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Discrete Optimization

Districting for salt spreading operations

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Abstract

The districting problem presented in this paper involves the partitioning of the road network in a region into sub-networks (or districts), to facilitate the organization of the operations to be performed within the region. Typically, each district contains one local center (depot) whose location is given, while the operations involve different types of routing, with routes starting and ending at the depot. For public sector applications like salt spreading and road maintenance, this partitioning is a real distinct stage in the organization and planning of services, and as is the case with location, districting is of a non-operational nature. Relevant characteristics of well-designed districts are: ability to support good routing, balance in workload, compactness of the sub-areas, centrality of the depot, etc. We present a heuristic procedure for our districting problem. First we partition the road network into small cycles, then we aggregate them into districts in two phases. Phase 1 uses an approach based on bin packing principles, while in Phase 2 a multi criteria approach is used. We illustrate the procedure and discuss its merits for the salt spreading operations in the province of Antwerp.

Keywords: Districting; Heuristics; Arc routing; Salt spreading; Elemental cycle approach

1. Introduction

Districting involves the partitioning of a large geographical region into sub-areas (districts) according to some criteria, to facilitate the organization of the 'operations' that have to be performed within the region. A common approach to tackle this problem is to subdivide the region into a large number of elemental areas (or units) – each elemental area has one or more attributes associated with it – and to aggregate these units into districts, while an objective function is optimized and some constraints are satisfied. In general, the constraints ensure the contiguity, compactness and balance of the districts, while it is usually not allowed to split a unit over several districts. Typical applications of this problem include the design of political districts (e.g. [12,13,18]), sales territories [11,22], emergency and health care districts [20], etc. In this paper we focus on the problem of districting for salt spreading operations on roads. In Section 2, we describe the

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underlying problem of districting associated with salt spreading operations and derive some guidelines for the design of good districts for these operations. Section 3 explains an approach to partition a road network into a large set of small cycles. The heuristic procedure in Section 4, subsequently uses these elemental cycles to build districts. Section 5 presents the results of a small case study where we applied the procedure to redesign the district borders in the province of Antwerp (Belgium). Conclusions follow in Section 6.

2. Salt spreading and districting

Salt spreading (or winter gritting) is common practice in wintertime in many countries with a moderate climate, and involves the spreading of salt on roads when frost, ice, or snow have made them slippery. The salt spreading problem can be most naturally seen as a capacitated arc routing problem (CARP). A CARP is a routing problem where demand occurs along the arcs of the network and in which the vehicles conducting the service have limited capacity (see [1,9,10] for recent surveys).

Several authors deal with the problem of salt spreading (see for example [4,6–8,15]), but in most cases the studies are limited to the actual routing, assuming that all other parameters and factors are given. In particular, it is assumed that depot location is fixed, and that the network on which salt spreading has to be performed is given. Lotan et al. [16] describe the problem of salt spreading in the province of Antwerp while emphasizing its location aspects. Location of depots for salt spreading operations is becoming a practical problem since on the one hand there is a general trend to reduce the number of depots and to save in overall managerial and administrative costs, and on the other hand secondary depots (silos) can currently be located at very low cost and enable refilling of trucks during a tour.

Routes for salt spreading are typically organized by districts and can be categorized into two main types: preventive gritting and curative gritting. Preventive gritting is done before roads have become icy, while curative gritting is done after icy conditions have occurred and requires double the amount of salt. Consequently, routes for preventive gritting can differ from those for curative gritting, but the same depots organize both types of operations within the same district borders. Moreover, in summertime other activities (such as road maintenance and inspection) are conducted from the same depots to the same districts.

Usually, a lot of attention is given to the problem of determining efficient operations within given district borders. Many times, however, significant long-term savings can be achieved if more careful attention is devoted to the determination of the district borders at the time of their conception. Cattrysse et al. [3] and Van Oudheusden et al. [21] discuss the interactions among location, districting and routing for salt spreading operations, and demonstrate the importance of districting for performing better routing. They define districting as the partitioning of the road network in a large region into sub-networks (districts) such that each district contains one local center (depot) and is independently responsible for the operations performed within its borders. The operations usually involve routing, with routes starting and ending at the depot.

The problem of districting, especially in the context of location and (arc) routing, is hardly addressed in the literature, or is assumed to be solved a priori, without explicitly taking into account its effects on the subsequent routing and other applications. One approach is to assume that districts are determined by the location of the depots. Following this approach, the partition into districts is done by assigning nodes and edges to the closest depot. Another approach is to perform multi-depot routing and to associate nodes and edges to the depot from which the tour containing them emanates. Levy and Bodin [14] describe an algorithm for a rather specific postmen routing application: the ‘arc oriented location routing problem’ with the following major steps: location of depots, allocation of arcs into ‘partitions’ (sub-networks), and routing. Each partition emanates from a depot node and corresponds to a (vehicle) route. They further require that a partition includes an Euler tour and that the total workload in each sub-network is within pre-specified bounds.

Hence, the collection of routes associated with the same depot can be seen as a district. The partitioning is, however, completely motivated by a specific routing problem, and evaluated according to: total dead mileage, their violation of bounds regarding total workload and the number of depots, which are all measures solely related to efficient routing for a specific instance, while the number of routes associated with a depot is determined by the degree of the depot node.

Male and Liebman [17] address a problem, which they denote 'districting', in the context of solid waste collection. However, a district is defined as the collection of the streets, serviced in a single tour. Thus, several districts are associated with the same depot and the partition into districts is sensitive to the parameters of the routing problem (vehicle capacity, dead mileage, etc.). Their definition of districts is more technical and does not correspond to our notion of districts as independent, geographical areas with (centralized) depots.

2.1. Guidelines for district design

From a planning point of view, there is a fundamental difference between districting and routing: whereas districting should be performed at the strategic and tactical level, routing is performed at the operational level. Thus districting should involve a more global view and is often related to the managerial and administrative levels. Apart from being a frame for routing, districts often serve administrative purposes. It is at the depot of a district that, over many years, useful, interrelated data are collected. Thus district borders should not change too frequently. They should be modified only when major changes take place, such as the construction of an important new road, the addition of secondary depots, the introduction of new district responsibility, etc.

Furthermore, the same districts should allow support to different types of routing and possibly other operations. Hence whereas routings are more sensitive to specific constraints (capacity, time, distance, etc.), districts should be more robust and not influenced by minor changes in the characteristics of the operations performed within the districts. Therefore, different guidelines should be used for districting and for routing, while keeping in mind the important interactions between the two operation levels.

In the literature, objective functions for evaluating good location and routing decisions are prespecified and usually relate to the optimization of some economic criterion. While location theory typically deals with minimizing maximal or total distance from the depot, routing schemes normally aim at optimizing the following measures: number of tours, total distance traveled, time spent on service and travel, dead mileage, compliance with capacity and time restrictions, etc. It appears to be more difficult to specify exact economical measures for performing good districting. Obviously, good districting should be able to support good routing; however, there are other requirements which do not relate directly to routing. Good districting should result in balanced and 'geographically' compact districts (somewhat circular or square in shape rather than elongated) whose borders define clear sub- areas. Thus, whereas vehicle routes can cross each other, it is reasonable to expect that districts would not overlap.

2.2. Dealing with multiple operations

Since districting should take into account the different tasks performed within the boundaries, careful enumeration of all planned activities and their related objectives is an important requirement. In the province of Antwerp mainly three types of operations are conducted from the same depots: preventive gritting, curative gritting and road maintenance.

When dealing with multiple operations, it is important to identify the most critical operation(s) in order to give it its proper weight. For the districts of the province of Antwerp it is clear that curative gritting has the most demanding requirements and should be considered as the most critical operation. Preventive gritting requires only half the amount of salt, and can be done by combining two tours into one, for example. For road maintenance and inspection operations, good districting allows every road to be reached from a depot within a certain distance and thus can be achieved

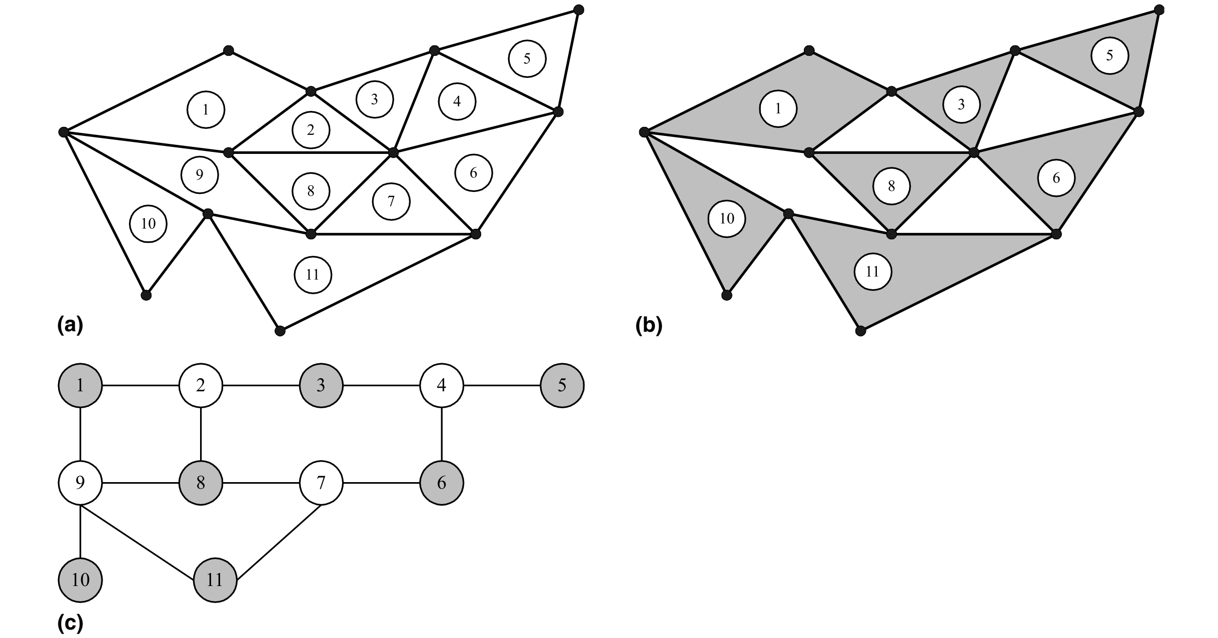

Fig. 1. (a) Finite faces of a planar Eulerian graph. (b) Checkerboard pattern (elemental cycles). (c) 2-Colorable dual graph.

by ensuring compact and balanced districts in which the depot is centrally located.

Clearly, the districting problem requires a multi criteria approach. Even if the focus is on a single kind of gritting, the problem is still of the multi criteria type. Indeed, several objectives should be considered, such as minimizing the number of trucks, minimizing dead mileage, minimizing total mileage, etc. Yet, in Antwerp, by performing a careful partition into compact districts and ensuring that all nodes in the sub-networks are of even degree (to enable tours with less dead mileage), the study can concentrate on the most important objective (minimizing the number of trucks) for the most critical operation (curative gritting).

3. The elemental cycle approach

We now present a method to define the building blocks used in our districting procedure. The approach, introduced by Male and Liebman [17] in the context of routing waste collection vehicles, is based on the partition of a planar topological (i.e. a planar representation), Eulerian graph into cycles using a 'checkerboard pattern'. This partition is characterized by a large number of small cycles where every edge belongs to exactly one of them as illustrated in Fig. 1. The contours of the different finite faces of a planar topological graph make up a fundamental basis of independent cycles [2]. By marking the faces according to a checkerboard pattern as shown in Figs. 1(a) and (b), we select a set of faces (with associated cycles) such that every edge belongs to exactly one cycle. We call the selected cycles *elemental cycles* and use them to design districts in Section 4.

A dual graph can be constructed such that every face of the primal graph (Fig. 1(a)) corresponds to a node in the dual (Fig. 1(c)). An edge between two nodes of the dual graph implies that, in the primal, the corresponding faces have at least one edge in common. The dual graph is obviously planar. Since the primal graph is Eulerian, all its nodes have even degree and, therefore, the dual graph has no circuits of odd length. For this reason, the dual graph is 2-colorable [2] and the partition of the primal graph into elemental cycles is unique.

Both Male and Liebman [17], and later Eglese [6] use the cycle approach to determine vehicle routes in the context of capacitated arc routing. In [8], Eglese criticizes the approach for the design of routes and provides examples that are far from optimal. The main drawback of the elemental cycles for arc routing problems emanates from the fact that a cycle has to be either included or excluded in a route. Since the construction of elemental cycles is independent of capacity considerations, the combining of cycles may involve additional tours and excessive dead mileage, as demonstrated in [8]. The construction of routes based on elemental cycles is often in contrast to the behavior of the more efficient capacitated arc routing algorithms (e.g. [19]) which construct tours around 'expensive' arcs, i.e. arcs that are far away from the depot, and in this way create 'large' tours which are typically very different from the routes constructed using the cycle approach. Li and Eglese [15] provide a clear graphical explanation as to why routings which serve the innermost zones first (this is the region in the close neighborhood of the depot) are bound to have more dead mileage than routings which serve first the outermost zones. Routing purely based on the elemental cycle approach clearly tends to serve the inner zones first, and is thus likely to incur more dead mileage.

On the other hand, some of the inherent characteristics of the elemental cycle approach can lead to good districting according to the measures mentioned in Section 2. The existence of relatively small and many different cycles allows aggregating them into districts so that districts are balanced and compact. A potential for good arc routing is maintained by ensuring that districts constitute an Eulerian graph. Moreover, districts are separable (by cycles) and each collection of connected cycles is also unicursal. Furthermore, the separability and additivity of cycles provide easy adjustment of the final partition to geographical, micro-climatical, or other 'secondary' constraints and considerations.

4. A districting procedure based on elemental cycles

The procedure builds all districts simultaneously by assigning elemental cycles to the depots and is divided into four stages: preprocessing, initial assignment, two-phase iterative assignment and improvement. We describe these four stages in Sections 4.1–4.4. It is assumed that the locations of the depots, where the vehicles are stationed and the salt is stored, are given and that they coincide with nodes of an undirected, planar road network. Furthermore, every edge of the graph has a length and a weight corresponding to the demand for salt.

4.1. Stage 1: Preprocessing

In the preprocessing, we partition the road network into elemental cycles and calculate the cycle weights and the ratios. The cycle weight CW_j corresponds to the amount of salt to service the roads in cycle j, while the ratio R_{ij} relatively measures the average distance to reach cycle j from depot i.

From Section 3 it is clear that the elemental cycle approach requires a unicursal graph as input. If the graph corresponding with the road network is not unicursal, we match the odd degree nodes at minimal cost (minimal total length) and augment the graph with the matching edges [5]. Next, a planar representation of the resulting graph is easily partitioned into elemental cycles.

Due to the matching, some edges appear twice: once with a weight equal to the demand for salt of that link and once with weight zero. This creates some freedom in the assignment of loads to cycles if the original and matching edge belong to different cycles. Bin packing theory shows that it is 'easier' to balance the load over a set of bins when a lot of small weight items are involved. Consequently, a lot of small weight cycles will facilitate the construction of balanced districts and thus, the cycle weights are determined such that cycles with largest and smallest possible weights are created.

The ratio R_{ij} is defined as the average shortest path distance $\bar{D}_{ij}$ to reach a node of cycle j from depot i, divided by the minimal average shortest path distance to reach cycle j from another depot, or, if CN_j is the set of nodes on cycle j and D_{ik} the shortest path distance from depot i to node k:

$$\bar{D}_{ij} = \frac{\sum_{k \in \mathrm{CN}_j} D_{ik}}{|\mathrm{CN}_j|}, \tag{1}$$

$$R_{ij} = \frac{\bar{D}_{ij}}{\min\limits_{i' \neq i} \{\bar{D}_{i'j}\}}. \tag{2}$$

A ratio value $R_{ij} \leqslant 0.5$ indicates that cycle j is considered very close to depot i. In that case, all ratios for that cycle with respect to other depots are larger than two, and thus the average distance to reach a link of that cycle from another depot, is at least twice as long. Note that for every cycle j there is at most one depot i for which $R_{ij} < 1$.

We further specify for every depot a benchmark ratio $0.5 < \mathrm{BR}_i < 1$ (e.g. $\mathrm{BR}_i = 0.7$), labeling cycles either as relatively 'close' to a depot or not. It also states which cycles are considered in either Phase 1 or 2 of the iterative assignment procedure (see Section 4.3).

4.2. Stage 2: Initial partial assignment

After preprocessing we initiate the district building process by assigning first the cycles that are adjacent to a depot and next, the cycles that are very close to a depot node (indicated by $R_{ij} \leqslant 0.5$). Mainly motivated by the criterion of compactness, these cycles can indeed be assigned immediately without an elaborate trade-off between different criteria.

To ensure the connectivity of the partially built districts during each cycle assignment, we only consider the cycles that are labeled eligible at that moment. A cycle is eligible for a certain depot, if it is not yet assigned to a depot (district), and has at least one node in common with a cycle already assigned to that depot.

By assigning cycles, other cycles may become isolated. For an isolated cycle (or a cluster of isolated cycles) all its neighboring cycles are already assigned to the same depot. Since the isolated cycles may become eligible for one depot only, we assign them immediately. In that way we reduce the number of cycles to be considered in the subsequent stages and consequently speed up the assignment procedure.

4.3. Stage 3: Two-phase iterative assignment

Stage 3 proceeds with the districting process in two phases, building all districts simultaneously in order to balance the total workload. The regions where Phases 1 and 2 are relevant, are influenced by the value of the benchmark ratio. They are shown in Fig. 2.

Phase 1 considers only cycles relatively close to the depots, indicated by a ratio value smaller than the benchmark ratio. Focusing on the construction of balanced and compact districts, these cycles are assigned in a rather straightforward procedure.

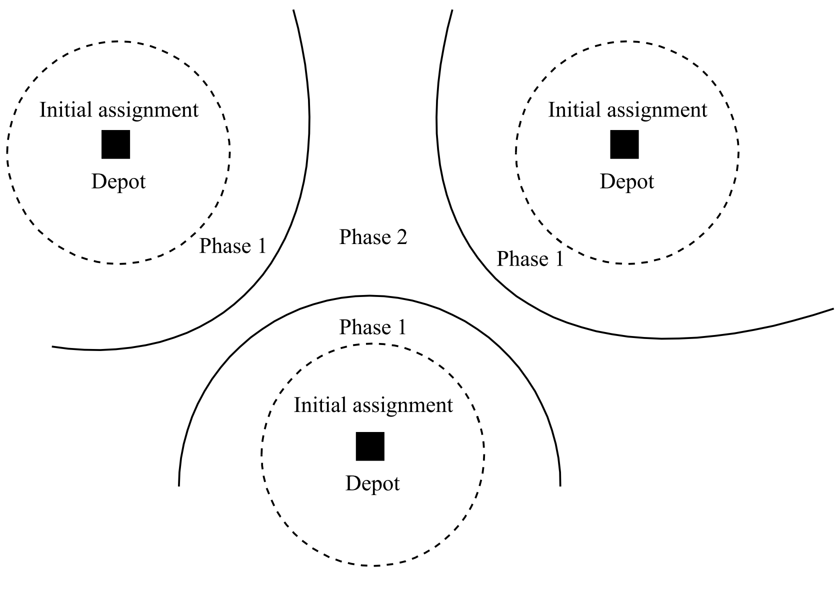

Fig. 2. Regions for Phases 1 and 2.

The Phase 1 procedure operates as a simple bin packing heuristic, and corresponds in some parts to the partitioning algorithm of [14]. At any point in the procedure, the district with the smallest workload assigned to it so far, becomes the next candidate district for expansion. The cycle added to this district is the one with the largest weight, adjacent to a cycle already assigned to that district and, with a ratio with respect to that depot smaller than the benchmark ratio. After each cycle assignment, isolated cycles are checked for and assigned, and the district workloads and eligible sets are adjusted.

Phase 1 ends when no cycle can be added to the district with the smallest workload. This means that for the assignment of the remaining cycles different (conflicting) objectives need to be considered, since otherwise the imbalance between the districts might become too large.

Therefore, Phase 2 uses a multi criteria approach. In each iteration of the Phase 2 procedure, we select for every depot i the largest weight eligible cycle j with a ratio smaller than BR_i. If no such cycle is available for a depot i, we select the eligible cycle j with the smallest ratio R_{ij}. Each candidate cycle depot assignment is evaluated on different criteria and a score is calculated. The candidate cycle depot pair with the lowest score is considered the winner of the test and is assigned next. Again, after a cycle assignment, isolated cycles are checked for and assigned, and the district workloads and eligible sets are adjusted.

Three criteria are defined here for Phase 2. It should be clear that they are only meaningful for salt spreading and road maintenance operations; other, appropriate criteria have to be defined when focusing on other applications. As a first criterion, district compactness is facilitated by assigning the nearest cycles first. Thus, we calculate for the candidate cycle depot assignments $\max\{\mathrm{BR}_i, R_{ij}\}$ and according to an a priori evaluation (e.g. see Table 1), give an appropriate score.

Minimizing the number of trucks is the most important objective for routing winter gritting vehicles. Its value is influenced by the final partition of the road network into districts. Considering this, we make, for the various candidate assignments, two estimates for the number of vehicles required in the final partition. Both estimates are based on a continuous approximation:

Table 1
A possible evaluation and score of the ‘closeness’ measure

Evaluation	Score
$\max\{\mathrm{BR}_i, R_{ij}\} \leqslant \mathrm{BR}_i$	0
$\mathrm{BR}_i < \max\{\mathrm{BR}_i, R_{ij}\} \leqslant 1$	1
$1 < \max\{\mathrm{BR}_i, R_{ij}\} \leqslant 1.5$	2
$1.5 < \max\{\mathrm{BR}_i, R_{ij}\} < 2$	3
$2 \leqslant \max\{\mathrm{BR}_i, R_{ij}\}$	Not allowed

$$\#\text{ vehicles} = \sum_i \left\lceil \frac{\text{total load district } i}{\text{vehicle capacity}} \right\rceil. \tag{3}$$

Assuming for each candidate cycle depot pair that the assignment is carried out, we divide the remaining (unassigned) cycles over the districts in the ‘best’ and the ‘worst’ way, such that the number of vehicles required is minimized, respectively, maximized. The sum of both estimates is used as the score to evaluate the candidate cycle depot pairs on the second criterion.

The third criterion evaluates the balance of the (partially built) districts: the addition of a cycle (together with possible isolated cycles) to a depot will either decrease or increase the imbalance, indicated, respectively, by a score of ‘0’ or ‘1’.

By comparing the individual scores of the different candidate cycle depot assignments, dominated assignments are removed first. Next, we calculate the weighted sum of the scores using e.g. the following three weights: $W_1 = 1$, $W_2 = 3$, and $W_3 = 2$, and choose the assignment with the smallest total score.

4.4. Stage 4: Improvement and user interaction

This stage shifts or interchanges cycles between the depots, taking the connectivity into account and trying to decrease the number of required trucks. In this step topographical, climatical or other secondary constraints are addressed. These considerations, obviously, may also slightly modify the decisions in the previous stages; for instance, they can play a role whenever a tie occurs in the cycle assignment. No specific exchange procedure was developed for Stage 4, but very likely, a user interactive procedure, as in [15], where the decision maker can suggest and evaluate certain changes, will give superior results. A visual representation of the districts will certainly help. It is also clear that there remains a lot of freedom; quite some trials can be explored, probably resulting in a more adequate partition. For example, other benchmark ratio values will influence the regions for both phases in Stage 3, and consequently affect the cycles considered. Other cycles or cycle weights are easily defined by considering other matching solutions (thus not necessarily the minimal cost matching) or by averaging the edge weights over the cycles, instead of creating cycles with the largest and smallest possible weights. Finally, in the multi criteria approach, the user can define the appropriate criteria for his own application, and carefully specify the evaluation scheme and the corresponding weights.

In the following section, we illustrate our procedure by redesigning the existing district borders for the winter gritting activities in the province of Antwerp.

5. Case: Districting in the province of Antwerp

Flanders, the northern part of Belgium, is divided into five provinces, Antwerp being one of them. The Flemish Government organizes its salt spreading and road maintenance activities per province, where each province is further divided into several districts. The Flemish administration is only responsible for these activities on the highways and regional roads, while the responsibility for the provincial and communal roads lies, respectively, with the provinces and communes. Unfortunately, there is no real co-operation between these authorities.

In 1996 the number of districts in Antwerp was reduced from nine to six, motivated by the need to reduce the operational costs. However, the new partition into districts did not explicitly take into consideration arguments related to the routing operations. Intuition was used, and district borders coincide to a large extent with irrelevant borders of communes.

Fig. 3 shows the regional road network and the current borders of four districts in the province of Antwerp. The districts are named according to the commune in which its main depot is located: Brecht, Vosselaar, Grobbendonk and Geel. All the roads allow traffic in two directions and are classified into two types: the larger (2×2) motorways and the smaller (1×1) regional roads. For motorways the two directions are spread separately, two lanes at a time. Consequently, they are modeled (initially) as pairs of oppositely directed arcs. Smaller roads allow gritting in one pass, servicing both directions together, and are represented as undirected edges. Hence, the (connected) graph corresponding with the road network is mixed.

Recall that the elemental cycle approach is designed for undirected networks and requires an undirected network as input. If the road network contains many one way streets, as is the case in some urban networks, the approach cannot work well. However the mixed network under consideration can, for our purposes, easily be replaced by an undirected network. A minimal cost Euler tour in the mixed network can be determined using a ‘decomposition approach’. Indeed, all arc pairs form circuits and, since the arcs in a pair have the same length, the shortest path distance between two nodes is independent of the direction of travel considered. Hence, ignoring directions and matching the odd degree nodes at minimal cost, determines the links to make the original graph Eulerian. In addition, once this undirected graph is partitioned into cycles, it is always possible to choose cycle directions appropriately (in accordance with the original road directions) and thus directions do not have to be considered explicitly.

The road network in the four districts is composed of 244 edges and 154 nodes; 92 of them have an odd degree. In total 806.5 km of two-lane roads need to be spread with salt, while a truck can spread approximately two lanes of 35 km.

Applying the procedure on this partial network of the province of Antwerp results in 95 elemental cycles. The initial assignment stage assigns 76 of them to the depots. Another 9 cycles are assigned during Phase 1 in three iterations and the re-

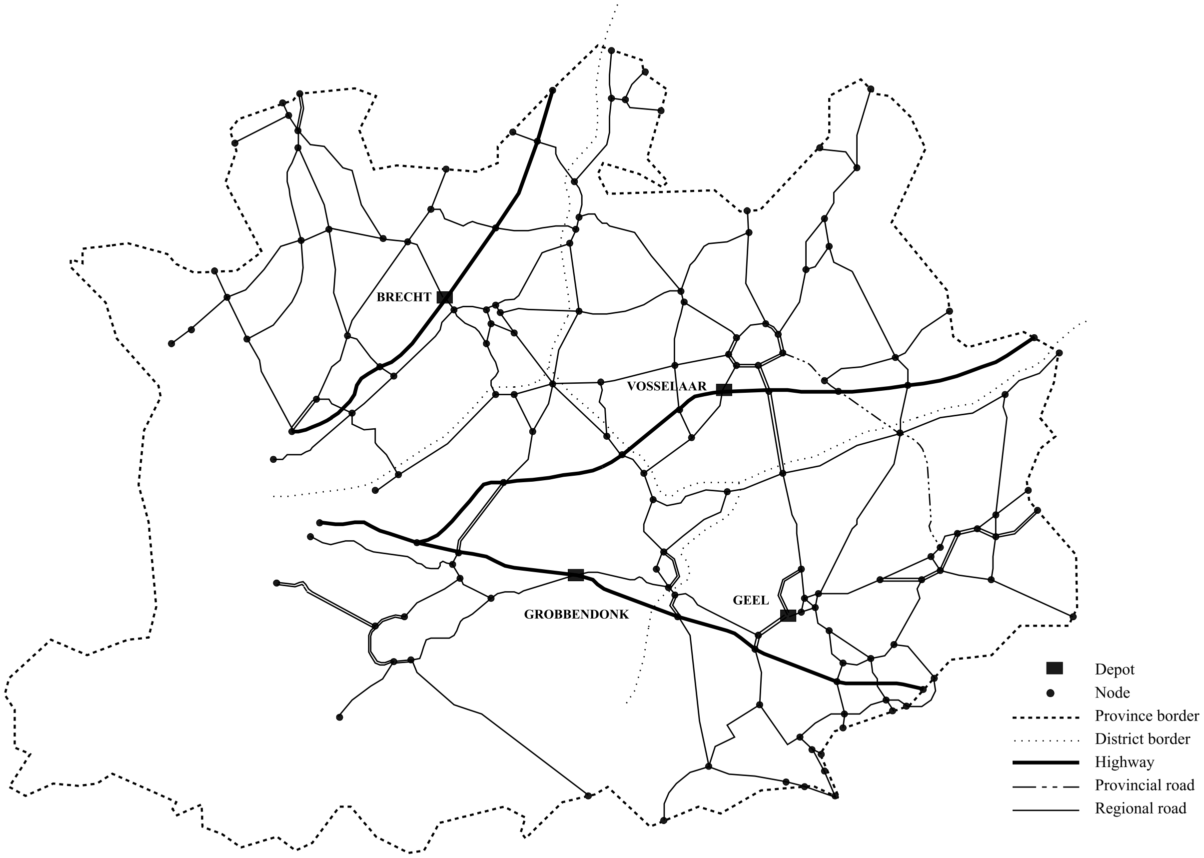

Fig. 3. The regional road network and the current district borders of four districts in Antwerp.

maining 10 cycles during Phase 2 in five iterations. The main reasons for this fast assignment are probably the rural characteristics of the road network and the occurrence of many isolated cycles. The final partition of the roads into districts is shown in Fig. 4, and seems (visually) quite reasonable. The actual performance of the districting procedure is evaluated by considering the underlying routing problem for curative gritting.

Table 2 includes the following data for the present districts of Antwerp: total distance to be serviced (load), a lower bound (LB) on the number of tours calculated by dividing the load of the district by an 'effective' tour length of 35 km (which is calculated considering standard road width and truck capacity), the number of existing tours at the district, and in the last column, the average distance gritted per tour. Considering the total load in each district, this partition is well balanced. It is also apparent that the average tour lengths in Brecht and Vosselaar are far below their 'optimal' potential length of 35 km. Hence, even without improving the partition into districts, it is possible to eliminate four tours, by making tours longer and closer to their capacity limits.

We first solved heuristically (based on [19]) the arc routing problem in the original districts and obtained the results of Table 3: column three shows the number of tours used in the improved routing solution and column four the total dead mileage. In total only 25 trucks were used and as can be seen in column five; average gritting distance per tour increased both in Brecht and Vosselaar.

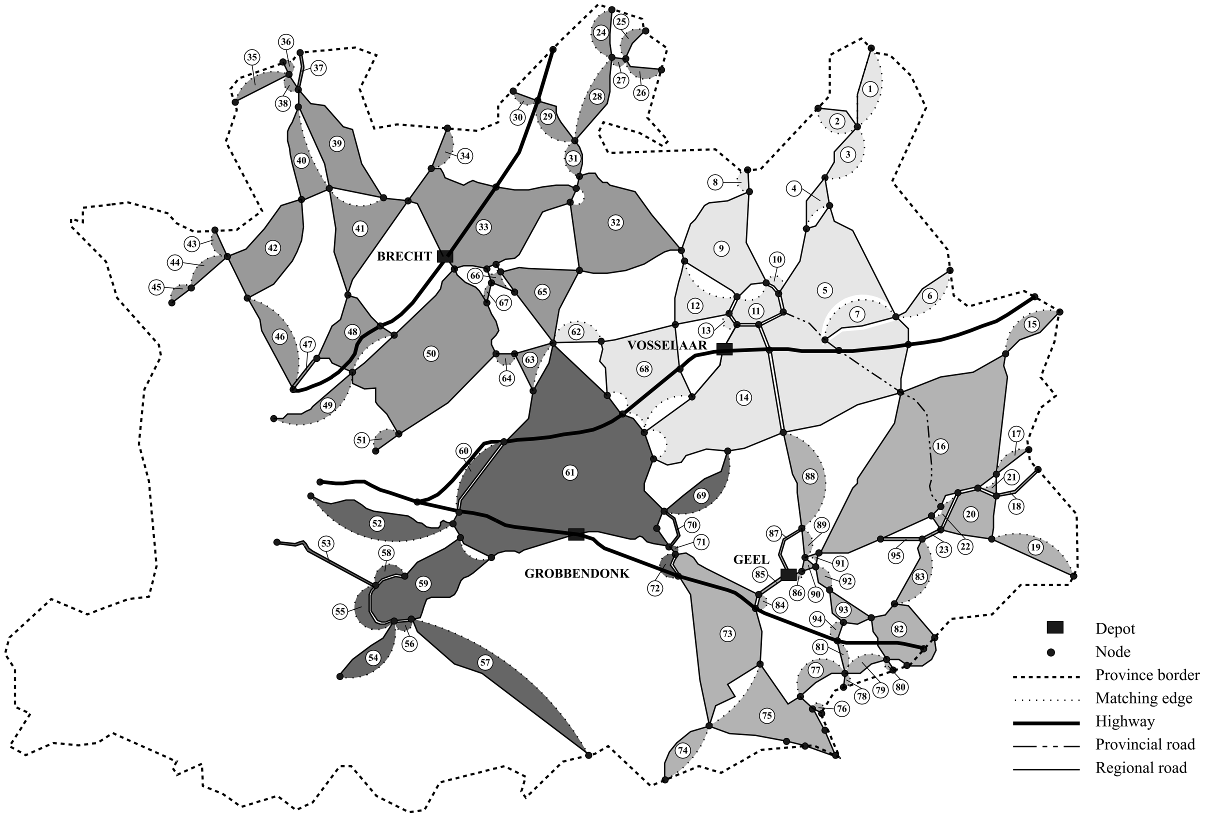

Fig. 4. Elemental cycles and new districts.

Table 2
Present districts and routing

District	Load (km)	LB (# tours)	# of tours	Avg. tour length (km)
Brecht	199.5	$\lceil 5.70 \rceil = 6$	9	22.2
Vosselaar	195.5	$\lceil 5.59 \rceil = 6$	7	27.9
Grobbendonk	178	$\lceil 5.09 \rceil = 6$	6	29.7
Geel	233.5	$\lceil 6.67 \rceil = 7$	7	33.4
Total	806.5	$\lceil 23.04 \rceil = 24$	29	

Finally, Table 4 summarizes the operational results of the districting procedure applied to the province of Antwerp. Obviously, this new partition is not so well balanced as the original one. However, with the same number of trucks (tours) as in the improved situation within the original districts (Table 3), we are able to grit the roads with approximately 80 km less deadheading, a reduction of about 14%. A further reduction in the number of tours (from 25 to 24) seems to be difficult. District Geel is probably the best candidate for exchanging a cycle (trying to decrease the number of tours to six). However, such an exchange has a negative impact on the other districts.

Comparing the new districts to the original situation in Table 2, it is clear that important savings can be obtained by using an appropriate districting and routing procedure. In addition,

Table 3
Present districts and improved routing

District (km)	Load	# of tours	Dead mileage (km)	Avg. tour length (km)
Brecht	199.5	6	157.5	33.3
Vosselaar	195.5	6	155.5	32.6
Grobbendonk	178	6	144.5	29.7
Geel	233.5	7	129	33.4
Total	806.5	25	586.5	

Table 4
New districts and routing

District	Load (km)	LB (# tours)	# of tours	Dead mileage (km)	Avg. tour length (km)
Brecht	262.5	$\lceil 7.50 \rceil = 8$	8	165.5	32.8
Vosselaar	172.5	$\lceil 4.93 \rceil = 5$	5	110	34.5
Grobbendonk	154.5	$\lceil 4.41 \rceil = 5$	5	117	30.9
Geel	217	$\lceil 6.20 \rceil = 7$	7	113	31.0
Total	806.5	$\lceil 23.04 \rceil = 24$	25	505.5	

this improvement is achieved by redesigning the inter-district borders of four districts only. Hence larger savings can be expected when larger problem instances are solved and more borders are involved.

6. Conclusions

In this paper, we discussed the problem of districting associated with salt spreading and road maintenance operations. Good districting for these operations is achieved by ensuring compact and balanced districts in which the depot is centrally located, while taking into account secondary considerations. Furthermore, districts should have the potential to facilitate efficient operations. Careful enumeration of all planned activities to be performed is thus an important requirement. Due to the similarity of these services, the different objectives of the activities are often not conflicting. Hence, many instances result in 'easy' multi-objective problems and often, as in the case for salt spreading in the province of Antwerp, only one or two objectives have to be considered explicitly.

The procedure described in this paper uses elemental cycles as building blocks for constructing districts. The approach is simple (cycles instead of nodes and edges) and flexible (due to the existence of many relatively small cycles). It has the capability to deal constructively with multiple objectives, namely: to minimize total distance and dead mileage, to minimize the number of trucks, and finally to create balanced districts. Many delicate trade-offs among total workload, capacity, distance and geographical inputs can be addressed by the decision makers during the different phases of the algorithm, especially during the last phase.

Finally, the procedure was tested for the salt spreading operations in the province of Antwerp, indicating considerable savings compared to the actual performance and proving the importance of a good districting procedure in the planning and organization of the services. Currently, attention is given to the further testing and fine-tuning of the algorithm.

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