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SALES TERRITORY DESIGN: AN INTEGRATED APPROACH*†

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A sales territory design procedure should solve the dual problems of boundary definition and call frequency. Furthermore, it should be possible to base the design on several workload and potential criteria. A procedure is presented which meets this specification. It employs recent developments in set-partitioning and includes a computer code which makes it possible to handle design problems of realistic proportions. The outcome of the design procedure, termed the market matrix, is a schedule which specifies which salesman calls on each customer, and the sales call frequencies which maximize the total sales from all territories. A case example illustrates how the procedure is applied.

1. Sales Territory Design

During the past decade the average cost of making an industrial sales call has nearly doubled to fifty-eight dollars. [1] Already a major part of the cost of doing business [18], personal selling expenditures continue to grow at an alarming rate. Such growth underlines the need to employ rigorous methods for utilizing this costly corporate resource. One possible solution is to design sales territories using mathematical procedures capable of identifying which customers to include in a territory, as well as the best call frequencies for individual customers. A new procedure for accomplishing this is described here.

Procedures for designing sales territories have appeared in the marketing literature in many forms. This is because managers can base their evaluations of sales territory designs on any one of several design criteria. One popular view distinguishes procedures which are based on *workload* from those which are based on *potential* [11]. The workload criterion produces territories which require a specified amount of selling time. The potential criterion produces territories which possess a specified amount of potential market demand. When sales calls are made two things happen: selling time is consumed *and* demand potential is tapped. This suggests that territory design should be based on both the workload criterion and the potential criterion, whereas most of the existing procedures are based on one or the other.

Another view of sales territory design procedures distinguishes between the *boundary* problem of determining which customers should be allocated to each salesman, and the *frequency* problem of determining how often each salesman should call on each customer. The solution to the boundary problem appears as management's territorial descriptions, whereas the solution to the frequency problem appears as the salesmen's call schedules. The two problems are commonly decomposed by the simple expedient

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[1]See [4] for an extended discussion. More recent speculations suggest that sixty-six dollars may be a more realistic average cost per call.

of having sales management describe territory boundaries, and then permitting salesmen to determine their call frequencies by trial and error. Perhaps because of this decomposition, existing design procedures are limited to solving one or the other of the boundary and frequency problems. Since the two problems are interrelated, however, it seems preferable for one territory design procedure to be able to solve both problems.

The overall objective of this study is to propose a territory design procedure which is sufficiently flexible to provide all of the above features. Specifically, the procedure can employ both the workload criterion and the potential criterion, and it can solve both the boundary problem and the frequency problem.

The scope of application can be global, as in the case of designing sales territories for an entire sales force, or it can be local, as in the case of the redesign of a small group of territories. The former has the advantage of determining a globally optimal solution, but, in practice, it may cause morale problems among the salesmen if widespread changes in sales territories are prescribed. The latter must be satisfied with local optimization, but local solutions are easier to implement, as well as making less severe computational demands.

Casual empiricism suggests that global changes are rarely undertaken; rather, firms typically proceed piecemeal by assessing territories individually or in small groups.[2] Furthermore, a recent survey of major U.S. corporations reveals that the average company has 87 salesmen each responsible for 159 customers [4]. Therefore, a typical local change in territory design might be expected to involve a few hundred customers. An example to be discussed in a later section consists of three territories containing a total of 500 customers.[3]

The design procedure implicitly assumes that the firm is employing the optimal number of salesmen. It is possible to modify the procedure to explore the sales force size but the present study will not include this. At the local level, the consequence of this omission is that the total amount of activity of an individual salesman (travelling, waiting, interviewing) is taken as given.

Thus, the specific problem which this study addresses is to allocate a given amount of the time of several salesmen to several hundred prospective customers so as to maximize sales.[4] The solution to the problem will specify which customers should be called on by which salesmen, prescribe the relative frequency of calling, and recognize a wide variety of relevant constraints such as preserving geographic compactness. Before stating the problem in a more rigorous fashion, it is necessary to review related work and indicate its relevance to the present study.

2. Antecedents

Early territory design procedures were for the entire sales force. Two such aggregate procedures were by Semlow [19] using sales potential, and by Talley [21] using

[2] The following remarks were attributed to the Vice-President of Marketing of Skil Corporation in a meeting of sales executives from large U.S. corporations [3]:

"Very few of us will ever be called on to design a national sales territory plan all at once. In most cases we are either trying to repair a poor sales territory, or else we are building the distribution of a new product on a market-by-market basis."

[3] A comment on the increase in problem complexity which results from increasing the number of territories and/or customers is provided in footnote 13.

[4] The maximization of sales is only one of several objectives which might be adopted. For example, one might multiply customer sales by a customer profitability factor to convert to a profit-maximizing objective.

workload. Subsequently, Lodish [13], Hess and Samuels [10], and Turner [23] employed procedures for designing sales territories for *individual* salesmen or small groups of salesmen. Changes over time were incorporated by Brown et al. [2], Schuchman [20] and Thompson et al. [22]. Designs based on stochastic response functions were incorporated by Lodish et al. [15]. Finally, the added complexity of a multiple-product line was recognized in procedures proposed by Montgomery et al. [16] and by Davis et al. [6].

Of the foregoing studies, those by Hess and Samuels [10] and Lodish [13] seem to be the most relevant to the procedure to be described here. Like the former study, the present procedure prescribes territory boundaries; that is, it determines which customers should be included in which territories. This necessitates simultaneous consideration of more than one salesman and more than those customers which *a priori* constitute a given sales territory. In contrast, the Lodish procedure takes the existing territorial boundaries as given, and concentrates on assigning each salesman's calling efforts within his respective territory.

However, the Hess procedure "... does not provide optimum sales territories" [10, p. P–53]. Its motivation is to equalize territories in terms of salesman activity, which is generally not the same as designing them so as to maximize sales. In contrast, the present procedure, like that of Lodish, does provide an optimizing solution. [5] Whereas Lodish specifies the optimal level of calling on each of a given set of customers, the present procedure does this as well as specifying which customers should be in each territory. [6]

Thus, the present procedure seeks to combine the best features of the Hess and Lodish approaches. As will be shown in the following sections, this is made possible by recent developments in integer-programming technology, specifically those associated with set-partitioning. In closing this review, it must be acknowledged that neither the dynamic aspects of territory design nor the effects of multiple interdependent products are accommodated. These problems must remain subjects for future research.

3. Market Matrix

The allocation procedure can best be introduced by considering its final product. This is a tabulation of the m prospects which have been designated to receive sales calls and the p salesmen who have been assigned to make the calls. A convenient way of viewing this tabulation is as an m by p matrix which will be termed the market matrix $E = (e_{ij})$, $i = 1, \ldots, m$, $j = 1, \ldots, p$, where e_{ij} is the effort (e.g., number of sales calls) which salesman j allocates to customer i. In the usual case of full-line selling, a maximum of one nonzero entry will appear in any row. The entries in each column indicate how one salesman's efforts should be allocated to his customers; that is, each column defines a salesman's territory. The typical allocation problem which was described in a previous section can be restated as that of determining the e_{ij} entries for $m = 500$ and $p = 3$.

Associate with E another matrix called the assignment matrix $A = (a_{ij})$ where

[5] Neither procedure finds an unqualified optimum. The Lodish procedure optimizes a modified problem, and the present procedure determines the optimum only of those territories which are proposed.

[6] In a recent paper, Lodish [14] implied that his procedure also provides guidelines for reallocating customers between territories, but insufficient information is available to evaluate this feature.

$$a_{ij} = 1 \quad \text{if } e_{ij} > 0,$$
$$\quad = 0 \quad \text{otherwise.}$$

Or, equivalently

$$a_{ij} = 1 \quad \text{if salesman } j \text{ calls on customer } i,$$
$$\quad = 0 \quad \text{otherwise.}$$

The particular column of A which defines the territory of salesman j is the intersection of A with another matrix which contains many more candidate territorial assignments for salesman j. Designate this the assignment matrix for salesman j: $A^j = (a_1^j, \ldots, a_{nj}^j)$, where a_k^j is a column vector representing the kth candidate territory for salesman j. Define a_{ik}^j as the (ik)th element of A^j such that

$$a_{ik}^j = 1 \quad \text{if the } k\text{th candidate territory for salesman } j \text{ includes customer } i,$$
$$\quad = 0 \quad \text{otherwise,}$$

for $i = 1, \ldots, m$, $j = 1, \ldots, p$, $k = 1, \ldots, n_j$.

Column j of A can be characterized by R^j, the expected total sales response obtained from salesman j expending the effort designated by matrix E on his customers. A procedure for estimating this will be described in a later section. Similarly, the expected total sales response R_k^j can be estimated for each candidate territory in the assignment matrix A^j of salesman j. These R_k^j measures provide the means for selecting which of the candidate territories appears in the market matrix for salesman j.

The above concepts permit the procedure for designing the market matrix to be described in four steps. These are summarized in Figure 1. The individual steps will be

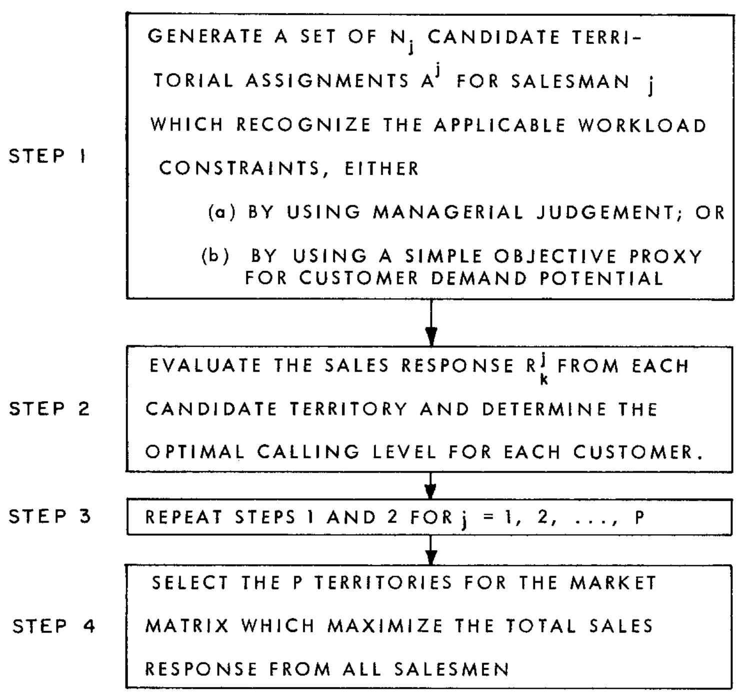


FIGURE 1. Procedure for Designing the Market Matrix

described in detail in each of the following sections. Before concluding this overview, however, it may be helpful to relate the steps to existing procedures which were described in the previous section.

Step 1 provides candidate solutions to the boundary problem for the territory of salesman j. It corresponds conceptually to the Hess and Samuels [10] procedure with two important exceptions. First, it identifies n_j of the best solutions in terms of a crude activity measure. These satisfy the applicable workload constraints as well as many other logical constraints. In contrast, Hess and Samuels select the single best solution for salesman j based on a single activity measure. Second, an innovative method based on obtaining a number of the best solutions to a mathematical programming problem can be employed, as well as a transportation-programming approach similar to that of Hess and Samuels.

Steps 2 and 3 determine the optimal sales response evaluation for each of the candidate territories. They correspond to the Lodish [13] procedure with one exception. These steps evaluate customers which may be beyond existing territorial boundaries, and thereby make provision for structural changes between sales territories.

Step 4 has no analogue among existing sales territory design procedures. It considers candidate territories having boundaries which are generated by workload criteria, and selects that set of territories with the calling time assigned so as to maximize total expected sales response. Thus, Step 4 performs an important integration function. It integrates the above two kinds of criteria, and it integrates the above two kinds of problems encountered in sales territory design. This is made possible by recent set-partitioning developments which greatly increase the capacity of integer-programming procedures.

4. Candidate Territories

The number of possible candidate territories which can be generated from the total customers in two or three sales territories is typically very large.[7] The object of Step 1 is to reduce this to a more mangeable number. In general, n_j will be a few dozen, or at most a few hundred, for each salesman.

Two methods are proposed to accomplish this reduction. The first employs the judgments of experienced persons such as sales supervisors or other managers. Presumably, those managers who have "survived" the experience of being a salesman can distinguish good territories from poor territories. By proposing candidate customer assignments, these managers can reduce the number of assignments to be considered. This method is particularly applicable when existing territories are undergoing moderate revisions.

Some managers can generalize their experiences beyond the proposing of specific changes, to the stating of design heuristics. By employing certain heuristics as territory design constraints, they can be incorporated into the optimization of some workload or potential criterion. For example, the repeated observation that salesmen dislike inequities in travel time might suggest the constraint that in all candidate territories the aggregate distance between customers must not exceed some specified upper limit.

[7]The number of ways of assigning m customers to p territories is [7]:

$$\binom{m+p-1}{p-1} = 125{,}751$$

for the typical example described in the previous section, where $m = 500$ and $p = 3$.

By incorporating explicit heuristics into a constraint set, this method is suitable for generating candidate territories via mathematical programming. For example, one approach consists of mazimizing demand potential,[8] subject to a set of constraints such as the above travel time constraint. A number of candidate territories can be obtained by employing a procedure developed by Piper and Zoltners [17] for finding the best n_j solutions to an integer program. The relevant integer program may be stated formally as:

Maximize

(1) $$\sum_{i=1}^{m} d_i a_i$$

subject to

(2) $$Ga \leqq b, \qquad a_i = 0 \text{ or } 1 \text{ for } i = 1, \ldots, m,$$

where

d_i = demand potential of customer i;

$a = (a_1, \ldots a_i, \ldots a_m)$ is a vector of variables such that

$$a_i = 1 \quad \text{if salesman } j \text{ calls on customer } i, \qquad = 0 \quad \text{otherwise;}$$

G is a linear constraint matrix;

b is a column vector of constants.

The contents of matrix G reflect various explicit constraints which management may wish to impose on the makeup of the n_j candidate territories. The procedure is sufficiently flexible to incorporate a wide variety of constraints. In the following paragraphs three illustrative workload constraints are suggested.

(a) *Call Frequency Constraints*

(3) $$\sum_i g_i a_i \geqq b_l,$$

(4) $$\sum_i g_i a_i \leqq b_u,$$

where g_i is call frequency norm for prospect i; b_l and b_u are lower and upper limits on the total number of calls each salesman can make.

The above norms and limits may only be implicit in the praise or criticism received from sales supervisors. In some situations, however, attempts are made to specify them explicitly.[9]

(b) *Stratification Constraint*

(5) $$\sum_{i \in U} a_i \geqq \pi_u m_u$$

where m_u is the number of prospects in stratum U; and π_u is the specified proportion of prospects which must come from stratum U. If certain salesmen can only call on customers within a particular stratum (e.g. wholesalers versus retailers), the following constraints must be added:

[8]Since calls have not been allocated to customers in Step 1, the objective cannot be to maximize sales response to calling. While demand potential is only an approximation for sales response, it seems appropriate for the gross screening which is required at this point.

[9]One large manufacturer estimates call frequency norms for individual prospects in various demand potential strata by employing regressions based on the call reports of their more productive salesmen.

$$\sum_{i \in U} a_i \leqq z_u m_u, \tag{6}$$

$$\sum_u z_u = 1, \tag{7}$$

where $z_u = 1$ if salesman j calls in stratum U; 0 otherwise.

(c) *Compactness Constraint*

$$\sum_{i=1}^{m} c_i a_i \leqq C \tag{8}$$

where c_i is the distance from a fixed point to customer i; C is an upper limit on travel distance.

If demand potential is not distributed evenly, it may be preferable to allocate to customer groups rather than individual customers. Heuristics can be incorporated into the routine for generating candidate territories, which treat remote groups as single customers for purposes of computing compactness.

The above three examples illustrate the descriptive richness which is available in linear integer constraints. For more information on additional types of constraints see [24].

5. Evaluation of Territories

Having generated n_j candidate territories for salesman j, it is necessary in Step 2 of the allocation procedure to develop a summary measure of the relative merit of each territory. One might argue that this was done in Step 1, since the n_j territories were chosen because of their having high aggregate demand potential. However, the demand potential measures are inadequate because they do not take into account the effect of salesman j's calling effort. In contrast, the R_k^j sales response measures which were referred to in an earlier section do incorporate the expected customer response to the salesmen calling at specified frequencies. The object of this section is to describe how these sales response measures can be developed.

It has been shown that the parameters of sales response functions covering small groups of customers [23] and individual customers [13] can be estimated using actual data on calls and sales and/or salesmen's judgments. However, a procedure for assigning salesmen from one territory to another is of questionable value if it depends on a salesman having already called on the customers before the desirability of his calling on them can be evaluated. Therefore it seems preferable to design the evaluation procedure so that it can employ subjective judgments about expected sales response, rather than requiring actual sales response results.

Since the persons who provide the subjective judgments are typically salesmen and supervisors who know the salesmen, some allowance is expected to be made for the identities of the salesmen who are expected to be making the calls. This idiosyncratic effect will be ignored, however, so that for purposes of exposition the j superscripts can be omitted. Then the sales response from any given assignment can be written: [10]

$$\sum_i r_i(x_i)$$

where $r_i(x_i)$ = sales response from customer i resulting from making x_i calls during the planning period.

[10] The concepts and terminology of this section follow Lodish [13] with some modifications, and are included here for completeness. One such modification is to view the judgments as nonidiosyncratic.

A procedure for eliciting the judgments which are necessary to estimate r_i has been described by Lodish [13] and will not be repeated here. [11] Essentially this takes place in an interview setting, in which the participants are encouraged to make realistic judgments which they would consider to be tolerable expectations. Provision must be made for reviewing and revising these judgments as calling experience is gained, particularly in the case of customers which are initially estimated by persons other than the salesmen to whom they are assigned.

The evaluation which is intended to characterize assignment k is the total sales response resulting from the salesman allocating his time optimally among the assigned customers. This optimization is typically achieved by first replacing r_i by a concave piecewise linear approximation r_i'. Assignment k can then be defined by the solution to the following program:

$$R_k = \max \textstyle\sum_{i=1}^{m} r_i'(x_i) \tag{9}$$

subject to the calling constraints

$$\textstyle\sum_{i=1}^{m} x_i \leqq X, \tag{10}$$

$$l_i \leqq x_i \leqq u_i, \tag{11}$$

where $X =$ total number of calls which a salesman can make during a planning period;

$l_i, u_i =$ lower and upper constraints on the number of calls which are made on customer i.

6. Optimal Territories

Having generated n_j candidate territories (Step 1), and evaluated them (Step 2) for each salesman (Step 3), the final step in the design of the market matrix consists of selecting p of the Σn_j candidate territories, such that each of the p salesmen has one territory, each customer is assigned to one salesman, and the total potential sales response is maximized. This can be formulated as the following set-partitioning problem:

Maximize

$$\textstyle\sum_{j=1}^{p} R^j y^j, \tag{12}$$

subject to

$$\textstyle\sum_{j=1}^{p} A^j y^j = e_m, \tag{13}$$

$$\textstyle\sum_{k=1}^{n_j} y_k^j = 1 \quad \text{for } j = 1, \ldots, p, \tag{14}$$

$$y_k^j = 0 \text{ or } 1 \quad \text{for } j = 1, \ldots, p \text{ and } k = 1, \ldots, n_j, \tag{15}$$

where

$R^j = (r_1^j, \ldots, r_{n_j}^j)$ is a vector of the potential sales response evaluations defined by (9);
$y^j = (y_1^j, \ldots, y_{n_j}^j)$ is a vector of variables associated with salesman j such that:

[11] The remainder of this section is intended primarily to complete the exposition. It could be replaced by the Lodish procedure, although the two are not identical.

$$y_k^j = 1 \quad \text{if salesman } j \text{ received assignment } k,$$
$$= 0 \quad \text{otherwise,}$$

e_m is a vector of m ones.

The objective function (12) ensures that the resulting market matrix yields the greatest feasible potential sales response. Constraint set (13) permits each customer to appear in one and only one territory. Note that this assumes that the market matrix is not too large to be assigned to p salesmen, given their individual effort constraints (3) to (8). Constraint set (14) limits each salesman to one of his candidate territories.

The above formulations satisfy conceptually the demands of the sales territory design problem as presented in this paper. The question of the feasibility of implementing the proposed formulations remains however. While that question cannot be completely resolved in this study, a description of some computational results is provided in the next section.

7. Case Example[12]

The sales manager of the Universal Supply Corporation has been planning for several months to redesign two sales territories in the northern part of the Indiana district. Anticipating the promotion of one of the incumbent salesmen, he had forewarned the other salesmen when hired last year, that some territory modifications would be made. Many of the 500 customers in the territories had not been receiving regular calls, and the sales manager believed that this was because both salesmen had too many customers to cover effectively. He plans to remedy this by redesigning the two territories so they will support three salesmen, and to implement this by hiring two replacements for the one promoted salesman. His design problem is to determine the best boundaries for the three new sales territories, and to assign the calling efforts within them.

The area in question is roughly the shape of a rectangle having dimensions 10 miles by 20 miles. It includes 8 towns which collectively account for less than 40 percent of the 500 customers. The distances are not great, but because most of the customers are spread out, the sales manager believes that the 3 salesmen should work out of different residence towns. There are 18 different combinations of 3 towns which are sufficiently removed from each other, so the sales manager considers these feasible residences when designing candidate sales territories.

Because travel time is not the prime consideration, the sales manager designs the candidate sales territories to have comparable demand potential. This is accomplished in two ways. First, the number of customers in each territory must be within 160–170. Second, the aggregate size of the customers, as measured by their cumulative factory employment, has to be approximately equal in all territories.

Two additional constraints involve the geography of the territories. For administrative reasons all parts of a territory should be contiguous, so that one salesman need not enter another salesman's territory to call on any of his customers. Also, the location of the residence town within a territory must be such that the aggregate "city block" distance from it to all customers in the territory is minimized.

Using managerial judgment and a transportation LP algorithm (Step 1(a)), 129 feasible sets of candidate territory designs are generated which satisfy the above

[12]This case is hypothetical but it contains elements from similar problems encountered by the authors in two real companies.

constraints. The sales manager wants to identify that design which will yield the maximum sales, under an assumption that each salesman will make an average of 8 calls per day or 2000 calls per year. This means that an optimal assignment of calls, and the resulting sales, must be evaluated for each candidate territory. Based on these evaluations of 129 candidate territories for each of three salesmen, that set of three territories which maximizes total sales is the desired design. In the resulting solution, each customer must receive calls from one salesman.

To evaluate the sales from each candidate territory requires an estimate of the sales response of individual customers. The sales manager knows that each customer will probably respond differently to sales calls. Since customer assignments are not known in advance, however, it is impossible at this stage to count on having the response characteristics estimated by the salesman to whom the customer will be assigned. Therefore, the sales manager has to rely on a procedure which approximates each customer by one of several sales response stereotypes. These are simple piecewise linear functions of sales versus calls. Differences in the magnitude and rate of sales response, as might be caused by differences in competition or demand potential, are provided by the orientations of the linear segments of the various functions. The stereotype which best suits each customer is determined by a sales supervisor who has limited familiarity with the customers, with some help from one or more salesmen in adjacent territories.

The optimal assignment of calls within territories can be accomplished using an LP algorithm. In addition to the call capacity constraints, the sales manager specifies that each customer must receive at least two calls and no more than twenty calls per year.

The final step consists of selecting the optimal set of three sales territory designs. This employs the set-partitioning algorithm[13] as described in the previous section, which selects from 129 candidate designs, the one which maximizes total sales. The resulting territory design is shown in Figure 2. The sales manager has confidence in this solution because

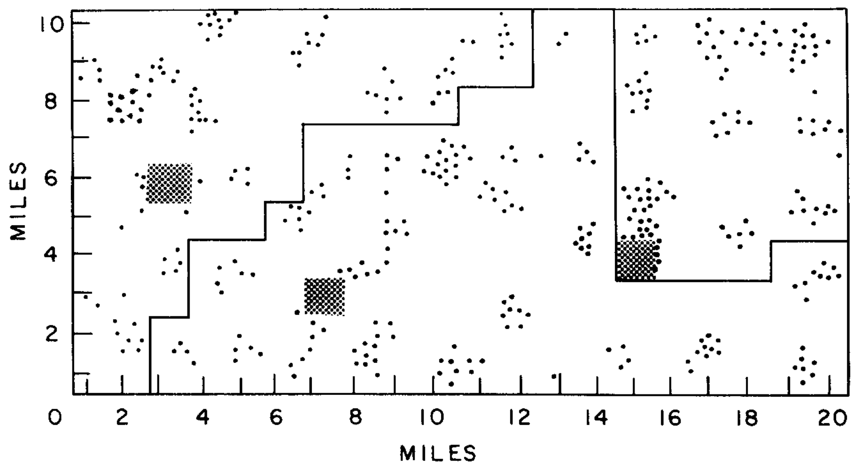
(1) the territories will be considered equitable by the salesmen, since the demand potential is approximately the same in each territory; in fact, the percentages of customers in each territory are 33.2 for the territory on the left, 33.6 for the territory in the center, and 33.2 for the territory on the right;

(2) the total sales response from the three territories is a maximum; of the $3,115,353 total sales response, 31.2 percent comes from the left-hand territory, 35.0 percent comes from the center territory, and 33.8 percent comes from the right-hand territory;

(3) the 500 by 3 market matrix, which is not shown because of space limitations, permits the sales manager to suggest the number of calls which each customer should receive to achieve the maximum total sales response.

Inspection of Figure 2 suggests that the salesman in the center territory will have to do relatively more travelling than the salesman in the right-hand territory. This inequity is expected to be tolerable, however, because the mileages are small and relatively little of the time of any salesman will be spent travelling.

[13] Using the Bowman-Starr [1] algorithm, the computations required 0.2 seconds of CPU time on a Univac 1108 computer. This time is relatively insensitive to increases in the number of salesmen (rows), but relatively sensitive to increases in the number of candidate territories (columns). For example, larger problems solved by other researchers required up to 75 seconds when 500 columns were involved. Thus, it may be possible to double or treble the number of salesmen with only a nominal change in computation time; corresponding increases in the number of candidate territories take substantially longer but are still computationally feasible.



Notes: Dots represent individual customers. Shaded blocks represent optimal resident locations. Solid lines represent optimal sales territory boundaries.

FIGURE 2. Optimal Boundaries and Resident Locations for Three Sales Territories.

The existence of differences in sales response can be expected to persist. In practice, some salesmen will work harder than others because of the opportunity to demonstrate greater commitment. The question of the degree to which such differences should be encouraged by the compensation plan is beyond the scope of this study. Given the tentative nature of the estimation of sales response characteristics for individual customers, however, it seems preferable at this stage to overlook the initial territorial sales response differences.

8. Conclusions

The designing of sales territories should permit more than one criterion to be employed, and it should permit variation in boundaries and in the amounts of calling effort assigned to individual customers. A procedure has been described which accomplishes all of these objectives, by building upon existing work and by introducing certain recently-developed computational techniques. A case example has been presented which describes how the procedure can be applied.

Further extensions are needed in three directions. The existence of carryover effects from sales calls suggests that the procedure be extended to provide dynamic solutions over time. The proliferation of product lines suggests that the procedure be extended to multiple-product situations. Finally, the heroic assumption that the optimal number of salesmen is known suggests that the procedure be extended to encompass determination of the sales force budget. All of these extensions are necessarily topics for future research.

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