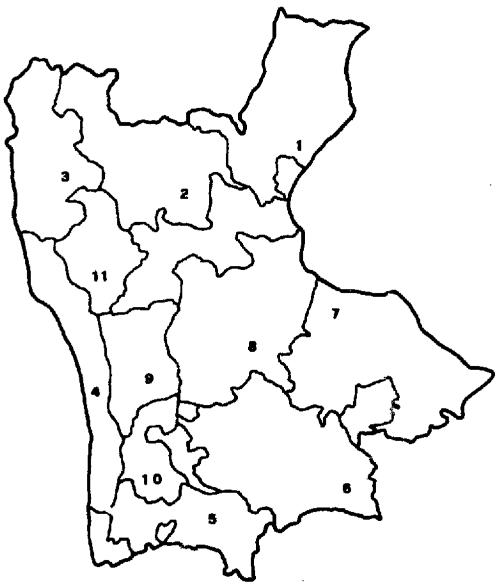

Fig. 3. Province of Cosenza: mountain communities.

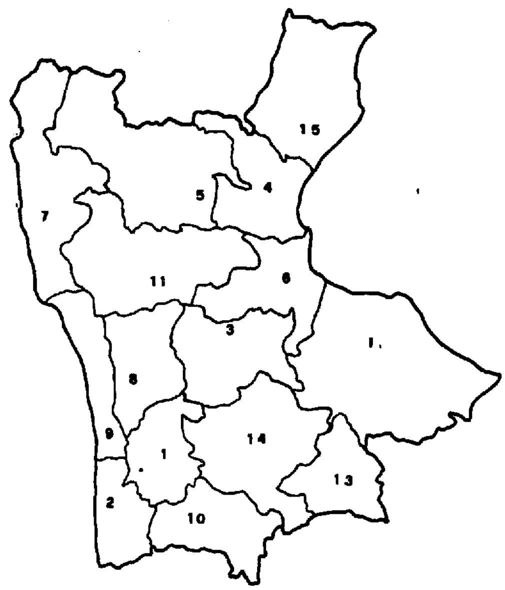

Fig. 4. Province of Cosenza: school districts.

In the province of Cosenza, there are already 11 mountain communities (a sort of administrative district) and 15 school districts (see Fig. 3 and 4).

For the counties belonging to the same mountain community and school district, the attraction coefficient γ has been taken equal to 0.5. Once fixed $\beta = 1$, α^* comes out to be equal to 278 (greater than $\bar{\alpha} = 171$).

By solving the transportation problem, one find the nuclei and 10 'unassigned' counties. By using the algorithm proposed in Section 4, one can obtain the 'optimal' hospital districting.

In a similar way, it is possible to obtain the optimal hospital districting for the more significant β values (i.e. for $\beta = 0$ the objective function takes into account only the inconvenience associated with the distance).

The limits considered are $p_{\min} = 50000$ and $p_{\max} = 200000$ for all the LHDs, with the exception of the LHD of Cosenza, which could be larger than the upper bound, and the LHD of St. Giovanni in Fiore which might not satisfy the lower bound.

By using the algorithm discussed in Section 5, one obtains six feasible partitions in LHDs. The

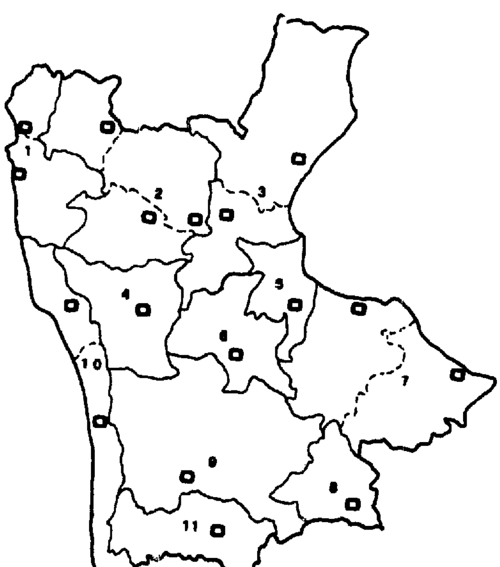

Fig. 5. Province of Cosenza: territorial delimitation of LHD ($\beta = 0$); ------ delimitation of the hospital districts.

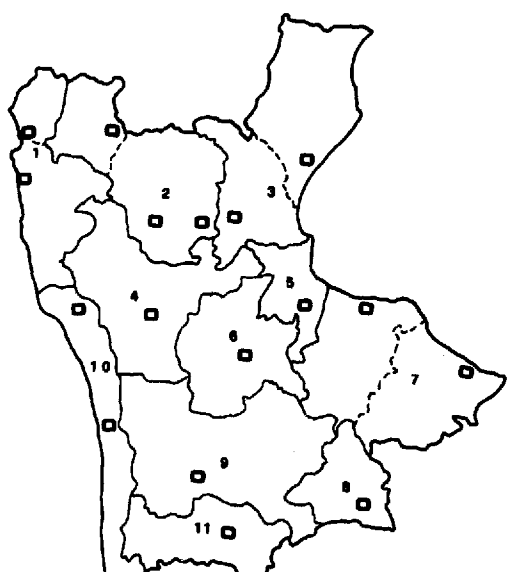

Fig. 6. Province of Cosenza: territorial delimitation of LHD ($\beta = 1$); ------ delimitation of the hospital districts.

optimal partition in LHDs is shown in Fig. 5 and 6 respectively for $\beta = 0$ and $\beta = 1$.

The results have been analyzed for some particularly significant β values (0; 0.25; 0.50; 0.75; 1.00).

7. Analysis of the results and suggestions for possible restructuring

In order to discriminate the optimal solutions, among the feasible partition in LHDs (Problem (B)), some possible evaluation indices are proposed. If

$$\bigcup_{l=1}^{r} n_l = \{n_1 \cup n_2 \cdots \cup n_r\}$$

is the partition in LHDs with r being the number of LHDs found ($r \leqslant m$), and

$$\bigcup_{i=1}^{v} u_i = \{u_1 \cup u_2 \cdots \cup u_v\}$$

is the previous partition in v zones (i.e. school districts), one may define the following indices:

— accessibility index of the lth LHD:

$$T_{n_l} = \frac{\sum_{j \in n_l} \sum_{k \in \bar{n}_l} p_k t_{kj}}{\sum_{k \in n_l} p_k}$$

where $\bar{n}_l$ is the subset of the counties with at least one hospital and n_l is the subset of all the counties belonging to the lth LHD, p_k the population of the kth county, t_{kj} a measure of the minimal distance between the county k and the county j where there is a hospital;

— percentage index of consolidation between the lth LHD and a previous ith district

$$\Omega_{li} = \frac{\sum_{k \in \delta_{li}} p_k}{\sum_{k \in u_i} p_k} \cdot 100$$

where $\delta_{li} = n_l \cap u_i$;

— index of availability of beds in the lth LHD

$$\alpha_{n_l} = \frac{\sum_{k \in n_l} p_k}{\sum_{j \in \bar{n}_l} b_j}$$

which represents the ratio: people in the lth LHD over hospital beds. This index can be disaggregated for single hospital specialities (i.e. general medicine, general surgery, etc.)

To evaluate the degree of over- or under-dimensioning of the partitioning of the LHDs, one has to compute for each LHD its deviation in the ratio populations/beds from a planned value $\alpha_{st.}$. For the lth LHD, it is:

$$D_{n_l} = \sum_{j \in \bar{n}_l} b_j \cdot \frac{\alpha_{n_l} - \alpha_{st.}}{\alpha_{n_l}}.$$

For each LHD, one has calculated:

— the internal accessibility index, measured in weighted kilometers (with the weights 1.0; 1.2; 1.5 and 2.0 respectively for turnpikes, highways, state and provincial roads) (see Table 1).

— the consolidation index (see Table 2), computed with reference to the following correspondences:

LHD	Mountain community/ies	School district/s
1	3	7
2	2	5
3	1	15–4
4	11	11
5	8	6
6	8	3
7	7	12
8	6	13
9	9–10	1–8–14
10	4	9–2
11	5	10

— The dimensional entity of each LHD computed by assuming $\alpha_{st.} = 114$. This value corresponds to the effective national average value of inhabitants for each hospital bed (see Table 3).

From Tables 1 and 2, it appears that for increasing β the total average accessibility of the service worsens. On the other hand, the total average consolidation index improves with respect to the existing districting. The choice of an optimal β depends on the objectives of the planner.

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Table 1
Indices of internal accessibility

β \ LHD	1	2	3	4	5	6	7	8	9	10	11	total average
0	22.2	12.6	27.5	26.0	9.0	10.4	21.4	-	11.7	17.7	27.3	18.5
0.25	22.2	11.9	27.5	28.0	9.0	10.4	21.4	-	11.7	17.7	27.3	18.7
0.50	22.2	10.3	28.6	30.0	9.0	10.4	21.4	-	11.7	17.7	27.3	18.8
0.75	22.2	9.4	27.5	30.7	9.0	17.1	21.4	-	10.7	17.7	27.3	19.3
1.00	25.8	8.6	26.9	40.5	4.7	21.1	21.4	-	10.2	17.1	27.3	20.3

Table 2
Percentage consolidation indices computed jointly with respect to mountain communities and school districts

β \ LHD	1	2	3	4	5	6	7	8	9	10	11	total average
0	100	75.4	84.4	59.8	87.7	80.6	10.8	100	25.4	23.4	67.2	66.2
0.25	100	80.5	80.9	64.1	87.7	80.6	94.9	100	25.4	83.4	66.2	66.8
0.50	100	83.4	82.6	65.6	87.7	80.6	94.4	100	25.4	83.4	66.2	67.8
0.75	100	84.7	82.6	65.5	87.7	84.7	94.9	100	26.7	83.4	67.2	68.8
1.00	100	89.9	90.1	68.5	100	85.9	94.9	100	27.4	94.7	67.8	71.1

Table 3
'Optimal' dimensioning of the LHDs

LHD	1	2	3	4	5	6	7	8	9	10	11
actual beds	312	701	247	108	191	96	595	155	1085	585	116
$\beta=0$	+52	−143	+394	+252	+182	+240	+131	+60	+723	+144	+259
$\beta=0.25$	+52	−162	+394	+273	+182	+240	+131	+60	+723	+144	+259
$\beta=0.50$	+52	−182	+391	+288	+182	+240	+131	+60	+723	+144	+259
$\beta=0.75$	+52	−200	+391	+307	+182	+331	+131	+60	+637	+144	+259
$\beta=1.00$	+135	−225	+338	+434	+137	+370	+131	+60	+590	+58	+266

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