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Contiguity Constraints for Single-Region Site Search Problems

This paper proposes an explicit set of constraints as a general approach to the contiguity problem in site search modeling. Site search models address the challenging problem of identifying the best area in a study region for a particular land use, given that there are no candidate sites. Criteria that commonly arise in a search include a site's area, suitability, cost, shape, and proximity to surrounding geographic features. An unsolved problem in this modeling arena is the identification of a general set of mathematical programming constraints that can guarantee a contiguous solution (site) for any 0-1 integer-programming site search formulation. The constraints proposed herein address this problem, and we evaluate their efficacy and efficiency in the context of a regular and irregular tessellation of geographic space. An especially efficient constraint form is derived from a more general form and similarly evaluated. The results demonstrate that the proposed constraints represent a viable, general approach to the contiguity problem.

Site search problems represent a challenging class of location problem. The broad goal in this context is to search a study region for the best area to locate a given land use. Criteria that commonly arise in a search include a site's area, suitability, cost, shape, and proximity to surrounding geographic features. Site search problems can be considered in the larger context of site selection problems (Malczewski 1992; Arentze, Borgers, and Timmermans 1996) with the defining characteristic that there are no candidate sites from which to select the best site. They arise in a number of application areas including locating a biological reserve (Williams and ReVelle 1996), landfill (Minor and Jacobs 1994), residential subdivision (Baerwald 1981), or hazardous waste site (Van Zee and Lee 1989).

When addressing site search problems in a computational domain, the landscape is generally represented as a spatial tessellation (Wright, ReVelle, and Cohon 1983; Gilbert, Holmes, and Rosenthal 1985; Diamond and Wright 1991, Minor and Jacobs 1994; Williams and ReVelle 1996). For this reason, the computational site search

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problem is to assemble nonoverlapping land units into a unified site according to a set of competing objectives. A fundamental requirement in many problem contexts is that the site be contiguous. A site is contiguous if it is possible to walk from any point in the site to any other point in the site without leaving it. In some application contexts, site contiguity may not be a factor (Cocks and Baird 1989), or it may be a desirable quality but not a necessity (Williams and ReVelle 1996).

This paper proposes and evaluates a set of explicit contiguity constraints as a general approach for addressing the contiguity problem in site search modeling. The term "general" in this context refers to the goal of identifying a constraint form that can be included in any 0-1, single-region, site search model. The overall strategy is to decompose a global site search problem into a set of smaller, local problems. Spatial decomposition establishes a context wherein the formulation of tractable contiguity constraints becomes an option. A general contiguity constraint set will greatly simplify the task of guaranteeing a contiguous solution to a model and allow an analyst to focus on more substantive objectives and constraints associated with a particular search. In this way, the constraints in this paper can be considered a foundation from which to formulate a variety of site search models.

The paper begins by describing site search problems in general and reviewing the modeling approaches that have been utilized to address contiguity as a factor. The subsequent section presents a means for decomposing a global site search problem into a set of smaller, local problems. Two mathematical constraint forms for guaranteeing a contiguous solution in a site search model are presented. The constraints are analyzed in the context of representative data sets using techniques in enumeration and integer programming. The paper concludes with a discussion of the results and directions for further research.

1. SEARCHING FOR AN OPTIMAL CONTIGUOUS SITE

The general site search problem is to seek out new land resources to meet a given planning need. As noted, there are a variety of criteria that commonly arise in the search for a best site. Generally, there are a set of competing objectives and associated constraints that guide the search. The constraints define a space wherein a theoretically best site exists. Figure 1 depicts an example site represented in continuous geographic space, a regular tessellation (raster), and an irregular tessellation. To identify the best site for a given purpose, imagine a contiguous site that is free to move across the landscape and change its area, shape, orientation, and proximity relationships with other geographic features to capture the optimal site and situation. The

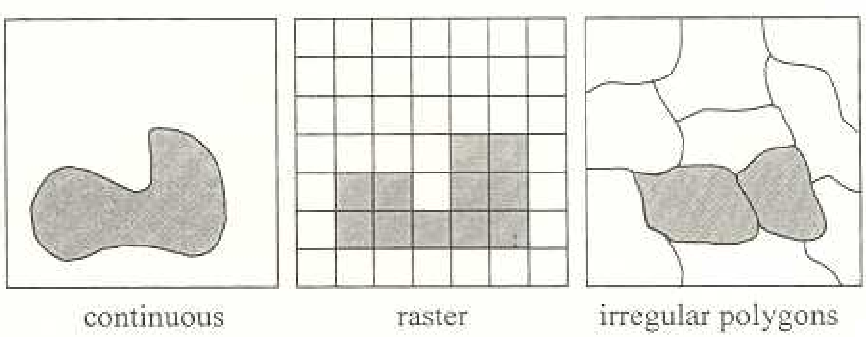

FIG. 1. Site Search Problems Represented in Continuous and Discrete Space

underlying surface may be a suitability surface, cost surface, or any combination of surfaces.

Table 1 depicts sample objectives and constraints that might arise in a typical problem. The criteria can be divided into two classes, attributes and spatial characteristics, where each criterion may be expressed as an objective or constraint. GIS-based methods for addressing site search problems such as suitability mapping (Hopkins 1977) and land screening (Dobson 1979) focus on site attributes and do not generally support objectives or constraints associated with spatial site characteristics such as shape. This is because spatial site characteristics refer to aggregations of land units, and GIS-based methods emphasize attributes of individual land units. The focus of this paper is on single-region, site search models that allow explicit spatial site characteristics to be included in a search.

One spatial site characteristic that has a significant effect on the search for a best site is contiguity. If a contiguous solution is required, then a site must be represented as a connected aggregation of land units. Connected land units in a tessellation are generally defined as those that share an edge or point. Contiguity has a profound effect on the inherent difficulty in solving a site search model. In essence, it places a critical constraint on the spatial arrangement of the land units that constitute a site. If contiguity is not required, then the spatial arrangement of the land units in a site is free to vary, and the search for the best site can often be reduced to a search for the n best land units. This might be accomplished by ranking them according to certain criteria and taking the top n units. However, the n best by any criteria often results in a noncontiguous site. The following sections review two approaches that have been developed to include contiguity as a factor in site search problems.

1.1 Model Structures That Encourage Contiguity

One approach for including contiguity as a factor is to formulate a model that encourages this property. Two specific strategies have been developed to this end. The first is to formulate a model that allows an analyst to constrain the perimeter of the site (Wright, ReVelle, and Cohon 1983; Minor and Jacobs 1994). A site with a shorter perimeter is likely to result in a contiguous solution, if the site must be of a minimum area. However, as Wright, ReVelle, and Cohon and Minor and Jacobs note, noncontiguous solutions are frequently encountered using this method. A contiguous solution can only be guaranteed when a site's perimeter is constrained to its minimum length for a given site area. The second approach is to require a buffer of land units around a site that surrounds an internal core of land resources (Williams and ReVelle 1996). Williams and ReVelle demonstrate that if the objective is to minimize the cost of purchasing a site, the solution will often be contiguous, as these sites have fewer cells and, thus, a lower total cost. This is similar to encouraging compactness by con-

TABLE 1

Site Attributes and Spatial Characteristics as Objectives and Constraints

	attributes	spatial characteristics
objectives	cost (min) proximity (max/maximin) suitability (max/maximin) environmental impact (min) area (max/min)	contiguity (max) compactness (max) buffer width (min/max) fragmentation (min)
constraints	cost $\leq$C proximity $<, >, =$ P suitability $>$ S area $<, >, =$ A	contiguity (guarantee) compactness $<$ C buffer width = B largest patch of type i $>$ P

trolling a site's perimeter, but perimeter is expressed as a buffer rather than as edge length. These models represent unique approaches to incorporating spatial and non-spatial characteristics into a single site search formulation, but they are not a general strategy for guaranteeing contiguity in a model.

1.2 Solution Methods That Enforce Contiguity

A second approach for including contiguity as a factor is to develop a solution procedure that guarantees a contiguous solution. In this approach, contiguity is not mathematically formulated as part of the model. Rather, it is stated as a necessary property of any valid solution. The key is to develop an algorithm that guarantees this property. Two algorithmic approaches have been developed to this end. The first is to design an implicit enumeration algorithm that searches for the optimal, contiguous solution to a model (Gilbert, Holmes, and Rosenthal 1985; Diamond and Wright 1991). An implicit enumeration algorithm relies on an underlying binary tree to effectively enumerate all feasible site patterns in a spatial tessellation (Nemhauser and Woolsey 1988). Another approach is to design an heuristic, region-growing algorithm that begins at a seed land unit and adds units that are contiguous to the current site (Brookes 1997). Both of these approaches are effective, but a unique algorithm must be tailored for each new site search problem encountered. This can take a significant amount of time and effort and requires a commitment by an analyst to a particular model formulation.

2. CONTIGUITY CONSTRAINTS AND SPATIAL DECOMPOSITION

The goal in this paper is to pursue a set of explicit, tractable contiguity constraints that can guarantee a contiguous solution for any single-region site search model regardless of the tessellation. This constitutes a general approach to the contiguity problem. A number of authors have called for research into the explicit representation of this factor (Wright, ReVelle, and Cohon 1983; Diamond and Wright 1989; Williams 1994). A benefit of contiguity constraints is that they facilitate solving a model using a mixed integer-programming (MIP) solver like CPLEX™, LINDO™, or XMP™. Identifying a tractable contiguity constraint form has proven to be problematic. As MacMillan and Pierce (1994) note regarding the related contiguity problem in political districting (see also Horn 1995), "The nastiness of this problem springs partly from its size, both in terms of its dimensionality and number of constraints, but mainly from the fact that the space defined by the constraints is non-convex" (p. 228).

2.1 Contiguity Constraints in One Dimension

A missing element that may have precluded the formulation of a tractable set of contiguity constraints is a root land unit. If a given root is constrained to being in a site, then contiguity can be defined in relation to this root. This substantially reduces the problem of formulating tractable constraints that guarantee a contiguous site. In short, a root can be utilized to define a precedence by which land units must be included in any solution to ensure contiguity. Diamond and Wright (1991) and Brookes (1997) both relied on the notion of a root (or seed) unit to develop a specialized site search algorithm. Here we utilize the same concept to formulate general contiguity constraints in a mathematical programming context. Figure 2 shows a sample study area and a unit r that must be in the best solution. If the solution must be contiguous, then unit 7 cannot be included unless unit 6 is in the solution. Similarly, if unit 3 is in the solution, then unit 4 must be included.

Given an upper bound on the area of a site, a rooted approach allows a *feasible neighborhood* to be defined within which the best contiguous site must exist. An ex-

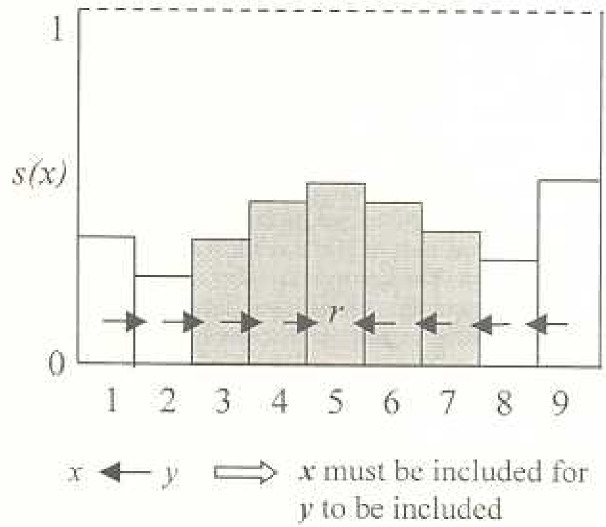

FIG. 2. One-Dimensional Contiguity Precedence for a Rooted (r) Site Search Problem in a Discrete Suitability Map $s(x)$

ample neighborhood is shown in gray in Figure 2 for the root, assuming that a site may comprise a maximum of three land units. A constraint set that represents this approach to contiguity can be formulated as follows:

Constraint: A land unit cannot be selected unless its adjacent neighbor closer to the root is selected.

$$x_i \leq x_j \qquad \forall\, i, j \in N_r \text{ where } p_{jr} = p_{ir} - 1 \tag{1}$$

where $x_i = \begin{cases} 1 \text{ if land unit } i \text{ is selected for the site} , \\ 0 \text{ otherwise} ; \end{cases}$

p_{ij} the path length (in land units) from i to j ;
r the root land unit ;
N_r $\{i \mid i$ is in the feasible neighborhood of $r\}$.

To identify the optimal solution for the one-dimensional study region in Figure 2, a model that relies on these constraints would have to be solved nine times, once for each potential root unit. This is similar to passing a neighborhood operator (Tomlin 1990; Worboys 1995) or convolution filter (Richards 1993) over a study region, where the feasible neighborhood, and associated optimal solution, changes for each root. An example site search neighborhood operator to solve for the maximally suitable site in a study area R can be formulated as follows:

$$z_r \leftarrow \left\{ \max \sum_{i \in N_r} s_i x_i : \sum_{i \in N_r} x_i = a, x_r = 1, (1) \right\} \quad \forall r \in R \tag{2}$$

where z_r is the objective value achieved for root r, s_i is the suitability score for land unit i, a is the area of the site, and (1) is the constraint set in equation (1).

2.2. *Contiguity Constraints in Two Dimensions*

General Contiguity Constraints. Transferrring these constraints into two dimensions is not straightforward. In one dimension, there is a single precedence by which the land units surrounding a root must be included to guarantee contiguity. In two dimensions, a single precedence does not exist. Figure 3 depicts three potential sites that include the root r. In each case, unit a was approached from a different adjacent land unit. Therefore, there is more than one path from which a land unit can be reached from a given root. The number of paths between a root unit and another land unit depends on the definition of adjacency, the pattern of land units, and the maximum allowable path length.

Another approach to formulating contiguity constraints must be pursued. A useful observation is that every land unit in a contiguous solution has a shortest path (in land units) to the root that uses only units in the site. For example, consider the three site patterns in Figure 3. In the example on the left of the figure, the path from cell a to the root is five cells, but in the center, the shortest path from cell a to the root is three cells. This leads to the following general proposition:

> *Every land unit i that is contiguous to a root unit r must have at least one selected adjacent land unit that is 1 unit closer to the root using only selected land units.*

This proposition can be used to develop a set of general contiguity constraints that include every possible means of approaching a given land unit in a regular or irregular tessellation.

Constraints: A land unit cannot be part of a site unless it is contiguous to the root by a path of length j in selected land units.

$$x_i = \sum_{j=l_{ir}}^{u_{ir}} c_{ji} \qquad \forall i \tag{3}$$

A land unit cannot be contiguous to the root by a path of length j (in selected land units) unless it has at least one selected adjacent land unit that is contiguous to the root by a path of length $j - 1$ (in selected land units).

$$c_{ji} \le \sum_{k \in A_i} c_{(j-1)k} \quad \forall i \text{ and } j = l_{ir} \ldots u_{ir} \tag{4}$$

The root is contiguous to itself by a path of length 0.

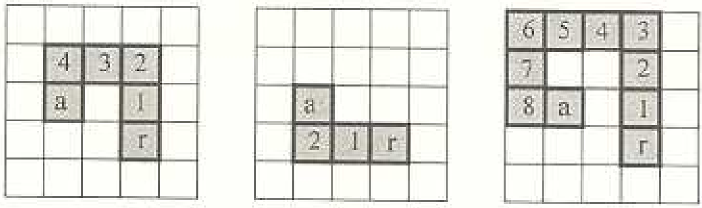

FIG. 3. Contiguous Paths between a Root Unit (r) and a Land Unit (a)

$$c_{0ir} = 1 \tag{5}$$

where $x_i = \begin{cases} 1 \text{ if unit } i \text{ is selected ,} \\ 0 \text{ otherwise ;} \end{cases}$

$c_{ji} = \begin{cases} 1 \text{ if unit } i \text{ is } j \text{ units from the root using only selected land units ,} \\ 0 \text{ otherwise ;} \end{cases}$

l_{ir} the shortest path length to the root from i in land units (lower bound) ;
u_{ir} the maximum path length that i can be from the root in selected land units (upper bound) ;
A_i $\{k \mid k \text{ is adjacent to } i\}$.

These constraints allow a given land unit to be any path length, in selected land units, from a root unit, between a lower and upper bound. The lower bound l_{ir} is the unit's shortest possible path to the root, and the upper bound u_{ir} is the maximum allowable path length in selected land units to the root. These constraints add a considerable number of variables to a given site search model, as each land unit requires a "stack" of contiguity variables c_{ji} in addition to its binary selection variable x_i. The c_{ji} variables can be relaxed linear variables with an upper bound of 1 to reduce the number of binary variables in a model and improve solvability. Overall, the inclusion of additional contiguity variables within a root's feasible neighborhood is not a very powerful approach to formulating contiguity constraints. While a model that includes these constraints can be solved with a standard MIP solver, the number of required variables leads to a condition where only the smallest of feasible neighborhoods can be addressed. The primary benefit of this constraint set is that it represents a general, unambiguous formulation of the rooted contiguity problem in a spatial tessellation.

Shortest Path Contiguity (SPC) Constraints. To reduce the number of contiguity variables in a model, we can constrain the allowable paths between a given land unit and the root by lowering the value of u_{ir} in constraints (3) and (4). For example, a relatively constrained formulation is to limit each land unit to being its shortest path from the root and perhaps its next shortest path from the root. We define this case as shortest path contiguity-2 (SPC-2). This can be generalized to SPC-k, where k is the number of contiguity variables associated with each land unit in the model. An increase in k increases the feasible paths between a root unit and another land unit and adds another layer of contiguity variables within the root's feasible neighborhood. We define shortest path contiguity (SPC) as the special case where a land unit can only be its shortest path (in selected land units) from the root. In this case, constraint (3) reduces to the following form:

$$x_i = c_{ji} \tag{6}$$

where j is land unit i's shortest path (in land units) to the root. This implies that the contiguity variable c_{ji} for each land unit is redundant, as it is equal to the selection variable. Therefore, we can eliminate all contiguity variables and express contiguity to the root using only the binary selection variables:

$$x_i \le \sum_{j \in C_i} x_j \quad \text{where } C_i = \{j \mid p_{jr} = p_{ir} - 1\} \text{ and } x_i \in \{0, 1\} \tag{7}$$

where p_{ij} is the shortest path length from land unit i to j. This constraint states that a land unit can only be selected if at least one of its adjacent land units closer to the root has been selected. The similarity of this constraint to the one-dimensional version in equation (1) is clear, as it imposes a precedence on the order in which land units may be included in a site. However, unlike the one-dimensional case, there is more than one way to reach a given land unit from a root. The factors that affect this precedence are the definition and pattern of adjacency. Figure 4 depicts an example of this precedence for a regular and irregular tessellation. This greatly reduces the number of variables and constraints necessary to guarantee contiguity, as these constraints add no variables to a model and only one constraint per land unit. If adjacency is defined to include a shared point between land units, the set C_i in equation (7) increases in size, but no additional variables or constraints must be added to a model to accommodate this expanded definition of adjacency and contiguity.

An important aspect of shortest path contiguity constraints (SPC) is that not all contiguous land unit patterns are feasible in a model that relies on these constraints. In other words, some contiguous patterns are overlooked. Figure 5 depicts examples of contiguous patterns that would be included and overlooked in a model that includes SPC constraints. Essentially, SPC constraints do not allow a pattern that doubles back toward the root in any direction. Section 4 explores the contiguous patterns that are included and overlooked by SPC constraints in more depth.

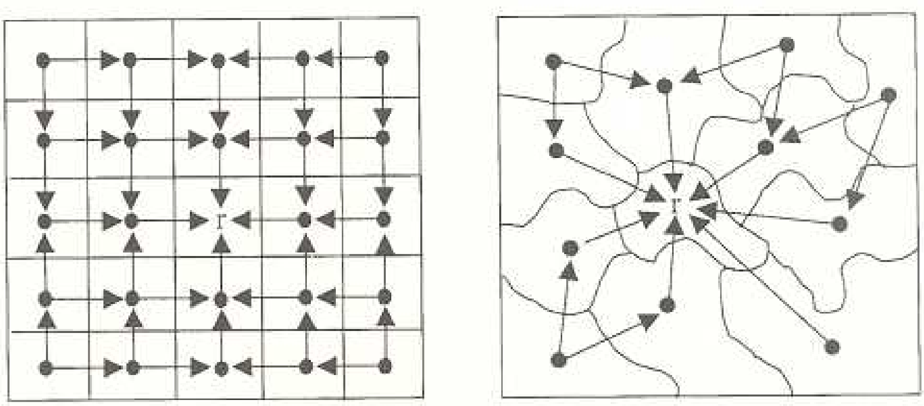

FIG. 4. Shortest Path Contiguity (SPC) Land Unit Precedence for an Example Regular and Irregular Tessellation (*r* is the root land unit)

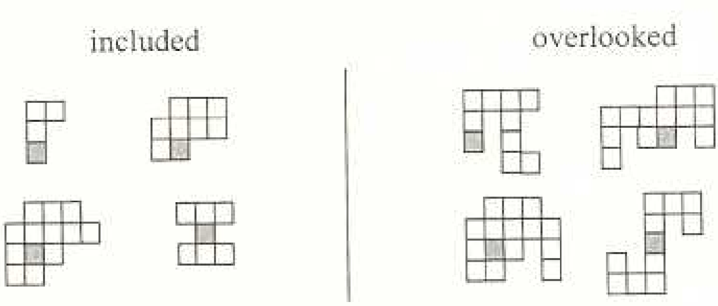

FIG. 5. Contiguous Patterns That Are Included and Overlooked by Shortest Path Contiguity Constraints (root unit in gray)

3. FEASIBLE NEIGHBORHOOD PROBLEMS

Before evaluating the proposed constraints, an important step is solving the feasible neighborhood problem. In two dimensions, this neighborhood is an area around a root unit. The desired site area and compactness are important factors in determining this neighborhood. A search for a larger site dilates the feasible neighborhood, and a search for a smaller one contracts it. Similarly, the search for a less compact site dilates the feasible neighborhood, and a search for a more compact site contracts it. This is important because discrete, site search models generally include a binary decision variable for each potential land unit, and the number of binary variables is the primary determinant of how difficult a 0–1 integer programming model is to solve optimally (Nemhauser and Woolsey 1988). Thus, the number of land units in any given problem instance should be minimized to improve solvability. We describe two strategies for solving the feasible neighborhood problem: exact and heuristic.

3.1 Exact Feasible Neighborhoods in a Raster

In a regular tessellation, it is worth pursing the exact minimum feasible neighborhood because neighborhood shape is the same for every land unit. Extra land units that cannot possibly be in a root's best site add unnecessary variables and constraints to a problem instance. The primary factor that influences the size of the feasible neighborhood is the maximum allowable number of land units in the site. For example, if there is a constraint of this type in a model, then the equation for the number of cells in the feasible neighborhood for a root cell in a raster is

$$n = a^2 + (a - 1)^2 \tag{8}$$

where a is the maximum number of cells in the site and n is the number of cells in the root's feasible neighborhood. This functional relationship was derived though observation, and it can be used to determine the number of binary decision variables (cells) in the feasible neighborhood in a raster, given no other spatial constraints on the site.

A second factor that influences the size and shape of a root's feasible neighborhood is the desired site compactness. A compactness constraint can be used to substantially reduce the feasible neighborhood, as most practical problems are for very compact sites. There are a number of quantitative shape compactness measures in the geographic literature (Austin 1984; MacEachren 1985; Horn, Hampton, and Vandenberg 1993). Consider the measure that relies on the ratio of a site's area to its perimeter squared. The perimeter can be multiplied by a constant k to place the measure on a 0–1 scale. If k is set to .25, the measure will score a square site in a raster as the most compact shape with a value of 1. Placing an adjustable lower bound on this ratio allows an analyst to control the site's minimum compactness. It also facilitates calculating a site's maximum perimeter length as follows:

$$\lambda \leq \frac{A}{(kP)^2}, \text{ so } \; P \leq \frac{1}{k}\left(\frac{A}{\lambda}\right)^{1/2} \tag{9}$$

where P is the perimeter of a site in cell edges, A is the site's area in cells, k is a constant, and λ is a lower bound on the site's compactness. An upper bound on a site's perimeter is useful because it can be used to further reduce the size of the feasible neighborhood. By solving for the minimum perimeter necessary to reach each cell from a root cell, we can effectively rule out cells that cannot be reached within the defined perimeter limit. If the search is for a very compact site, then the perimeter will be highly constrained, and the feasible neighborhood will be correspondingly small.

Given the above compactness measure, the minimum feasible neighborhood can be identified by solving for the minimum perimeter necessary to reach a given cell from a root cell. This can be formulated as a 0–1 integer-programming problem. The following formulation can be used to identify the exact shape of the feasible neighborhood, given the site area and perimeter limit.

Objective: Find the minimum perimeter necessary to reach a cell from a root land unit.

$$\min \sum_{i \in N_r} \sum_{j \in A_i} y_{ij} \tag{10}$$

Subject to: The site must be a cells in size.

$$\sum_{i \in N_r} x_i = a \tag{11}$$

The site must be contiguous by shortest path contiguity.

$$x_i \leq \sum_{j \in C_i} x_j \qquad \forall i \in N_r \tag{12}$$

If land unit i is selected but j is not, then their shared edge is part of the perimeter.

$$x_i - x_j \leq y_{ij} \qquad \forall i \in N_r \text{ and } j \in A_i \tag{13}$$

If land unit j is selected but i is not, then their shared edge is part of the perimeter.

$$x_j - x_i \leq y_{ij} \qquad \forall i \in N_r \text{ and } j \in A_i \tag{14}$$

The root and the cell in question must both be in the solution.

$$x_i = 1 \qquad i \in \{r, k\} \tag{15}$$

where $x_i = \begin{cases} 1 \text{ if unit } i \text{ is selected for the site ,} \\ 0 \text{ otherwise ;} \end{cases}$

$y_{ij} = \begin{cases} 1 \text{ if adjacent land unit } i \text{ or } j \text{ is selected, but not both ,} \\ 0 \text{ otherwise ;} \end{cases}$

a the area of the site (in cells) ;
r the index of the root cell ;
k the index of the cell in question ;
p_{ij} the shortest path (in land units) between i and j ;
C_i $\{j | p_{jr} = p_{ir} - 1\}$;
A_i $\{j | j \text{ is adjacent to } i, j > i, \text{ and } j \in N_r\}$;
N_r an area larger than the exact feasible neighborhood of r .

The objective of this model is to identify the minimum perimeter necessary to reach a cell from a root. Note that the contiguity constraints presented in the prior section are used to help solve this problem. Due to the inherent symmetry of a raster neighborhood around a root, this problem only needs to be solved for a very small number of cells. An initial approximation of the feasible neighborhood for a root must be determined. The model above can be used to reduce the neighborhood to its exact minimum.

Figure 6 shows the neighborhood for a nine-cell problem with the minimum perimeter values necessary to reach each cell in edges. The darkest area is the feasible neighborhood for a site with a maximum perimeter length of twelve edges. In other words, if a site is constrained to having a maximum perimeter of 12, then cells outside the area marked "12" cannot possibly be in the root's feasible neighborhood for this problem instance. As the perimeter is allowed to expand, and the constraint on compactness is relaxed, the feasible neighborhood essentially dilates (progressively lighter shades of gray).

Through observation, it was noted that two parameters are necessary to define a neighborhood that has been reduced by a maximum perimeter constraint in a regular grid. The first parameter is the maximum Manhattan distance that a cell can be from the root, and the second is the maximum Euclidean distance that a cell can be from the root. A cell must meet both of these constraints to be in the root's neighborhood. For example, assume that the search is for a nine-cell site and the minimum compactness of the site is .50 (1 is maximally compact). Equation (9) would look as follows:

$$P \leq \frac{1}{.25}\left(\frac{9}{.50}\right)^{1/2} = 16.97\,. \tag{16}$$

Therefore, the longest the site's perimeter can be is sixteen cell edges. In Figure 6, the only cells that can be reached with a perimeter length of 16 are the cells with a maxi-

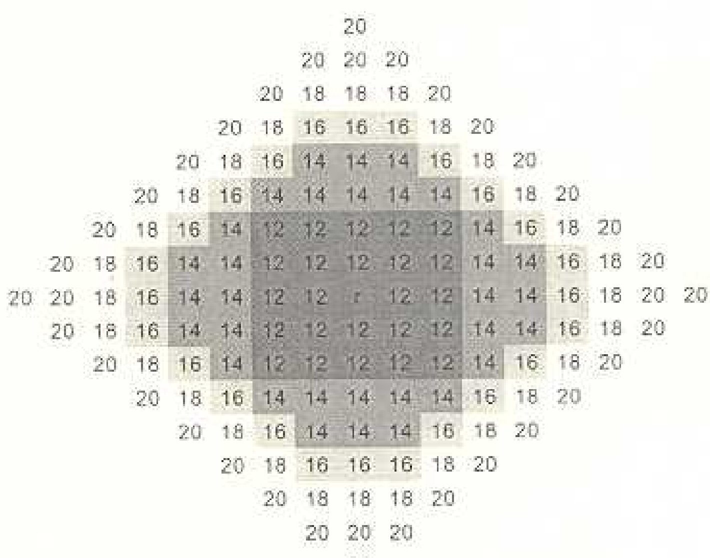

FIG. 6. Solution Values to the Perimeter-based Feasible Neighborhood Problem for a Nine-Cell Site in a Raster

mum Manhattan distance of six cells from the root and a maximum Euclidean distance between cell centers of five cells. It suffices to store the values of 6 and 5 for these two parameters in a table that has placeholders for the neighborhood size for a nine-cell problem when the perimeter must be less than sixteen edges. Table 2 shows a variety of site areas and perimeter lengths and the corresponding maximum Manhattan and Euclidean distances to define the minimum feasible neighborhood. The values in this table were derived using the model formulated in equations (10) – (15). Note that this table is constant for a raster and need only be derived once prior to solving future site search problems.

3.2 Approximating the Feasible Neighborhood

A less intensive approach to the feasible neighborhood problem is to approximate it with a heuristic rule for either a regular or irregular tessellation. In the context of an irregular tessellation, this is the only sensible option, as every land unit has a uniquely shaped exact feasible neighborhood. Consider the following approximation method for the perimeter-based measure of shape compactness previously described. In this problem context, a land unit with a centroid greater than one-half the maximum perimeter from the root's centroid, in Euclidean distance, cannot possibly be in the feasible neighborhood of the root unit. It is simply not possible to identify a site that contains both land units and meets the compactness (maximum perimeter) constraint. A perimeter-based exclusion distance can be computed for an irregular tessellation given a perimeter based compactness measure as follows:

$$\lambda \leq \frac{A}{(kP)^2} \quad \text{so} \quad P \leq \frac{1}{k}\left(\frac{A}{\lambda}\right)^{1/2} \quad \text{and} \quad d_{ri} \leq P/2 - w \tag{17}$$

where d_{ri} is the Euclidean distance from the root to the centroid of any land unit in the feasible neighborhood, and w is the minimum width of the site. Setting k to .282

TABLE 2
Feasible Neighborhood Definition Parameters for Constrained Perimeter Problems in a Raster

Area (cells)	Compact. (0 – 1)	Perimeter (edges)	Maximum Manhattan	Maximum Euclidean
25	1.00	20	8	4
	0.83	22	9	6
	0.69	24	10	8
	0.59	26	11	9
36	1.00	24	10	5
	0.85	26	11	8
	0.73	28	12	9
	0.64	30	13	11
	0.56	32	14	12
49	1.00	28	12	6
	0.87	30	13	9
	0.77	32	14	10
	0.68	34	15	12
	0.60	36	16	13
	0.54	38	17	14
64	1.00	32	14	7
	0.89	34	15	10
	0.79	36	16	12
	0.71	38	17	13
	0.64	40	18	15
	0.58	42	19	16

places the measure on a 0–1 scale and scores a circle as the most compact shape with a value of 1 (MacEachren 1985). This approach is similar to Diamond and Wright's (1991) exclusion distance embedded within their implicit enumeration algorithm. While this will not result in the minimum neighborhood for a given root unit, it is simple to calculate and can greatly reduce the number of land units in the neighborhood depending on the values of λ and w. Identifying new methods for approximating the feasible neighborhood for various problem instances is an area for futher research.

4. CONSTRAINT ANALYSIS AND EVALUATION

The purpose of this section is to evaluate the proposed contiguity constraints. The questions that guide this analysis are:

- How does the number of contiguous patterns that SPC constraints overlook vary with the area and compactness of a site?
- What is the relationship between the desired area and compactness of a site and the efficacy and efficiency of a site search neighborhood operator?
- What strategies can be developed to improve the utility of the constraints and their efficiency?

4.1 Constraint Efficacy

The purpose of this part of the analysis is to evaluate the utility of the proposed constraints. Figure 5 demonstrates that it is relatively easy to identify contiguous site patterns that SPC constraints overlook (that is, do not consider feasible). However, a more systematic method of evaluation is required to answer the questions above. One approach is to utilize an enumeration algorithm to search all contiguous land unit patterns (Goodchild and Hosage 1983). An enumeration algorithm can be used to evaluate every pattern as to whether it would be feasible or overlooked by SPC constraints. This allows absolute site pattern counts to be established that can be classified by their spatial properties like perimeter and area.

An enumeration algorithm was developed based on Diamond and Wright's (1991) notion of *restricted branching*. The algorithm relies on a branch-and-bound tree, where restricted branching implies that only contiguous patterns are enumerated. One difference between our algorithm and Diamond and Wright's is that we have a root unit in a spatial tessellation to establish the root node of the branch-and-bound tree. The only bound on the depth of our tree is the maximum number of land units in a site. This algorithm can be used to explore questions posed for relatively small problems, where problem size is a function of the maximum number of land units in a site. For larger site search problems, the computational burden of enumerating and evaluating all patterns that are contiguous to a root becomes prohibitive. The initial computational experiments were performed using the raster data set in Figure 12 because spatial regularility leads to more interpretable benchmark statistics.

Area Analysis. The goal in this phase is to isolate the number of contiguous patterns that SPC constraints overlook as a function of the desired area of a site. The enumeration algorithm was used to count the number of contiguous patterns for a rooted site search problem, as well as the number that SPC constraints overlook across a range of site areas. Table 3 shows the results of this analysis. It is clear that the number of potential patterns grows exponentially as a function of the desired site area, as does the number of contiguous patterns that SPC constraints overlook. The percentage of site patterns overlooked by SPC constraints increases approximately linearly as the number of land units in the site is increased. For a site as small as twelve cells, the percentage of contiguous patterns that SPC constraints overlook exceeds the percentage of contiguous patterns that the constraints consider feasible.

Compactness Analysis. To assess the effect of compactness on the utility of SPC constraints, we used an area-to-perimeter-based measure of shape compactness. A site's perimeter is the sum of the lengths of the shared edges between adjacent land units when only one of these land units is selected for the site. The enumeration algorithm was augmented to calculate the perimeter of every site pattern examined. Table 4 depicts the results of this analysis. Site area ranges from four to fifteen cells, and the perimeter ranges from the most compact contiguous site to the least compact contiguous site for each area. As noted by Wright, ReVelle, and Cohon (1983), only even number perimeters need to be considered in a raster, as a site pattern cannot have an odd perimeter length. A noteworthy trend in Table 4 is that the percentage of patterns that SPC constraints overlook decreases as the perimeter of the site decreases. Therefore, a site search model that utilizes SPC constraints includes a much higher percentage of the compact patterns than noncompact patterns. This is a desirable

TABLE 3

The Number of Rooted, Contiguous Site Patterns in a Raster, and the Number and Percentage of Patterns That SPC Constraints Overlook

area (cells)	Neighborhood size	Potential Patterns	SPC overlook	% SPC overlook
1	1	1	0	0.00
2	5	4	0	0.00
3	13	18	0	0.00
4	25	76	0	0.00
5	41	315	8	2.54
6	61	1296	96	7.41
7	85	5320	744	13.98
8	113	21800	4688	21.50
9	145	89190	26208	29.38
10	181	364460	135696	37.23
11	221	1487948	666296	44.78
12	265	6070332	3147556	51.85
13	313	24750120	14439722	58.34
14	365	100868236	64748736	64.19
15	421	410919990	285107948	69.38
	Total	**544579083**	**368277698**	**67.63**

TABLE 4

The Percentage of Rooted, Contiguous Patterns That SPC Constraints Overlook in a Raster Stratified by Site Area (cells) and Perimeter (cell edges)

area \ perimeter	8	10	12	14	16	18	20	22	24	26	28	30	32
4	0.00	0.00											
5		0.00	2.91										
6		0.00	0.00	9.20									
7			0.00	3.40	17.64								
8			0.00	0.75	10.86	26.81							
9			0.00	0.00	5.28	20.43	35.90						
10				0.00	1.83	13.78	30.28	44.50					
11				0.00	0.94	7.66	23.99	39.67	52.64				
12				0.00	0.00	4.51	17.10	34.08	48.42	59.92			
13					0.00	2.25	11.79	27.74	43.49	56.36	66.35		
14					0.00	1.06	7.73	21.76	37.93	52.09	63.39	71.93	
15					0.00	0.35	5.02	16.40	32.30	47.25	59.78	69.51	76.69

quality, as most site search problems are for relatively compact sites, because non-compact sites are sinuous and often contain holes.

Improvement Strategies. A key concept that can be used to improve the utility of SPC constraints is that a contiguous pattern that is not feasible from one root using these constraints may be feasible by another root using the same contraints. Figure 7 shows a pattern where one root (a) would overlook the pattern with SPC constraints, but another unit in the same pattern would not (b). As long as one land unit in a contiguous pattern sees the pattern using SPC constraints, the pattern is globally feasible for the entire study area.

In order to understand the benefit of this observation, the enumeration algorithm was augmented to test whether a pattern that would not be feasible using SPC from one root cell would be feasible from any of its constituent cells. If the pattern was feasible using SPC constraints for any constituent cell, then the pattern can be considered globally feasible for the study area. Table 5 depicts the percent of patterns that would not be feasible by SPC constraints from any constituent land unit using SPC constraints. The patterns in the table with compactness score greater than .46 for the measure given in (9) are outlined. Note that all patterns within this compactness constraint are globally feasible (that is, feasible by some constituent root cell) for a site between nine and fifteen cells. This means that a model that includes SPC constraints could guarantee the optimal solution in this instance, if every locally rooted problem were solved and this compactness condition were included.

The result of this portion of the analysis is that SPC constraints are a viable, general solution to the contiguity problem in some problem instances. The utility of the constraints increases as the desired site compactness is increased. A caveat regarding this table is that it was derived for a site that ranges in area from nine to fifteen cells, and there is no simple way of knowing whether this trend holds for larger sites. In other words, the question as to whether SPC constraints would include the same percent-

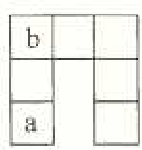

FIG. 7. A Pattern That Is Not SPC Feasible by One Root (a) May Be by Another (b)

TABLE 5

The Percentage of Patterns That SPC Constraints Overlook in a Raster When Every Land Unit Is Used as a Root for a Local Site Search Problem

area \ perimeter	12	14	16	18	20	22	24	26	28	30	32
9	0.00	0.00	0.00	0.00	0.00						
10		0.00	0.00	0.00	0.26	2.44					
11		0.00	0.00	0.00	0.06	1.15	4.61				
12		0.00	0.00	0.00	0.00	0.48	2.84	7.28			
13			0.00	0.00	0.00	0.14	1.61	5.21	10.24		
14			0.00	0.00	0.00	0.04	0.76	3.53	8.04	13.29	
15			0.00	0.00	0.00	0.00	0.36	2.13	6.08	11.09	16.25

age of patterns in a model for a larger area (for example, one hundred cells) under similar compactness bounds remains an assumption.

4.2 Efficiency Analysis

The purpose of this part of the analysis is to evaluate the efficiency of the proposed constraints and associated spatial decomposition method. To achieve this end, a series of computational experiments was perfomed. The most significant factor regarding the efficiency of a site search neighborhood operator is the number of land units in the feasible neighborhood. Each additional land unit increases the number of variables and constraints in a problem instance. The prior section articulated the effect that desired site area and compactness have on the size and shape of the feasible neighborhood for an example compactness measure. The following experiments test the effect of varying site area and compactness on the efficiency of models that include the proposed contiguity constraints. All experiments were conducted using CPLEXMIP™ on a Sun Spark 10.

The prior section also noted that SPC constraints overlook relatively noncompact contiguous site patterns. However, a constraint form was presented in (3) to (5) to progressively include more patterns in a search by adding layers of contiguity variables. This constraint form proved to work for very small problems but was inefficient for moderately sized problems, even in the minimal case where each land unit is assigned two contiguity variables in addition to its binary selection variable (SPC-2). For this reason, we bypass demonstrating SPC-2 constraints and focus on applying SPC constraints (no contiguity variables) in the context of both a regular and irregular tessellation.

Area, Compactness, and Efficiency. The first experiment tests the effect of varying site area and compactness on the efficiency of a site search operator for a regular tessellation. An example single-objective operator is given below to identify the most suitable, contiguous site. Site suitability is defined as the sum of the individual land unit suitabilities.

Objective: Maximize the suitability of the site.

$$\max z_r = \sum_{i \in N_r} s_i x_i \tag{18}$$

subject to: The area of the site must meet minimum area requirements.

$$\sum_{i \in N_r} a_i x_i \geq A_l \tag{19}$$

The area of the site must not exceed maximum area requirements.

$$\sum_{i \in N_r} a_i x_i \leq A_u \tag{20}$$

The perimeter of the site cannot exceed an upper bound.

$$\sum_{i \in N_r} \sum_{j \in A_i} e_{ij} y_{ij} + \sum_{i \in N_r} e_{ib} x_i \leq P \tag{21}$$

If land unit i is selected but j is not, then their shared edge is part of the perimeter.

$$x_i - x_j \leq y_{ij} \qquad \forall i \in N_r \text{ and } j \in A_i \tag{22}$$

If land unit j is selected but i is not, then their shared edge is part of the perimeter.

$$x_j - x_i \leq y_{ij} \qquad \forall i \in N_r \text{ and } j \in A_i \tag{23}$$

To select a land unit, at least one of its adjacent land units closer to the root must be selected.

$$x_i \leq \sum_{j \in C_i} x_j \qquad \forall i \in N_r \tag{24}$$

The root land unit must be part of the site.

$$x_r = 1 \tag{25}$$

where $x_i = \begin{cases} 1 \text{ if land unit } i \text{ is selected ,} \\ 0 \text{ otherwise ;} \end{cases}$

$y_{ij} = \begin{cases} 1 \text{ if } i \text{ or } j \text{ is selected (but not both),} \\ 0 \text{ otherwise ;} \end{cases}$

r the index of the root ;
a_i the area of land unit i ;
s_i suitability of land unit i ;
z_r the objective value achieved for root r ;
P the maximum perimeter length of the site ;
p_{ij} the shortest path length (in land units) between i and j ;
C_i $\{j | p_{jr} = p_{ir} - 1\}$;
N_r $\{i \mid i \text{ is in the feasible neighborhood of } r\}$;
A_i $\{j | j \text{ is adjacent to } i, j > i, \text{ and } j \in N_r\}$
e_{ij} the edge length between land units i and j ;
e_{ib} the edge length between land unit i and the boundary of the feasible neighborhood or study area ;
A_l a lower bound on the area of a site ;
A_u an upper bound on the area of a site .

This formulation is very similar to the one presented to define the feasible neighborhood for a raster in (10) to (15). But here we are solving for the best contiguous site within a defined feasible neighborhood that includes a root unit and meets the area and compactness constraints.

The raster suitability data set in Figure 12 was utilized to test the efficiency of this site search operator. Ten root cells were randomly selected from this data set (that is, 120, 838, 697, 436, 481, 766, 822, 557, 33, 573), and the operator was executed on each root across a series of site areas ranging from twenty-five to sixty-four cells. In the formulation above, the lower bound on the area of the site and the upper bound on the perimeter control the desired site compactness. The upper bound on the area of the site in constraint (20) is not required but can be useful in approximating the feasible neighborhood. For the raster-based experiments, the upper and lower bounds on the site area in (19) and (20) were set to the same value. This reduces the two constraints to a single equality constraint. Compactness was defined using equation (9), so the most compact shape in a raster is a square. Figure 8 shows the mean solution time for the ten sample root cells across a range of area and compactness values. As expected, solution time increases as the compactness constraint is relaxed or the area constraint is increased. The compactness scores range from 1 to .59 using the measure in (9). Below .5, the feasible neighborhood dilates to a point where the benefits of the proposed spatial decomposition become negligible. For some of the cells, the complete feasible neighborhood is not present in the data set, as the cell may be near the data set boundary. This leads to an efficiency gain that would arise in any bounded study region.

The irregular suitability data set in Figure 13 was also used to test the sample operator in (18) to (25). The data set has 238 land units with an average area of 23.28 square kilometers and an average perimeter of 36.72 kilometers. Figure 9 shows the mean solution time for ten root land units randomly selected from the tessellation.

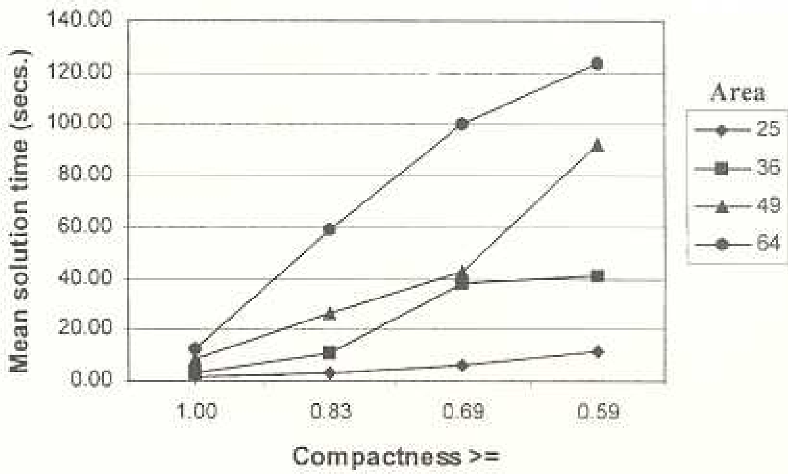

FIG. 8. Compactness, Area (cells), and Mean Solution Time (regular tessellation)

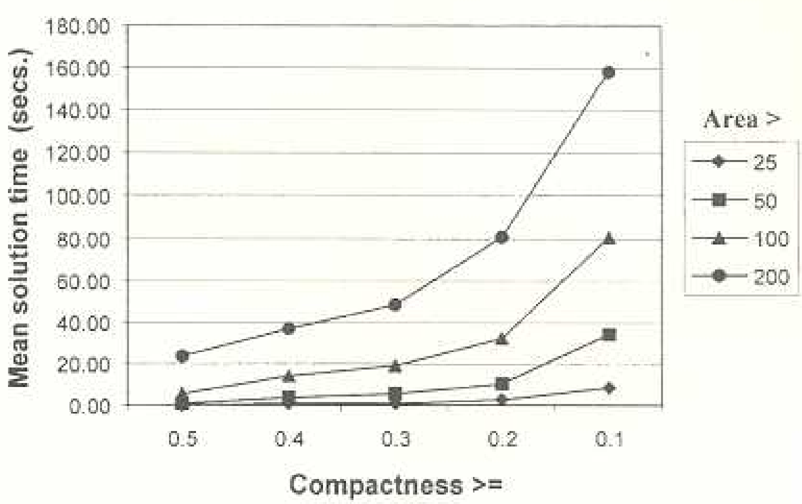

FIG. 9. Compactness, Area (km²), and Mean Solution Time (irregular tessellation)

The two independent variables in the plot are the minimum area of the site (km^2) and its minimum compactness. Site compactness and the approximate feasible neighborhood were defined using equation (17). The minimum site width w was set to 0 to assess the efficiency of the worst-case feasible neighborhood approximation in each case. A similar relationship is evident in this plot, where relaxing the compactness constraint or increasing the minimum area of the site increases the solution time necessary to solve a problem instance. The compactness values range from .1 to .5, where compactness values greater than or equal to .6 in this tessellation are not feasible. This is because a perfect circle has a value of 1 by this measure, and it is difficult to assemble irregular land units into a circular shape.

Improvement Strategies. An assumption to this point is that a site search neighborhood operator must be applied to every land unit in a spatial data set. In many problem contexts, this is not necessary, as screening methods can be used to remove land units from consideration. There are two types of screening that may be performed. The first is *attribute-based screening* or land screening (Dobson 1979). In this case, a land unit is removed from consideration based entirely on its attributes. An example of this type of screening is a constraint where a site may not include a given land use type. Attribute-based screening is powerful but must be exercised with care, as removing a land unit from consideration removes all potential sites that might utilize it as an element. If a large number of scattered land units are screened in a study area, it can be difficult to meet the spatial requirements for a site. For this reason, arbitrary attribute screening thresholds should be avoided (for example, suitability $< x$).

A second type of screening is *contextual screening*. This involves analyzing the contents of a land unit's feasible neighborhood prior to solving a local site search model. For example, given a maximum site area constraint, if the p best noncontiguous land units in the feasible neighborhood do not meet a minimum threshold, there is no point in solving for the root's best contiguous site. This can be accomplished by sorting the land units in each root's feasible neighborhood by some criteria and aggregating the p best to derive an upper bound for the root's best site. Root land units with a best noncontiguous site that do not meet a minimum threshold can be screened from consideration. A central problem is deriving the threshold. One approach is to sort the roots by their best noncontiguous site and solve the top one in the list for its best contiguous site. This provides an initial lower bound whereby any root with a best noncontiguous site below this lower bound can be screened. By proceeding through the sorted list of potential roots, each time a better contiguous site is found, the lower bound improves, and potential roots from the bottom of the list can be screened. Contextual screening can significantly reduce the time to solve for the optimal site in a particular problem instance, as every land unit that is screened from consideration represents one less local optimization problem that must be solved. The benefit of this type of screening varies according to the spatial structure of the input maps and the size of the feasible neighborhood. Identifying effective contextual screening methods in various problem instances is an area for further research.

4.3 Generating Noninferior Trade-off Curves

Contextual screening was used to aid in generating example noninferior trade-off curves between competing objectives. An inferior solution in multiobjective programming is one that can be improved for all objectives (Cohon 1978). Figure 10 depicts a series of noninferior trade-offs between the competing objectives of maximizing site suitability and maximizing compactness for the raster data set. The trade-offs are entirely for contiguous solutions. It would not have been possible to produce these plots using the above model without including the proposed contiguity constraints, as relaxing the compactness constraint would have resulted in a noncontiguous site. The plots show a general trade-off, where increasing the compactness requirement results in a lower total suitability for a site of equal area. The plot also shows that, regardless of the compactness constraint, increasing the area of a site always increases its total suitability in a regular tessellation when suitability values are positive.

Figure 11 depicts a similar set of noninferior tradeoff curves for the irregular data set. In the context of an irregular tessellation, the area and compactness of a site must be allowed to vary. Constraint (19) is the lower bound on the site area, and constraint (21) is the upper bound on the site's perimeter. Together these two constraints control the compactness of the site using equation (17). No upper bound on the area of the site was set. The bound on the perimeter served as an upper bound on the area

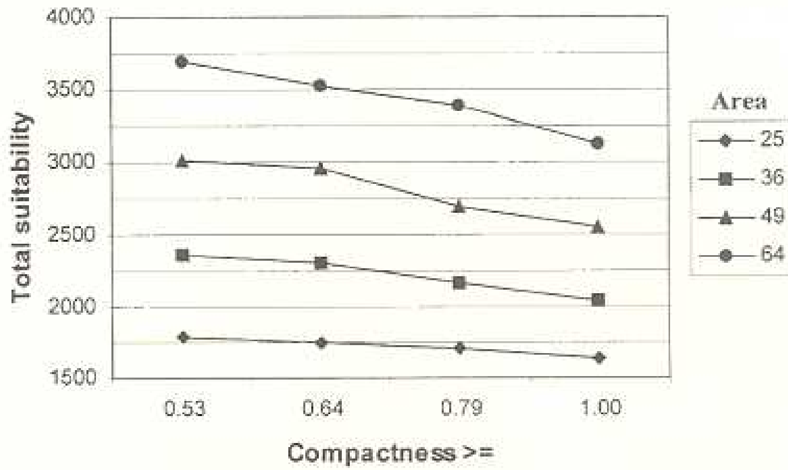

FIG. 10. Noninferior Trade-off Curves for Total Suitability versus Compactness (regular tessellation)

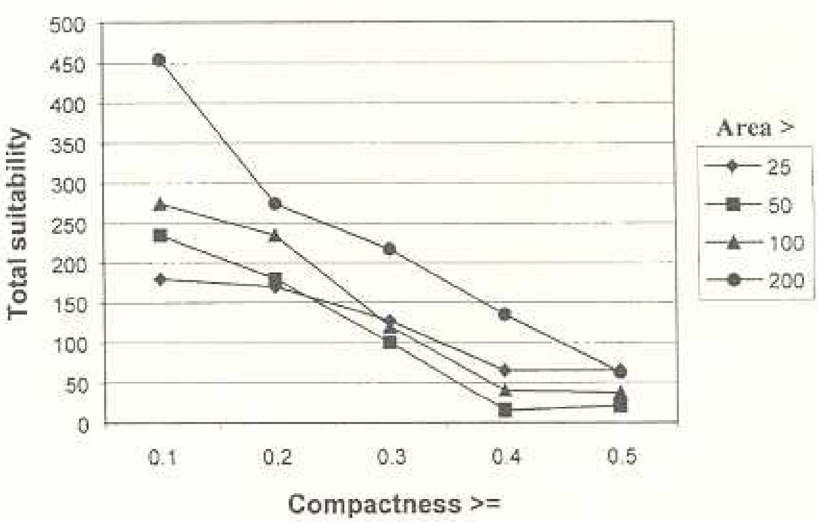

FIG. 11. Noninferior Trade-off Curves for Total Suitability versus Minimum Compactness (irregular tessellation)

so, as the compactness value is relaxed, the site area may get larger in this problem instance. A similar trend to the regular tessellation case is evident, where increasing the desired compactness of a site decreases the total suitability that can be achieved. The plot also shows that when the compactness constraint is minimal, a larger minimum area constraint will result in a site with a greater total suitability. However, as the compactness constraint increases, a site with a larger minimum area (for example, 100) may achieve a total suitability score less than that of a site with a smaller minimum area (for example, 25). In essence, to meet the minimum area and compactness constraints, a site with a larger minimum area may have to migrate to an area of lower suitability to meet these spatial constraints.

On a final note, a constraint was added to the formulation in (18) to (25) to improve the efficiency in generating the noninferior curves. In short, total suitability cannot decrease as site compactness is relaxed from any noninferior point. A simple constraint can be added to the model to quickly rule out a root land unit as possessing a potential noninferior solution. The total suitability for the last identified noninferior solution was used as a lower bound on the total suitability for the next problem. This is a final form of screening where CPLEX™ abandons a 0-1 problem if the relaxed linear solution for the problem instance cannot meet the following constraint:

$$\sum_{i \in N_r} s_i x_i \geq S \tag{26}$$

where S is the total suitability of the last noninferior solution in progressing from a compact solution to a less compact solution.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
0	23	30	13	11	6	9	7	21	90	80	25	4	18	10	5	9	10	14	7	52	36	28	12	12	13	3	13	17	28	20
30	18	23	14	10	4	14	28	18	49	93	51	25	7	10	18	8	8	6	82	58	54	18	19	48	11	5	18	32	33	8
60	9	35	28	12	4	7	10	21	36	29	27	10	7	2	18	10	10	12	11	12	52	22	32	60	55	33	36	27	11	7
90	9	13	4	12	10	13	8	7	3	9	8	3	3	2	3	7	2	9	14	22	75	38	30	60	88	85	22	12	10	4
120	2	4	3	10	6	2	1	2	3	11	3	2	3	1	3	12	9	7	35	83	94	52	32	47	91	49	10	12	18	9
150	2	2	2	9	12	3	2	3	2	1	1	2	2	8	3	5	17	48	64	94	91	86	36	30	38	20	7	3	9	3
180	3	13	3	14	13	3	10	1	2	2	1	7	6	1	1	14	19	55	58	87	94	85	34	33	19	28	8	19	3	1
210	12	17	12	2	28	23	11	8	1	1	1	1	1	3	3	21	27	48	36	88	91	63	35	28	28	17	8	14	3	14
240	28	30	18	23	14	6	3	2	1	1	1	2	2	1	3	10	23	41	51	64	64	55	38	14	25	37	2	3	2	1
270	53	29	50	14	34	11	8	13	1	1	1	1	1	1	1	1	5	13	27	22	33	80	55	23	23	17	18	3	3	2
300	83	34	17	6	11	22	11	9	6	2	2	1	3	3	3	3	2	14	17	26	20	19	14	2	10	8	5	2	14	3
330	44	36	19	17	12	28	22	10	1	1	1	1	1	8	20	7	6	4	10	30	14	8	2	2	3	8	3	3	2	1
360	79	54	36	17	13	18	9	3	2	1	2	1	3	20	35	12	13	11	25	45	8	2	2	2	3	3	2	2	1	2
390	56	21	33	24	10	7	13	11	3	3	2	7	12	73	57	80	88	30	34	30	8	10	1	1	2	3	3	3	3	6
420	41	39	27	19	25	10	5	10	8	3	2	11	27	52	79	94	94	91	49	28	9	10	6	2	1	2	2	1	10	17
450	53	55	83	36	62	30	10	6	8	2	23	19	27	8	52	77	58	30	80	86	25	10	2	1	1	1	2	1	2	10
480	50	76	87	71	79	3	6	2	6	3	12	19	38	19	79	36	30	34	28	23	23	20	10	8	1	1	1	2	3	17
510	7	10	49	91	51	2	7	7	23	3	9	27	14	13	33	28	20	14	16	7	13	11	13	8	2	3	3	1	5	3
540	14	81	27	79	19	14	8	30	14	10	5	14	49	14	30	18	10	10	7	10	9	14	13	3	5	5	2	2	3	2
570	30	55	26	36	64	19	1	11	34	52	16	85	17	23	21	43	13	3	9	13	3	6	2	14	5	1	9	3	4	10
600	58	76	63	79	50	27	7	27	52	11	48	27	30	19	49	43	33	9	18	14	12	9	3	3	8	8	13	21	9	17
630	65	24	89	55	36	25	8	4	7	18	18	17	25	35	18	56	19	25	9	3	2	3	3	6	10	3	3	18	37	11
660	27	41	35	14	14	7	3	2	3	5	3	6	10	4	14	8	30	9	2	1	1	3	10	5	3	2	3	10	7	8
690	80	83	87	50	18	12	3	2	8	3	3	10	8	11	8	11	12	17	3	3	7	14	4	5	10	18	3	3	1	19
720	81	84	91	33	9	8	10	9	1	10	1	1	9	8	10	2	3	3	2	1	3	2	1	9	14	8	17	2	2	1
750	90	82	92	34	18	6	5	2	2	1	3	1	2	13	6	1	1	1	1	1	1	1	1	2	5	16	14	8	3	3
780	29	65	87	65	54	9	2	2	1	11	1	1	3	2	3	1	1	1	1	1	1	1	1	1	1	5	8	8	3	2
810	19	19	55	49	11	13	1	1	1	1	3	1	3	3	2	10	1	1	1	1	1	1	1	1	7	1	5	1	2	2
840	3	18	33	10	7	1	1	1	3	1	1	1	2	2	1	2	1	1	1	1	1	1	1	1	1	3	9	3	2	1
870	3	8	13	10	30	2	2	1	1	1	1	1	2	1	1	2	1	1	1	1	1	2	2	1	1	8	2	3	2	3

FIG. 12. Regular Suitability Data Set (900 spatial units)

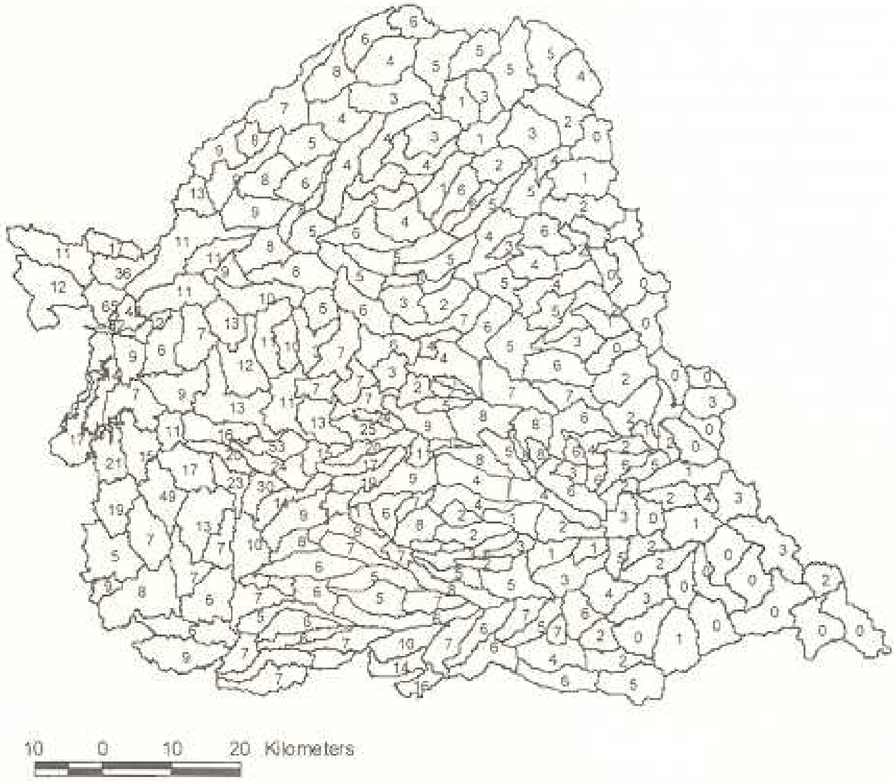

FIG. 13. Irregular Suitability Data Set (258 spatial units)

5. CONCLUSION

The constraints proposed in this paper represent a viable, general approach to the contiguity problem in site search modeling. The associated spatial decomposition method uncouples the historic relationship between the total number of land units in a spatial data set and a model's solution time. However, the proposed approach leaves the difficulty in solving a problem instance a function of the number of land units in the feasible neighborhood. For this reason, the efficiency of this approach improves as the desired site area is decreased and the desired site compactness is increased. Although most practical siting problems are for relatively compact sites, the benefit of these constraints is probably not in solving applied siting problems but in experimenting with new site search problem formulations and benchmarking the solution quality of heuristic algorithms.

SPC constraints are a very efficient set of contiguity constraints, as they add no variables to a model and only one constraint per land unit. However, they overlook some noncompact contiguous patterns in a tessellation and cannot guarantee the optimal solution in all problem instances. The likelihood that they identify the optimal, contiguous solution increases as the desired site compactness increases. The degree to which this is an issue depends on the problem context. In the least, they offer the opportunity for spatial analysts to rapidly test experimental formulations without developing a unique solution algorithm for each model.

We presented two improvement strategies to aid in efficiently searching for the optimal contiguous site using SPC constraints. The first is attribute-based screening, where land units that cannot possibly be in any site are removed from the search process. A local site search model does not need to be solved for these land units, and they should never appear in another root's best site. The other improvement strategy is contextual screening, which involves preprocessing potential root land units to determine if the overhead associated with solving an optimal site search model for a land unit is warranted. The benefit of these two screening strategies can be substantial but varies according to the problem instance.

There are a number of interesting directions to pursue in the context of this research. Experimenting with new formulations that utilize the constraints proposed in this paper is one obvious area. This might involve new shape measures like elongation or perforation (Wentz 2000), new methods for representing geographic features and the landscape in general (Goodchild 1992), or new applications. Another area in need of research is general heuristic methods for solving site search problems like Interchange, TABU, and region-growing. Solution quality can be assessed with a model that includes the constraints in this paper, and a good algorithm might find a better contiguous solution in noncompact cases. Heuristic algorithms are the key to integrating site search models with a GIS. The constraints may also have value in approaching multi-region site search problems as well as volumetric searches in a three-dimensional spatial framework. Finally, the notion that the most efficient proposed constraints (SPC) overlook a subset of non-compact site patterns highlights the need for a tractable constraint form that captures all contiguous site patterns regardless of spatial characteristics.

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