OR PRACTICE

DECISION SUPPORT SYSTEM FOR THE SCHOOL DISTRICTING PROBLEM

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For a school board with several schools in its territory, the School Districting Problem is to specify the groups of children attending each school. A decision support system used to help administrators is described in this paper. It includes several heuristic procedures to assign edges of the network to the schools. The color graphics display is extensively used to assess the quality of the solution and to provide interactive functions for modifying the solution.

For a school board with several schools in its territory, the School Districting Problem is to specify the groups of students attending each school. School and class capacity constraints must be satisfied, and several *social* objectives are desirable: Students should attend the same school from year to year and school sectors should be contiguous to allow students from the same neighborhood to be assigned to the same school. The contiguity of the sectors also facilitates busing of children whenever necessary.

Administrators must solve this problem on a yearly basis, and whenever a major reorganization is required (opening a new school, closing one, modifying the capacities, etc.), several scenarios have to be analyzed. It is difficult to carry out this analysis manually. On the other hand, because other less tangible social objectives may be considered, it seems unrealistic to use a completely automatic computerized procedure to generate a solution without any interaction with administrators. The purpose of this paper is to present a decision support system to deal with the school districting problem. The system offers two major options:

1. the user can generate interactively a first draft of the districting of the territory using several available heuristic procedures;
2. the user can interactively modify a districting of the territory by referring to its graphics display. The system automatically updates all reports relative to school capacities after each modification.

To assess the quality of the solutions, three different types of output are available:

- color graphics displays that illustrate school sectors in different colors;
- information on school capacities, edge populations, and the number of students assigned to the different schools are available on a separate monochrome display where the menus are also presented to the user;
- hard copies of the reports and geographical maps are also available via the printer.

The system is implemented on a microcomputer (IBM PC type). This choice of equipment entailed the selection of fast heuristic procedures for generating the first draft of the districting problem.

This problem has received attention before. The school districting problem has been analyzed from the viewpoint of racial balance in Clarke and Surkis (1968), Heckman and Taylor (1969), Belford and Ratliff (1972), and Franklin and Koenigsberg (1973). Models and methods for designing sectors for new and existing schools have been presented in Ploughman, Darnton and Heuser (1968) and Holloway, Wehrung and Zeitlin (1975). On the other hand, in Mine, Ohno and Miyaji (1982), Gauvreau (1982) and Plante (1985) students from the same neighborhood are grouped into small units, and then these units are assigned to schools. In Guénette (1987) and in this paper, the unit is taken as the students

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located on a street segment between two intersections. This problem is related to the political districting problem (e.g., Garfinkel and Nemhauser 1970) where contiguous sectors with specified characteristics have to be generated.

Our approach to this problem has evolved through time. First, a computer program running in batch mode without any interface with the user was designed in Gauvreau. We were sensitive to the need for a color graphics display to assess the quality of the solution generated. An earlier version of an interactive computer system is presented in Plante. Finally, the end result of our effort is this decision support system that allows the user to maintain full control over the whole districting operation.

In Section 1, we present a model and the heuristic procedures available in the decision support system to generate the first draft. The system is presented in Section 2. In a concluding section we indicate the efficiency of the system, its use and potential.

1. MODEL AND SOLUTION PROCEDURES

Let $G = (N, A)$ be the road network for a school board that includes several schools. The edges of the network are the street segments, and the nodes are their intersections and the school locations. For each edge, the number of students in each grade is also specified. The school districting problem is to assign each edge (with students located on it) to a school.

Denote by $\bar{A} \subset A$ the subset of edges with students located on them. Let

I = the number of edges in $\bar{A}$;
J = the number of schools;
K = the number of grades (or levels).

The following input data are assumed to be available:

α_j^k = the number of classes of grade k $(1 \leq k \leq K)$ available at school j $(1 \leq j \leq J)$;
ϕ_k = the upper bound on the number of students in a class of grade k $(1 \leq k \leq K)$;
r_i^k = the number of students of grade k $(1 \leq k \leq K)$ on edge ai $(1 \leq i \leq I)$.

The decision variables are denoted by

$$x_{ij} = \begin{cases} 1 & \text{if edge } a_i \ (1 \leq i \leq I) \\ & \quad \text{is assigned to school } j \ (1 \leq j \leq J) \\ 0 & \text{otherwise.} \end{cases}$$

The model includes the following constraints.

Assignment Constraints

$$\sum_{j=1}^{J} x_{ij} = 1, \quad 1 \leq i \leq I \tag{1}$$

Capacity Constraints

$$\sum_{i=1}^{I} r_i^k x_{ij} \leq \alpha_j^k \phi_k, \quad 1 \leq k \leq K, \quad 1 \leq j \leq J. \tag{2}$$

To cope with the contiguity constraint, we introduce the notion of distance between an edge a_i and a school j, denoted d_{ij}. For instance, d_{ij} can be defined as the distance between the node where school j is located and the end-node of a_i that is closer to this node. We also introduce the notion of the *walking distance* w used to specify which students have to be bused to their schools; i.e., if $d_{ij} > w$ and $x_{ij} = 1$, then students on edge a_i have to be bused to school j. Conversely, if $d_{ij} \leq w$, we say that a_i is within walking distance of school j. Then partition $\bar{A}$ as

$$\bar{A} = \left(\bigcup_{j=1}^{J} W_j \right) \cup Z \cup B$$

where

$$W_j = \{a_i \in \bar{A}: d_{ij} \leq w \text{ and } d_{il} > w \text{ for all } 1 \leq l \leq J, l \neq j\}$$

$$Z = \{a_i \in \bar{A}: d_{ij} \leq w \text{ for more than one index } j,\ 1 \leq j \leq J\}$$

$$B = \{a_i \in \bar{A}: d_{ij} > w \text{ for all } 1 \leq j \leq J\}.$$

To reduce the number of students to be bused, the following walking constraints are introduced.

$$\text{For any edge} \quad a_i \in \left(\bigcup_{j=1}^{J} W_j \right) \cup Z \quad \text{then} \quad x_{ij} = 1 \quad \text{only if } d_{ij} \leq w. \tag{3}$$

Hence, if $a_i \in W_j$, then $x_{ij} = 1$. Of course, constraints 2 and 3 may be conflicting in the sense that there may not be any solution that satisfies both types of constraints simultaneously.

The edges in Z should be assigned to their closest school as long as capacity constraints 2 can be satisfied. Furthermore, to improve contiguity of the sectors, priority should be given to edges that are closer to their closest school. For the edges in B, contiguity of the sectors facilitates the busing operation, and hence, represents a desirable objective.

Finally, the following measure is used during the assignment process to assess how well the solution generated complies with the capacity constraints.

Excess Capacity Measure

$$\mathrm{ECM} = \sum_{j=1}^{J} \sum_{k=1}^{K} \max\left\{0, \sum_{i=1}^{I} r_i^k x_{ij} - \alpha_j^k \phi_k\right\}.$$

The assignment process includes the following procedures.

Procedure *W*-edges

For $1 \leq i \leq I$, $1 \leq j \leq J$ if $a_i \in W_j$ then $x_{ij} = 1$.

Procedure *Z*-edges

Order the edges in Z in increasing order of their distance to their closest school. In order, assign each edge $a_i \in Z$ to the closest school j such that $d_{ij} \leq w$ and constraints 2 remain verified. If this is not possible, then assign a_i to the closest school (even if some constraints 2 are not verified).

Procedure *B*-edges

Order the edges in B in increasing order of their distance to their closest school.

In order, treat each edge $a_i \in B$ and determine S_i, the set of schools to which adjacent edges to a_i (that is, edges with an end node in common) are assigned. If $S_i = \phi$, then take $S_i = S$, the set of all schools. Assign a_i to the closest school j in S_i such that constraints 2 remain verified. If this is not possible, then assign a_i to school j in S_i inducing the smallest value ECM.

The most natural approach to deal with the problem is to use the procedures in the following order: procedure ***W*-edges**, procedure ***Z*-edges** and procedure ***B*-edges**.

2. DECISION SUPPORT SYSTEM

The menu-driven system is implemented on a microcomputer (IBM PC type) with two monitors: the color graphics display allows assessment of the contiguity aspect of the solutions; information on school capacities, edge populations, the number of students assigned to the different schools, and the menus available are given on a separate monochrome monitor.

Four menus are offered to the user: *Main*, *Districting*, *Graphics*, and *Modification*. Their specific content is summarized in Tables I to IV. The *Districting* menu includes the procedures presented in Section 1 and additional functions that provide information about the problem data and the schools' attendance. The *Graphics* menu includes several functions to improve the graphics display, to analyze the solution in specific regions, and to print information concerning the solution. Finally, the *Modification* menu allows the user to modify the solution and the data of the problem, and to obtain information about specific edges and schools.

The road network is shown on the color graphics display. A different color is associated with each school and the edges assigned to it. When using the *Modification* menu, the user can select an edge (pointing to it with the cursor) and change the school to which it is assigned. A similar function is available to modify the assignment of all edges within a rectangle (specified with the cursor) to a specified school. The color of the edges is modified accordingly and the reports are updated automatically. Thus, these operations are executed interactively.

This interactive system should be regarded as a tool for the user to obtain a suitable solution. All functions included in the menus are available at all time and can be called whenever required.

To illustrate a typical analysis, we refer to the problem shown in Figure 1. The problem includes over 4,000 students, 7 schools offering kindergarten and grades 1 to 6. The network includes more than 1,250 nodes and 1,700 edges. The 7 schools are located at the circled nodes on Figure 1.

First, the procedure ***W*-edges** is used. The sequential assignments of these edges to their school are shown on the color graphics display as the edges take the color of the school where they are assigned. In Figure 2, we illustrate the edges assigned to the three schools in the upper right corner of the network during execution of procedure ***W*-edges**. Broken and dotted line edges are assigned to the upper left and the lower right schools, respectively. Note that no edge has been

Table I
Main Menu

F1 Help	F2 Save	F3	F4	F5 End	F6	F7	F8 Distr

Help User manual for the different functions available.
Save Create a file for the solution generated.
End Terminate the program, but first verify whether the user wishes to save the solution.
Dist Move to the *Districting* menu.

Table II
Districting Menu

F1 *W*-edges	F2 *Z*-edges	F3 *B*-edges	F4 ChgPss	F5 Zero	F6 Tables	F7 Cap	F8 Graph

***W*-edges**	Procedure ***W*-edges**.
***Z*-edges**	Procedure ***Z*-edges**.
***B*-edges**	Procedure ***B*-edges**.
ChgPss	Modify the schools' capacities.
Zero	Reinitialize the solution process.
Tables	On the monochrome display are shown tables that indicate for each grade in each school the number of students acceptable and assigned.
Cap	On the monochrome display are indicated the number of classes per grade available in each school and the student/teacher ratio for each grade.
Graph	Move to the *Graphics* menu.

assigned to the middle school because all edges within its walking distance are also within walking distance of at least one of the other two schools. Furthermore, once the procedure is completed, the number of students in each grade assigned to each school is updated, and these statistics together with the capacities are available on the monochrome monitor.

Then procedures ***Z*-edges** and ***B*-edges** are called successively to generate an initial solution. As before, the sequential assignments of the edges are shown on the color graphics display and the statistics are updated. The results after procedures ***W*-edges** and ***Z*-edges** are applied successively are illustrated in Figure 3. Broken line edges are assigned to the upper left school, heavy line edges to the middle school, and dotted line edges to the lower right school. Figure 4 shows the assignment of the edges in the lower left corner of the network in Figure 1 after applying procedure ***B*-edges**. Since these edges are not within walking distance of any school, the assignments are not homogeneous.

Referring to Figures 3 and 4, it is obvious that contiguity and homogeneity need to be improved. Furthermore, in some cases, school capacities can be exceeded for some grades. Hence, the user needs interactive tools to modify this solution into a suitable one. For this purpose, functions **Mod** and **Mgr** allow the modification of the assignment of one edge at a time or the assignment of all edges within some rectangle, respectively. To complete these modifications the user works with the cursor on the color graphics display. Furthermore, to foresee the consequences of such a modification on the schools' capacities, function **Edge** generates information on the number of students per grade located on a specific edge.

If the solution is not fully suitable or other scenarios are to be analyzed, then function **Chg Pss** allows modification of the school capacities before repeating the process.

Note that, in some cases, it might be interesting to specify interactively the assignment of some ***B*-edges** before calling the procedure ***B*-edges** in order to bend the assignment into one amenable for busing, for instance. Indeed, recall that in this procedure the assignment of adjacent edges may influence the assignment of a specific edge.

Finally, at any stage of the process, the user can obtain (on the printer) hard copies of the network indicating the edges assigned to schools or copies of the statistics and tables summarizing the results.

Table III
Graphics Menu

F1 Print	F2 Zoom	F3 Full	F4 Color	F5 Walking	F6 PrTb	F7 PrCap	F8 Modif

Print	A copy of the network is printed and the edges assigned to a school (specified by the user) are shown with larger strokes.
Zoom	Zoom on a region of the network.
Full	Return to the full network.
Color	Modify the colors of the schools and their edges to obtain a better view of the solution.
Walking	Illustrate the subsets of edges within walking distance of specified schools.
PrTb	Print the TABLES described in the *Districting* menu.
PrCap	Print the CAP described in the *Districting* menu.
Modif	Move to the *Modification* menu.

Table IV
Modification Menu

F1 School	F2 Edge	F3 Mod	F4	F5	F6 Mgr	F7	F8 Main

School	Indicate the number of students per grade acceptable and assigned for a specific school pointed with the cursor.
Edge	Indicate the number of students per grade located on a specific edge pointed with the cursor.
Mod	An edge selected with the cursor is reassigned to another school also selected with the cursor.
Mgr	All edges within a specified rectangle are reassigned to another school selected with the cursor.
Main	Move to the *Main* menu.

3. RESULTS AND CONCLUSION

The system was coded in *C*-language together with the graphics software GKS. It was implemented on an IBM-PC AT with the following equipment.

- 30 Mb hard disk
- 640 kb of user memory
- EGA color display and adaptor
- Monochrome display and adaptor
- Graphics printer
- Serial mouse with an RS-232C serial port
- 80287 math coprocessor.

The microcomputer is powerful enough to support the system since, for the preceding application, it takes about 12 seconds to execute procedures ***W*-edges**, ***Z*-edges**, and ***B*-edges**. Furthermore, the network is

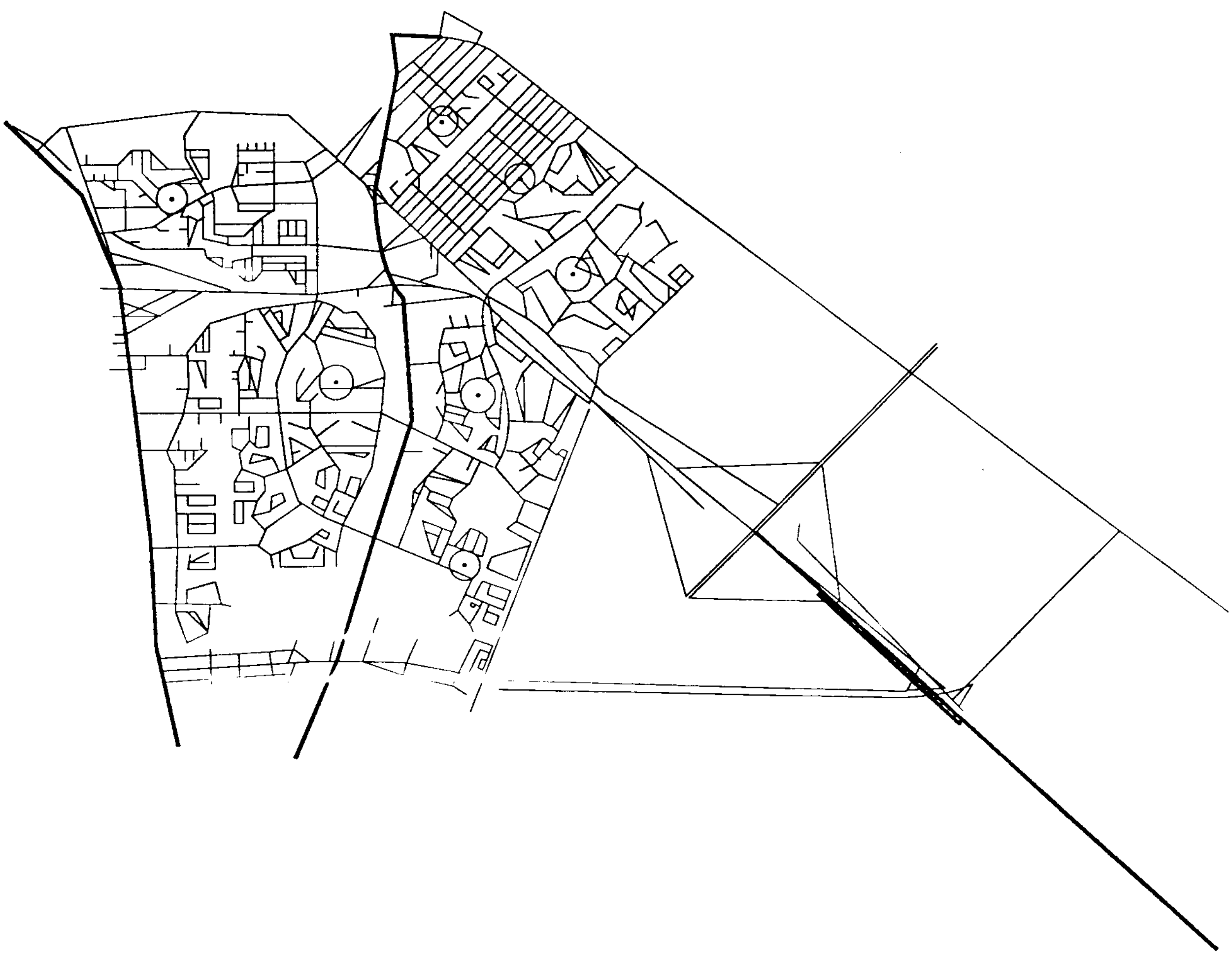

Figure 1. Road network.

Figure 2. Assignments after applying procedure **W-edges**. (___) Assigned to upper left corner school; (• • • • •) assigned to lower right corner school.

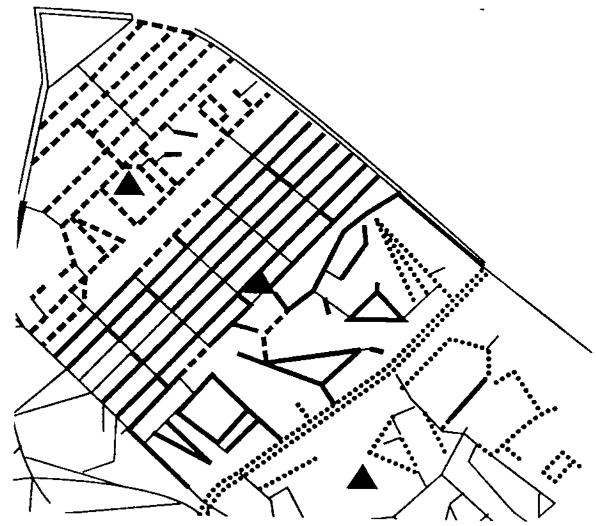

Figure 3. Assignments after applying procedures **W-edges** and **Z-edges**. (___) Assigned to upper left corner school; (——) assigned to middle school; (• • • • •) assigned to lower right corner school; (——) no children living on the edge.

drawn on the graphics display in about 21 seconds. Finally, the average time required to obtain information for a school is 0.1 seconds and 1 second for an edge.

To specify the physical network, basic information on nodes, arcs, and streets have to be input. Each node is characterized by its coordinates and a pointer to the first arc in the set of arcs adjacent to it. For each arc, the end-nodes are specified together with the next arc adjacent to each of these end-nodes. Each street is characterized by a name and a set of arcs corresponding to its street segments. With such a structure, a path between two nodes is easily specified.

These data were made available to us by the school board transportation department. They could have been obtained from Statistics Canada, a federal organization. It was relatively easy to generate the network. The memory required to store these data is allocated dynamically. For instance, a network with 2,000 nodes, 3,000 arcs and 500 streets uses approximately 70 kb of user memory.

The system has been shown to several school board administrators who were quite satisfied with the quality of the starting solution generated with the heuristic procedures. Indeed, this solution was, in general, close to the one obtained manually and easily modified into a suitable one.

The system is of particular interest whenever a major reorganization is required because a school has to be closed or opened, or school capacities are modified. Administrators may then analyze several scenarios very rapidly.

The collaboration of Gilbert Boissy, Brossard school board administrator, was essential for the success of our development. He is using the system. Furthermore, he has fully benefitted from using it because the school board recently faced a major reorganization. It

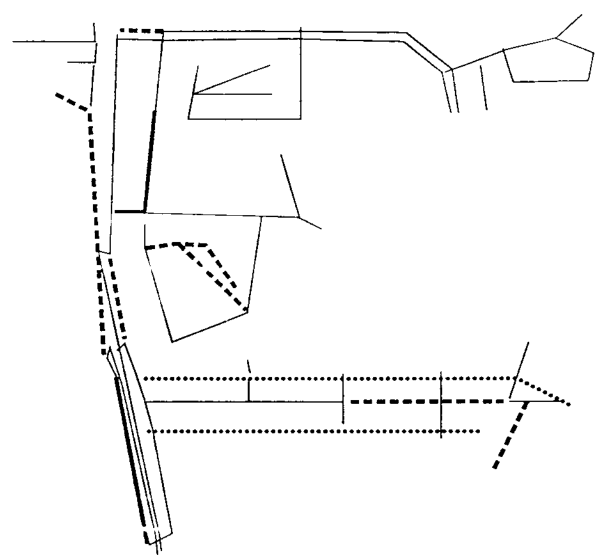

Figure 4. Assignments after applying procedure **B-edges**.

was then possible to analyze several scenarios to identify the most suitable solution for the majority of the population.

By using the system, the time required to analyze a scenario and to obtain the appropriate reports is reduced by a factor of more than fifteen. Furthermore, the analysis is much easier and more precise. Indeed, when the districting operation is completed manually, the user has to refer to several files to trace the street segments, their population and their assignment to schools. The analysis is cumbersome, and the human factor could be a source of errors. Finally, the information can be transferred directly to the main databank of the school board.

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