A MULTICRITERIA DISTRICTING HEURISTIC FOR THE AGGREGATION OF ZONES AND ITS USE IN COMPUTING ORIGIN-DESTINATION MATRICES

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ABSTRACT

Territorial aggregation is an important issue in many traditional applications, such as political districting, plant location, health care zoning, and travel demand study. In particular, in travel demand analysis one is interested in estimating travel flows between origins and destinations. The number of trips performed between origin i and destination j is represented by the (i, j)-th entry of an Origin-Destination (OD) matrix. In order to define the OD matrix, a discrete set of origins and destinations is needed, and territorial aggregation methods are used precisely to choose these origins and destinations. Typically, a survey is performed to collect data on travel between the selected origins and destinations. However, when the survey OD matrix is large and sparse, problems in the estimation of travel demand may arise. This drawback can be avoided by further aggregating the origins and destinations on which travel demand is evaluated and by obtaining a new and smaller OD matrix where each cell entry carries a greater value. The corresponding map is then exploited to calibrate the statistical model used to estimate the original disaggregated travel flows. In this work, we first suggest a set of optimality criteria which can be used to find "good" territorial aggregations and we adopt the Old Bachelor Acceptance heuristic to identify them. In order to understand the trade-off between these criteria, we introduce appropriate optimality indexes and we minimize their weighted combinations with several different sets of weights. Our application refers to the city of Rome and the actual study, which involved statisticians, engineers and operations researchers, was supported by the local government (Comune di Roma). Our experimental results on the city of Rome show that the Old Bachelor Acceptance heuristic finds aggregations with low values for all the indexes simultaneously. In particular, our results lead to the conclusion that certain sets of weights must be adopted if one wants to be sure to find "good" aggregations.

Keywords: Territorial aggregation, local search, multicriteria optimization, OD matrices.

RÉSUMÉ

L'agrégation territoriale est une question importante dans de nombreuses applications telles que le découpage électoral, la localisation de structures industrielles, la répartition en zones du système sanitaire, et l'étude de la demande de déplacements. En particulier, en ce qui concerne l'analyse de la demande de déplacements, nous voulons estimer les flux entre les origines et les destinations de déplacements. Le nombre de déplacements entre une origine i et une destination j est représentée par l'entrée (i, j) d'une matrice Origine-Destination (OD). Afin de définir la matrice OD, il faut disposer d'un ensemble discret d'origines et de destinations, et les méthodes d'agrégation territoriale sont utilisées à cette fin. Habituellement, une enquête permettra de collecter des données sur les déplacements entre les origines et les destinations choisies. Toutefois quand la matrice OD de l'enquête est grande et creuse, des problèmes dans l'estimation de la demande de déplacements peuvent se produire. Cet inconvénient peut être évité en agrégeant ultérieurement les origines et les destinations sur lesquelles la demande est évaluée, en obtenant une nouvelle matrice OD plus petite où les valeurs des entrées sont plus grandes que dans la matrice initiale. La carte d'origines et destinations obtenue est exploitée pour calibrer le modèle statistique utilisé pour estimer les flux de déplacements desagrégés originaux. Dans ce travail, nous suggérons premièrement une série de critères qui peuvent être utilisés pour trouver de "bonnes" agrégations territoriales et nous adoptons l'algorithme heuristique *Old Bachelor Acceptance* pour les identifier. Afin de comprendre le *trade-off* entre ces critères, nous introduisons les index correspondants et nous minimisons leurs combinaisons pondéreés avec plusieurs séries différentes de poids. La ville de Rome est l'objet de notre applica-

Recd. Dec. 2002, Rev. Oct. 2003, Acc. Apr. 2004

tion, issue d'une étude qui a impliqué la collaboration de statisticiens, d'ingénieurs et d'experts en recherche opérationnelle et a été soutenue par le gouvernement local (la commune de Rome). Nos résultats expérimentaux sur la ville de Rome montrent que les agrégations trouvées par l'algorithme heuristique *Old Bachelor Acceptance* produisent des valeurs basses pour tous les index simultanément. En particulier, nos résultats mènent à la conclusion que certaines séries de poids doivent être adoptées pour être sûr de trouver les "bonnes" agrégations.

Mots-clés : agrégation territoriale, recherche locale, optimisation multi-critère, matrices origines-destinations.

1. INTRODUCTION

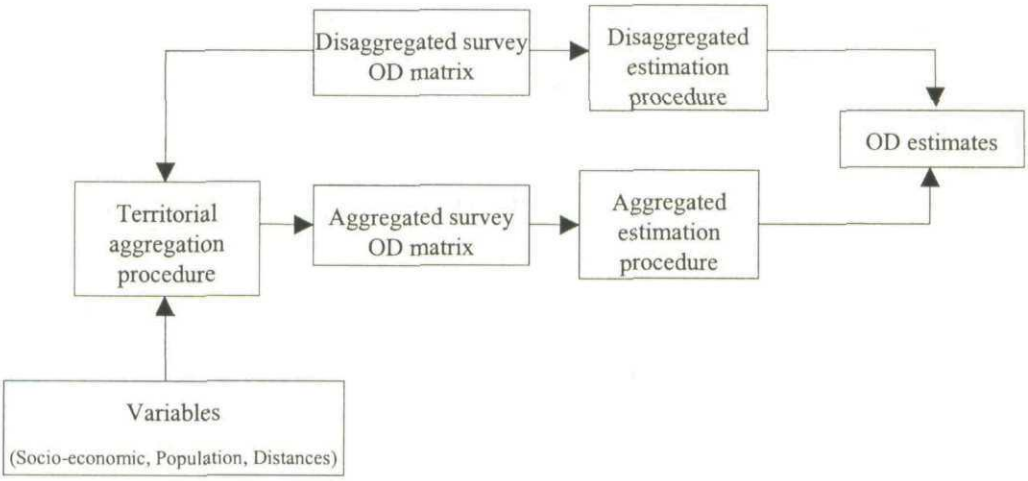
Territorial aggregation plays an important role in many traditional applications, such as political districting, plant location, health care zoning and travel demand study [2, 4, 8, 12]. Different territorial aggregation algorithms have been suggested in literature, but recently the attention of many authors has moved to local search algorithms, such as Tabu Search, Simulating Annealing, and others [4, 9, 10, 11, 12, 18]. Following the results of a comparison between different local search algorithms presented in [18], we suggest to use the heuristic known as the Old Bachelor Acceptance algorithm [14] for travel demand models. In travel demand analysis one is interested in estimating travel flows between origins and destinations. The number of trips performed between origin i and destination j is represented by the (i, j)-th entry of an Origin-Destination (OD) matrix. In order to define the OD matrix, a discrete set of origins and destinations is needed, and territorial aggregation methods are used precisely to choose these origins and destinations. Typically, a survey is performed to collect data on travel between the selected origins and destinations. However, when the survey OD matrix is large and sparse, problems in the estimation of travel demand may arise. This drawback can be avoided by further aggregating the origins and destinations on which travel demand is evaluated and by obtaining a new and smaller OD matrix where each cell entry carries a greater value. In general, when territorial aggregation is needed, it is performed by one of the traditional cluster analysis techniques based on a single criterion [1].

The use of local search techniques in this field is a new approach. To our knowledge, there is not much experimental work on territorial aggregation for OD models, although the level of aggregation is very important in trip distribution models estimation [3, 5]. Some interesting remarks about this problem are given in [22].

One of the technical issues related to the analysis of travel demand is the definition of individual choice sets, representing the alternative destinations each person may choose to reach. Since individuals cannot be taken into account one by one, the territory under study is partitioned into small homogeneous areas, called "micro-zones", which represent the origins and destinations of the flows. Territorial aggregation procedures are generally applied to identify these micro-zones. Once the micro-zones are known, travel flows are estimated on the basis of an observed OD matrix by the use of traditional techniques, such as the maximum likelihood method. We will refer to this case as the "disaggregated estimation procedure". However, there are situations in which territorial aggregation must be applied twice. In fact, when the survey OD matrix is large and sparse, problems in the estimation of travel demand may arise. To solve these problems, one may choose to increase the sample size, but this implies additional economic costs and may generate serious computational problems. The alternative is to adopt a territorial aggregation procedure which merges the micro-zones into larger macro-zones [3, 8]. This produces a new and smaller OD matrix where the cell entries carry greater values, thus providing a less sparse survey OD matrix without increasing the sample size. The aggregated survey OD matrix is then exploited to estimate the disaggregated travel flows. We will call this procedure the "aggregated estimation procedure". Figure 1 shows both the aggregated and disaggregated procedures used to estimate travel flows.

When adopting an aggregated estimation procedure one must accept the disadvantages due to the fact that the model looses some of its original explanatory power, given that the number of

Figure 1: Aggregated and disaggregated procedures for OD flows estimation



observations available will be smaller and each observation will refer to a larger zone (which are called "macro-zones"). However, the use of some "optimality" criteria while aggregating can help in minimizing the loss of information between the original and new observations, thus allowing for a better estimate of the model's parameters.

We suggest using the following three optimality criteria while performing the territorial aggregations: *compactness*, *population equality* and *inner variance*. Compactness and population equality are typically adopted in territorial aggregation problems, such as political districting [4, 18, 19]. In particular, compactness requires the aggregated macro-zones to be as round as possible, while population equality aims at macro-zones as balanced as possible with respect to their population. On the other hand, inner variance is introduced in order to guarantee that the macro-zones are as homogeneous as possible, with respect to a given set of socio-economic variables, such as the population, the number of schools, hospitals, shopping centers, etc. It is well known that if the total inner variance of the macro-zones is equal to 0, then the explanatory power of the aggregated estimates is the same as the one of the disaggregated estimates, whatever the actual aggregation adopted [3, 8]. Therefore, it is worth believing that minimizing inner variance may help to reduce the loss of explanatory power due to the aggregated model.

In a recent work related to territorial aggregation for the estimation of travel demand in home-to-work models [20], the authors compare the performance of the aggregated and disaggregated estimation procedures when the survey OD matrix is sparse. The three above-mentioned criteria are taken into account and the resulting aggregations are defined "good" or "bad" according to their capability of leading to reliable parameters' estimates. The experimental results show that, when the survey OD matrix is sparse, "good" aggregations correspond to cases where the corresponding optimality indexes are optimized simultaneously. In these cases the estimates obtained by the aggregated estimation procedure are more reliable than those obtained by the traditional disaggregated procedure. However, [20] does not suggest how these "good" territorial aggregations can be found.

This paper is aimed to provide a method to obtain "good" aggregations, with particular attention to the inner variance criterion. The objective function we consider is a convex combination of three indexes measuring the lack of compactness, the lack of population equality and the inner variance, respectively, with different sets of weights. It is worthy to note that the criteria adopted are *conflicting*, i.e., it is difficult to reach the minimum value for all of them at the same time. Our experimental results show that some sets of weights provide better solutions than others.

In this work, we adopt a graph model to represent the territory of the city of Rome on which the experimentation is performed.

The remainder of this paper is organized as follows. Section 2 is devoted to the definition and measurement of the optimality criteria. In Section 3 the graph theoretic model is described, while in Section 4 the local search algorithm is introduced. Section 5 describes the experimental plan, while Section 6 provides the experimental results. Finally, some conclusions are reported in Section 7.

2. CRITERIA FOR THE TERRITORIAL AGGREGATION

There are two main groups of criteria which play different roles in the aggregation procedure. The first group concerns the very nature of a macro-zone and comprises criteria such as *integrity, contiguity* and *absence of holes*. Integrity guarantees that no micro-zone is split up between two or more macro-zones. Contiguity requires that it is possible to walk from every point to every other point in a macro-zone without leaving it, as if it were a single land parcel. Absence of holes means that if one draws any closed curve C in a given macro-zone, all points within the inner domain of C belong to the same macro-zone. In other words, no macro-zone can be fully surrounded by another macro-zone. As we will see later (see Section 3), these constraints are naturally satisfied when the territory is represented as a graph G and each macro-zone is seen as a connected subgraph of G.

The second group of criteria is concerned with compactness, population equality and inner variance. As we already noticed, the first two are widely accepted as basic principles in traditional aggregation problems, while the third one is *ad hoc* for territorial aggregation in travel demand models. Compactness is a key element in territorial aggregation. This intuitive concept is difficult to define and to measure. Consider a graph model, where a macro-zone is a set of adjacent nodes. The *center* of a given macro-zone is the node which minimizes the sum of the distances from all other nodes of the same macro-zone. The distance between two macro-zones is given by the distance between the corresponding centers. When macro-zones are perfectly round (compact), the distance between pairs of micro-zones located in different macro-zones is reasonably approximated by the distance between the centers of the corresponding macro-zones. Therefore, finding the most compact possible macro-zones makes the spatial distribution of the micro-zones change as little as possible. The population equality principle is guaranteed if each macro-zone has the same population size. Finally, minimizing inner variance guarantees macro-zones as homogeneous as possible with respect to a set of social, economic and demographic variables. In the following subsections each criterion will be formalized through an appropriate indicator.

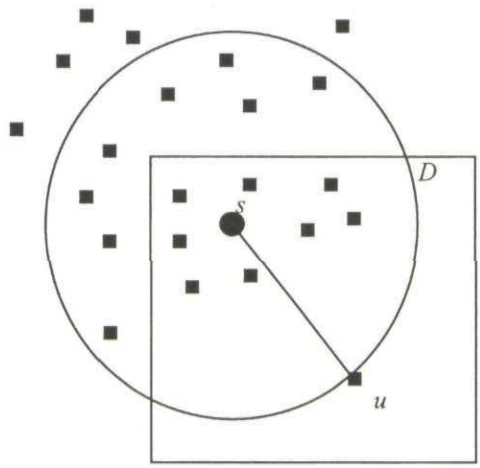
2.1 Population Equality and Compactness

The most popular indexes of population equality are global measures of the distance between the populations of the macro-zones and the mean population. Many different indicators of population equality can be adopted [19]. Our index, which is based on the L_1 norm and normalized by the average population, is defined as follows

$$\frac{\sum_{j=1}^{k} |P_j - \bar{p}|}{k\bar{p}}, \tag{1}$$

where P_j is the population of macro-zone j, $\bar{p}$ is the mean population over all macro-zones and k is the total number of macro-zones. This index measures the average deviation of population of the macro-zones from the average population as a percentage of the average population itself. Many other indexes of this kind can be built simply by replacing the L_1 norm by other norms. Index (1) is equal to 0 when there is perfect population equality, but its maximum value is equal to $2(k-1)/k$. In fact, when each macro-zone population is perfectly equal to $\bar{p}$,

Figure 2: The circle centered in s and passing through u locates all the units that are no farther from the center than u



the index value is 0. On the other hand, when one macro-zone contains the whole population $P = k\bar{p}$, and the other $k - 1$ macro-zones are empty, the index value is given by

$$\frac{|P - \bar{p}|}{k\bar{p}} + (k-1)\frac{|0 - \bar{p}|}{k\bar{p}} = \frac{2(k-1)}{k}.$$

The last is a very unusual case since it corresponds to extremely unbalanced zonings. In practice, if the map is not grossly unbalanced, the index value is smaller than 1. However, a normalized version of the index can be considered.

Intuitively, a macro-zone (or in general a figure) is compact if it spans a round region, but it is hard to say what compactness exactly means. The problem is that the criterion has such a wide meaning that it cannot be completely formalized as an index. Many different definitions have been proposed [13, 16, 17, 21, 24, 26] and each of them seems to be correct, but not exhaustive. There are measures based on length versus width, others based on the perimeter of the macro-zones, others which compare the area of each macro-zone with the area of a "canonical" compact figure (generally a circle), others refer to the moment of inertia. In our application we build a non-compactness index by extending the ideas suggested in [2], in order to take into account both the geographic location of the micro-zones and their population.

Consider a macro-zone and its weighted center s which is calculated by weighting each micro-zone with its population. Let u be the farthest territorial unit from s within the same macro-zone. Among all the units that are no farther from the center than u, we calculate the percentage of those that do not belong to the macro-zone, and the corresponding percentage of population. The index is close to 0 for round macro-zones and it is close to 1 for elongate or indented macro-zones. Figure 2 shows the index computation for macro-zone D, where s is assumed to be the weighted center of D.

With respect to macro-zone j we denote by P_j^c the sum of the population of the micro-zones within the circle. Then, the index for macro-zone j is given by

$$\varphi_j = \frac{P_j^c - P_j}{P_j^c}. \tag{2}$$

A global index is obtained as the average of φ_j over the k macro-zones.

Notice that both compactness and population equality indexes are equal to 0 when the corresponding criterion is perfectly satisfied. These are, in fact, non-compactness and non-population equality indexes, respectively, and our aim is to minimize them simultaneously.

2.2 Inertia

In territorial aggregation particular attention must be paid to the inner variance criterion. In this application it was measured by *total inner inertia.* Let $\{C_1, C_2, \ldots, C_k\}$ be the set of the k macro-zones in the map. Total inner inertia of the map is given by the sum of the inner inertia of the single macro-zones. Let I_{TOT} denote the total inner inertia and $I(C_j)$ the inner inertia in macro-zone C_j, then we get

$$I_{TOT} = \sum_{j=1}^{k} I(C_j), \tag{3}$$

where $I(C_j)$ is defined as follows:

$$I(C_j) = \sum_{d \in C_j} \|x_d - g(C_j)\|^2, \tag{4}$$

and where $\|v\|^2 = v_1^2 + v_2^2 + \ldots + v_r^2$ is the squared L_2 norm of the r-vector $(v_1, v_2, \ldots, v_r)$, x_d is the vector of variables for each micro-zone d in C_j, while $g(C_j)$ is the center of gravity of C_j. Denote by Q the number of variables and by $x_d(i)$, $i = 1, 2, \ldots, Q$, the i-th component of a point $x_d \in R^Q$, it follows that

$$g_i(C_j) = \frac{1}{|C_j|} \sum_{d \in C_j} x_d(i), \quad i = 1, 2, \ldots, Q. \tag{5}$$

Notice that when $|C_j|$ is constant for all j, $I(C_j)$ is proportional to the inner variance of macro-zone C_j. Consider a map composed by only one macro-zone which includes all the micro-zones. Let $I(1)$ be the corresponding total inner inertia value. We obtain our total inner inertia index as the ratio between I_{TOT} and $I(1)$. Since $I(1)$ refers to the most heterogeneous map, it is equal to the maximum possible total inner inertia value. Therefore, index $I_{TOT}/I(1)$ takes values in the [0,1] interval.

In this application we consider a set of 24 socio-economic variables, including the population, the number of schools, hospitals, public offices and shopping centers. Other variables related to the transportation network, such as the number of underground and bus stations, or the number of airports, and services in general, such as the number of trading licenses issued to shopkeepers are considered, too.

The best result of territorial aggregation would be to have all criteria satisfied, but, in general, this is impossible because the different criteria are in conflict. In fact, we will never have a map which is optimal with respect to all criteria simultaneously, but, rather, a set of solutions which cannot be compared one to another. The best one can do is finding the set of all *Pareto-optimal* solutions. However, in general, finding the Pareto-optimal solutions is computationally difficult. Thus, we adopt a heuristic method in order to find solutions which provide a good compromise between the values of the different indicators.

3. FORMALIZING A MODEL ON A GRAPH

Graph theory provides a simple and clear way to represent a territory and its characteristics. It allows us to visualize the problem and to handle it efficiently. In fact, graph models provide a very compact and simple way of coding the elements of a territory. This is especially useful when working on a computer. Following [23], we formulate a graph model for the territorial aggregation problem in the city of Rome. Each node i of the graph corresponds to a micro-zone and an edge (i, j) exists if and only if node i and node j are neighboring micro-zones[1]. Assume

1 In this application two micro-zones are considered neighboring if they share a portion of border, but they are not if they touch themselves in just one point.

Figure 3: The urban district map of the city Rome

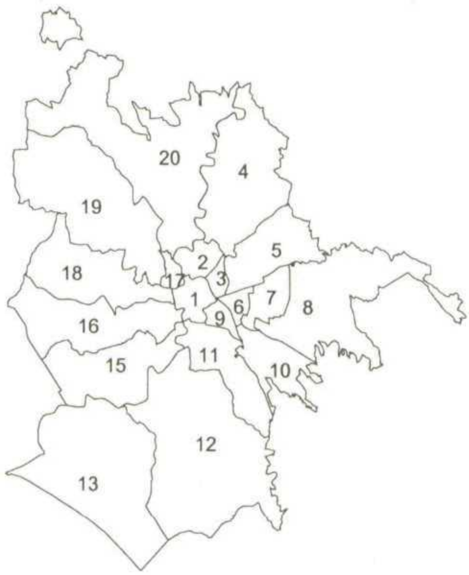
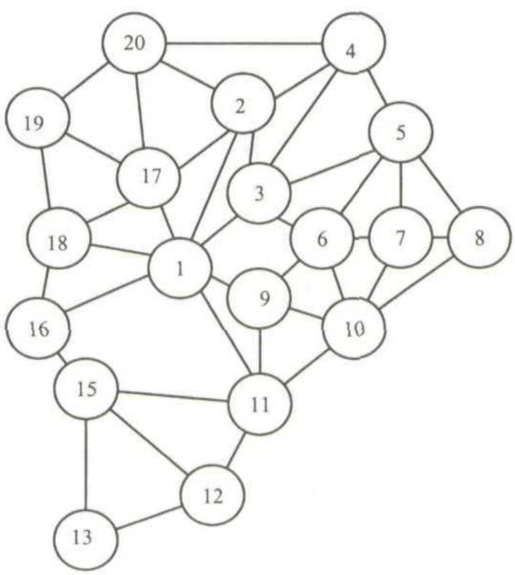


Figure 4: The graph of the city Rome



that a micro-zone coincides with an urban district, then the territory of the city of Rome is partitioned as in Figure 3 and the corresponding graph model is the one in Figure 4[2].

Actually, in this application the territory was subdivided into 458 micro-zones which are much smaller than the urban districts of Figure 3. In our graph model the weights of the edges are given by the distances between pairs of micro-zones, calculated as the Euclidean distances

2 Notice that urban district (node) 14 is missing. It corresponds to the airport district which was not considered as part of the city in our application. In fact, traffic flows from and towards this district were regarded to as part of the total flow between the city and the neighboring area which surrounds it.

between the centers of the micro-zones. According to our graph representation, a macro-zone can be defined as a subset of nodes such that for every pair of nodes there is a path joining them in this subset and each node of the path belongs to the subset, too. Therefore, a macro-zone is a subset of nodes of graph G which induces a connected subgraph on G. Consider, for example, the graph in Figure 4: districts 5, 6, 7, 8 and 10 could form a macro-zone because they are all connected to each other; on the contrary, districts 3, 4, 5, and 9 cannot be aggregated in a macro-zone because district 9 is not geographically contiguous to any other district of that group. To be more precise, consider a connected graph $G(N, E)$. Let $n = |N|$, $m = |E|$, and let k be an integer such that $1 \leq k \leq n$. A partition $\pi = \{C_1, C_2, \ldots, C_k\}$ of V into k subsets (briefly, a k-partition) is said to be *connected* if, for each $h = 1, 2, \ldots, k$, the subgraph $G(C_h)$ induced by C_h is connected. The set of all connected k-partitions of G will be denoted by $\Pi_k(G)$. As mentioned above, if the graph G represents the territory and k is the number of macro-zones, then each element $\pi \in \Pi_k(G)$ represents a possible aggregated map satisfying the contiguity and the integrity constraints. We assign weights also to the nodes of the graph. In particular, each node is associated to a weight vector given by the values of the variables selected to describe each territorial unit. On the other hand, each edge (i, j) is characterized by the distance between units i and j. Using these elements, for each possible aggregation of micro-zones into macro-zones we are able to calculate the corresponding values of the indexes of non-population equality, non- compactness and inertia. Let $f_1(\pi)$, $l = 1, 2, 3$, be the functions of such indexes, then the territorial aggregation problem can be formulated as the following minimization problem:

$$\min_{\pi \in \Pi(G)} f(\pi) = w_1 f_1(\pi) + w_2 f_2(\pi) + w_3 f_3(\pi) \tag{6}$$

where $0 \leq w_l \leq 1$, $l = 1, 2, 3$, and $\sum_l w_l = 1$.

We are interested in minimizing the convex combination (6) of the three indexes and, possibly, the value of each of them. Notice that by taking as objective the convex combination not all Pareto-optimal solutions of the multiobjective problem are found.

4. APPLYING LOCAL SEARCH

Given an initial map of the territory, we can improve it locally by using a local search technique. "Locally" means that the map is modified just on the borders of each macro-zone, in order to find zonings with better values for the criteria considered. To start a local search algorithm, at least an initial solution must be available. When such a solution is not at hand, it is necessary to generate it and this is generally done by randomization. Moreover, the contiguity constraint must always be taken into account. Thus, to apply local search, one needs an initial partition of the territory into the prescribed number of clusters k such that each cluster is a connected subgraph of the whole graph G. Generally, for aggregation problems, the set of possible solutions is so wide and complicated that it is very recommendable to use an automatic procedure rather than solving the problem by hand. By using a local search algorithm with a mixed objective function, it is possible to find good solutions also in the multicriteria sense [2, 4, 18, 19]. We chose to apply a recent local search algorithm called Old Bachelor Acceptance [14]. Basically, Old Bachelor Acceptance is a threshold acceptance method with a special threshold adjusting mechanism. In fact, there is a threshold value defining the maximum acceptable worsening of the objective function at each iteration. This implies that the objective may improve, but it also may worsen up to a certain extent. At each iteration, the threshold adjusts itself following a non-monotone schedule and even negative threshold values can be reached. In particular, the threshold decreases each time the algorithm improves the current solution, while it increases when a disadvantageous step is performed. This strategy allows avoiding bad premature local optima and it is also able to find new descent directions when we are very far from a minimum. Someone may recognize that Old

Bachelor Acceptance is very similar in spirit to the well known Simulated Annealing algorithm [6, 15]. However, there are important differences between the two. First of all, Simulated Annealing is a hill-climbing method which escapes from local minima by probabilistically accepting a worsening in the objective function. The probability of acceptance depends on both the size of the worsening and the value of the *temperature* parameter at the current iteration. In Old Bachelor Acceptance there are no probabilities because the accepting procedure is completely deterministic. In addition, in Simulated Annealing the temperature parameter decreases at each iteration. On the contrary, in Old Bachelor Acceptance the threshold increases at each failure of the algorithm in finding a better solution, but it decreases when a better solution is reached. The strategy of this algorithm consists in alternating *dwindling expectation*, in order to escape a local minimum, to *ambition*, in order to explore the solution space and find new local minima. A detailed description of Old Bachelor Acceptance and Simulated Annealing can be found in [14] and [6, 15, 25], respectively. Here, we briefly describe only Old Bachelor Acceptance. By *initial solution* we refer to the map from which the algorithm starts. A *feasible solution* is a map that satisfies integrity, contiguity and absence of holes. A *move* brings a node (micro-zone) from its macro-zone to an adjacent one Therefore, at each iteration the algorithm modifies the current map locally, because only one micro-zone is involved in the move and only the borders of two macro-zones are modified. In Old Bachelor Acceptance, the functions *decr*(T) and *incr*(T) define the threshold schedule and can be defined in different ways, according to the specific problem under study. The main steps of Old Bachelor Acceptance are described in the following.

OLD BACHELOR ACCEPTANCE

1 *Choose an initial solution* π *and an initial threshold* T.

2 *Until the stopping condition holds:*

2.1 *select a random feasible solution in the neighborhood of* π, *say* π'
(each feasible solution is generated by selecting a feasible move);

2.1.1 *if* $f(\pi') \leq f(\pi) + T$,
set $\pi = \pi'$;

2.1.1.1 *if* $f(\pi') < f(\pi)$,
set $T = T\text{-}decr(T)$;

2.1.2 *otherwise* $[f(\pi') > f(\pi) + T]$,
set $T = T + incr(T)$;

3 *return the best* π *found up to now.*

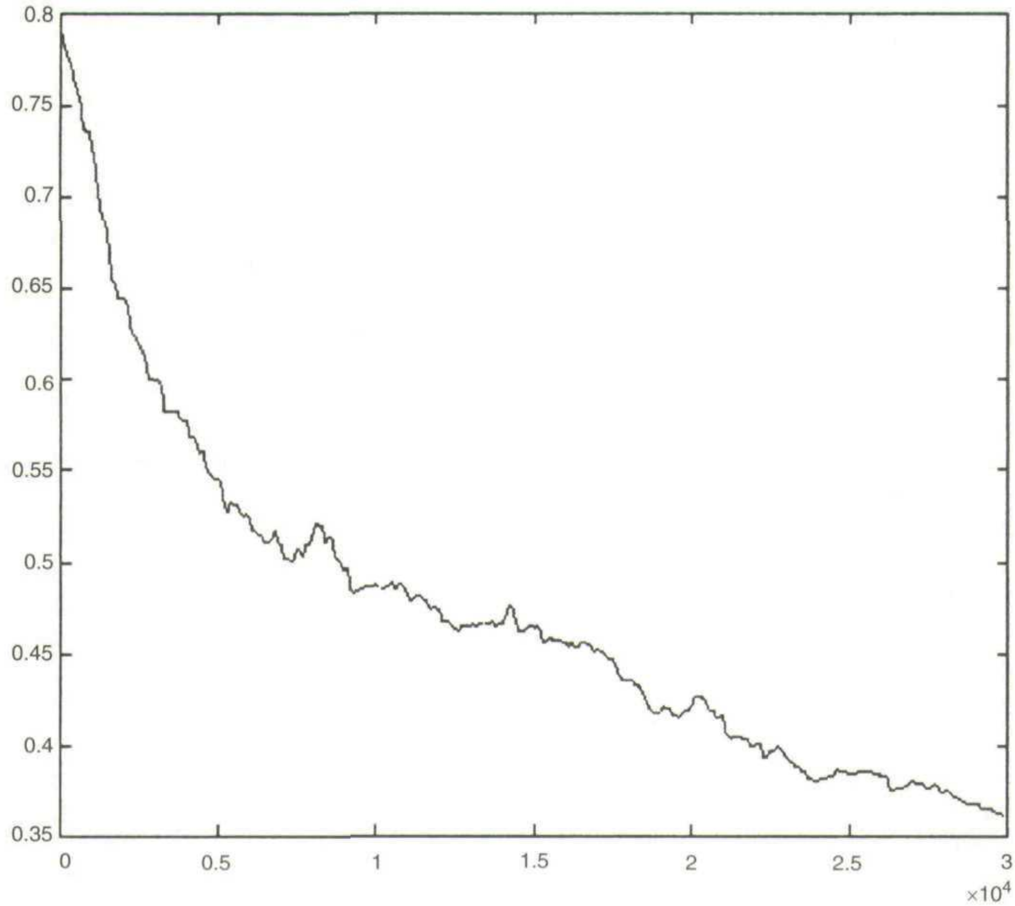
We adopted a multiple stopping condition, setting the maximum number of iterations and an upper bound on the number of consecutive moves which do not improve the objective function. Obviously, the algorithm also stops when it is not able to escape from the last local minimum found. In order to understand how Old Bachelor Acceptance works, the diagrams in Figure 5, 6 and 7 show the direction of search of the algorithm. The three figures refer to cases with a single objective function: inertia is considered in the first case, compactness in the second case and population equality in the third one.

In order to evaluate the general performance of Old Bachelor Acceptance for territorial aggregation in trip distribution models we chose to test our procedure for several times, starting from different initial solutions. The set of initial solutions was obtained by the following cutting strategy: first, a random spanning tree of the graph is selected by using a breadth-first search technique; secondly, the tree is cut into k sub-trees (the number of macro-zones) by randomly selecting their roots. The use of breadth-first search instead of depth-first search is due to the fact that the former produces macro-zone maps more compact than the latter.

5. EXPERIMENTAL PLAN

In this section we briefly analyze the experimental plan and the methodological choices made for the application of Old Bachelor Acceptance to the territorial aggregation of the city of Rome.

Figure 5: The sequence of solutions Old Bachelor Acceptance finds when inertia is the objective function



Starting from different initial aggregated maps, we apply local search trying to generate a better map in terms of population equality, compactness and, above all, inertia[3]. According to our graph model, the left and the right side of the Tiber are considered separately since we do not want macro-zones to be disconnected by the river. On the right side of the city (Rome-Vatican) we have to aggregate 126 territorial units (micro-zones) into 17 macro-zones; in the other side (Rome-Capitol) there are 332 micro-zones and we have to draw 38 macro-zones (see [8]). The corresponding graphs are described in the following table.

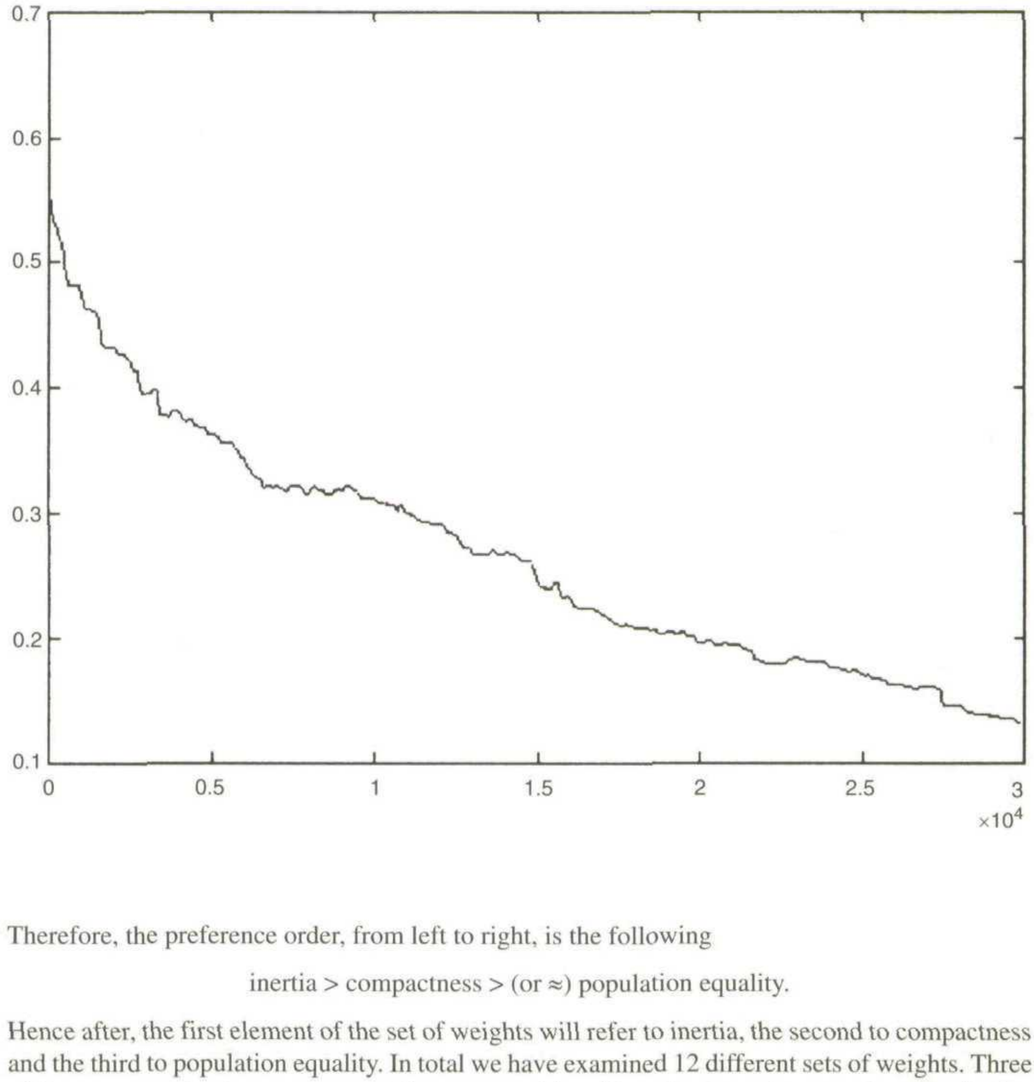
Notice that the two graphs have the same density. Therefore, the main difference between our two territories is given by their size, that is, the number of nodes of the corresponding graph. As we will see in the next section, the size of the graphs affects the performance of the algorithm.

Different sets of weights are considered in the multi-objective function. The weights are chosen considering the different importance of each criterion. Actually, we are not interested in the complete set of solutions, since there is a preference order on the set of criteria. Inertia minimization is the main objective and compactness is generally preferred to population equality[4].

3 An initial map for this problem was created for sampling purposes in a preliminary phase of the study, but it was produced only on the basis of maximizing population equality. This map would have been adopted for the estimation of the trip distribution model if the value of inertia had not been so bad. As we will show in the following pages, not only it is easy to remarkably improve inertia, but it is also possible to improve compactness, with a worse but still acceptable value for population equality.

4 This is the opinion expressed by the workgroup appointed by the City of Rome, where statisticians, transportation engineers and operations researchers have co-operated in this project.

Figure 6: The sequence of solutions Old Bachelor Acceptance finds when compactness is the objective function



Therefore, the preference order, from left to right, is the following

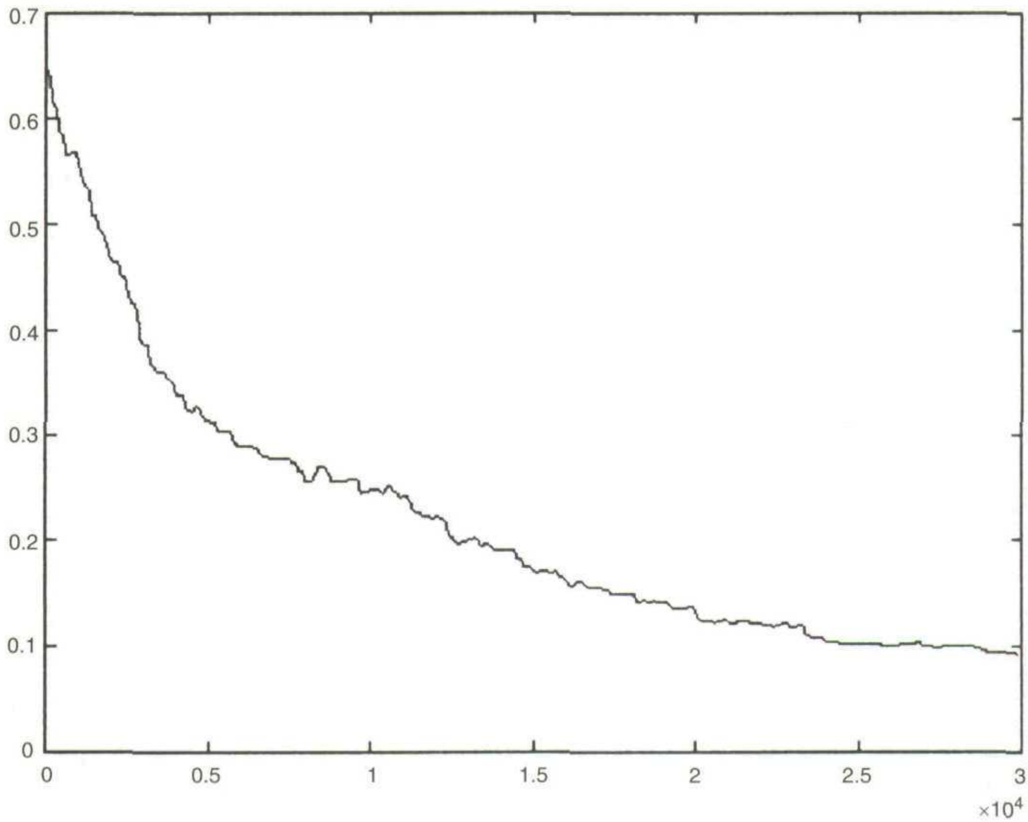
$$\text{inertia} > \text{compactness} > (\text{or} \approx) \text{ population equality.}$$

Hence after, the first element of the set of weights will refer to inertia, the second to compactness and the third to population equality. In total we have examined 12 different sets of weights. Three of them represent extreme cases and they are considered in order to test how much each criterion can be improved when optimized alone. The set (1, 0, 0) is therefore adopted to optimize inertia, (0, 1, 0) to optimize compactness and (0, 0, 1) to optimize population equality. We also consider the set (0.34, 0.33, 0.33) which distributes the total weight among the three objectives equally. These four sets have been analyzed mainly for completeness and comparison purposes.

Table 2 shows all the different sets of weights considered. It should be noticed that in some cases population equality is assigned a weight equal to 0. These sets of weights were considered in order to understand the relation between inertia and population equality and to investigate whether the elimination of population equality could turn out to be particularly profitable in terms of inertia. In fact, inertia and population equality are very conflicting criteria, therefore, we are inclined to ignore the latter if this can ensure small final values for the former.

We believe that these 12 cases are enough to analyze the region of the set of feasible solutions which we are interested in. Notice that our intention is not to provide *the* formula for the definition of the set of weights in territorial aggregation for trip distribution models. In fact, there are many uncontrollable factors which affect the probability of finding good solutions, such as the number of nodes and the number of arcs, which make the problem at hand different each time.

Figure 7: The sequence of solutions Old Bachelor Acceptance finds when population equality is the objective function



We want to provide a method to obtain good macro-zone plans according to the three criteria, even if the set of weights may change from case to case. As we will see in Section 6, it is impossible to fix an optimal vector of weights, but some vectors definitively provide better solutions than others.

The application consists of a total of 600 runs: for both territories, Rome-Capitol and Rome-Vatican, 5 runs were performed in order to analyze the effect of randomization for each of the 12 preference or weight profiles and starting from 5 different initial maps. The evaluation of each profile was made on the basis of the average case over these five runs.

6. RESULTS

This section is devoted to the analysis of the solution found with the different sets of weights. Notice that, in general, we are not able to find the optimal value of the objective function, even when a single criterion is considered. Actually, our heuristic approach was motivated by the fact that aggregation problems similar to ours are generally difficult to solve. In particular, it can be shown that the problem of finding a partition of a graph into k connected components which minimizes the L_1-norm (1) is NP-hard even if the graph is a tree with at most one vertex of degree ≥ 3 and diameter ≤ 4 [7].

Moreover, as stated above, the structure of the territory affects the optimization procedure. In fact, the final local minimum is reached after a small number of iterations (generally 3000 or 4000 iterations are sufficient) when Rome-Vatican is considered, while the whole procedure (30000 iterations) is necessary for Rome-Capitol. This is mainly due to the different size of the problem since the first graph is about 3 times smaller than the second. A direct consequence is that the running times related to Rome-Vatican are shorter than the ones observed for Rome-

Table 1: Parameters of the Rome-Vatican and Rome-Capitol graphs

	Rome-Vatican	Rome-Capitol
Number of nodes	126	332
Number of arcs	316	834
Density	2.51	2.51

Table 2: Different combinations of weights adopted in the mixed objective function.

Different Combinations	Weights		
	Inertia	Compactness	Population Equality
Case 1	0	0	1
Case 2	0	1	0
Case 3	1	0	0
Case 4	0.34	0.33	0.33
Case 5	0.80	0.10	0.10
Case 6	0.80	0.20	0
Case 7	0.70	0.20	0.10
Case 8	0.70	0.30	0
Case 9	0.60	0.30	0.10
Case 10	0.60	0.40	0
Case 11	0.50	0.25	0.25
Case 12	0.50	0.40	0.10

Table 3: Values of the three indexes in the initial solutions.

Indexes	Rome-Capitol				
	Solution 1	Solution 2	Solution 3	Solution 4	Solution 5
Inertia	0.79	0.81	0.75	0.79	0.83
Non-Compactness	0.55	0.52	0.52	0.58	0.46
Non-Population equality	0.65	0.72	0.58	0.64	0.88

Capitol. In any case, the running times are always very encouraging because they are about one minute for Rome-Vatican and between five and six minutes for the other territory. In the following discussion we refer only to Rome-Capitol which represents the most interesting case. Table 3 shows the values of the three indexes for the different initial solutions generated.

In Table 4 we report on the average improvement of the objective function over all runs performed with a single index, starting from each initial solution. The table should be read by rows. The three rows refer to the set of weights (1, 0, 0), (0, 1, 0) and (0, 0, 1), respectively. The variations corresponding to indexes with weight equal to 0 are ignored. Since our objectives must be minimized, the improvement of an index corresponds to a decrease of its value (negative sign). Moreover, since the indicators are measured on non-homogeneous scales, per cent variations are considered.

For example, when *Solution 1* is considered, minimizing inertia leads to an average improvement about 57% with respect to the 5 repeated runs performed starting from this solution, while minimizing compactness and population equality provides 74% and 85% average improvements, respectively. We can observe that the average improvement of inertia is always smaller

Table 4: Average per cent variations for runs with a single objective, starting from the 5 different initial solutions.

	Rome-Capitol				
Indexes	Solution 1	Solution 2	Solution 3	Solution 4	Solution 5
Inertia	–57%	–54%	–57%	–52%	–55%
Non-Compactness	–74%	–74%	–71%	–74%	–74%
Non-Population equality	–85%	–72%	–89%	–84%	–62%

Table 5: Average variation in case of weights (0.34 0.33 0.33).

	Rome-Capitol				
Indexes	Solution 1	Solution 2	Solution 3	Solution 4	Solution 5
Inertia	–53%	–39%	–57%	–50%	–32%
Non-Compactness	–48%	–45%	–39%	–46%	–51%
Non-Population equality	–40%	–45%	–36%	–42%	–45%

than the average improvement of compactness which is, in turn, generally smaller than or equal to the improvement of population equality. This happens even if the initial value of inertia is higher than the one of population equality. Population equality is easier to optimize than inertia because it involves only one variable (population) while inertia involves many variables (population, number of schools, number of hospitals, etc.) which can be very differently distributed on the territory. In particular, for Rome-Capitol inertia, compactness and population equality improve on average about 55%, 73.6% and 78.4%, respectively when considered alone.

Table 5 illustrates the average per cent variation of each objective function in the case of equally distributed weights (0.34, 0.33, 0.33), for all the initial maps. The improvement of the indexes is generally smaller than in Table 4 because the improvement of one index is constrained by the others.

When equally distributed weights are considered, we observe the simultaneous decrease of all three indicators in both the territories considered and in each of the repeated runs. For Rome-Capitol there are substantial variations for all the indexes: population equality improvement is between 36% and 45% (41.6% on average), compactness improves on average about 45.8%, while the average improvement of inertia is 46.2%. Therefore, in this case inertia improvement is considerable and we could be tempted to choose the set of equally distributed weights for our mixed objective function. Unfortunately, when equally distributed weights are considered, no criterion is preferred to the others and the search goes on depending only on the random choice of the next neighboring solution. The high values of the variances associated with the average improvements of Table 5 show the dangerous lack of stability in these results. In particular, the coefficient of variation calculated with respect to inertia is too high (0.20). In order to reach good and stable values for inertia, with small variability, the weight of this index must be increased. However, in this case we have to check what happens for population equality and compactness.

Consider the remaining sets of weights. Population equality worsens in all cases in which it has weight equal to 0 and sometimes this worsening reaches values around 40–45%. Since population equality values were very bad in the initial solutions, such worsening implies that the index reaches values close to 1 in the final solution. For this reason, weights of *Case 6*, *8* and *10* were dropped. Among the five remaining sets of weights, (0.50, 0.25, 0.25) and (0.50, 0.40, 0.10) can be ignored since the runs performed do not always provide good results for inertia. Even if the

Table 6.a: Average final values for inertia, non-compactness and non-population equality in Rome-Capitol when the weights are (0.80, 0.10, 0.10).

	Rome-Capitol				
Indexes	Solution 1	Solution 2	Solution 3	Solution 4	Solution 5
Inertia	0.35	0.37	0.37	0.38	0.37
Non-Compactness	0.40	0.36	0.41	0.40	0.33
Non-Population equality	0.55	0.62	0.55	0.61	0.74

Table 6.b: Average final values for inertia, non-compactness and non-population equality in Rome-Capitol when the weights are (0.70, 0.20, 0.10).

	Rome-Capitol				
Indexes	Solution 1	Solution 2	Solution 3	Solution 4	Solution 5
Inertia	0.36	0.37	0.37	0.38	0.40
Non-Compactness	0.34	0.32	0.36	0.37	0.25
Non-Population equality	0.57	0.63	0.54	0.56	0.77

Table 6.c: Average final values for inertia, non-compactness and non-population equality in Rome-Capitol when the weights are (0.60, 0.30, 0.10).

	Rome-Capitol				
Indexes	Solution 1	Solution 2	Solution 3	Solution 4	Solution 5
Inertia	0.38	0.39	0.38	0.41	0.41
Non-Compactness	0.30	0.28	0.31	0.32	0.22
Non-Population equality	0.58	0.61	0.60	0.61	0.83

improvement observed for compactness is remarkably high when weights (0.50, 0.40, 0.10) are considered, it is preferable to assign a higher weight to inertia in order to guarantee better and more stable outcomes for the value of this index which remains our main objective.

The last three sets of weights are (0.80, 0.10, 0.10), (0.70, 0.20, 0.10) and (0.60, 0.30, 0.10). All of them assign 0.10 to population equality, but this is not sufficient to guarantee that its value improves. In fact, there are cases in which population equality worsens, even if the worsening is negligible (3%). The residual weight is allocated differently between inertia and compactness. In general we expect a positive relation between the weight assigned to an index and the value of its improvement. This is what happens in our experiment, although changing the weight for inertia from 0.60 to 0.80 does not create a big difference in its final value. For each initial solution in Rome-Capitol, Table 6 reports on the average final values of our three indexes over the five repeated runs for the last three sets of weights, respectively. The average absolute values of the indexes in the final solution are shown rather than their average per cent variations in order to keep an eye on the real final value of the indexes while making comparisons between solutions.

Notice that *Solution 5* always provides better final values for compactness and worse final values for inertia and population equality when compared to the other four cases. This is a consequence of the values of the indexes in the corresponding initial solution. In fact, as can be seen in Table 3, *Solution 5* has a very good value for compactness and bad values for the other two indexes. However, in all the three tables the variability of the average final values obtained for inertia is very low (the coefficient of variation is always about 3–4%), guaranteeing stability of our results. Obviously, the values of the other two indexes have higher coefficients of variation (between 8% and 13% for compactness and between 11% and 14% for population equality). If we compare Table 6.a and Table 6.b, or Table 6.b and Table 6.c, there is a very slight difference in the final values of the three indexes. However, the comparison between Table 6.a and Table 6.c shows that inertia worsens on average from 0.37 to 0.39 (6%). For population equality there is an average worsening of 5%, while for compactness there is a substantial improvement (about 25%), providing an average final value of 0.29. Therefore, compactness shows to be highly sensitive to a difference of 0.20 in its weight, while the values obtained for inertia seem not to be affected as much by changes in the weight assigned. This may also depend on the fact that a change from 0.10 to 0.30 (for compactness) means that the weight assigned is three times greater, while from 0.60 to 0.80 (inertia) it is only one third more. These three sets of weights seem to provide good solutions for our problem. From a trade-off point of view, it could however be preferable to choose (0.60, 0.30, 0.10) since the final values for compactness are the best, while assigning a higher weight to inertia does not make it sensibly improve. Moreover, when (0.80, 0.10, 0.10) is considered, we must take into account a small loss in compactness with respect to the previous set of weights, but, in some cases, also a slight worsening of population equality with respect to its initial values. This is acceptable only if inertia is considered so important that we cannot renounce to an average 0.02 improvement with respect to the previous case. In such cases it would be useful to intensify the experimentation on a specific subset of weights in order to gain some more per cent points for inertia. For example, one may analyze vectors obtained by small modification of (0.60, 0.30, 0.10), (0.70, 0.20, 0.10) and (0.80, 0.10, 0.10).

7. CONCLUSIONS

In travel demand models territorial aggregation usually concerns the preliminary phase of the study, that is, when origin and destination zones must be designed. However, it is also necessary when the survey OD matrix is sparse and reliable statistical estimates of the flows cannot be guaranteed. In this application we suggested a multicriteria approach, where inertia is the main criterion, but compactness and population equality are also taken into account. The core of the application is the analysis of different sets of weights to assign to the criteria in order to reach solutions with good compromises between them. By applying a specific local search technique called Old Bachelor Acceptance, we found aggregations with very low values for indexes which measure inertia, non-compactness and non-population equality, even if it is well known that the corresponding criteria are usually in conflict. Our main result was finding a set of good maps with very low total inner inertia and, simultaneously, good values for the other indexes. In particular, our algorithm provides aggregated maps where micro-zones belonging to the same macro-zone are very homogeneous under the socio-economic point of view. According to [20], such aggregations are useful to find reliable estimates of OD flows when the OD survey matrix is sparse. This application also provides results which show the stability of the outcome of the Old Bachelor Acceptance algorithm for this particular kind of problem. We would like to see more applications of this promising algorithm to different combinatorial problems and we wish it can progress at least as much as Tabu Search which is, at the moment, by far the most applied and efficient local search heuristic.

ACKNOWLEDGMENTS

The author would like to thank Prof. Bruno Simeone for the numerous conversations on this subject and for his many suggestions during the preparation of the work and she is grateful to Dr. Aline Pennisi and Dr. Andrea Scozzari for their precious support.

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