Clustering for routing in densely populated areas

Luc CHAPLEAU, Jacques-A. FERLAND
and Jean-Marc ROUSSEAU
University of Montreal, Montreal, Quebec, Canada

Abstract. This paper introduces a new approach for generating school bus routes in a dense urban area. First, a districting algorithm is used to determine clusters including appropriate numbers of students. Then, for each cluster, a route and the stops along this route are determined. Numerical results are reported and compared with those obtained previously. Although the algorithm has been developped and tested in a specific context, it could easily be extended to more general vehicle routing problems.

Keywords: Route selection, algorithm, heuristic

1. Introduction

In this paper the school bus routing problem in an urban area is reexamined. More traditional approaches to this problem were tested and have been reported previously in [6], but recently a new approach has been developed and much better results have been obtained.

Let $G = (N, A)$ be the road network for a city including several schools. The edges of the network are the street segments, and the nodes are their intersections and the schools locations. For each school in this network, the set of students to be transported and their locations on the edges of the network are specified. The problem is to determine a set of routes for each school (or group of schools) independently.

In an urban area, it is assumed that the students walk to the stops where they are picked up and dropped off.

This paper was published thank to a contribution from the F.C.A.C. fund for research support.

Received October 1983; Revised April 1984

North-Holland
European Journal of Operational Research 20 (1985) 48–57

The problem includes four major constraints:

(i) upper bound on the total distance that a student has to walk;

(ii) upper bound on the number of students on each route (route capacity);

(iii) upper bound on the number of stops on each route;

(iv) upper bound on the length of each route.

Note that, in a dense urban area, the second constraint becomes binding before the last two constraints.

In [6], the following approach was used to solve the problem:

Step 1. Each student is assigned to a node near his location.

Step 2. For each school (or group of schools), the stops are determined.

Step 3. The students are assigned to these stops to satisfy constraint (i).

Step 4. A set of routes is determined for each school or group of schools independently.

With this approach, the stops and the number of students assigned to them were selected independently of the routes to be generated and their capacity. Furthermore, it was not possible to split up a stop into many stops and redistribute the students among several routes to equalize the load.

To take advantage of the interaction between the selection of stops and routing in this new approach, Steps 2, 3 and 4 of the preceding approach are replaced by Steps 2′ and 3′.

Step 2′. For each school (or group of schools), determine a minimum number of 'clusters' (each cluster including a number of students approximately equal to the upper bound specified in (ii)).

Step 3′. Each cluster specifies a 1-route problem for which the stops must be selected and a route generated.

Note also that the procedure in Step 2′, tends to reduce the number of routes by maximizing the number of students on each of them.

This approach uses the 'group first, route second' routing strategy as in the Sweep method proposed by Gillet and Miller [9] or as in the mathematical programming based technique of Fisher and Jaikumar [7]. In the Sweep method, however, the clustering technique is rigid and cannot easily take into account the distance in the underlying road network. Also, route shape cannot be controlled. In the Fisher and Jaikumar technique, a seed (stop) must first be assigned manually or otherwise to each cluster. Travel costs are then calculated approximately for the potential inclusion of each stop in each cluster. A generalized assignment algorithm is then used to assign each stop to a cluster respecting its capacity constraint. While these authors have obtained very good results for a distribution problem, we believe that the clustering strategy described in this paper is much more flexible in that the shape of the route generated can be controlled; it is thus easier to adapt to any context including passenger transportation as will be seen later.

The procedure for assigning students to street intersections (mini-stops) is summarized in Section 2. In Section 3, our initial approach is summarized. Section 4 is devoted to the new clustering approach. Finally, concluding remarks are summarized in Section 5.

2. Location

In our network, students are located on edges (street segments) and must first be assigned to nodes (referred to as mini-stops using the Bodin and Berman notation [1]). This simplifies the computation of walking distance and the analysis of the distribution of students in the network. Note that the walking distance of a student is taken to be equal to the distance between the mini-stop and the stop to which he is assigned.

In an earlier version, reported in [6], the edges of the network were scanned successively, and the students on each of them were assigned to the incident node (or the nearest mini-stop) with the largest number of students already assigned to it. The purpose of this was to reduce the number of mini-stops with students assigned to them. A disadvantage of this procedure was that it generated different assignments according to the order in which the edges were scanned.

The procedure was later modified to take into account the potential locations of the stops (specified by the user) from which the actual stops are selected. Here, each student is assigned to the incident node of his edge which is the closer to a potential stop. Ties are broken randomly, but all students on an edge are assigned to the same incident node of this edge. The average walking distance was reduced with this approach.

3. Initial approach

This section summarizes the approach presented in [6]. In Step 2, the stops for each school (or group of schools) are specified by the user or determined according to the following procedure: Select the potential stop with the largest number of students within walking distance from it; assume that these students are assigned to it, and repeat the procedure with the remaining students and the remaining potential stops. Once the stops are selected, Step 2 is completed by assigning each student to the nearest stop. If upper bounds are specified on the number of students at some stops, and if these bounds are exceeded when the preceding procedure is applied, then a straightforward transportation problem is formulated and solved. The underlying objective is to reduce the number of stops required and to minimize the total walking distance.

Several methods for determining the routes are given and analyzed in [3,6]. Two of these were retained for potential use: Clarke and Wright [3] and Insertion [11]. These methods are combined with a 2-OPT exchange procedure [4,10] to improve the routes.

This approach does not guarantee a solution with the minimum number of routes because the underlying goal of these routing methods is to minimize the total mileage covered by the vehicles. A more important objective would be to maximize the number of students on each route in order to minimize the number of routes. The numerical results reported in [6,8] demonstrate the necessity of repeating these methods with a wide range of parameters to increase the chances of obtaining a fair solution.

The main drawback of the approach is that it is difficult to group together several stops with large numbers of students assigned to them without exceeding the capacity bound of the routes.

4. The proposed clustering approach

In this new approach, a minimum number of clusters is first identified and then, for each cluster, stops are selected and the route is generated.

Given the subgraph of G specified by a school (or group of schools) and the mini-stops of its students, a minimum number of clusters of mini-stops are determined in Step 2′ of the procedure. This minimum is calculated to be

$$\mathrm{Ncl} = \left]\frac{\text{Number of students}}{\text{route capacity}}\right[$$

where $]a[$ is the smallest integer greater than or equal to a.

4.1 The definition of distance

First, we introduce the definition of distance used to determine the clusters. Referring to Figure 1, the distance between two points x and y is taken to be

$$\mathrm{dis}(x,y) = \begin{cases} \overline{xy}^2\left\{1+\lambda\left[1-\dfrac{\left(\overline{xy}^2-\overline{yz}^2+\overline{xz}^2\right)}{2\cdot\overline{xy}\cdot\overline{xz}}\right]\right\} \\ \quad \text{if } \overline{xy}\leqslant\overline{xz} \\ \overline{xy}^2\left(1+2\lambda\dfrac{\overline{yz}}{\overline{xy}}\right) \\ \quad \text{if } \overline{xy}>\overline{xz}, \end{cases}$$

where $\overline{ab}$ is the shortest distance between a and b in the network, x is assumed to be further from z than is y (i.e. $\overline{xz} > \overline{yz}$), and z is the school location.

To understand the definition suppose first that the straight line distance between the points is used instead of the shortest distance on the network to define $\overline{ab}$. Then $\mathrm{dis}(x,y)$ could be rewritten as:

$$\mathrm{dis}(x,y) = \overline{xy}^2(1+\lambda\sin\theta) \quad \text{if } \overline{xy} \leqslant \overline{xz}$$

where θ is the angle defined in Figure 1. Thus, if x, y and z are on the same line, then $\sin\theta = 0$ and $\mathrm{dis}(x,y)$ is simply the square of the straight line distance. Also, since $\sin\theta$ increases with θ, it follows that the penalty factor $(1+\lambda\sin\theta)$ increases with θ. (Of course $\theta < 90°$ and so $(1+\lambda\sin\theta) \leqslant (1+\lambda)$.)

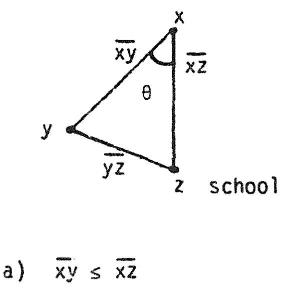
The second part of the definition of $\mathrm{dis}(x,y)$ considers the case where the school z is located between nodes x and y. In this case the angle θ becomes irrelevant, and the penalty increases with the distance of the closest node y from z.

A penalty factor is introduced into the computation of the distance between two points because the criterion for including two points in the same cluster is the distance between them. Since the points in a cluster will belong to the same route, the penalty factor estimates the potential detour induced by the inclusion of a new node.

Different cluster shapes are generated for different values of the parameter (for instance circular clusters are generated with $\lambda = 0$). Moreover this definition of distance could be generalized by taking λ inversely proportional to the distance of x to the school, for instance. Then the penalty for detour would increase near the school. Note that such a modification would be similar in spirit to the Sweep method [9], In our experiment, good results (using elliptic clusters) were obtained with $\lambda = 1/2$.

4.2 The districting algorithm

At this stage, the only relevant constraint is the route capacity constraint (ii) in Section 1. A districting algorithm, based on the distances as defined in Section 4.1, is used to generate Ncl elliptic clusters with their longest axis in the direction of the school location. These Ncl clusters are built sequentially allowing the capacity constraint to be violated to some extent. Then, exchange algo-



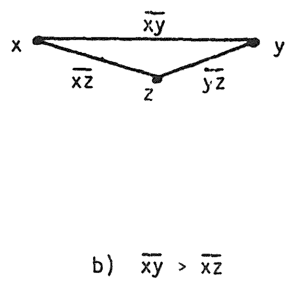


Figure 1. The definition of distance.

rithms are used to modify the clusters first so that capacity constraints are satisfied and then to improve a measure of compacity.

First, a *compacity measure* for each cluster is defined as follows:

$$\mathrm{Comp}_r = \sum_{m \in M_R} \mathrm{dis}(m, m_r^*), \quad 1 \leqslant r \leqslant \mathrm{Ncl},$$

where M_R is the set of mini-stops included in the r-th cluster and m_r^* is the centroid of the cluster satisfying the property

$$\sum_{m \in M_R} \mathrm{dis}(m, m_r^*) = \min_{n \in M_r} \left\{ \sum_{m \in M_R} \mathrm{dis}(m, n) \right\}.$$

The total compacity value is equal to $\mathrm{Comp} = \sum_{r=1}^{\mathrm{Ncl}} \mathrm{Comp}_r$.

Also, a *capacity measure* is specified as the sum over all clusters of the square of the number of students in excess of estimated vehicle capacity (which may include a small tolerance over its real capacity to take into account a percentage of no-show).

The procedure is to select the furthest mini-stop from the destination as the reference node for a cluster. Include in the cluster the reference node and the closest (using dis as the distance) mini-stops to the reference node until the route capacity is reached or would be exceeded if the next node were to be introduced. Repeat the procedure with the remaining mini-stops until Ncl clusters have been specified. If some mini-stops do not belong to any cluster at the end of the procedure, then select the cluster which includes the smallest number of students and introduce the closest (using dis) such mini-stop into it. Repeat this until all mini-stops are within some cluster. At the end of this initial step, Ncl clusters are defined but route capacity may be exceeded for some of them.

The initial solution is then modified through an exchange procedure to obtain a solution satisfying the route capacity constraint. If some cluster includes a number of students exceeding the route capacity, select the cluster with the largest number of students. Call this cluster the reference cluster.

Two types of exchange are then used to try to improve the solution. First a one-level exchange where a mini-stop is transferred from the reference cluster to one of its neighbours is attempted. Two-level exchanges are then tried; these consist first of transferring a mini-stop from the refence cluster to one of its neighbours, say cluster C, and then transferring one or several mini-stops from C to the neighbours of C. The exchange is performed if the capacity measure is improved. If the capacity measure is not improved but not worsened, the exchange could be performed to improve the compacity measure.

If the capacity constraint is respected in all cases, the exchange process may go on to improve the compacity without allowing the capacity to be exceeded in any case.

Note that when an exchange takes place, new centroids and a new compacity values are determined for the modified clusters and a new reference cluster is selected. When no more exchanges are possible, the process stops. If the capacity is still exceeded in some cluster, the decision maker may either accept the solution or restart the process with one more cluster. Other exchange mechanisms could also be devised.

For each of the cluster, a route including the bus stops along this route must be generated.

4.3. Route generator

Initially, each cluster is treated separately. The first step is to determine a set of stops for a cluster. The procedure used is similar to Step 2 of the previous procedure (Section 2): Select the potential stop still available with the largest number of students in the cluster within walking distance from it; assume that these students are assigned to it, and repeat the procedure with the remaining students of the cluster and the remaining potential stops. Note that once a stop has been selected for a cluster it is not available for any other cluster. Once all the stops have been selected for a cluster, the students are assigned to their nearest stop.

Then a route through the stops of the cluster is determined using a 2-OPT procedure [1,4]. Any heuristic or exact procedure for the traveling salesman could also be used.

An exchange procedure between routes then takes place to reduce the total distance. One level exchanges (transferring a stop from one cluster to another) and two level exchanges (exchanging two stops in two different clusters) are implemented until no improvement in total distance is possible.

The final operation is aimed at reducing the length of each route using the available potential

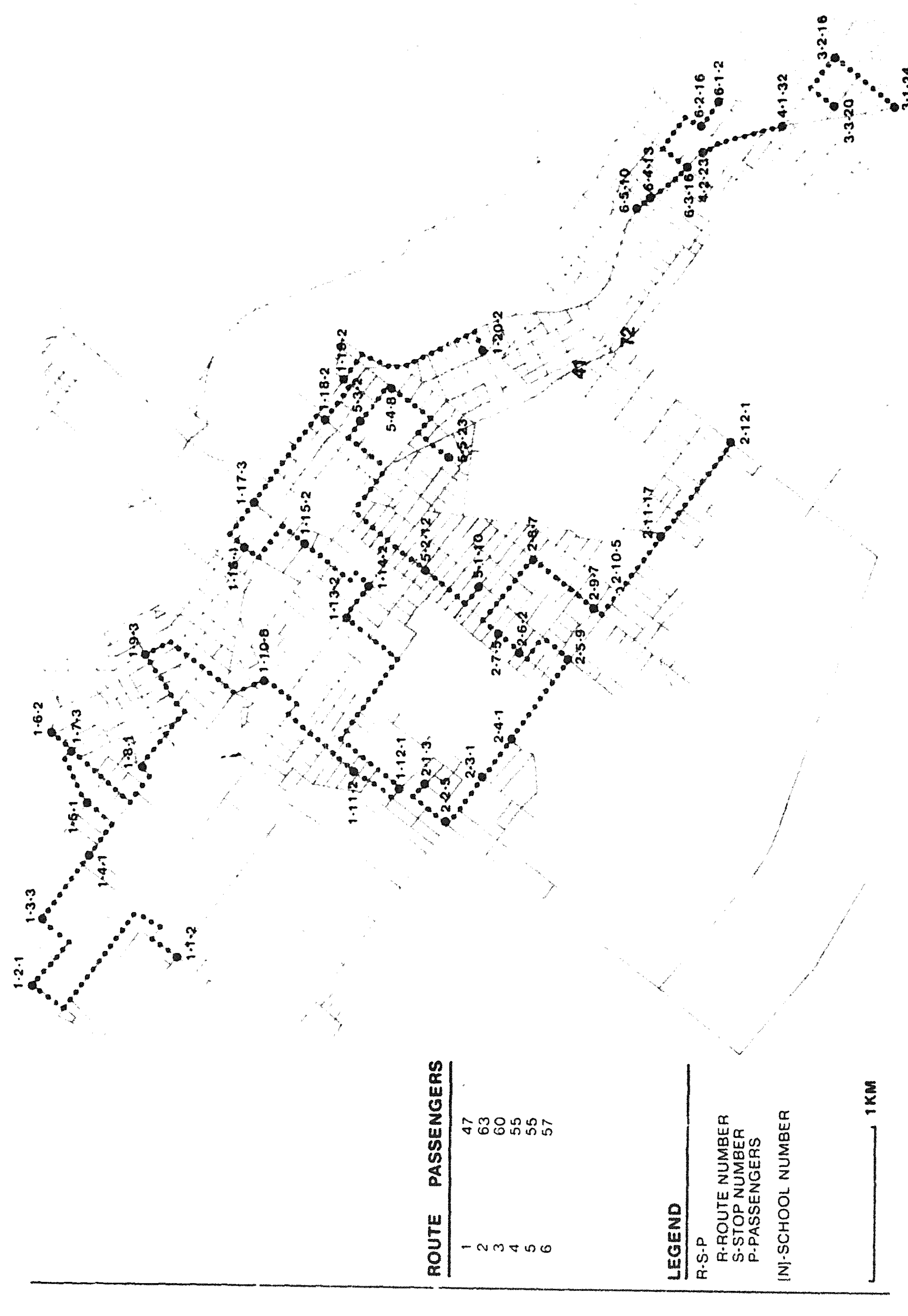


Figure 2. Clustering (6 routes).

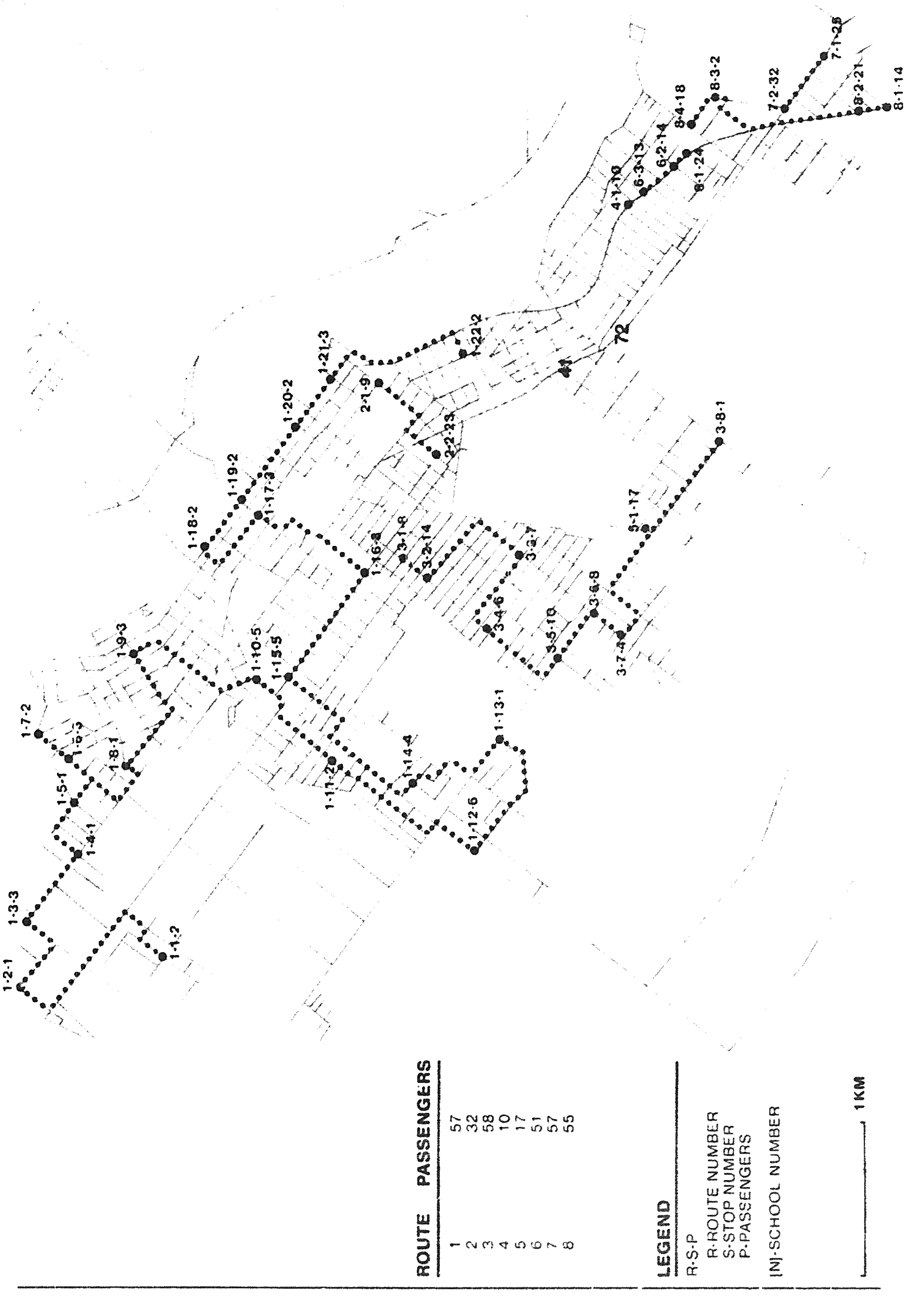


Figure 3. Insertion (8 routes)

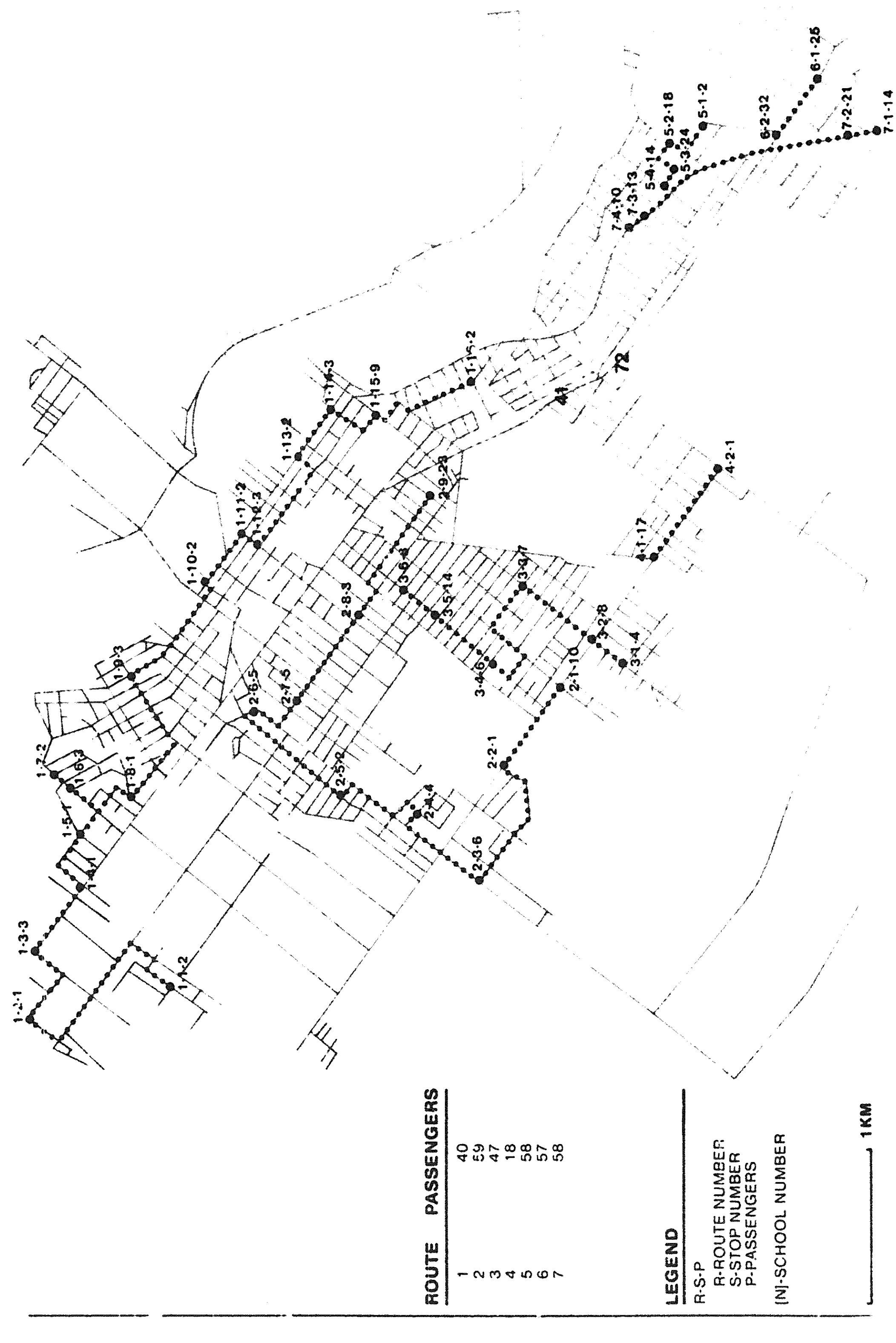


Figure 4. Clarke and Wright (7 routes).

Table 1
Route specifications related to algorithms in dense urban area

			Problem A	Problem B	Problem C	Problem D	4 Problems together
Clarke and Wright							
	Number of students		337	482	355	905	2079
	Number of stops		43	57	47	63	210
	Number of routes		7	9	7	18	41
	Average students per route		48.1	53.6	50.7	50.3	50.7
	Total length	(metres)	35884	50584	37115	85988	209571
	Average length per route	(metres)	5126	5620	5302	4777	5111
	Average length per student	(metres)	106	104	104	95	100
	Average walking per student	(metres)	183	179	175	176	177
Insertion							
	Number of students		337	482	355	905	2079
	Number of stops		43	57	47	63	210
	Number of routes		8	9	7	17	41
	Average students per route		42.1	53.6	50.7	53.2	50.7
	Total length	(metres)	40386	54288	39030	85149	218853
	Average length per route	(metres)	5048	6032	5575	5008	5337
	Average length per student	(metres)	119	112	109	94	105
	Average walking per student	(metres)	183	179	175	176	177
Clustering							
	Number of students		337	482	355	905	2079
	Number of stops		47	62	56	76	241
	Number of routes		6	8	6	15	35
	Average students per route		56.2	60.3	59.2	60.3	59.4
	Total length	(metres)	36358	48112	39972	87362	211804
	Average length per route	(metres)	6059	6014	6662	5824	6051
	Average length per student	(metres)	107	99	112	96	101
	Average walking per student	(metres)	179	162	175	154	163

Table 2
Condensed route specifications

Algorithm	C.P.U. seconds	Number of students	Number of stops	Number of routes	Average students/	Total length (metres)	Average length/ route	Average length/ student	Walking distance (metres)
Clarke and Wright	343	2079	210	41	50.7	209571	5111	100	177
Insertion	402	2079	210	41	50.7	218853	5337	105	177
Clustering	574	2079	241	35	59.4	211804	6051	101	163

C.P.U. time measured on Control Data model Cyber-173

stops. Each stop is selected sequentially and a potential route not using it is determined. All the students assigned to this stop are reassigned (if the walking distance constraint is satisfied), to other stops of the same route. Remaining students are then assigned to the potential stop nearest to the potential route. If the new route has a reduced distance compared with the previous route, the new route is selected and students are reassigned to their nearest stop on this route.

4.4 Discussion

The main improvement of this method over the initial approach is that it tends to generate the minimum possible number of routes. This was not guaranteed by the previous approaches since Clarke and Wright and Insertion methods tend to minimize the total distance rather than generating the minimum number of routes. Another quality of the solution is the disjointness of the routes determined. This is an important practical consideration for school transportation because when two routes are too 'close', students may switch from one route to another, creating additional problems for the authorities. In fact we have retained the principle advantage of the Sweep approach (i.e. that it attempts to utilize transportation capacity to the maximum) but using a much more general clustering approach.

Figures 2, 3 and 4 indicate an improvement in the overall design of the routes generated with this new approach. Furthermore, referring to Tables 1 and 2, the results indicate a decrease in the number of routes with the new approach and this gain is not made at the expense of the students since the average walking distance and the average route length per student (i.e. total distance travelled divided by the number of students) do not increase.

Note that the constraints on the length and the number of stops on a route (mentioned in Section 1) are not included in this analysis since in a dense urban area, they are not relevant. Furthermore, if one of these constraints were not satisfied, then an exchange procedure could be used to generate a solution satisfying these.

5. Conclusion

Clearly, it appears that the routes generated using the new approach are more satisfactory in many respects. We have to admit, however, that the procedure of exchanging stops between routes to improve the total distance, and the design of the routes was not implemented for the results in Figures 3 and 4 (Clarke and Wright and Insertion). This procedure cannot in general reduce the number of routes and it is mainly in this respect that the proposed approach is superior.

The generalization of this clustering algorithm to more general routing problems is straightforward, using demand points instead of mini-stops and demand instead of number of students. The notion of stops is not essential to the algorithm; in this case, it leads to a reduction in route length. This approach is appropriate when it is desirable to use up vehicle capacity as much as possible thus reducing the number of vehicles required to perform a given delivery job. In this respect, it is much superior to the Clark and Wright or Insertion techniques that aim to reduce the distance rather than the number of vehicles. Also, it is much more flexible than the Sweep method with its rather rigid method of clustering clients. Is is also more flexible than the Fisher and Jaikumar algorithm because the stops of the routes can be controlled. The algorithm is also easier to implement and use, and does not require high computing capability. When the cost of operating a vehicle overshadows mileage cost, a feasible solution which operates at the minimum number of vehicles is certainly close to optimal.

References

[1] Bodin, L., and Berman, L., "Routing and scheduling of school buses by computer", *Transportation Science B* 13 (1979) 113–129.

[2] Bourjolly, J.-M., Laporte, G., and Rousseau, J-M., "Découpage électoral informatisé: Application à l'île de Montréal", *INFOR* 19 (1981) 113–124.

[3] Clarke, G., and Wright, J.W., "Scheduling of vehicles from a central depot to a number of derivery points", *Operations Research* 12 (1964) 568–581.

[4] Croes, G.A., "A method for solving travelling-salesman problem". *Operations Research* 5 (1958) 791–812.

[5] Dulac, G., "Affectation des étudiants aux arrêts d'un système de transport des écoliers", Rapport de Stage de maîtrise, Département d'Informatique et de Recherche Opérationnelle, Université de Montréal, 1978.

[6] Dulac, G., Ferland, J.A., and Forgues, P.-A., "School bus routes genrator in urban surroundings". *Computer and Opserations Research* 7 (1980) 199–213.

[7] Fisher, N. and Jaikumar, R., "A generalized assignment heuristic for vehicle routing", *Networks* 11 (1981) 109–124.

[8] Forgues, P.-A., "Génération des parcours pour le transport scolaire", Document de travail Nr. 101, Département d'Informatique et de Recherche Opérationnelle, Université de Montréal, 1978.

[9] Gillet, B.E. and Miller, L.R., "A heuristic algorithm for the vehicle dispatch problem", *Operations Research* 22 (1974) 340–349.

[10] Lin, S., "Computer solutions of the travelling-salesman problem", *Bell System Technical Journal* 44 (1965) 2245–2269.

[11] Rosenkrantz, D., Stearns, R. and Lewis, P., "Approximate algorithms for the travelling-salesman problem", *Proceedings of the 15th Annual IEEE Symposium on Switching and Automatic Theory*, 1974, 33–42.