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APPLICATION OF THE TRANSPORTATION MODEL TO A LARGE-SCALE "DISTRICTING" PROBLEM

PAUL G. MARLIN*,†
Factory Mutual Research, 1151 Boston-Providence Turnpike, Norwood, MA 02062, U.S.A.

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Scope and purpose—The task of partitioning a geographical area into sub-areas according to some criteria is referred to as a "districting" problem. A problem of this type is characterized by a large number of geographical locations, each having a specified "activity" level, and the need to assign each location to a sub-area or district. This paper deals with the question of assigning locations to a district in such a way that the total activity assigned to each district falls within specified upper and lower bounds and that the total travel cost to service the locations from a specified set of geographical points is in some sense minimized. The problem is of large enough scale such that any solution via methods which address each individual location—such as vehicle routing models—is impractical; and other methods are needed. The purpose of this paper is to describe the derivation of the mathematical model, to describe special features which aid its implementation and ability to model the actual situation, and to present results from an actual large-scale problem in the form of a series of maps.

Abstract—The "Application of the Transportation Model to a Large-Scale 'Districting' Problem" describes a method of partitioning an area containing many geographical locations, each with an associated activity or workload, into districts called "tours." The objective is to assign each location to a tour in such a way that the total workload assigned to each tour falls within specified limits and that the total cost to service the locations from a specified set of tour "centers" is minimized. The present approach is compared and contrasted with other approaches to the problem in several contexts. The actual situation motivating the present work and the derivation of the transportation linear programming model is described. Special attention is given to features which aid implementation and which more realistically model the actual situation by handling the user's implicit constraints, i.e. those not explicitly contained in the transportation formulation. Finally, the results of an actual large-scale problem are presented.

INTRODUCTION

This paper describes a procedure developed for a large loss prevention service organization. The company employs over 450 engineers as Loss Prevention Consultants (LPC's) who make approximately 70,000 inspection visits annually to clients located throughout the U.S. and Canada. The company is divided administratively into 18 regions. Each LPC is attached to a region and services only those risks within his region. Certain LPC's are assigned to the regional office located in a major city within the region while others, called "residents" are scattered throughout the region. The inspection visits are classified into two types: planned and unplanned. The former are performed on a regular basis—biannually, annually, or semiannually, depending on the characteristics of the risk. The time it takes to perform this type of inspection can be estimated based on its characteristics and past experience. The unplanned inspections include those following an insurance claim, inspections of candidates for insurance, and visits to risks with special problems. These occur randomly and constitute approximately 38% of the total workload. For planning purposes within each region, they are traditionally assumed to comprise a fixed percentage of the planned workload.

In conjunction with the implementation of an automated work assignment and scheduling system, it was necessary to partition each region into territories called "tours." After allowing for projected non-planned work, lost time, and room for a certain amount of growth, each tour constitutes one man-year of work. In this way, the total planned workload within a region

*Paul G. Marlin holds the BS in Mathematics from Spring Hill College, Mobile, Alabama, and the D.Sc. in Operations Research from The George Washington University. He is currently an operations research specialist with the Factory Mutual Research Corporation in Norwood, Massachusetts. He was previously with the Central National Bank in Cleveland, Ohio and the Institute for Management Science, the Center for Naval Analyses, and TRW Systems. He has published on queues, Markov Chains, and dynamic programming.

†Present address: School of Business Administration, University of Missouri–St Louis, 8001 Natural Bridge Road, St Louis, MO 63131, U.S.A.

dictates the number of necessary tours. In general, each resident is associated with a tour and the workload within it is serviced exclusively by him. On the other hand, the inspections within tours associated with the regional office are rotated among the regional office LPC's.

The process of designing tours comprises two steps:

(1) Assigning risk locations to a tour, i.e. partitioning the region;

(2) Sequencing the risks within each tour for scheduling and assignment purposes.

Conceptually, the process is analogous to a vehicle routing problem which can theoretically be solved in one step. However, a commitment on the part of management to a touring scheme, which required the geographic partitioning of regions, had already been made. It should be pointed out that Doll[4] advocates a similar two-step procedure for solving large-scale vehicle routing problems, where vehicle routes correspond to tours. However, little guidance on how to accomplish Step 1 is given.

The objective of this paper is to address Step 1 and to explore the problem: How can a region be partitioned into territories called tours in such a way that each tour contains an approximately equal workload while overall travel costs are "minimized"? The actual construction of tours within each territory and work scheduling are not addressed.

1. RELATED INVESTIGATIONS

Each region contains on the order of 2000–4000 risk locations. This magnitude, together with the organizational constraint mentioned above, precludes as a practical matter any tour construction scheme based on individual locations, such as a vehicle routing model. The general class of "districting" methods is more appropriate and has been investigated by several authors in various contexts: design of sales territories[3, 7, 10, 12 and 13], political districting[6, 8], design of territories or "turfs" for telephone service personnel[11], and refuse vehicle routing[1]. Each has its special features and objectives:

Sales territories —Maximization of profit with little attention to travel cost,*
Political districting —Compactness and connectedness of districts;
Telephone "Turfing' —Small geographic scale and detail modeling of road network;
Refuse vehicle routing—Orientation of districts to a fixed point (the disposal area) and the ability to create districts of various desired shapes.

In general these methods employ a sequence of optimization routines and/or heuristic algorithms and exhibit the following pattern:

(1) Definition of (N) basic geographic units or blocks, such as census tracts, zip code zones, or counties; and calculation of the workload or activity in each.

(2) Selection of (M) arbitrary blocks or geographic points as territorial "centers" and specifying their total workload or activity.

(3) Assignment of blocks to centers so as to minimize (or maximize) some objective function.

(4) Resolution of the "split block" or "activity constraint" problem.

(5) Redefinition of the territorial centers.

(6) Repeat of Steps 3–5 until no significant improvement is noted.

The split block problem occurs if the optimization algorithm used in Step 3 can assign portions of the activity in a basic block to more than one center. This is the case when the Transportation Linear Programming (TLP) model is employed[7, 8 and 11]. Another approach, zero-one programming[1, 3, 10 and 13], assigns each block to a unique center but does not necessarily satisfy activity constraints associated with each block or center.

The major difference between the TLP formulations of [7, 8] and that of [11] lies in the choice of the objective function. In the turfing problem of [11], the cost of servicing one unit in block j from center i, c_{ij}, is modeled as the distance represented by the shortest path through an adjacency graph from the ith center to the jth block. The adjacency graph has a node for each block and an arc connecting each pair of nodes representing adjacent blocks. This detail is no doubt motivated by the small geographic scale and the need to consider roadways and travel restrictions such as mountains, and rivers where no bridges exist. Hess and Stuart[7] and Hess *et al.*[8], on the other hand, model c_{ij} as d_{ij}^2, where d_{ij} is the straight-line distance between

*An exception is Hess and Stuart[7].

center i and block j. However, as Cloonan[2] points out, travel costs are more nearly proportional to d_{ij}. The d_{ij}^2 formulation places a premium on compactness and its use in [7] is most likely a carryover from the political districting problem[8] in which travel costs play little part.

The major differences between the problem outlined in the introduction and those of [7, 8 and 11] are the fixed nature of our tour "centers" and our emphasis of the travel costs between the centers and the geographic blocks. Our tour centers are defined by the location of the regional office and the residents. Thus, the application of the TLP is a more direct and natural one. Measures of compactness, such as the maximum distance between any two points within a territory, are less appropriate here; and in view of the comment in [2], d_{ij} is used as the unit cost. The question of redefining tour centers does not arise.

A further difference is our treatment of constraints on workloads assigned to a tour.

(a) The ideal or target workloads for the tours are not constrained to be equal.

(b) The tour workloads are constrained to lie within upper and lower bounds, which are specified as a percent deviation from the target.

Further, the split block problem is handled somewhat differently, as discussed below. Flexibility and the ability to model the implicit constraints of the user, i.e. those not included in the formal statement of the transportation model, are emphasized. The present approach resembles [11] in that manual intervention by the analyst and/or user is expected before subsequent iterations are performed.

2. DEVELOPMENT OF THE TRANSPORTATION LP MODEL

The following are assumed to be available as inputs:

M = number of tours;

N = number of basic blocks;

$I = \{i, i = 1, 2, \dots M\}$, the set of centers;

$J = \{j, j = 1, 2, \dots N\}$, the set of blocks;

w_j = required workload (man-hours per year) in block j, $j \in J$;

d_{ij} = distance from the block containing the "center" of Tour i to block j, $i \in I$;

a_i = weighing factor for ideal or target workload of Tour i;

f_i = maximum deviation from the target workload of Tour i, expressed as a fraction.

The target workload for the i-th tour is given by

$$t_i = \Big(\sum_{j \in J} w_j\Big)\Big(a_i \Big/ \sum_{k \in I} a_k\Big),$$

where setting $\{a_i\}$ to a constant yields the equal workload case. Lower and upper bounds on the workloads are

$$L_i = (1 - f_i)t_i \tag{1}$$

and

$$U_i = (1 + f_i)t_i, \tag{2}$$

respectively. The problem can be formulated as the linear program:

$$\text{Minimize } F = \sum_{i \in I} \sum_{j \in J} x_{ij} d_{ij}$$

st

$$\sum_{i \in I} x_{ij} \geq w_j, \quad j \in J;$$

$$L_i \leq \sum_{j \in I} x_{ij} \leq U_i, \quad i \in I \tag{3}$$

where $\{x_{ij}\}$ is the set of decision variables representing the amount of work in the jth block assigned to the ith tour. The objective function minimizes the total travel cost. The first set of constraints ensures that the workload in the jth block is satisfied; and the second set places upper and lower limits on the workload assigned to the ith tour.

The linear programming formulation assumes that the cost of servicing a block is proportional to its workload. While this assumption may not be completely realistic, we will demonstrate that it produces acceptable tours and that it is relatively easy to implement.

With the exception of the left-hand inequality in (3), this is a TLP. A tour center constitutes a source or supply point of the TLP formulation, while the basic geographical blocks represent destinations or demand points. The problem can be recast as a TLP (see [9] page 117) by partitioning the ith tour center (source) into two components; a primary component having a supply of L_i units and designated by the set of indices $\{i \in I\}$, and a secondary component with supply $U_i - L_i$ and designated by the set of indices $\{i = M+1, \ldots 2M\}$. In order to obtain feasibility by maintaining the equality between supply and demand, an artificial $(N+1)$-st block (destination) is created having demand

$$w_{N+1} = \sum \text{Supply} - \sum \text{Demand}. \tag{4}$$

The amount of work assigned to the jth block becomes $x_{ij} + x_{M+i,j}$. The resulting TLP is:

$$\text{Minimize} \sum_{i=1}^{2M} \sum_{j=1}^{N+1} x_{ij} d_{ij}$$

st

$$\begin{aligned} &\sum_{i=1}^{2M} x_{ij} \geq w_j, \quad j = 1, \ldots N+1, \\ &\sum_{j=1}^{N+1} x_{ij} \leq L_i, \quad i \in I; \\ &\sum_{j=1}^{N+1} x_{ij} \leq U_i - L_i, \quad i = M+1, \ldots 2M; \end{aligned} \tag{5}$$

with

$$d_{M+i,j} = d_{ij}, \quad j \in J;$$

and

$$d_{i,N+1} = \begin{cases} M^*, \text{ an arbitrarily large integer, } i \in I; \\ 0,\ i = M+1, \ldots 2M. \end{cases}$$

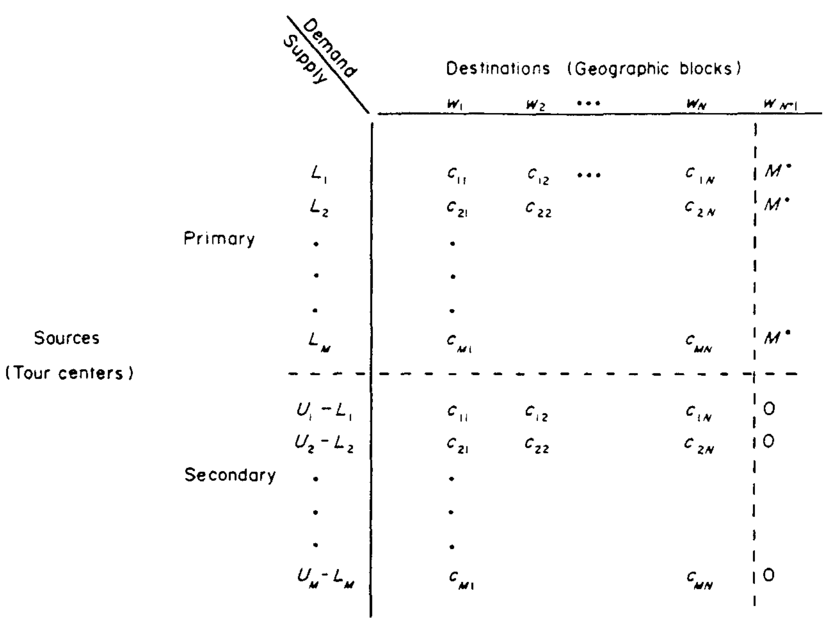

Fig. 1. Augmented transportation tableau.

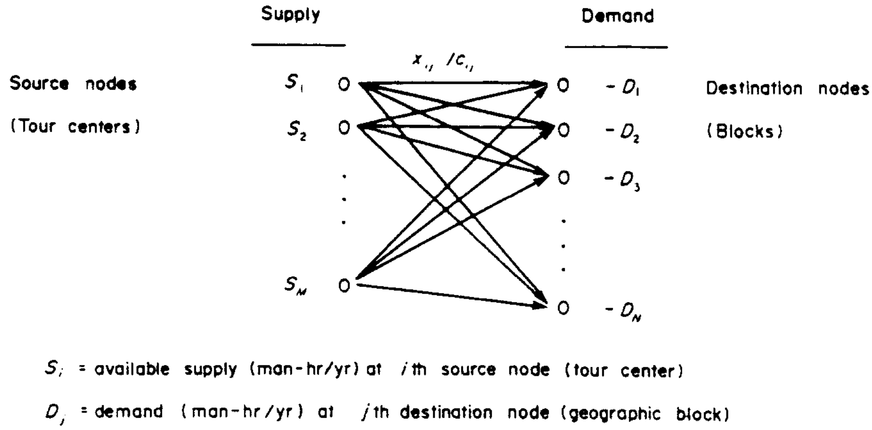

Fig. 2. Network formulation of TLP problem.

The augmented TLP tableau is pictured in Fig. 1. Defining $d_{i,N+1}$ in this way causes demands to be supplied first from the primary source and then from the secondary. The excess capacity from the secondary source is routed to the artificial destination. It can be shown that this formulation insures that the equality in (5) holds and that the total allocation from source i to block j satisfies (3).

The TLP problem is solved by reformulating it as a network and solving for the minimum cost flow through the network. The correspondence between the TLP and network formulation can be seen in Fig. 2. The nodes of the network represent the supply and demand points. A directed arc connects each source with each destination. The flow through each arc represents the amount of demand at a given destination supplied from a given source. The total flow out of a source node is the supply available at that node; while the total flow into a destination node is the total demand at that destination. The unit cost of flow is the $\{d_{ij}\}$ of the TLP problem. The network problem is solved by the Ford and Fulkerson out-of-kilter algorithm[5].

3. SPECIAL FEATURES

The following features were found necessary and/or useful in implementing the TLP model. Although the comments apply specifically to the example discussed below, they could apply to other problems on the same scale.

(a) The first consideration in implementing the model is choosing the definition of the basic geographical block. Too fine a definition would increase the data collection efforts and the computation time; selecting too large a block could result in poor tour design and would certainly compound the split block problem. The unit chosen was the county. Because of the scale of the problem, census tracts or individual zip codes are too fine a division and three-digit zip code areas too large. In several cases, counties could be combined into one block, especially in areas near the boundary of the region or in remote areas where there is very little work in given counties.

(b) The workload in each county or group of counties was assembled from a data base containing, among other items, the address and workload in man-hours of each risk to be visited. The zip code in the address was mapped into the county by means of a zip code—county correspondence file, available from the Postal Service.

(c) The out-of-kilter algorithm requires as inputs information concerning *each node*, one for each source and one for each destination in the TLP; and *arc*, one for each allowable source-destination combination. Two options are generally available. The first generates automatically an initial basic feasible solution by introducing artificial arcs and nodes. The second proceeds from an initial solution supplied, for example, by the "Northwest Corner" rule. Both options require for each arc an index pointing to the source node, an index pointing to the destination node, and the unit cost of flow. Option 1 also requires the net flow out of each node: the value of the supply constraint for source nodes and (-1) times the demand constraint for destination nodes. Option 2 requires the initial solution, i.e. the amount of flow through each arc, and the value of the dual variable for each node (which may be set to zero). The supply and demand constraints are contained implicitly in the solution. The network algorithm requires the supply and demand values to be integers. The workload in each geographic block, i.e. the demand at each destination node, is given as an integer in the present application. The supply at each source node as computed by equations (1) and (2) is rounded to the nearest integer; and any net gain or loss is accounted for by equation (4).

(d) The cost coefficients are determined by d^s_{ij}, the straight-line distance between the ith center and the jth block:

$$d^s_{ij} = \{(X_{c(i)} - X_j)^2 + (Y_{c(i)} - Y_j)^2\}^{1/2},$$

where

$c(i)$ = index pointing to the block in which the ith center is located;

and

(X_j, Y_j) = geographic coordinates of the jth block.

Rather than superimpose a rectangular grid on a map of the district and record $\{X_j, Y_j\}$ directly, a polar coordinate system was utilized as follows. An origin and east-west axis were established on a map of the region, which was then secured to a drafting table. The distance in centimeters ρ between the origin and the principle city or town in each block and the polar angle θ were read from a drafting machine. The rectangular coordinates were then calculated by

$$X = k\rho \cos \theta$$
$$Y = k\rho \sin \theta,$$

where k is the appropriate centimeters-to-miles scale factor.†

(e) With the straight-line distances as starting points, adjustments A_{ij}, expressed as a percentage of the straight-line distance, are used to reflect actual travel distances, i.e.

$$d_{ij} = d^s_{ij} \cdot A_{ij}/100.$$

Because of the scale of the problem, only large travel barriers need be considered. However, the adjustment feature is used primarily in modeling implicit constraints, resolving the split block problem, and in the post-optimality analysis as described below.

(f) If all blocks were connected to each center in Fig. 2, there would be $2M(N+1)$ arcs, or decision variables. The size of the network model is reduced in several ways. For example, a block containing a center would never be assigned to a tour centered in another block (no block's workload is greater than the total workload of a tour). Therefore, $2M(M-1)$ arcs can be eliminated. Also, there is a constraint of approximately 200 miles on the one-way distance that an LPC can travel by automobile, effectively reducing the number of arcs emanating from centers without reasonable scheduled air service. This is accomplished during the construction of the network model by setting a maximum length, K_i, for each arc; and eliminating an arc if

$$d_{ij} > K_i, \quad i \in I;$$

where $\{K_i\}$ is an input. Further, the geographic distribution of residents eliminates from consideration certain center-block combinations. This is accomplished by means of the maximum distance criterion or by requiring that blocks in a particular state not be assigned to tours centered in some other state, etc. Finally, implicit management constraints (such as not desiring Minneapolis residents to service locations in St. Paul and vice versa) eliminate certain center-block combinations. This is accomplished by applying an adjustment large enough to cause d_{ij} to fail the maximum distance criterion.

(g) There may be more than one tour centered in the same block. This will occur at the

†A quicker and more accurate procedure is to use an automatic digitizer such as the peripheral available with the HP-9821 desk calculator.

regional office city and in larger cities where more than one resident resides. If a block with a workload of W is the center for k tours, $k-1$ additional blocks are defined, each with a workload of W/k. The location of each block is placed on an imaginary circle around the edge of the city. In this way, each tour will be assigned an equal amount of "local" work with the remainder tending to form wedges radiating from the city. An alternate approach, depending on the requirements of the user, is to define as many strictly local tours within the multiple tour block as necessary and eliminate their workloads from the problem.

If it is desired to delineate the k tours within a metropolitan area, a smaller scale TLP problem can be formulated with zip codes or census tracts, for example, as the basic geographical blocks. In this case it is postulated that the modeling detail of [11] would not be required. A judicious choice of the distance adjustment coefficients by the user would suffice.

To summarize, the ability to redefine the basic blocks, to constrain the maximum center-block distances for each tour, to arbitrarily adjust distances, and to specify the target workload for each tour provides a great deal of *flexibility and provides the ability to capture the major features of the problem and to handle the implicit constraints of the user.*

4. THE SPLIT BLOCK QUESTION

The TLP produces at most $M-1$ split blocks, i.e. blocks whose workload is divided among more than one tour. Since the blocks are merely arbitrary sets of locations, the split blocks may be retained, or they may be removed for administrative convenience. In the latter case, the situation may be such that no more than one segment of the split block contains a large workload. In this case the assignment of the entire block to one tour can be made manually by the user without significantly violating the workload constraints. If this is not possible, adjustments are made to the distance coefficients and a new solution obtained. The adjustments A_{iij} are set so as to force the entire block to be assigned to the desired tour.

In certain cases the split blocks may be allowed to remain. The final allocations may be made manually by the user based on his knowledge of the territory. This is especially applicable when the block in question contains more than one distinct geographic concentration of work. If this is not the case, the block can be partitioned among the competing tours by means of a smaller scale TLP with zip codes, for example, as basic blocks. This may be desirable where large concentrations of work are located midway between two or more tour centers.

5. PILOT TEST IN MILWAUKEE REGION

The TLP model was tested using actual data from the Milwaukee region. Milwaukee is a geographically large region, requiring considerable travel. It includes the states of Wisconsin, Minnesota, North and South Dakota, the northern half of Iowa, the upper peninsula of Michigan, and the provinces of Manitoba and Ontario, west of Sault Ste. Marie. Inasmuch as the ultimate output of the model is a map delineating the tours, the discussion of the result will focus on maps of the region. As an aid in interpreting them, the following map symbols will be used.

Multiple Tour Blocks
Regional Office (5 tours)
Minneapolis–St. Paul residencies (4)

⊙ Location of Other Residencies

Milwaukee Tours
1 South
2 Southwest
3 Central
4 Northwest
5 North

Minneapolis–St. Paul Tours

A East (St. Paul)
B South
C Central
D North

┼┼┼ Split Block Boundaries†

The workload in Manitoba and Ontario equals approximately two man-years, which are assumed to be assigned to the Winnipeg and Thunder Bay residents, respectively. These two tours are eliminated from the discussion to follow. The total remaining workload required 18 tours (5 regional office and 13 resident) with an average of 1129 planned hours per tour.

To begin, equal size (workload) tours are examined; i.e. $\{a_i\} = \text{constant}$ and $\{f_i\} = 0$ in equations (1) and (2). The solution is shown in Fig. 3. The first point to notice in Fig. 3 is that the tours originating in Milwaukee and the Twin Cities form in general (in addition to their "local" component) elongated wedge-shaped areas emanating from their respective cities. This is advantageous in that major highways generally radiate in various directions from large cities and it facilitates the sequencing of locations for scheduling purposes (Step 2 in the construction of tours). Of course, the actual shape is dictated by the shape of the counties.

The second point to notice is that almost all of the tours exhibit a westward orientation, i.e. the "centers" are located in the far eastern part of the tour area. The St. Paul tour even extends to the northwest and southwest of Minneapolis. To some extent, this phenomenon is due to the extreme eastern location of the regional office and to the distribution of the residents relative to the workload. However, in order to determine to what extent it results from the assumptions of the model, several alternatives will be investigated.

The model divides central and western North and South Dakota, comprising almost one man-year, among three different Twin City tours. On the other hand, this work is currently serviced from Milwaukee. The model's decision is not illogical when one considers that when flying from Milwaukee to this area one must change planes in Minneapolis–St. Paul. This management decision is modeled by forcing the work in this area to be assigned to Milwaukee tours. This is accomplished by applying an arbitrarily large distance adjustment factor to the arcs connecting the Twin City tours with the two non-empty blocks in the area. A lack of air service in other resident sites eliminates them from consideration. The results in Fig. 4 indicate that the resident sites, with the exception of the southern Minnesota resident (Mower, MN) and the one just southwest of Milwaukee (Walworth, WI), are noticeably closer to their respective geographical centers. While the total travel cost, as measured by the objective function, is increased by 34%, the maximum trip length is reduced in some of the tours. This is a desirable feature from the standpoint of employee morale in that fewer overnight trips are required and these are of shorter duration on average. This is something that the user has to weigh against the increase in total travel.

The assumption of equal size tours may also have contributed to the westward orientation found in Fig. 3. The actual current work assignments for residents vary from the average by plus or minus 20%, i.e. the largest is 50% greater than the smallest. This is not an ideal situation, but is the accumulation over time of the effects of gradual shifts in business, the ability to hire residents, and historical work assignment practices. Reducing this variation by one-half, each tour was assumed to vary by only 10% from the average. This is accomplished by setting $f_i = 0.1$ in equations (1) and (2). The result is shown in Fig. 5. The assumption reduces the westward orientation somewhat, especially in Wisconsin. The model assigns the maximum amount of work to all of the Minnesota residents and to the two westernmost Wisconsin residents. On the other hand, all the Milwaukee and all but one of the remaining Wisconsin tours receive the minimum amount allowed by the constraints. It is interesting to note this distribution is much closer to that produced by the current work assignments. This relaxation of the supply constraints yields a decrease of 9% in the objective function.

†Segments of split blocks containing 10 or fewer hours were arbitrarily reassigned in the most practical way.

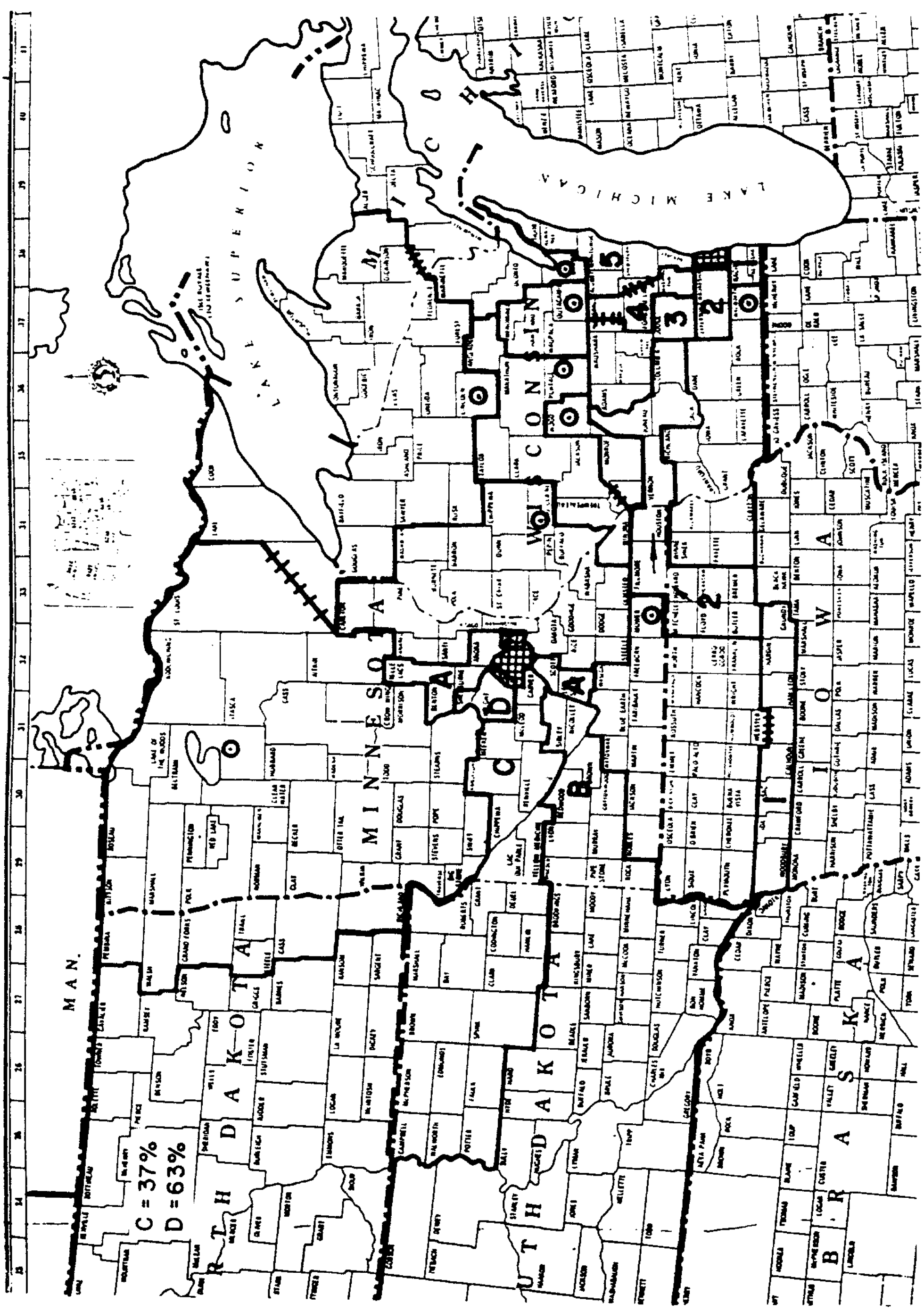

Fig. 3. Equal size tours.

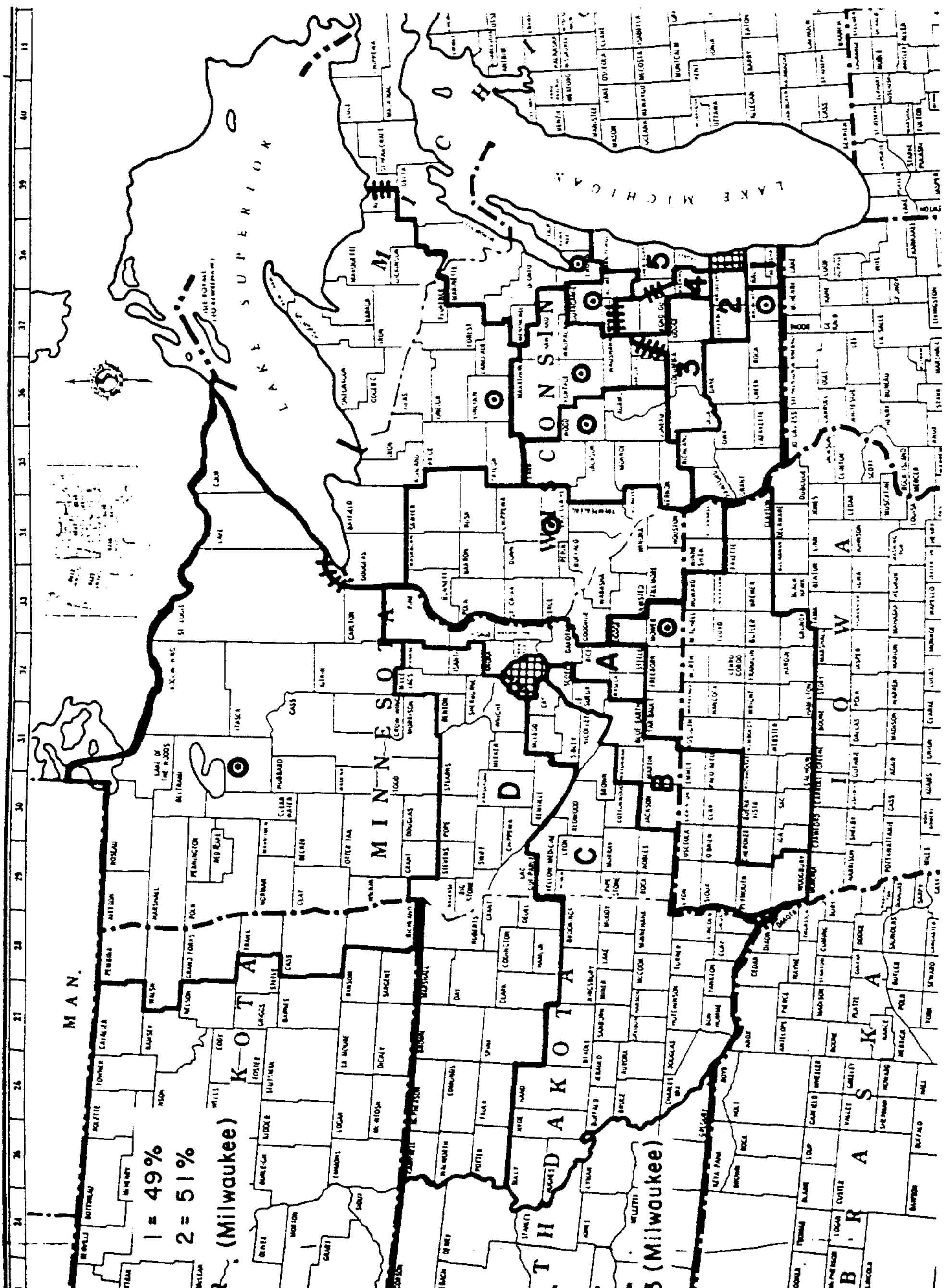

Fig. 4. N.D. & S.D. to Milwaukee.

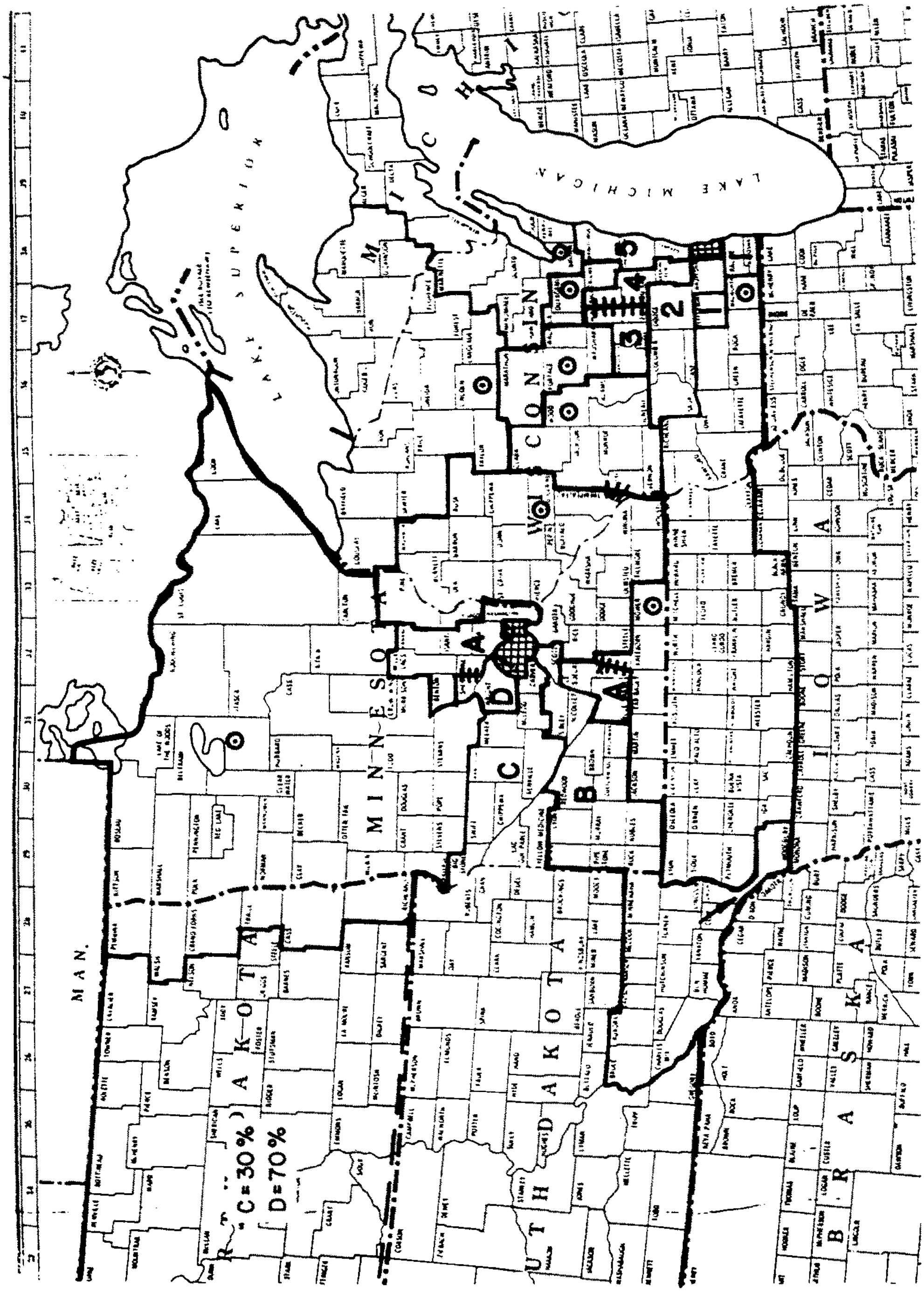

Fig. 5. Upper and lower bounds on tour size.

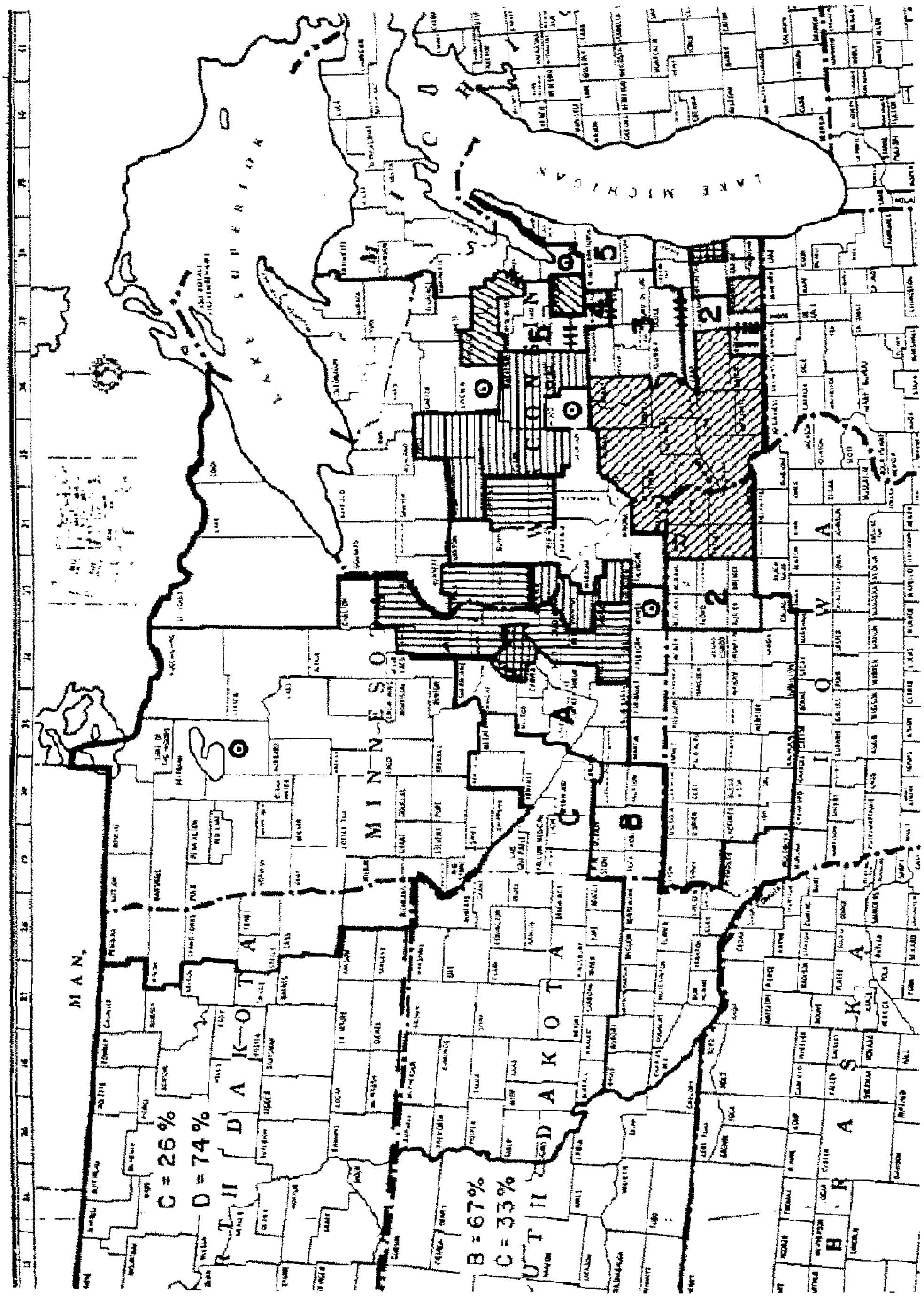

Fig. 6. 2 Unit cost = d_{ij}

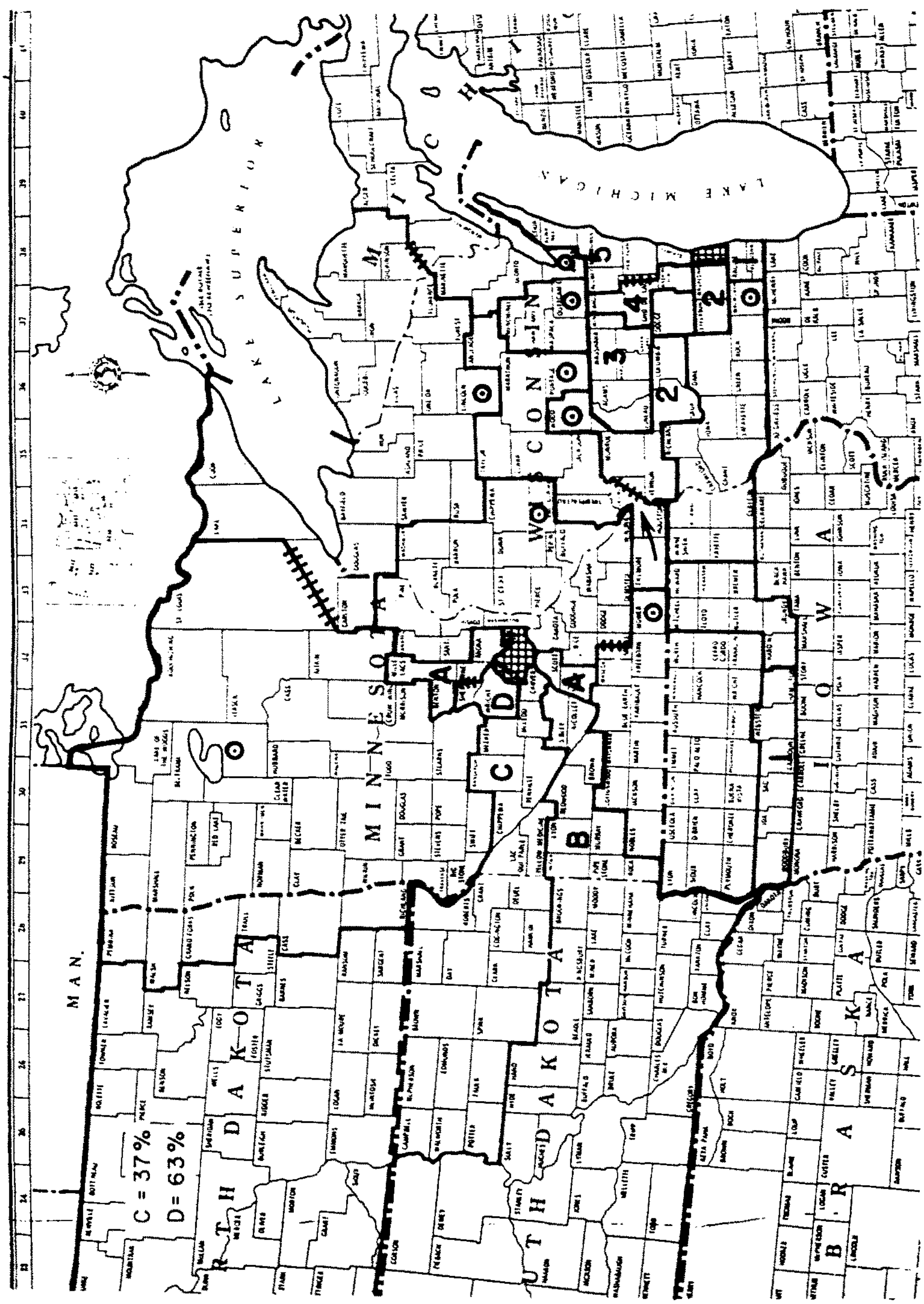

Fig. 7. Removal of split block.

It should be noted that the northern Minnesota tour covers a large geographical area, even in the equal workload case. This may be unacceptable if it causes an abnormal amount of auto travel time. Inasmuch as the model does not take this into consideration, the only way to account for it would be to set a smaller target workload by means of the tour size parameter a_i. Reducing the maximum allowable distance would produce an infeasible situation because of the low density of the business in that area.

As a final examination of the effect of the model's assumptions on tour shape, the use of d_{ij}^2 in the objective function as in [7, 8] is investigated. This formulation, shown in Fig. 6, produces compact segments of disconnected tours. The three Minneapolis tours are completely disconnected from the metro area by the St. Paul tour. Other disconnected resident tours are pictured in Fig. 6, in which each similarly shaded area represents the same tour. This result is obviously non-optimal from the standpoint of travel distance and would make the sequencing of risks for scheduling purposes difficult. Apparently, the success of the d_{ij}^2 formulation in [7, 8] is dependent on the ability to redefine territorial centers as the centroid of the TLP solution and resolve, and is not appropriate for the case of fixed territorial centers.

Finally, we demonstrate how split blocks can be resolved by means of the distance adjustment feature. Consider Fig. 3 in which Webster County, Iowa, is split between the southern Minnesota tour (Mower) and a Milwaukee tour (South). It will be assumed that Webster is assigned to the Milwaukee tour. This is accomplished by increasing the distance parameter for the arc connecting Webster with Mower. The result is shown in Fig. 7. The block is no longer split; and minor changes ripple through the entire system. This implicit constraint increased the value of the objective function by only 0.5%.

Although it is not presented among the results, the effects of seasonality can be handled readily. For example, the required workload during July and August in the Twin Cities area is such that there is an excess of approximately 100 hours over what its four residents can accomplish. This shortfall is currently being handled by the Eau Claire, Wisconsin, resident. This situation is easily modeled by subtracting 100 hours from the total demand in the Twin Cities blocks and simultaneously eliminating 100 hours from the supply available in the Eau Claire tour. The 100 hours is then assigned to the Eau Claire tour.

6. CONCLUSIONS

This paper has described the use of the transportation linear programming model for partitioning a large-scale geographic area into territories called tours in such a way that upper and lower bound constraints on the workload of each tour are satisfied and travel distance is minimized. Additional features which provide flexibility and the ability to model the actual situation according to the requirements of the user was also discussed. The method was demonstrated for a large-scale problem under varying assumptions using actual data.

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