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School redistricting: embedding GIS tools with integer programming

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The paper deals with a school redistricting problem in which blocks of a city must be assigned to schools according to diverse criteria. Previous approaches are reviewed and some desired properties of a good school districting plan are established. An optimization model together with a geographic information system environment are then proposed for finding a solution that satisfies these properties. A prototype of the system is described, some implementation issues are discussed, and two real-life examples from the city of Philadelphia are studied, one corresponding to a relatively easy to solve problem, and the other to a much harder one. The trade-offs in the solutions are analysed and feasibility questions are discussed. The results of the study strongly suggest that ill-defined spatial problems, such as school redistricting, can be addressed effectively by an interaction between objective analysis and subjective judgement.
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Introduction

School redistricting is the process of adjusting the boundaries of schools within a given school system. School redistricting may be done annually in response to overcrowded classrooms, projected growth and decline of enrolments, opening and closing of schools, modification of school capacities, etc. Countless hours are spent by school administrators, boards, and parents to create various redistricting alternatives, to examine their effects, and to agree (or at least compromise) on the final redistricting plan to implement.

A revision of school districts is often torn between two kinds of motivations that can contradict each other: overall efficiency and individual convenience. One type of efficient districting pattern is to have each student attend his/her nearest school. If school capacities do not permit this, overall efficiency could be achieved by minimizing the total distance travelled by all students. No matter how much administrative cost would be saved, however, this alternative does not necessarily satisfy every single individual. Those allocated to a school 'unfairly' distant may be unlikely to accept such an 'efficient' plan.

Geographic constraints also play an important role in school redistricting. First, contiguous districts are strongly preferred, if not required. Second, physical obstacles such as roads, rails, and bodies of water may prohibit some redistricting options; also, parents do not want to send their children across major streets with heavy traffic or hazardous sites. Third, various scales of geographic units need to be taken into account. In many instances, blocks—generally defined by streets—are the finest unit of granulation allowed during redistricting. Yet larger but more ambiguous units such as communities or neighbourhoods should be given some attention. A redistricting plan with careless division of neighbourhoods would be strongly opposed by the people living there.

There are other factors that are peculiar to American school systems. First, grade levels vary from one school to another. For instance, a single school system may have K(indergarten)-4(th grade) schools, K-8 schools, 5-8 schools, etc. As a consequence, some students are forced to transfer to other schools as they advance to higher grades. Second, students are racially diverse. Some local governments mandate or recommend that schools achieve a certain racial balance to eliminate potential educational disadvantages particular racial groups might have.

As many factors are involved, school redistricting is a technically as well as politically complex problem. Some of the technical complexity is, however, relieved by recent digital technological advances, such as geographic information systems (GIS). GIS generally facilitate preparation, interpretation, and presentation of spatial data. Each entity of spatial data has attribute(s), location, and possibly geometric and topological properties. Some school administrators may use a GIS simply to produce a paper map, on which they visually analyse the locations of schools and the distribution of students, and draw possible districting patterns. Others make a more intensive use, doing most of

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their analysis within the framework of a GIS since many commercial GIS applications are nowadays furnished with various tools for editing and analysing spatial data.

Certain optimization tools are even included as standard functions of commercial GIS packages. For example, the Arc/Info (ESRI) Network module adopts Dijkstra's algorithm for the shortest path problem with all arc costs nonnegative. A school redistricting problem requires a much more complex mathematical programming formulation than the shortest path problem, however, and solution techniques that are well beyond the capabilities of standard GIS tools. Nevertheless, integrating GIS tools with custom-made mathematical programming techniques can facilitate such solutions.

The application of mathematical programming to school redistricting problems certainly helps to understand the problem and generate rational solutions, but does not make the final decision. One reason for this is that a school redistricting problem is almost always ill-defined. While some districting criteria are relatively well suited for numerical treatment, others are too elusive to be quantified and may be overlooked (if not intentionally ignored). Thus, no matter how elegantly a school redistricting problem is formulated or solved as an optimization model, the generated solution usually cannot avoid objection or modification. If this modification cannot be done smoothly, the overall value of the school redistricting system may degrade significantly.

Accordingly, the purpose of this study is two-fold:

(1) to model a school districting problem so that it explicitly addresses many of the common school redistricting objectives and is simple enough to be solved by existing optimization algorithms; and
(2) to implement the model with an intuitive interface that allows the user to easily formulate and revise the model, and evaluate and modify the solution. We expect that such an interactive system will make the school redistricting process more efficient.

The remainder of the paper is organized as follows. The next section reviews the relevant literature and summarizes typical properties of good school redistricting patterns. Then we introduce a school redistricting model that explicitly accounts for those criteria. Our results are presented in the fourth section, where we describe the implementation of the model in a GIS environment and test it with actual data sets from the city of Philadelphia. The last section provides our concluding remarks.

Literature review and desired school districting properties

The school (re)districting problem can be seen as grouping small geographic units into clusters or districts, minimizing some distance measure or cost, and eventually complying with additional criteria. The problem is an old topic in the Operations Research/Management Science (OR/MS) community. Starting in the early 1960s, many linear programming (LP) approaches and solution methods have been proposed. Sutcliffe *et al*[1] summarizes, in a schematic way, most of the work published up to 1982. Papers that consider a single attribute objective function are, among others, by Clarke and Surkis,[2] Koenigsberg,[3] Heckman and Taylor,[4] Belford and Ratliff,[5] Franklin and Koenigsberg,[6] Liggett,[7] Holloway *et al*,[8] McDaniel,[9] McKeown and Workman,[10] Jennergren *et al*,[11] Bovet,[12] and Bruno *et al*.[13] After 1982, fewer attempts have been reported in the OR/MS literature, among them were Schoepfle and Church,[14] Ferland and Guénette,[15] Taylor *et al*,[16] and Lemberg and Church.[17] We refer the reader to the latter for further references.

The two features that are common to all school districting models are: first, the geographic units where students live must be assigned to schools; second, the students assigned to a school cannot exceed the available space. Most models minimize an objective function that represents an aggregate block–school distance measure, and many also consider minimum capacity usage, racial balance and/or other attribute equilibrium issues.

Among previous school districting studies, only Liggett[7] and extensions to his model[17] consider explicit integer (binary) variables, for requiring that each geographic unit be assigned as a whole to the same school. Ferland and Guénette[15] also deal with binary assignment decisions even though they do not present a formal optimization model. The other approaches work with continuous decision variables and geographic units can be 'split' into different schools. Splits are generally undesirable because they oppose the sense of neighbourhood and because they create the additional problem of deciding which students of the split unit must go to which school. Most authors are aware of this fact, but either they argue that in their case studies splits were 'quite few', or they apply post-optimal heuristics or hand adjustments to fix it. Taylor *et al*[16] proposes a nonlinear penalization function to encourage zero–one results, however did not use it because a re-definition of the units was preferred.

Until now, few LP studies have accounted for individual grades separately. In most cases, school assignments are done ignoring the students' grade. This would be reasonable if all schools had the same grade structure and if the 'grade blend' were homogeneous among the geographic units. When these assumptions are not satisfied, more attention should be placed on the grade attribute. In Belford and Ratliff[5] and Taylor *et al*,[16] schools are classified as elementary school (kindergarten through fifth grade), middle school (sixth through eighth grades), or high school (ninth through 12th grades) and different model runs were done for each category. But again, within each category, there is a grade homogeneity assumption, and further problems could also arise if there was a school with, for example, kindergarten through seventh grade. McKeown and Workman[10] are more drastic and develop an individual model for each grade. In this way, there is no grade homogeneity assumption but a post-optimal heuristic is needed to fix the geographic units that are 'split by grade' (ie when students living in the same block but of different grades are assigned to different schools). The model proposed by Franklin and Koenigsberg[6] is more comprehensive than any other as its decision variables account for both grade and racial types. They were, however, unable to solve it given the computational resources available at the time.

Many authors declare contiguity as a desirable property but, to our knowledge, none of the previous studies add district contiguity as an explicit constraint. At most, in some cases, like in Franklin and Koenigsberg[6] or Holloway *et al*,[8] the objective function is made up of squared block–school distances in order to emphasize the creation of compact districts. However, neither this goal nor contiguity is guaranteed. It is true that, due to geographic segregation, contiguity may conflict with other desired properties (for example racial balance), but models are useful for providing insight into the tradeoffs.

From an optimality point of view, most school districting papers that solve a continuous LP or a derived transportation problem get optimal or near-optimal solutions (usually with 'splits'). But again, due to computational limitations, the case studies are rather small (and many are not real). Belford and Ratliff[5] solve the biggest instance with approximately 300 geographic units and 11 schools. Liggett[7] applies an implicit enumeration algorithm to a real problem with 140 geographic units and 11 schools, but there is no discussion about the optimality of the final solution.

The school districting problem is closely related to other general districting problems treated extensively in the OR/MS literature, including sales territory alignment or creation of political voting tracts. In these problems, the objective is also to group small units into larger districts, but they have more degrees of freedom because usually there are no geographic points equivalent to the schools with the role of natural centres.

For the sales districting problem, previous work includes that of Hess and Samuels,[18] Segal and Weinberger,[19] Zoltners and Sinha,[20] and Fleischmann and Parachis.[21] Q1 Works of Zoltners and Sinha[20] is particularly interesting because it provides a good review and defines four reasonable properties that identify a good sales territory alignment. Based on these properties, they propose a methodology and an integer (binary) model that incorporates explicit contiguity, workload balancing constraints, and compatibility with geographic considerations. They apply their procedure to three real cases (the biggest with 280 sales units to be assigned) and obtain solutions that violate the workload balance. They accept violations below 5%. To solve the model, they use a Lagrangean relaxation procedure, and because their sub-problems have the integrality property[22] the objective value obtained is the same as the LP relaxation value.

For the political districting problem, many mathematical programming approaches can be found in the OR/MS literature. Here, the use of mathematical tools is even more important because it gives an objective procedure for generating voting districts (free of partisan influence). Hess *et al*[23] are among the first in using a facility location model to address this problem. Garfinkel and Nemhauser[24] report a two-stage enumerative procedure that minimizes the maximum deviation from the desired district average size. In the first stage, they generate a certain amount of feasible districts and, in the second stage, they propose a tree search algorithm to solve the optimization problem. Contiguity is considered as one of the district feasibility conditions. However, the procedure fails to solve an instance with 55 voting units to be allocated in five districts. Hojati[25] also proposes a two-stage approach, he first uses Lagrangean relaxation to determine district centres (again the sub-problems have the integrality property), and then voting unit assignment is carried out with a transportation model. Other network optimization models can be found in George *et al*[26] and the references therein. Mehrotra *et al*[27] present a promising Branch & Price approach. Their model is quite similar to that of Garfinkel and Nemhauser, but the feasible districts are generated through an optimization sub-problem instead of through enumeration. The complete procedure has pre- and post-processing, and a real case study with 46 voting units and six desired districts is described.

We proceed in a way similar to that of Zoltners and Sinha[20] for the sales territory alignment problem. Based on the above literature review and personal conversations with representatives of the School District of Philadelphia, we identified seven desirable properties that a 'good' school districting should satisfy:

- (P1) Each block (for each grade) is assigned to exactly one school.
- (P2) School assignments must not exceed each grade's capacity and may need to be balanced relative to other attributes.
- (P3) Each school district must be contiguous. We define a district to be contiguous if any pair of blocks that belong to the district can be linked with a 'path' of blocks that are also part of the district.
- (P4) No school boundary can cut across such geographic obstacles as railroads, rivers, or streets with heavy traffic.
- (P5) The total distance travelled by all students is minimized, but no student should travel more than a specified maximum distance.
- (P6) All students in a block must go to the same school unless that school has no classrooms for the corresponding grades. This property makes the redistricting

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plan more rational and also decreases each student's chance of future school transfer.

(P7) A new districting pattern must maintain a certain degree of similarity to the existing one. This property is relevant when the school redistricting is visualized in a longer time frame since it is unrealistic and impractical to think of creating whole new school districts every year.

From a modelling perspective, properties (P1)–(P7) have never been treated simultaneously, at least to our knowledge. A multi-objective or a goal-programming model such as those proposed by Knutson *et al*[28] or Sutcliffe *et al*[1] go a long way, but still the problem of setting appropriate weights for different attributes remains. Based on the conversations with the School District of Philadelphia, our approach is therefore to minimize a single attribute objective function (total travelled distance) subject to properties (P1)–(P7).

To partially justify the selection of these seven properties and the modelling contribution of our study, we present (in Table 1) a summary of all the school districting papers mentioned here in the literature review (which is by no means exhaustive). They are sorted by year of appearance and a mark is placed if that paper somehow considers the respective property. We are not interested in implementation details, but rather concentrate on whether the authors considered the property to be relevant. A few comments must be made:

(i) We stated properties (P1)–(P7) in terms of city blocks, but many papers allocate other units, for example, census tracks, grid blocks, bus stops, etc. The choice of the appropriate unit depends on the available information and the desired level of aggregation.

(ii) In the case of (P2), we only checked if the paper considered school capacities and/or race balance. Few papers go beyond these two attributes due to lack of data or model complexity. Taylor *et al*[16] mention some exceptions where socio-economic and busing burden balance is also required. Another example is Sutcliffe *et al*[1]'s goal-programming model, that also tries to minimize total travel difficulty and

Table 1 Presence of (P1)–(P7) in previous school districting papers

School districting papers	*Year*	*(P1), assign one-to-one*	*(P2), attribute constraint*		*(P3), contiguity*	*(P4), Geo. feasibility*	*(P5), Min. total dist. and restrict worse case*		*(P6), No grade-split*	*(P7), limited reassignment*
			Cap.	*Race*			*Min.*	*Max.*		
Clarke and Surkis[2]	1968		Y	Y/S				Y		
Koenigsberg[3]	1968		Y	Y/S			Y			
Heckman and Taylor[4]	1969		Y	Y			Y			
Belford and Ratliff[5]	1972		Y	Y/S	D		Y	Y		
Franklin and Koenigsberg[6]	1973		Y	Y/S			Y		D	
Liggett[7]	1973	Y	Y	Y	D		Y	D	Y	
Holloway *et al*[8]	1975	Y	Y		D	Y	Y		Y	Y
McDaniel[9]	1975		Y	Y/S			Y			
McKeown and Workman[10]	1976	Y	Y				Y		Y	
Jennergren and Obel[11]	1980	D	Y			Y	Y	D	D	D
Knutson *et al*[28]	1980		Y	Y/S			Y			
Bovet[12]	1982	Y	Y				Y	Y	Y	
Bruno and Anderson[13]	1982		Y	Y				Y		
Sutcliffe *et al*[1]	1984		Y	Y/S		Y	Y			
Schopfle and Church[14]	1989		Y	Y/S			Y			
Ferland and Guénette[15]	1990	Y	Y		D		Y		Y	
Taylor *et al*[16]	1999	Y	Y	Y			Y			D
Lemberg and Church[17]	2000	Y	Y	Y	D		Y		Y	Y

Abbreviations: Y—the property is considered explicitly in the model; S—race-splits are allowed; D—the property is mentioned as 'desirable' but is not explicitly considered in the model.

deviations from the average sex and reading-age retarded proportions.

(iii) In the case of (P5), we checked if some distance metric is minimized (Euclidean distance, network distance, squared Euclidean distance, etc), and if fairness is considered via an imposed worst case bound, that is, a maximum allowed distance for a block assigned to a specific school

(iv) We regard these properties as necessary conditions for a good school districting plan. They are far from being sufficient since there are also subjective criteria involved.

To conclude this section, it is important to state that previous interactive decision support systems (DSS) based on mathematical programming techniques for the aforementioned districting problems have been reported in the OR/MS literature. Among them, in the school districting case, Ferland and Guénette[15] document a successful attempt. They programmed a menu-driven DSS that runs on a PC. Capacities per grades and district contiguity are considered. The DSS generates a starting solution based on a heuristic that assigns street segments to schools. The proposed allocation can be graphically displayed and modified by the user. In a different way, Taylor *et al*[16] also take advantage of up-to-date graphical tools. In their integrated planning system, geographic units are created through an interactive computer interface and several GIS-based output maps are generated to illustrate districting solutions or demographic statistics.

School redistricting model

A general redistricting model is presented below. Indices i, k, and n represent blocks, grades, and schools, respectively. The binary variable x_{ikn} equals 1 if grade k students of block i are assigned to school n, and 0 otherwise. S_{ik} is the number of grade k students in block i, D_{in} is the distance from block i to school n, A_{kn} is the k-th grade capacity of school n, $N(i,n)$ is the set of blocks adjacent to block i that are closer to school n, and $C(i)$ is the closest school to block i. B_n is the maximum walking distance allowed for students assigned to school n. For each grade k, $R(k)$ is the set of (i,n) pairs representing the current block–school allocations, and $(1-P)$% of these pairs must be kept.

$$(M_{SD}) \qquad \min \quad z = \sum_i \sum_k \sum_n S_{ik} D_{in} x_{ikn} \qquad (1)$$

subject to :

$$\sum_n x_{ikn} = 1 \qquad \forall i, k \qquad (2)$$

$$\sum_i S_{ik} x_{ikn} \leqslant A_{kn} \qquad \forall n, k \qquad (3)$$

$$x_{ikn} \leqslant x_{i(k+1)n} \quad \forall i, k, n \quad \text{s.t.} \quad A_{(k+1)n} \neq 0 \qquad (4)$$

$$x_{ikn} \leqslant \sum_{j \in N(i,n)} x_{jkn} \quad \forall i, k, n \quad \text{s.t.} \quad N(i,n) \neq \phi \qquad (5)$$

$$x_{ikn} = 0 \quad \forall i, k, n \quad \text{s.t.} \quad N(i,n) = \phi \quad \text{and} \quad n \neq C(i) \qquad (6)$$

$$D_{in} x_{ikn} \leqslant B_n \quad \forall i, k, n \qquad (7)$$

$$\sum_{(i,n) \in R(k)} x_{ikn} \geqslant (1-P) * |R(k)| \quad \forall k \qquad (8)$$

$$x_{ikn} \in \{0, 1\} \quad \forall i, k, n \qquad (9)$$

The objective function (1) represents to the total walking distance (or equivalently the average walking distance). Constraint (2), together with the integrality condition (9), guarantees that each block–grade pair is assigned to exactly one school, eliminating 'splits'. Constraint (3) ensures that no school violates its capacity by grade. If necessary, a minimum capacity usage can be expressed in the same manner. Constraint (4) says that if the k-th graders of block i are assigned to school n, then the $(k+1)$-th graders of that block must be assigned to the same school, unless school n does not provide grade $(k+1)$. Finally, constraints (5) and (6) deal with contiguity. In a recursive way, constraint (5) says that, for each grade, in order to assign block i to school n, there must be a 'path' of blocks also assigned to the same school that connects block i with school n. On the other hand, if there is no way of building such a connecting path, then constraint (6) prohibits the assignment of block i to school n, unless n is the closest school to block i (ie $C(i)=n$). Note that the effectiveness of constraints (5) and (6) depends on how the sets $N(i,n)$ and the parameters $C(i)$ are calculated. This can be done in several ways with different consequences, as will be seen later.

As the objective function reflects the average walked distance, in order to encourage individual equity a maximum walked distance is added in constraints (7). This constraint also helps to build more compact districts, as in this form the resulting patterns will be consolidated rather than spread out.

In a redistricting case, as stated in property (P7), only a certain proportion of blocks should be reallocated. This can be done by imposing constraints (8), which say that, for each grade, at least a proportion $(1-P)$ of blocks must remain assigned to their current school. If $P=1$ then the new districts are established from scratch. In contrast, if $P=0$, the actual school division is preserved.

If some other attribute balance is desired (as those mentioned in Taylor *et al*[16] and Sutcliffe *et al*[1]), then the following additional constraints can be added:

$$m_l \sum_i \sum_k S_{ik} x_{ikn} \leqslant \sum_i \sum_k T_{lik} x_{ikn} \leqslant M_l \sum_i \sum_k S_{ik} x_{ikn} \, \forall l, n \tag{10}$$

where T_{lik} is the number of grade k students of block i that have attribute l (for example, are females). M_l and m_l are the maximum and minimum desired proportions, respectively, of students with attribute l.

The optimal solution of model (M_{SD}) plus the interaction with the GIS interface to be presented below yield a school districting plan that satisfies properties (P1)–(P7). Solving model (M_{SD}) can be difficult, but many cases are easy. The next section shows the results for two real instances. Due to population, geographic, and school structure considerations, one is easy and the other is very complicated.

System implementation and computational results

We built a system aimed at supporting school redistricting by coupling the optimization models described in the preceding section with a commercial GIS package (ArcView 3.2, from ESRI). In this system, the optimization model and GIS software do not share a common data structure, but are loosely coupled[29,30] through the transfer of input/output data as ASCII files. Since this data transfer is conducted behind the scene, users do not notice that different applications are being used. We have chosen this form of linkage based on our conversations with the School District of Philadelphia in order to maintain the GIS functionality and interface familiar to potential users. To check the usefulness of the system, we tested it with actual data provided by the School District of Philadelphia.

Interface of the system

As stated earlier, the value of the system does not lie only in the underlying 'optimization' model but also in the interface through which the user interacts with the model. Since school redistricting is a trial-and-error process rather than a well-defined problem, smooth interaction between the user and the model is crucial. It is very likely that during a redistricting process, existing criteria are modified and new criteria emerge. To streamline the process, the user can interact with the model in five different manners as follows. Each mode of interaction is associated with one or two items of a menu called 'Redistricting' (Figure 1), which is the only control added to the original GIS application.

School and block maps selection. The first task the user performs is the selection of an area (cluster) to redistrict. To do so, after loading maps of schools and blocks encompassed by the area of interest, the user clicks on the first item of the menu called 'Select School Theme' and selects the school map. Then the user clicks on the second item called 'Select Block Theme' and selects the block map.

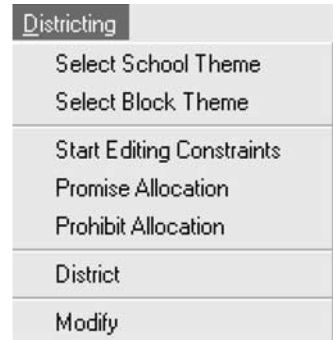

Figure 1 Redistricting menu.

Constraint parameters specification. The user specifies in a table the right-hand-side parameters of the model's constraints (ie lower and upper limits on the enrolment number of each grade for each school, maximal allowable travel distance for each school, upper limits on the percentage of reallocated blocks, etc). Clicking the third item called 'Start Editing Constraints' of the menu calls the constraint table and makes it editable. Upon finishing editing, the user needs to click again on the same item that has been renamed 'Stop Editing Constraints'.

Pre-allocation. If the user knows, before running the redistricting model, which blocks are promised or prohibited to be allocated to particular schools for any reason other than defined constraints, (s)he can pre-determine where to (or not to) allocate particular blocks. To do so, the user selects blocks graphically using existing functions of the GIS program, and selects the fourth item called 'Promise Allocation' or the fifth item called 'Prohibit Allocation.' Then, a dialog box appears and requests the user to select a school where to (or not to) allocate the selected blocks. This pre-allocation function is useful when there are criteria that are difficult to explicitly formulate or would complicate computation. For example, this function may be used to prevent students from crossing rails. This tool also helps to reduce the size of the optimization model.

Allocation. Once the right-hand-side values of the constraints are specified and pre-allocation is done (if necessary), the redistricting process can proceed. By selecting the sixth item called 'Redistricting', an external optimization routine is called to solve the model. Then the results are transferred back to the GIS application to be summarized in tabular form (ie the number of students of each grade, gender, and race for each school; total, average, and maximal distance travelled for each school; number and percentage of blocks reallocated) and are visualized in cartographic form.

Post-allocation. After solving the model, if the user finds any school district unacceptable, s(he) can modify it at their discretion. To do so, the user selects blocks graphically, using an existing function of the GIS, and selects the seventh item called 'Modify Allocation'. Then, a dialog box appears and urges the user to select a school to which the selected blocks are reallocated. An updated summary table and a redistricting map follow. This post-allocation function is useful since the model cannot enumerate all criteria in advance. For example, if a school district does not have smooth boundaries, this can be fixed by reallocating the blocks that caused rough boundaries. Of course, such a post-allocation may lead to a sub-optimal or even an infeasible solution to the model, but such trade-offs between objective criteria and subjective judgement should be justified in practice.

Data

The City of Philadelphia is divided into 22 regions called clusters for school administrative purposes. We applied the system to two of these clusters, namely, the Fels cluster encompassing six schools (one K-5 school and five K-8 schools) and 487 blocks (Figure 2(a)), and the Olney cluster encompassing 12 schools (six K-4 schools, one K-7 school, two K-8 school, three 5-8 schools) and 617 blocks (Figure 2(b)). We have chosen the Fels cluster as an example because it is one of the most homogeneous in terms of grade structure, and the Olney cluster for the opposite reason. Considering the larger number of blocks encompassed by the Olney cluster and its more complicated grade structure, we anticipated that this cluster would provide a computationally harder problem than the Fels cluster.

The student data include four attributes: block of residence, grade, gender, and race. The block of residence is a key value for linking attribute and location data. The data of individual students were then aggregated to block level to compute the number of students by grade, gender, or race in each block. These tasks were done using existing functions of the GIS package and concretely give values for parameters S_{ik} and T_{lik} of the model.

The school data include 10 attributes: street address and capacity of each grade, kindergarten through eighth grade (see Tables 2 and 3). According to section 206 of the School District of Philadelphia Board of Education Policies, although a student should, in principle, 'attend the school within whose boundary lines the legal residence of the parent or local guardian is located', (s)he is allowed to 'attend any school in which there is room regardless of boundary lines, provided (s)he can meet the entrance requirements, if any, and provided established procedures are followed'. As a result, a significant number of students (nearly 20 and 12% of the students who live in the Fels and Olney clusters, respectively) attend schools outside the cluster where they live. Following the original spirit of school districting (ie that each student attends a school that is in the same cluster as where (s)he lives), we did the computational experiments based on the assumption that all schools from Cluster X are filled only with the students that live in Cluster X. This way each cluster can be solved as a separate instance. We defined the capacity per grade to be the current enrolment numbers

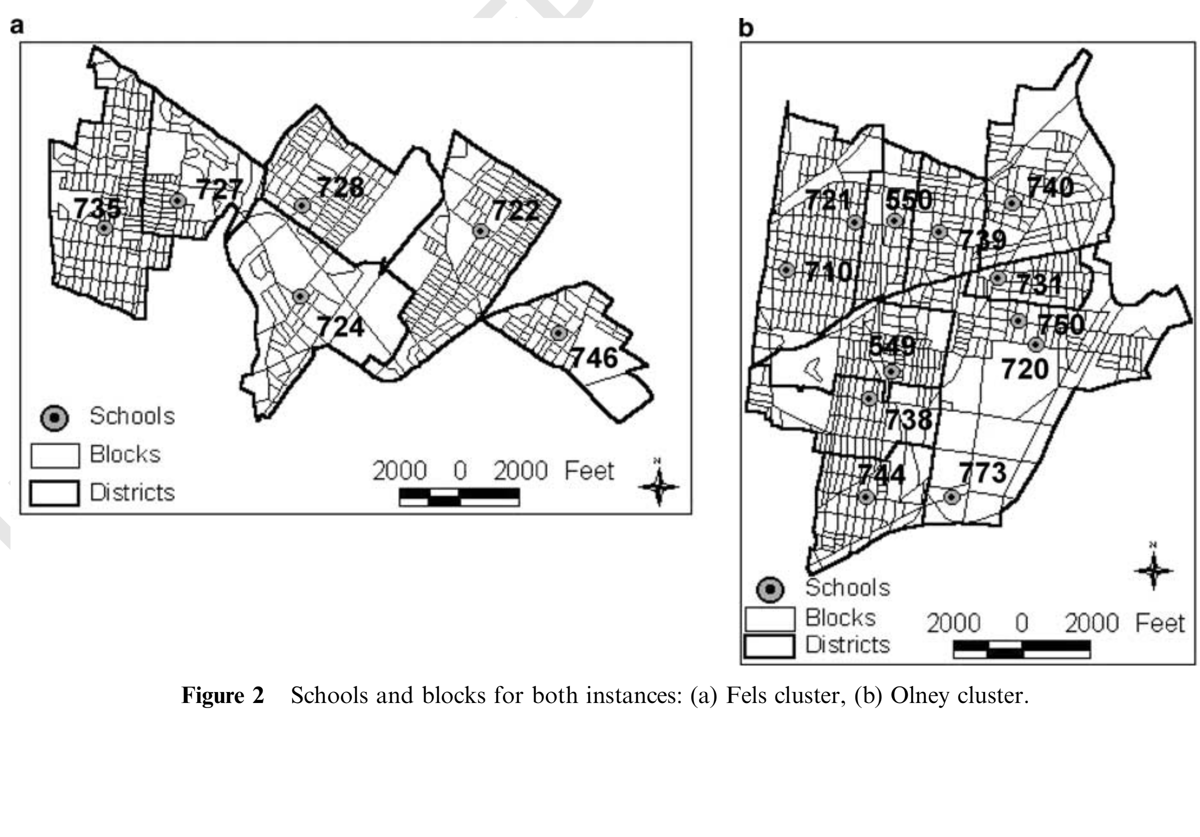

Figure 2 Schools and blocks for both instances: (a) Fels cluster, (b) Olney cluster.

Table 2 Capacity per grade of schools in Fels cluster

School	*k*	*g01*	*g02*	*g03*	*g04*	*g05*	*g06*	*g07*	*g08*
722	167	181	122	137	142	149	137	136	136
724	94	89	98	101	92	110	112	106	112
727	89	128	158	127	151	117	140	138	142
728	101	119	92	109	98	77	108	90	89
735	193	216	249	208	193	198	0	0	0
746	34	40	38	35	35	32	45	35	45

Note: 0 indicates the corresponding grade is not available.

Table 3 Capacity per grade of schools in Olney cluster

School	*k*	*g01*	*g02*	*g03*	*g04*	*g05*	*g06*	*g07*	*g08*
549	126	136	88	100	90	0	0	0	0
550	93	119	85	84	80	84	95	78	0
710	0	0	0	0	0	152	214	227	284
720	150	178	177	175	164	0	0	0	0
721	165	190	214	183	165	0	0	0	0
731	101	110	108	107	112	0	0	0	0
738	183	140	139	155	126	0	0	0	0
739	98	115	132	142	115	102	92	61	94
740	116	101	77	99	74	75	73	59	62
744	141	157	43	129	119	0	0	0	0
750	0	0	0	0	0	256	252	260	218
773	0	0	0	0	0	279	318	301	333

Note: 0 indicates that the corresponding grade is not available.

of each grade regardless of where students come from. Therefore, capacity values are by no means definite or rigid. Rather, their determination or modification is left to the planners' discretion.

After recording the locations of schools on a street map, the distance from each block to each school was calculated in the following way. First, for each block i, a geographic centroid was determined. The centroid was then 'projected' to the closest street, and finally, the network distance from that point to each school n was established as D_{in} (the distance from block i to school n). Once all the D_{in} values were measured, the elements of each set $N(j,n)$ were identified as all those blocks i that share a boundary segment with block j such that $D_{in} \leqslant D_{jn}$. Then we have $C(i) = argmin_n \{D_{in}\}$, and again, the described tasks were done using existing functions of the GIS package.

It is important to note the approximations implicit in the previous calculations. The D_{in} values may somewhat misrepresent the true student travel distance when the centroid of a block is projected away from where the majority of the block's students are actually located. It is not hard to think of more adequate measures, for example, the distance from block i to school n could be defined as the average of the individual distances of each student that lives there, but this would require intense computations with limited payoffs. What is really crucial is the fact that the sets $N(i,n)$ were constructed based on the D_{in} values. The computation is simple but the desired contiguity of the districts can be

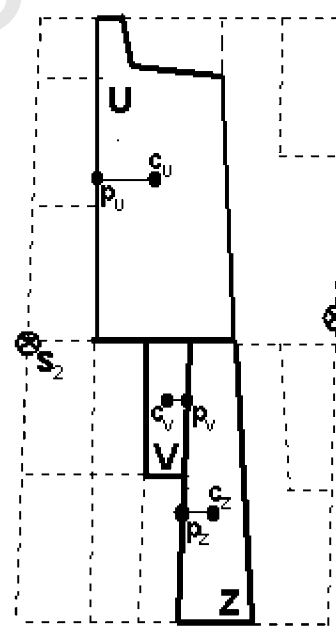

Figure 3 D_{in} calculation example.

affected in a minor way. In fact, consider Figure 3, which is a small example extracted from the Olney cluster.

Points $\mathbf{s_1}$ and $\mathbf{s_2}$ represent two different schools. Blocks **V**, U, and **Z** are 'neighbours' with centroids $\mathbf{c_v}$, $\mathbf{c_u}$, and $\mathbf{c_z}$, respectively. Points $\mathbf{p_v}$, $\mathbf{p_u}$, and $\mathbf{p_z}$ are the projections of the controids to their closest street. Hence, the distance from block **V** to school $\mathbf{s_1}$ is the network distance from point $\mathbf{p_v}$ to point $\mathbf{s_1}$. Since the network distance $\mathbf{p_v s_1}$ is shorter than $\mathbf{p_z s_1}$, as well as $\mathbf{p_u s_1}$, then by definition, block **V** has no neighbouring block closer to school $\mathbf{s_1}$ (ie $N(\mathbf{V}, \mathbf{s_1}) = \phi$). However, since school $\mathbf{s_1}$ is the closest school to block **V** (ie $C(\mathbf{V}) = \mathbf{s_1}$) then, according to constraints (5) and (6), it is possible to assign block **V** to school $\mathbf{s_1}$ and all its surrounding neighbours to school $\mathbf{s_2}$, which in our model (M_{SD}) would correspond to a feasible non-contiguous district for school $\mathbf{s_1}$. Therefore, given the 'block to school' distances described here, constraints (5) and (6) are not sufficient to rule out non-contiguous districts. In the appendices, we briefly discuss when constraints (5) and (6) are sufficient to ensure contiguous districts.

Test of the system

The system was tested on the following five scenarios with the aforementioned two cluster data sets:

Scenario (0): base scenario that duplicates the current districting pattern.

Scenario (1): redistrict so that the capacities per grade are not violated.

Scenario (2): for the Olney cluster, add a maximum walking distance constraint.

Scenario (3): redistrict so that the proportion of race 'type 1' students in each school is within a pre-specified range of the cluster's ratio.

Scenario (4): redistrict assuming that all schools are K-8.

The numerical results for the Fels and Olney clusters are summarized in Tables 4 and 5, and each scenario is analysed in the following subsections.

It should be noted that the figures shown below illustrate the districting patterns only for grades K-4 since in the School District of Philadelphia the school boundaries are defined only for these lower grades. For the same reason, and in order to have a fair comparison with the current situation, we did not impose the contiguity constraints (5) and (6) for the higher grades 5–8.

Regarding the parameter P of constraint (8), it was set to 1 in all the runs, except for the base scenario (0). Also, if the

Table 4 Numerical results for the Fels cluster

	Scenario (0)	*Scenario (1)*	*Scenario (3)*	*Scenario (4)*
Average travelled distance (ft)	2067	2040	2250	2040
Number of overcrowded schools	0	0	0	0
Worst travelled distance (ft)	6115	6115	7441	6115
% Students that travel 0–0.5 miles	70.4%	71.7%	69.1%	71.7%
% Students that travel 0.5–1 mile	28.9%	28.1%	26.0%	28.1%
% Students that travel 1–1.5 miles	0.7%	0.2%	4.9%	0.2%
% Students that travel 1.5 + miles	0.0%	0.0%	0.0%	0.0%
% Students that go to closest school	93.6%	97.6%	89.1%	97.4%
% Students that go to closest or second closest school	100.0%	100.0%	96.0%	100.0%
% Blocks reallocated	—	6.6%	11.9%	9.0%
CPU time (s)	—	100	515	7

Note: 1 mile = 5280 ft.

Table 5 Numerical results for the Olney cluster

	Scenario (0)	*Scenario (1)*	*Scenario (2)*	*Scenario (3)*	*Scenario (4)*
Average travelled distance (ft)	2037	2145	2045	2259	1666
Number of overcrowded schools	4	0	2	5	0
Worst travelled distance (ft)	5810	8791	5984	10 845	4630
% Students that travel 0–0.5 miles	76.2%	75.4%	76.5%	70.9%	83.4%
% Students that travel 0.5–1 mile	21.3%	18.7%	19.8%	22.9%	16.6%
% Students that travel 1–1.5 miles	2.5%	5.9%	3.7%	5.9%	0.0%
% Students that travel 1.5 + miles	0.0%	0.0%	0.0%	0.3%	0.0%
% Students that go to closest school	59.0%	59.2%	63.5%	54.5%	76.0%
% Students that go to closest or 2nd closest school	83.7%	82.9%	85.2%	78.6%	93.5%
% Blocks reallocated	—	15.2%	17.3%	34.0%	42.6%
CPU time (s)	—	2901	604	36 012	402

grade capacities made the problem integer-infeasible, given that there is some flexibility in defining these capacities (refer to the end of the previous section) they were all increased by a small amount (for example, room for five more students) and the model was resolved. We defined a school to be 'overcrowded' if the final number of students allocated exceeded the initial capacity by more than 5%.

The test runs were done on a dual 866 MHz Pentium III PC with 256 MB of RAM. The model was written in GAMS and solved using parallel Cplex 6.6. Basic model reductions were applied, for example, given that all schools that have grade K also have grades 1–4, constraint (4) implies that only one binary variable x_{ikn} was needed for all grades k from K to 4. In the end, the Olney instance was modelled with 18 236 equations and 25 491 binary variables. In the Fels case, there were 10 525 equations and 8541 binary variables. Owing to the precedence arising from constraint (4), in the Branch & Bound process lower-grade variables were assigned a higher branching priority. In general, the running times are reasonable (in this paper we did not concentrate on algorithmic efficiency). The only exception is Scenario (3) in the Olney case, which will be further discussed.

Scenario (0): replicate each cluster's current districting pattern. Scenario (0) is obtained by setting $P = 0$. It does not involve any optimization and can be considered as a benchmark. Note that in reality, as previously mentioned, students do not necessarily attend schools of their residence. This explains why in our framework the current districts of the Olney cluster cannot be followed, unless four overcrowded schools are allowed (with two of them exceeding the original capacity by more than 10%). The corresponding districting maps are the same as those seen in Figure 2.

Scenario (1): redistrict so that the capacities per grade are not violated. As can be seen from Tables 4 and 5, in Scenario (1) there is a small decrease in average travelled distance for the Fels cluster and a small increase for the Olney cluster. In the last case, this was the price paid for eliminating the capacity violations.

Scenario (2): redistrict the Olney cluster with a maximum walking distance constraint. From Table 5 it can be seen that, when redistricting the Olney cluster subject to no capacity violation (Scenario (1)), the worst travelled distance increases by more than 50%. Therefore, we added a maximum distance constraint saying that no student should walk more that M ft ($B_n = M$ for all n). The results shown in Table 5 (for Scenario (2)) correspond to $M = 6000$ and Table 6 reports results for other values of M.

From Table 5, it can be seen that lower values of the worst travelled distance are only possible if some overcrowded schools are allowed (recall that the grade capacities are increased only if the model is integer-infeasible). The last two lines confirm this trade-off, but in both cases only one of the schools exceeds the original capacities by more than 10%; therefore, it can be argued that they still represent better solutions than the base Scenario (0).

Figure 4(a) shows the districting map for the last run of Table 6. We observed that these districts were more compact compared to those generated under Scenario (1). Compactness is a property that is hard to define and measure in an explicit manner, but is often in the planners' mind.[6,8,17] Limiting the maximum distance is an alternative to achieve this property. Another elusive yet highly desired characteristic of school boundary is smoothness. Smooth boundaries generally make school districts clearer and seemingly less biased. Figure 4(a) does not have this property, but its smoothness can be improved by using the post-allocation tool mentioned at the beginning of this section. Figure 4(b) shows the same solution as Figure 4(a) but after manually reallocating 14 blocks. Six of these blocks had no students; therefore, the impact of the post-allocation on the feasibility and/or objective value was relatively insignificant. In general, there are many blocks where no students reside and the planner using the post-allocation tool can trivially reallocate them at his convenience.

Notice that this scenario was not considered in the Fels case because, from Table 4, Scenario (1) achieves the best maximum walking distance (in fact, there is a block whose closest school is 6115 ft away).

Scenario (3): racial balance. The objective of this scenario is to redistrict each cluster so that the percentage of students of race 'type 1' in each school is within the range $[R_1 - p_1,\ R_1 + p_2]$ where $p_1,\ p_2 \in [0, 1]$ are parameters and R_1 is the cluster's ratio (ie number of students of race 'type 1' divided by the total number of students in the

Table 6 Scenario (2) tested with different values of M

Max. distance (M) in ft	*Worst travelled distance (ft)*	*Average travelled distance (ft)*	*Overcrowded schools*
None	8791	2145	0
7500	7350	2172	0
6000	5984	2045	2
5000*	6691	1999	3

*The maximum distance constraint was imposed only to a sub-set of schools.

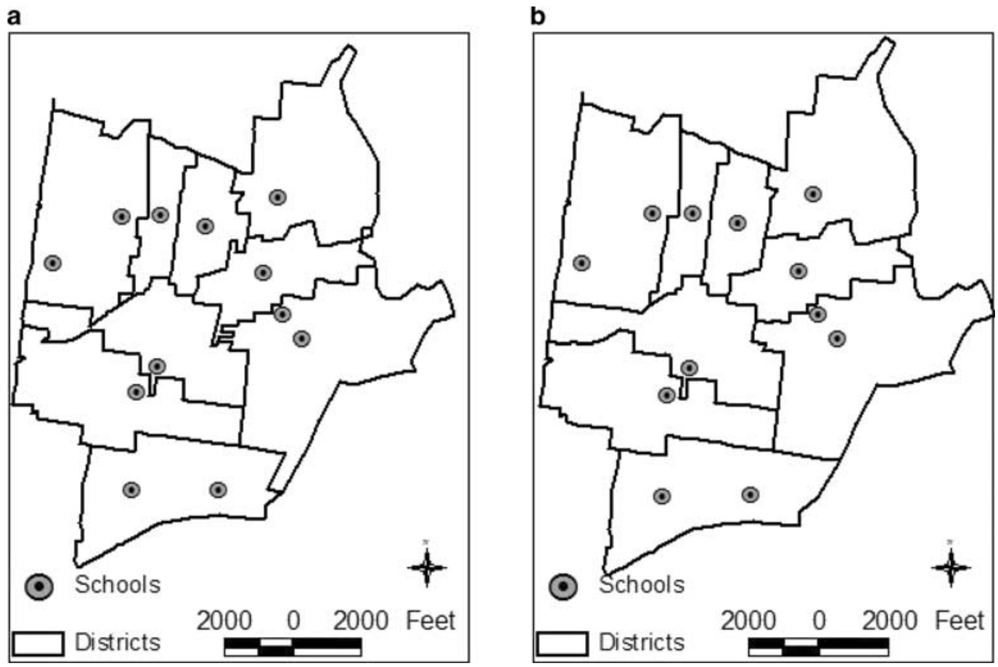

Figure 4 Redistricting Olney cluster with a maximum walking distance constraint: (a) before post-allocation, (b) after post-allocation.

cluster). The results for this scenario shown in Tables 4 and 5 (for Fels and Olney, respectively) consider $p_1 = p_2 = 15\%$.

For the Fels cluster, an additional maximum walking distance constraint (of 7500 ft) was imposed in order to avoid overly spread out districts. The average travelled distance increased by 10% (see Table 4) and the resulting districts have some irregular borders, but again this could be fixed using the post-allocation tool as explained in the previous section. Note that the running time still has the same order of magnitude.

Finding an optimal solution for this scenario in the Olney case turned out to be relatively hard. The running time reported in Table 5 is above 10 h. To deal with this difficulty, we used the pre-allocation tool. Basically, all the blocks that were clearly closer to one particular school (relative to other schools) were *a priori* assigned (fixed) to that school. For example, all the blocks in the extreme northeast or southwest corners of Olney cluster were treated in this manner. We then fixed approximately 28% of the blocks, and the total running time decreased by almost 80% (to nearly 2 h). In further research, it would be interesting to analyse the structure of the model in order to improve the solution time, but at the moment we are mostly interested in the application rather than developing efficient algorithms for the more difficult cases.

The difficulty in solving Scenario (3) for the Olney cluster can be explained by the racial distribution of the students. Even though the percentage of race 'type 1' students is high ($R_1 = 47\%$), they are not uniformly distributed. Hence, a feasible pattern is hard to find; moreover, the existing solutions have districts with extremely irregular shapes. Figure 5(a) shows the districting map where the proportion of race 'type 1' students must be in the range [0.25, 0.75]. In addition, a maximum traveled distance constraint of 5000 ft was imposed (only for some schools), and the solution has three overcrowded schools (with two exceeding the original capacity by more than 10%). It can be seen that the districts of Figure 5(a) and 4(a) have similar shapes. This similarity is clearly lost in Figure 5(b), where the range of race 'type 1' students was narrowed to [0.32, 0.62]. This last figure has no maximum distance constraint; otherwise, the problem was integer-infeasible (regardless of the school capacities).

In addition to the extremely elongated shape of some districts in Figure 5(b), there is one particular district that is not contiguous (it is split in two). This is a direct consequence of how the D_{in} parameters and the $N(i,n)$ sets were calculated, as discussed before.

Scenario (4): redistricting assuming all schools are K-8. This scenario represents an interesting 'what if…' question and shows the potentiality of the system. The total capacity of every school was divided proportionally for each grade according to the cluster's student-per-grade ratios (ie the total number of students per grade divided by the total number of students in the cluster), and then a 5% surplus was added. We included an additional constraint expressing that the total number of students allocated to a school should be no more than the original total capacity (before the surpluses per grade were added).

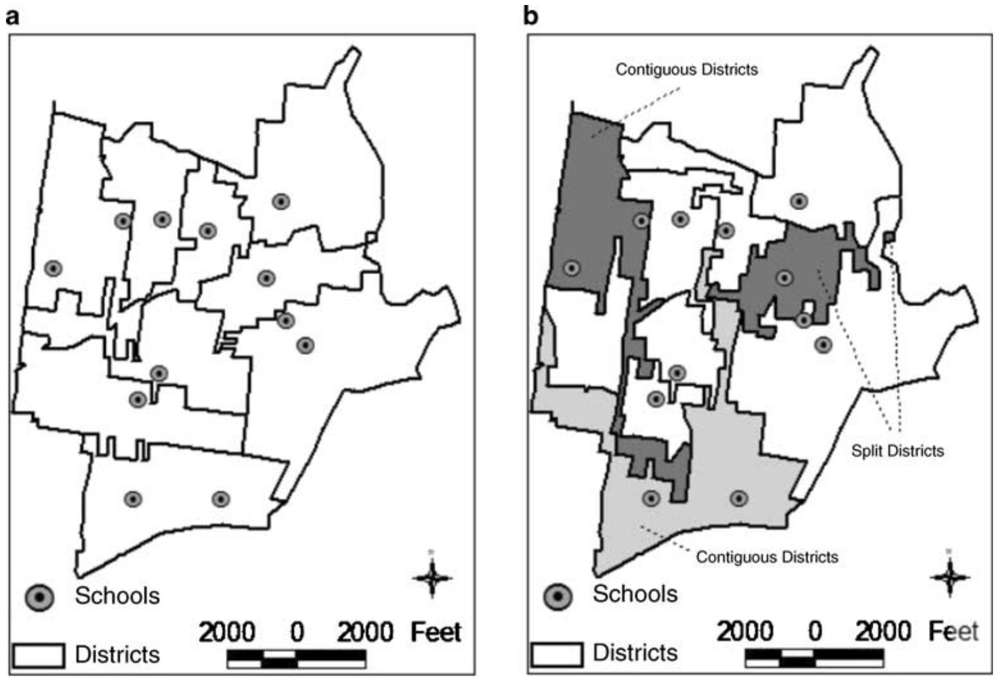

Figure 5 Redistricting Olney cluster with racial balance: (a) racial range = [0.25, 0.75], (b) racial range = [0.32, 0.62].

Since seven of the eight schools in the Fels cluster were K-8, the solution barely changed (see Table 4). This grade restructuring, however, improved the Olney cluster substantially, as the average travelled distance was decreased by 20% and there were no overcrowded schools (see Table 5).

Conclusions

This paper has presented an interactive school redistricting system coupling a commercial GIS with an exact optimization model to solve the school redistricting problem. The model explicitly considers common quantitative properties of a good school redistricting plan as found in the literature, and other qualitative desirable properties are covered through the GIS interactive interface. The solutions found with this system improve on existing solutions, and even more importantly, provide insight into the trade-offs involved. The results are intuitively clear: for example, as the capacities per grade become more stringent, the walking distance increases; if the students are not uniformly distributed by race (which is usually the case), there is a clear trade-off between achieving certain racial balance and keeping contiguous (and compact) districts. The model, in an iterative procedure, can help find an adequate equilibrium. The capability for analysing multiple scenarios is evident; this was done for answering the question 'what if all the schools had the same grade structure?' Clearly, this is a theoretical answer. Such a modification would involve many other issues not considered in the model. To mention a few, there is no cost analysis, teachers' availability and opinion are not considered, nor is the parents' point of view taken into account. Nevertheless, the solution provided by the model would help set any further discussion on a solid ground. Similarly, the system could be extremely helpful for locating new schools and/or analysing the impact of a school shutdown.

In general, school redistricting criteria, whether quantitative or qualitative, cannot be enumerated in advance, but rather are established and modified during a trial-and-error redistricting process. Therefore, the major implication of this study is that GIS can be an effective tool connecting a mathematical model's ability to handle complexity and human's intuition and experience to solve highly subjective ill-defined spatial problems. The results obtained seem to indicate that the presented approach could be a valuable tool for school planners.

Future research could concentrate in studying multiple periods with enrolment forecasts and the combinatorial structure of the model in order to reduce its computational time.

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Appendix: Ensuring contiguous districts

Considering again the example in Figure 3, the problem is that, given the 'block to school' distance measure described in the implementation section, constraints (5) and (6) are not sufficient to rule out non-contiguous districts. Note that this problem remains if instead we define the 'block to school' distance to be the Euclidean distance from the centroid of the block to the point that represents the school. Note also that if we define $C(i) = NULL$ for every block i except those that are directly next to a school (where $NULL$ represents a value different from any of the existing schools), then this problem would be fixed. However, this condition is excessively strict and would rule out not only non-contiguous districts but also many contiguous ones. For instance, in the example of Figure 3, if we set $C(\mathbf{V}) = NULL$ then constraints (6) would dictate that block **V** could never be assigned to school $\mathbf{s_1}$.

One approach to overcome this problem is to measure the distance between a school and the nearest part of a block rather than the block's centroid. More formally, regarding block **V** as a set of points, the canonical distance L between a school **s** and block **V** can be defined as follows:

$$L(\mathbf{s}, \mathbf{V}) = \min\{Eucl(\mathbf{s}, \mathbf{q})/\mathbf{q} \in \mathbf{V}\}$$

where **s** is a point representing a school, **V** is a set of points representing a block, and $Eucl(i,j)$ is the Euclidean distance between points i and j.

Given this distance measure, if every block can be connected to its closest school by a straight line contained in the area (cluster) being studied, then constraints (5) and (6) ensure contiguous districts. A sketch of the proof for this claim would be as follows. Let $\mathbf{p} = argmin_{\mathbf{q} \in \mathbf{V}}\ \{Eucl(\mathbf{s},\mathbf{q})\}$. If $\mathbf{p} \neq \mathbf{s}$, then $N(\mathbf{V}, \mathbf{s}) \neq \phi$, that is, **V** has an adjacent block that is closer to **s** (because the line that connects **p** and **s** is part of the cluster). Therefore, constraints (5) ensure that there is a path from any point of block **V** to school **s**. If $\mathbf{p} = \mathbf{s}$, then $C(\mathbf{V}) = \mathbf{s}$, that is, block **V** and school **s** are next to each other. Therefore, neither constraint (5) nor (6) applies so that block **V** can be assigned to school **s** without any further requirement.

Calculating the canonical distance from a set to a point is often computationally intensive. However, since the GIS works with the block information in vector format, that is, each block is typically represented by a polygon consisting of a finite set of point (rarely more than 20), a surrogate distance measure would be

$$L(\mathbf{s}, \mathbf{V}) = \min\{Eucl(\mathbf{s}, \mathbf{q})/\mathbf{q} \in GIS(\mathbf{V})\}$$

where $GIS(\mathbf{V})$ is the set of points that represent block **V** in a GIS vector format.

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