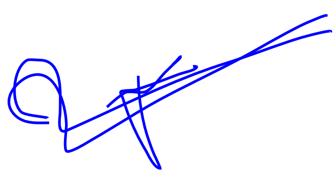
Locating and Dispatching Two Types of Ambulances Considering Partial Coverage: Stochastic Integer Programming Models and Heuristics

By Beatriz Garcia Ramos



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WORD COUNT 19354
TIME SUBMITTED 18-MAR-2025 06:18PM
PAPER ID 115229008

Universidad Autónoma de Nuevo León

Facultad de Ingeniería Mecánica y Eléctrica

Subdirección de Estudios de Posgrado

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como requisito parcial para obtener el grado de

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FACULTAD DE INGENIERÍA MECÁNICA Y ELÉCTRICA
SUBDIRECCIÓN DE ESTUDIOS DE POSGRADO

Los miembros del Comité de Tesis recomendamos que la Tesis *Locating and Dispatching Two Types of Ambulances Considering Partial Coverage: Stochastic Integer Programming Models and Heuristics*, realizada por el alumno Beatriz Alejandra García Ramos, con número de matrícula 1550385, sea aceptada para su defensa como requisito parcial para obtener el grado de Doctora en Ciencias en Ingeniería de Sistemas.

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San Nicolás de los Garza, Nuevo León, 18 de marzo de 2025

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Acknowledgment

Agradezco a Dios principalmente porque durante mi vida ha puesto en mi camino a las personas correctas y me ha regalado los momentos correctos para mejorar personal y académicamente. Agradezco que me permita tener salud y que me demuestre su amor directamente y a través de las personas que están a mi alrededor.

Agradezco a mis padres Beatriz Ramos Larralde y Jesús García Gámez que en cada etapa de mi vida me han apoyado para seguir creciendo como mujer y como profesionista. Gracias por su acompañamiento en mi camino, por sus consejos, su amor y su protección, y por siempre preocuparse por el bienestar de mi hermana y mío para que tengamos una gran calidad de vida y valoremos cada instante.

Agradezco a mi hermana Karina Guadalupe García Ramos que ha sido mi compañera de desvelos y distracciones y que me ha impulsado a recordar que la vida no es cien por ciento el trabajo, sino que también hay que disfrutar de los buenos momentos en compañía de la familia y los amigos.

Agradezco a mis amigos desde maestría Alberto, Delavy, Gabriela, Mayra, Alan, Astrid, Citlali, Pablo y Yessica, quienes han sido parte importante de mi vida. Ellos han sido de gran apoyo cuando necesito hablar sobre mis problemas académicos porque son quienes más me entienden, y además de eso siempre están para mi en las buenas y en las malas. Agradezco también a mis amigos Obed, Arturo, y Paola, que me han acompañado en muchos años de mi vida y quienes son para mí como una familia. Y especialmente agradezco a mi mejor amigo Víctor Domínguez, con quien desahogo mis penas y con quien comparto mis alegrías día con día a pesar de vivir a ciento tres kilómetros de distancia, tú has sido mi soporte, mi paño de lágrimas y quien pone color a mi vida.

Agradezco al Consejo Nacional de Humanidades Ciencias y Tecnología (CONAHCyT) por la beca de manutención que me fue otorgada, con la cual pude facilitarme mis estudios al utilizar ese ingreso en transporte, comida e incluso en la investigación que durante cuatro años estuve realizando. Además, parte de mi investigación fue financiada por el Proyecto CF2023-I-880, del Programa de Ciencia de Frontera del CONAHCyT.

Agradezco a los Doctores que se encuentran en el Programa de Posgrado en Ciencias en Ingeniería en Sistemas por ser parte de mi crecimiento académico y por darme sus enseñanzas sobre lo que un investigador y un docente puede llegar a ser. En particular, agradezco a los miembros de mi comité de tesis, la Dra. Iris Abril Martínez Salazar, la Dra. María Angélica Salazar Aguilar, el Dr. Vincent André Lionel Boyer, la Dra. Yajaira Cardona Valdés y la Dra. Irma Delia García Calvillo, por haber leído mi tesis y haberme apoyado en las correcciones de la misma. De manera especial, agradezco a mi asesor Roger Z. Ríos Mercado y a mi coasesora Yasmín A. Ríos Solís que durante toda mi investigación estuvieron apoyándome y dirigiendo la misma.

Agradezco a la Facultad de Ingeniería Mecánica y Eléctrica (FIME) y a la Universidad Autónoma de Nuevo León (UANL) por ofrecer un gran programa académico en el Posgrado en Ciencias en Ingeniería de Sistemas en el cual pude concluir una etapa académica que me ayudará a superarme a mí misma y por la beca que se me proporcionó para llevar a cabo mis estudios. Agradezco también las instalaciones que proporcionan a nuestro posgrado para que podamos tener un laboratorio en donde trabajar y salones donde podamos tomar las materias que se nos imparten.

Abstract

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Título del estudio: Locating and Dispatching Two Types of Ambulances Considering Partial Coverage: Stochastic Integer Programming Models and Heuristics.

Número de páginas: 59.

The thesis aims to study a decision-making problem on the Emergency Medical Service (EMS) systems and implement algorithms to solve it. The objective is to improve Mexico's 911 system by locating and dispatching ambulances to maximize patient attention at the minimum response time possible. The problem we studied is referred to as the the Emergency Vehicle Covering and Planning Problem (EVCP), which is modeled as a two-stage integer stochastic program. In the first stage, the location of the ambulances must be determined prior to the occurrence of the accidents. Then, after the accidents become known, the ambulance dispatching decisions are taken in the second stage.

The proposed solution methodology is to determine the location and dispatch of the ambulance based on scenarios. These scenarios show how the system works. That is, whether a demand point, which is a place where a patient could need attention, has to be served by one ambulance or more than one ambulance. In this investigation, we study a finite number of scenarios to determine where to locate ambulances and how to dispatch them to demand points according to the system.

The study method analyzes integer stochastic models to adapt some ideas for a practical solution. We are interested in improving Mexico's EMS system, which is different from first-world EMS systems. These differences lead us to be unable to use mathematical models as we find them in the literature; nevertheless, we can build an integer stochastic model based on combining ideas proposed before and new concepts from us.

One of the contributions, from the modeling perspective, is to introduce partial rate coverage in the objective function. Typically, partial coverage is used in deterministic models because of its simplicity. Another contribution is to propose a feedback approach to solve the ambulance location and use it as an input for the proposed stochastic programming model.

The solution method was fully assessed in a wide collection of problem instances.

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Chapter 1

Introduction

Emergency Medical Services (EMS) systems provide medical care for people who suffer a medical incident. These systems control emergency call services received at the emergency number established for emergencies, commonly 9-1-1. These systems have two phases. As we can see in Figure 1.1, the first phase is the response to an emergency call: an operator responds to the call and identifies the type of emergency, such as medical, security, or fire. The operator asks some questions to identify the type of emergency (dismissing prank calls) [35]. If the patient needs medical attention, the operator contacts an ambulance (usually the nearest) and asks for attention at the emergency scene. The second phase is the response of an ambulance: paramedics prepare to go to the emergency scene, the ambulance is equipped with the material resources needed to attend to the patient, the ambulance leaves its base, arrives at the scene, treats the patient, leaves the scene and arrives at a hospital (commonly the nearest) if necessary, and finally returns to its base to wait for another emergency call [20].

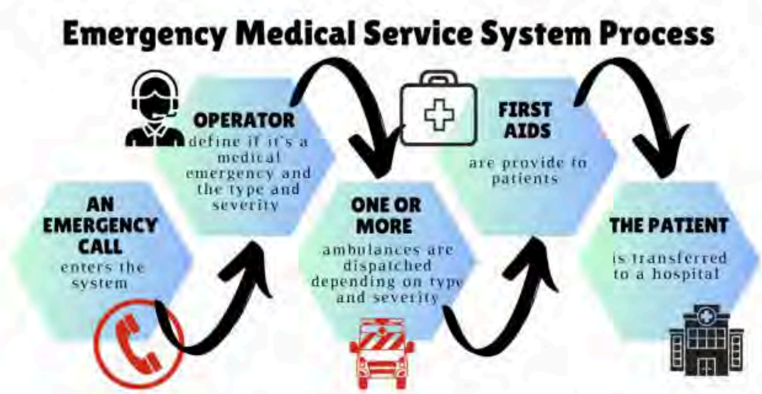


Figure 1.1: An Emergency Medical Service System Process.

EMS systems have significantly impacted operational research and medical investigations in the last decades [1, 7, 45]. Scientists are concerned about the impact of calls emergency' average response time for attending a patient who suffers a medical incident. Moreover, the cost of purchasing material resources, medical vehicles, or building a new medical center, among other things, can limit patient service [31]. Lack of human and material resources can cause insufficient attention in patients [2, 26, 33].

The most studied problem is reducing the average response time when an emergency call arrives at a call center and someone needs medical attention [14]. The objective is to provide, as soon as possible, the initial treatment for a patient who has a medical problem caused by an accident, trauma, or a natural disaster to reduce the mortality of the patients. For a short response time, it is more likely that people will survive. Another objective that EMS system problems consider is to maximize coverage to handle all emergency calls that enter the system [19]. In addition, there are some problems that consider improving patient survival or reducing the patients' mortality [56].

Our interest is in the EMS systems of Mexico. In Mexico, there exists the 9-1-1 number controlled by the C-5 organization (Centro de Coordinación Integral, de Control, Comando, Comunicaciones y Cómputo del Estado), which receives emergency calls. Some calls are for medical emergencies, others for police emergencies, and still others for fire emergencies. When a call enters the system and an operator decides that it is a medical emergency, the operator has to determine if it is necessary to send an ambulance or not. In addition, a doctor can continue the call to guide the person on the phone if the patient needs immediate attention while the ambulance arrives. Paramedics can then attend to the patient and transfer the patient to a hospital [24].

We propose a two-stage stochastic programming model with recourses for ambulance location and dispatching, considering two service providers to obtain a coordinated EMS system to solve those problems. The following sections present the background investigations about EMS systems (Chapter 2) and the usually used models. We describe the problem and the factors that affect the EMS system in Mexico (Chapter 3). In addition, we describe (Chapter 4) the instance generation and experimental assessments. Then, we solve the problem and define the model (Chapter 5) used to do the experiments. Finally, we show conclusions (Chapter 6) that we obtain from experiments described in the previous section. In this section, we also propose future work.

1.1 MOTIVATION

Our interest is to improve the Emergency Medical Service System in Mexico, particularly in Nuevo León. World Health Organization establishes that there must be four ambulances per 1,000 people, which is not available in all states of our country.

Due to the lack of available ambulances, emergency calls are answered late. However, buying more ambulances so that there are more available to distribute is not sufficient [21]. Improving the distribution of ambulances and locating and dispatching them in a better way could improve the EMS systems.

1.2 PROBLEM DESCRIPTION

We address a problem where we have to locate a limited number of two heterogeneous types of ambulances in different city points and dispatch them to the sites where accidents occur. Our problem considers the uncertainty of the accident (demand) points. Our goal is to maximize the total and partial coverage and the response time in which the patients receive medical first aid. We propose a two-stage quadratic stochastic program for this problem. In the first stage, the location of the limited number of two types of ambulances is decided. In the second stage, the dispatching of the ambulances to accidents is determined. This stochastic model allows partial coverage of the accidents by the ambulances based on a decay function. Given that the model is intractable even for medium-sized instances, we propose a location-allocation methodology that relies on the solution of an auxiliary surrogate model, which is faster to solve. This location-allocation heuristic consists of two phases. In the location phase, the location of the ambulances is obtained by solving the surrogate model. Then, this information is the input for the allocation phase, where the original model is solved. Experimental results show the effectiveness and efficiency of this proposed approach, obtaining high-quality solutions in reasonable times.

1.3 HYPOTHESIS

This investigation hypothesizes that we can model the Emergency Vehicle Covering and Planning problem as a stochastic programming model with resources based on different scenarios. These scenarios consider accident types at each demand point; many of them can help to know what to do when a situation occurs in the 9-1-1 system. The ambulance location and dispatching in the system are optimized.

1.4 Objectives

This investigation aims to improve an Emergency Medical Services System considering partial coverage. The main idea is to obtain an optimal ambulance location and optimal policies for ambulance dispatching. The system we consider for solving the problem includes different factors that affect the system. Those factors are:

- Various types of accidents and variations on maximal response times depending on accident types;
- Different ambulances types, which are ambulances for basic life support (BLS) and ambulances for advanced life support (ALS);
- There is variation in demand points depending on the day of the week and the hour of the day, which can be considered different scenarios.

The objective of solving the problem is to create a scenario-based stochastic programming model with resources considering more than one service provider involved in the system to respond to incoming emergency calls.

1.5 Scientific dissemination

Over these years, this investigation has been presented at different national and international conferences. The national conferences are CSMIO (Congreso de la Sociedad Mexicana de Investigación de Operaciones) in the years 2021 (online), 2022, and 2023; CSMM (Congreso de la Sociedad Matemática Mexicana) in the years 2021 (online) and 2023. Also we participated in ELAVIO (Escuela Latinoamericana de Verano en Investigación Operativa) in the year 2022; the *Coloquio VNL de Gráficas, Combinatoria y sus Aplicaciones* in the year 2023 and some seminars in one middle school (Preparatoria 7 Puentes) in the year 2022 and two high schools (Facultad de Ciencias Físico Matemáticas and Facultad de Ingeniería Mecánica y Eléctrica), in the year 2021 and 2024, of the UANL (Universidad Autónoma de Nuevo León). The international conferences are CLAIO (Latin-Iberoamerican Conference on Operations Research) in the year 2024; and the INFORMS ALIO-ASOCIO International Conference in Medellín, Colombia, also in the year 2024.

An article of this investigation has been submitted and is under review.

Chapter 2

Background

Emergency Medical Services (EMS) systems provide basic but urgent in-situ medical care for people who suffer a medical incident and then transport patients to hospitals [5, 9, 46]. When scientists talk about EMS systems, many terms are used to explain the problem. Two of these terms are demand points and potential sites. Demand points are sites where an emergency call is usually made. Typically, there is a different demand for each point, depending on the number of calls made within a period. Potential sites are places where a vehicle (ambulance) could be located if necessary to cover some demand points, either statically or dynamically.

The first phase of an EMS is the response to an emergency call by an operator that identifies the emergency type: accident, medical, security, fire, etc. The second phase is dispatching one or several ambulances to the emergency scene to provide urgent medical care. Some emergency situations, such as a multiple-car accident, may involve several people; thus, more than one ambulance could be needed. Moreover, different types of ambulances may be required in an emergency: Basic Life Support (BLS), usually with two Emergency Medical Technicians (EMTs), and Advanced Life Support (ALS) units with an EMT, an advanced EMT, and one or two paramedics. The third phase involves the treatment of the patients by paramedics and their transport to a hospital [5].

EMS systems in developing countries, as is the case in Mexico, lack around 30-60%[1] of the number of ambulances suggested by the World Health Organization (WHO), which is at least four ambulances per 100,000 people [16]. For the Red Cross, an EMS operating with this small number of ambulances is considered similar to a war situation[1]. Thus, one of the main contributions of this work is to deal with the problem of deciding whether an emergency will be totally or partially covered. Sadly, some emergencies may remain uncovered by an emergency unit.

[1] Anonymous interviews done by the authors.

In general, the main objective of emergency vehicle planning problems is to reduce the average response time of a patient's initial treatment administered by a paramedic in an emergency [3, 8, 50, 51]. In fact, the speed and number of ambulances dispatched to accidents are crucial. Each ambulance has a response time for travel from the potential site where it is located to the demand point where the patient will be cared for. Every minute of delay in treatment in a cardiac patient reduces the probability of survival by 24% [44].

There are many models to solve the problems of EMS systems divided into deterministic, probabilistic, and stochastic problems, which use different solution methods to solve them [17]. The first problem that we studied are the static ones.

2.1 Static models

These models are used to solve a system that only considers a particular point in time. When these models are used to solve EMS systems, it refers to allocating ambulances that will not be moved from the base.

There are two early models for static problems: Location Set Covering Model (LSCM) and Maximal Covering Location Problem (MCLP), which are problems focused on covering the maximal demand points in the entire zone. However, over time, these problems evolved according to the needs of the Emergency Medical Services, as will be defined below.

2.1.1 Deterministic models

Deterministic models were proposed to solve static problems because sometimes emergency calls need to be attended for different vehicle types. Most of them are covered once, like the Backup Coverage Problem or the Double Standard Model, which use two different radii of coverage [36]. Alternative deterministic models are the tandem equipment allocation model or the Facility-Location Equipment-Emplacement Technique, which consider two types of vehicles (one for BLS and another for ALS), or the fact that sometimes more than one ambulance has to be located on a potential site to maximize that a demand point is covered twice.

In the thousands was introduced by Berman et al. [11] a decay function to classify coverage as full, none, and partial coverage in a generalized MCLP model. They added a weighted demand for each node covered, considering the distance between facilities and demand points. The objective aims to maximize the total demand weight covered by all facilities when a determined number of facilities are located.

A year later, Karasakal and Karasakal [34] introduced partial coverage to the MCLP problem. This problem aims to maximize the coverage level for all demand points, deciding where to locate a certain number of facilities within the available potential sites. The model was based on a p-median formulation and classified coverage into three levels: totally covered, partially covered, and not covered. They defined a monotonic decay function that decreases according to the distance between the facility and the demand point for partial coverage. The distance between a facility and a demand point must be less than or equal to the maximum full coverage distance established to consider total coverage. Demand points are considered not covered for a facility if the distance between it and the demand point is greater than or equal to a maximum partial coverage distance. To solve large-size problems, they used a Lagrangian relaxation.

A decade later, Wang et al. [53] used an extension of the MCLP Problem to maximize coverage for fire emergencies establishing a travel cost between potential sites and demand points. This extension considers a partial distance and quantity coverage for multi-type vehicles to locate and dispatch them. The partial distance is calculated with a decay function that decreases according to the increase in vehicle response time. Quantity coverage determines whether an emergency is fully served or not, comparing the number of vehicles dispatched with the necessary quantity. For this problem, they have to consider demand priority to know where vehicles must be located and the patient's classification to decide how to dispatch them.

As an extension of Double Standard Model, Dibene et al. [22] created the Robust Double Standard Model. They added demand scenarios to the original Double Standard Model problem. These scenarios divide weeks into workdays and weekends, divided into four periods: night, morning, afternoon, and evening. They added eight scenarios applied to optimize the Red Cross Tijuana, Mexico system, increasing the coverage of demand points to more than 95% locating ambulances at different points of the city that are not the original bases.

For us, it is imperative to gather all this information for our project as it provides information on the different types of accident coverage and the improvement in the location of ambulances. A thorough understanding of these models and their effectiveness will enable us to optimize our ambulance location strategies.

2.1.2 PROBABILISTIC MODELS

In the 1980s, some researchers thought about the problems involved with probabilities. One of these probabilities involved in EMS systems is the probability of an ambulance being busy responding to an emergency call. This probability is called the *busy fraction*. The Maximum Expected Covering Location problem uses this probability. An extension of this model is the Maximum Expected Covering Location problem with time variation, which considers travel speed variations during the day. Another extension is the Adjusted Maximum Expected Covering Location model, which considers different busy probabilities for each potential site to locate ambulances. All these models can use the hypercube queueing model to calculate the busy fraction [25].

Other models were proposed to maximize the coverage of the demand points with a probability α used to calculate the busy fraction; one of them is the Maximal Availability Location problem I, which considers the busy fraction is the same for all potential sites. Another model is the Maximal Availability Location problem II, which uses the hypercube model to assume different busy fractions for each potential site.

There exist more probabilistic models created in the nineties. The first is an extended version of the LSCM called Rel-P; this version considers that more than one ambulance can be located at the same potential site, but each potential site has a probability to have ambulances that are available to respond to a call and considers the probability of the busy fraction, too.

The second model is the two-tier model, which considers two types of vehicles to allocate at potential sites (BLS and ALS), considering two different coverage radii and having an associated probability for the combination of how many ALS vehicles can be located at the radius A, how many ALS can be located at the radius B, and how many BLS vehicles can be located at the same radius B for each demand point.

Laura Albert and Maria Mayorga researched the EMS systems of Hanover, Virginia. All these investigations about Hanover, Virginia, were applied to this county to obtain practical solutions, but all models can be used to any other EMS system changing data inputs.

The first research focuses on considering a new approach to calculate the response time threshold (RTT), a class of EMS performance measures [38]. The approach uses the patient survival rate, considering that patients have cardiac arrest and random response times that depend on the distance between demand points and potential sites instead of patient outcomes, which is most used. Then, they use these measures on a hypercube model to evaluate different RTTs needed to input a model that considers fire stations and rescue stations to be potential sites where ambulances could be located and distributed in Hanover's rural and urban areas. This model optimizes the location of ambulances on potential sites to maximize patient survival.

Later, Albert and Mayorga et al. used the performance measures as input to the performance measure dispatching problem [23]. Based on the survival rate of the patient, they used a Markov decision process that identifies the best and most robust RTT to maximize the level of coverage, prioritizing the location of the patient. The research concludes that the optimal survival rate is obtained when the system has an RTT of eight minutes [39]. However, this time for RTT does not apply to Hanover because of the number of ambulances they have, so they started a pilot program called the quick response vehicle to have more vehicles for patient attention, obtaining a nine-minute RTT; these new vehicles are ALS vehicles without transporting patients to the hospital, only attending patients at the scene, and BLS ambulances transport patients if necessary [40]. The idea of including these quick response vehicles is to minimize the need to use ALS ambulances.

When talking about optimizing EMS systems, one can also speak about dispatching. Bandara et al. [8] considers demand priorities for the different emergency calls arriving at the system. The objective is to maximize the patient's survival probability when an ambulance is dispatched to demand points calculating a reward for each dispatch. They used a Markov Decision Process model formulation to determine the optimal dispatching strategies for an EMS system.

Toro-Díaz et al. [50] involves location and dispatching decisions for EMS vehicles in the same mathematical model with two focuses, minimizing the mean response time that takes since an emergency call is received and maximizing the expected coverage demand, using a continuous-time Markov process to balancing flow equations needed to control the busy fraction for each ambulance. Balancing these equations takes exponential time, and authors consider a genetic algorithm to obtain some solutions and combine them to create new solutions to reduce computational time. This genetic algorithm was applied to Hanover, Virginia, and when they have midsize problems, the nearest dispatch rule is the best solution. It can vary depending on the zone where it is applied.

Amorim et al. [3] involve a simulation inputting an initial solution to decide if ambulances have to stay at the potential sites establishes when the mathematical model is solved or if some of them have to be moved to another potential site. To decide how to proceed, they used different day's period times when traffic in the city is changing on each week's days, which they called *scenarios*, to maximize the patient's survival.

Transitioning from probabilistic models to scenario-based models in the management of EMS systems is imperative, as scenarios provide a more robust framework for addressing uncertainty. Probabilistic models frequently require assumptions regarding the likelihood of various events, such as the busy fraction of ambulances, which can be challenging to estimate accurately and may not adequately capture real-world complexities. In contrast, scenario-based models allow for the incorporation of various demand and traffic conditions, enabling more realistic and flexible planning. Using scenarios, researchers can ensure a more reliable and adaptive emergency response system, improving dispatching decisions as we can see in the next section.

2.1.3 Stochastic models

Recently, ambulance location, allocation, and dispatching problems involved uncertainty at demand points to have a more realistic model. This uncertainty is caused because it is impossible to know when the system will receive an emergency call.

In Boujemaa et al. [15], a two-stage stochastic model with recourse is proposed. The first stage of the model determines where to open ambulance stations with a fixed cost to open them. For the second stage, allocation is determined considering the expected traveling cost from ambulance stations to demand points. A demand point is considered covered if an ambulance station is within a threshold value. And some important factors that they included are two different demand types: life-threatening calls and non-life-threatening calls; two ambulance types: ALS and BLS; and scenarios structured by two data for each demand point: number of life-threatening calls and number of non-life-threatening calls respectively. This problem minimizes the ambulance location-allocation cost and is solved by a Sample Average Approximation algorithm that allows computing lower and upper bounds for problem solutions and providing the corresponding optimality gaps.

Later, Bertsimas and Ng [12] implemented stochastic and robust formulation for ambulance deployment and dispatch for a problem constructed as a graph. These formulations were compared with Maximum Expected Covering Location problem and Maximal Availability Location problems and aimed to minimize the fraction of late-arrivals without requiring ambulances to be repositioned, sending to demand points the closest available ambulance, and maintaining a call at a queue if there are no ambulances available at the system. The demand has the problem's uncertainty, which was constructed by four demand types: single for each demand point, local for the demand point and the nearest points, regional for a region of the entire zone, and global for the whole area. They determined a deterministic equivalent model to solve the stochastic formulation, and for the robust formulation, they did a column and constraint algorithm.

Recently, Yoon et al. [55] studied a two-stage stochastic problem for locating and dispatching two types of emergency vehicles: ALS and BLS. The first stage locates the ambulances at potential sites, while the second stage dispatches ambulances from the places where they were located to demand points when a call arrives. The objective is to maximize expected coverage considering a penalty when a call is not serviced. One difference from other problems is that the system manages multiple emergency call responses, divided into high-priority and low-priority calls. Any vehicle type can serve low-priority calls. However, high-priority calls have two options for the service: the first option is that these calls can be responded by an ALS ambulance. The second option is that a nearby BLS ambulance can service the call first, followed by an ALS ambulance that is not necessarily closed. A Sample Average Approximation deterministic equivalent formulation solved this problem for small data, while a Branch-and-Benders-Cut Solution solved a large-scale problem. And they did another problem version considering non-transport vehicles that can attend patients without translating them to hospitals.

Some works propose stochastic programming models based on call-arrival scenarios as a bundle of calls, the total number of emergency calls in each demand node during a given period. As in this work, a two-stage stochastic program deploys the ambulances in the first stage and dispatches them to respond to demand in the second stage. Beraldi and Bruni [10] and Noyan [43] induce a reliability approach using probabilistic constraints. Nickel et al. [42] minimize the total cost of locating ambulances while ensuring a minimum level of coverage. By considering a bundle of calls, they address the volume of calls during a short period, such as the Friday night hours. Bertsimas and Ng [12] implemented stochastic and robust formulations for ambulance deployment and dispatch to minimize the fraction of late arrivals without requiring ambulances to be relocated, sending to demand points the closest available ambulance, and maintaining a call at a queue if there are no ambulances available in the system.

2.2 HEURISTICS, METAHEURISTICS AND MATHEURISTICS

Scenario based stochastic programming problems have a significant advantage over deterministic approaches due to the variety in their objectives of the decision process, and their constraints and their relationship with uncertainty in the systems they study [13]. However, recent advancements in the field of metaheuristics have significantly influenced the approach to stochastic combinatorial optimization problems, which are defined by their inherent uncertainty and discrete nature.

In the past decades traditional optimization methods often fall short in effectively addressing these challenges; hence, metaheuristics have emerged as a powerful alternative. Recent trends highlight the development of hybrid metaheuristics that combine multiple approaches or integrate exact methods, adaptive metaheuristics that dynamically adjust parameters based on intermediate results, and multi-objective optimization techniques to manage conflicting objectives [29].

Emergency Medical Service systems also involved these procedures to improve them. Chanta et al. [18] presents a novel approach to optimize the location and dispatch of emergency response units. The proposed methodology combines Tabu Search with an embedded queuing model to address both the spatial placement of units and the dynamic dispatching decisions necessary to respond to emergencies effectively. By incorporating queuing theory, the model accounts for the stochastic nature of emergency incidents and response times, resulting in a more responsive and efficient emergency service system. This hybrid approach demonstrates significant improvements in minimizing response times and maximizing coverage, which is crucial for effective emergency management.

Later, Juan et al. [32] reviewed on simheuristics highlights the integration of simulation techniques with metaheuristics to address stochastic combinatorial optimization problems. This approach enables realistic modeling of systems with significant uncertainty. Advances include hybrid models, adaptive mechanisms, and multi-objective frameworks, further enhanced by modern computational power. These developments have expanded the applicability of simheuristics to real-world problems in logistics, supply chain management, and network design, demonstrating their potential for robust and efficient solutions in uncertain environments.

Recently, Schermer et al. [47] had innovative solutions for the Vehicle Routing Problem by incorporating the use of drones alongside traditional delivery vehicles. The study introduces a matheuristic approach, which combines mathematical programming techniques with heuristic methods, to efficiently solve the Vehicle Routing Problem with drones. The proposed matheuristic demonstrates high performance in optimizing delivery routes, reducing total operational costs, and improving service levels in logistics operations.

Finally, Gruler et al. [28] focuses on addressing the complex multiperiod in - ventory routing problem where demands are stochastic and uncertain. The proposed solution combines variable neighborhood search with simulation-based techniques.

The combination of these methods aims to provide robust and efficient strategies for managing inventory and routing over multiple periods in the presence of demand variability.

All the information gathered will be used collectively to formulate a novel problem that can integrate and utilize the knowledge and strategies mentioned above. This approach aims to provide an advanced and comprehensive method of optimizing EMS systems.

2.3 Contribution

The novelty in this work lies in additionally maximizing the coverage of emergency situations and considering different types of ambulances. When a BLS ambulance is dispatched to an emergency that requires ALS, it can reduce the patient's survival. Thus, this work considers that ALS ambulances can be used as BLS units, but the contrary is not allowed [6]. There are a few works dealing with different types of ambulances as we do in this work. McLay [37] determines how to optimally locate and use ambulances to improve patient survivability and coordinate multiple medical units with a hypercube queueing model. Grannan et al. [27] determine how to dispatch multiple types of air assets to prioritized service calls to maintain a high probability of survival of the most urgent casualties in a military medical evacuation by a binary linear programming model. In Yoon et al. [55], two types of vehicles are considered, but one of them is a rapid vehicle that cannot offer the first care services of an ambulance. Moreover, neither of these works considers partial coverage of the calls.

We denote our problem as the *Emergency Vehicle Covering and Planning* (EVCP) problem which consists of locating the limited number of two heterogeneous types of ambulances in different city points and dispatch them to accident points, considering the uncertainty of accident points, so as to maximize coverage (even if partially) with short medical first aid response time. Usually, the location and dispatching decisions are made separately [9, 22, 54]. In the EVCP problem, these two interrelated decisions are determined simultaneously as in Amorim et al. [3], Ansari et al. [4], Toro-Díaz et al. [51].

We propose a novel two-stage stochastic program for the EVCP problem. The stochastic program locates the limited number of heterogeneous types of ambulances in the first stage, and in the second stage, the dispatching of ambulances to accidents is determined. The EVCP stochastic model allows partial coverage of accidents by ambulances based on a decay function [53]. Similarly to Yoon et al. [55], we generate the call-arrival scenarios by sampling emergency call logs to use them in the second stage of our stochastic model. In this manner, we address the volume of calls during a short period, such as on Friday night. Thus, time is not explicitly measured and it is assumed that a vehicle can be assigned only once during this high ambulance demand period [58]. Boujemaa et al. [15] use a bundle of calls but do not consider a heterogeneous ambulance fleet.

Another contribution of this work is the methodology to solve the EVCP stochastic model. In fact, the proposed model can only be solved for relatively small instances with a restricted number of scenarios. Thus, instead of decomposing the model with Bender's methods as is usually done [49, 57], we propose a location-allocation methodology [48, 52] that relies on the solution in an auxiliary surrogate model, which is faster to solve. We name this method *an scenario-based feedback approach* because the location of the ambulances obtained by this surrogate model is used as input to the original model. Thus, we obtain high-quality solutions in a reasonable time with an off-the-shelf solver without complex decomposition techniques.

Some works use metaheuristic methods to solve their stochastic models. Toro-Díaz et al. [50] integrate location and dispatch decisions for EMS vehicles to minimize the mean response time of an emergency call and maximize the expected coverage demand, using a continuous-time Markov process to balance flow equations that control the busy fraction of each ambulance. A genetic algorithm can solve midsize instances. Some others, such as Amorim et al. [3], use simulation to decide whether ambulances stay at potential sites established by a mathematical model or must be moved to another potential site to maximize patient survival. They work on a complete day period, while we focus on high-demand periods of some hours. Moreover, we do not need a metaheuristic due to the high-quality solutions that we obtained with the scenario-based feedback approach. However, we would like to propose a matheuristic to improve the solution obtained from this approach.

Chapter 3

Problem description

The Emergency Vehicle Covering and Planning problem (EVCP) locates a limited number of two heterogeneous types of ambulances in different city points and dispatches them to emergency scenes, considering the uncertainty of the emergency locations, to maximize the emergency total and partial coverage and the response time in which patients receive medical first aid.

3.1 Description and assumptions

Let us formally describe the Emergency Vehicle Covering and Planning problem. Let set I include the possible demand points where patients may need medical attention in a city or region. This set can be very large, so we consider all the demand points observed in the historical data. In our case study, $|I|$ can be as large as 1500 demand points. Set L provides the potential sites or ambulance stations where ambulances could be located, such as hospitals, firehouses, malls, or similar places where the ambulance and paramedics can wait for emergency calls. We consider instances with up to 30 potential sites for the experimental results. Set K contains the two types of ambulances available in the system: the BLS (labeled with index $k = 1$) and ALS ambulances (labeled with index $k = 2$), which are limited by a known parameter η_k for each type $k \in K$. These ambulances must be allocated to a potential site $l \in L$ and dispatched toward a demand point $i \in I$ if there is an emergency situation.

The travel time of any type of ambulance from a potential site $l \in L$ to a demand point $i \in I$ is given by r_{li}. Ideally, ambulances should arrive in less than τ minutes in a life-threatening emergency. Usually, τ is a fixed value in the range $[8, 15]$. This work also considers that the emergency is not covered if an ambulance takes more than a maximum time $\tau_{\max}$ to arrive. In this case, unfortunately, the accident has probably been dealt with by other means.

Since the aim of the EVCP problem is to reduce the response time of the patient's first medical aid, even if it is in a partial or late way, we define a benefit decay function that only depends on the response time of a location $l \in L$ to any demand point $i \in I$:

$$c_{li} = \begin{cases} 1 & \text{if} \quad r_{li} \leq \tau, \\ 1 - \frac{r_{li} - \tau}{\tau_{\max} - \tau} & \text{if} \quad \tau < r_{li} < \tau_{\max}, \\ 0 & \text{if} \quad r_{li} \geq \tau_{\max}. \end{cases}$$

3.2 Information related to the scenarios

The operational level is represented by a set of scenarios S with a bundle list of arriving calls. Each scenario $s \in S$ represents the realization of accidents at the demand points. Thus, a scenario is represented by the number and type of ambulances needed at each demand point. Recall that an ALS ambulance can be sent instead of a BLS ambulance, but not the other way around. Thus, each scenario $s \in S$ indicates if there is an accident at a demand point $i \in I$ and provides the value a^s_{ki} related to the number of ambulances required of type $k \in K$.

For each scenario $s \in S$, let $I^s \subseteq I$ contain only the demand points $i \in I$ where ambulances are needed, that is, where $a^s_{ki} \neq 0$ for any $k \in K$. We define five different types of ambulance coverage related to the response times cases for each demand point $i \in I^s$:

- Total: the a^s_{ki} required ambulances of each type k are dispatched to i, and all arrive in less than τ time.
- Total-late: the a^s_{ki} required ambulances of each type k are dispatched, but at least one arrives between $(\tau, \tau_{\max})$ time.
- Partial: at least one of the a^s_{ki} required ambulances is not dispatched, for $k \in K$, but all the dispatched ones arrive in less than τ time.
- Partial-late: at least one of the a^s_{ki} required ambulances is not dispatched, for $k \in K$, but at least one of the dispatched arrives between $(\tau, \tau_{\max})$ time.´]
- Null: none of the a^s_{ki} required ambulances arrives in less than $\tau_{\max}$ time, for $k \in K$.

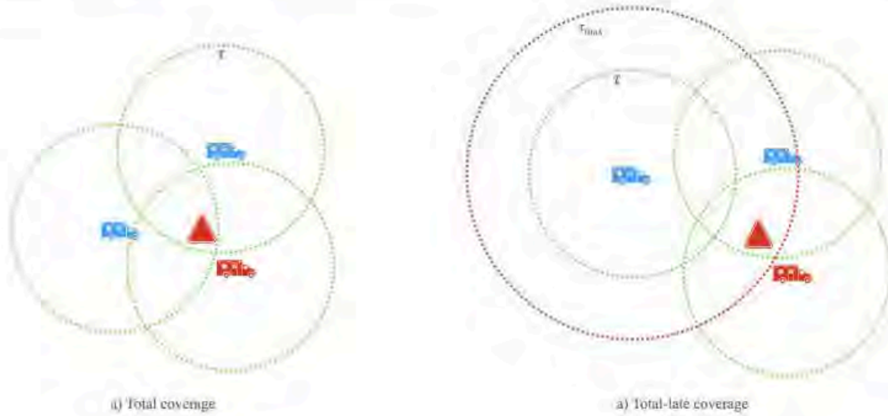


Figure 3.1: Two different coverage cases for a scenario $s \in S$ where $i \in I^s$ requires $a^s_{1i} = 2$ basic ambulances (blue) and $a^s_{2i} = 1$ advanced ones (red). Total coverage in the left: all ambulances arrive in less than the ideal time τ. Total-late coverage in the right: at least one of the ambulances arrives between $(\tau, \tau_{\max})$.

Figure 3.1 illustrates two different coverage cases for a scenario $s \in S$ where $i \in I^s$ requires $a^s_{1i} = 2$ basic ambulances (indicated in blue) and $a^s_{2i} = 1$ advanced one (indicated in red). All ambulances arrive in less than the ideal time τ for the Total coverage (left-hand-side figure). In the Total-late coverage line, at least one of the ambulances is late since it arrives between $(\tau, \tau_{\max})$ (right-hand-side figure). In the Partial coverage, the number of required ambulances is not met, but at least they arrive in less than the ideal time τ. In the Partial-late coverage, not only are there not enough ambulances to cover the demand point, but they arrive late, that is, between $(\tau, \tau_{\max})$. In the Null coverage, ambulances may be dispatched to the demand point, but since the arrival times are larger than $\tau_{\max}$, the demand point is considered uncovered.

Table 3.1 summarizes the sets and parameters used to describe the EVCP problem.

3.3 Maximum expected coverage stochastic formulation for the EVCP problem

The Maximum Expected Coverage (MEC) formulation is a stochastic integer quadratic programming model in which the first stage variables x_{lk} correspond to the

I	set of possible demand points (possible accident places)
L	set of possible ambulance location sites
K	set of ambulance types
η_k	total number of ambulances in the system of type $k \in K$
r_{li}	response time from potential site $l \in L$ to demand point $i \in I$
τ	ideal response time to give the patients the first medical aid in an emergency
τ_{max}	maximum response time to cover an accident
c_{li}	benefit from traveling from potential site $l \in L$ to demand point $i \in I$
S	set of scenarios
a^s_{ki}	number of needed ambulances of type $k \in K$ at demand point $i \in I, s \in S$
I^s	set of demand points for $s \in S$ with at least a value $a^s_{ki} \neq 0$ for $i \in I, k \in K$

Table 3.1: Sets and parameters to describe the EVCP problem.

number of ambulances of type $k \in K$ located at $l \in L$, and the second-stage variables correspond to the ambulance dispatching decisions at each demand point for each scenario $s \in S$:

$$y^s_{lki} = \begin{cases} 1 & \text{if an ambulance of type } k \in K \text{ in location } l \in L \\ & \text{is dispatched to demand point } i \in I^s, \text{ for scenario } s \in S, \\ 0 & \text{otherwise.} \end{cases}$$

We defined the following binary variables related to the *total* and *total-late* coverages related to the response times of the ambulances to the demand point $i \in I^s, s \in S$:

$$f^s_i = \begin{cases} 1 & \text{if demand point } i \in I^s \text{ has a } \textit{total} \text{ coverage,} \\ 0 & \text{otherwise,} \end{cases}$$

$$g^s_i = \begin{cases} 1 & \text{if demand point } i \in I^s \text{ has a } \textit{total-late} \text{ coverage,} \\ 0 & \text{otherwise.} \end{cases}$$

The following sets of binary variables are for the *partial* and *partial-late* coverages of the ambulances to the emergencies:

$$h^s_i = \begin{cases} 1 & \text{if demand point } i \in I^s \text{ has a } \textit{partial} \text{ coverage,} \\ 0 & \text{otherwise,} \end{cases}$$

$$w^s_i = \begin{cases} 1 & \text{if demand point } i \in I^s \text{ has a } \textit{partial-late} \text{ coverage,} \\ 0 & \text{otherwise.} \end{cases}$$

Finally, to indicate a null coverage of a demand point, we define

$$z_i^s = \begin{cases} 1 & \text{if active demand point } i \in I^s \text{ has a } \textit{null} \text{ coverage,} \\ 0 & \text{otherwise.} \end{cases}$$

The MEC formulation is as follows.

$$\max_x \mathbb{E}_{s\in S}[\mathcal{Q}^s(x)] \tag{3.1}$$

where

$$\mathcal{Q}^s(x) = \sum_{i\in I^s}(\alpha_1 f_i^s + \alpha_2 g_i^s + \alpha_3 h_i^s + \alpha_4 w_i^s - \phi z_i^s)$$

$$\text{s.t.} \quad \sum_{l\in L} x_{lk} \leq \eta_k \qquad k \in K \tag{3.2}$$

$$\sum_{i\in I} y_{lki}^s \leq x_{lk} \qquad l \in L, k \in K, s \in S \tag{3.3}$$

$$f_i^s \sum_{k\in K} a_{ki}^s \leq \sum_{l\in L}\sum_{k\in K} c_{li} y_{lki}^s, \quad a_{2i}^s f_i^s \leq \sum_{l\in L} c_{li} y_{l2i}^s \qquad i \in I^s, s \in S \tag{3.4}$$

$$g_i^s \sum_{k\in K} a_{ki}^s \leq \sum_{l\in L}\sum_{k\in K} y_{lki}^s, \quad a_{2i}^s g_i^s \leq \sum_{l\in L} y_{l2i}^s \qquad i \in I^s, s \in S \tag{3.5}$$

$$g_i^s \leq M\left(\sum_{l\in L}\sum_{k\in K} y_{lki}^s - \sum_{l\in L}\sum_{k\in K} c_{li} y_{lki}^s\right) \qquad i \in I^s, s \in S \tag{3.6}$$

$$h_i^s \leq \sum_{k\in K} a_{ki}^s - \sum_{l\in L}\sum_{k\in K} y_{lki}^s, \quad h_i^s \leq a_{2i}^s - \sum_{l\in L} y_{l2i}^s \qquad i \in I^s, s \in S \tag{3.7}$$

$$\sum_{l\in L}\sum_{k\in K} y_{lki}^s h_i^s \leq \sum_{l\in L}\sum_{k\in K} c_{li} y_{lki}^s \qquad i \in I^s, s \in S \tag{3.8}$$

$$w_i^s \leq \sum_{k\in K} a_{ki}^s - \sum_{l\in L}\sum_{k\in K} y_{lki}^s, \quad w_i^s \leq a_{2i}^s - \sum_{l\in L} y_{l2i}^s \qquad i \in I^s, s \in S \tag{3.9}$$

$$w_i^s \leq M\left(\sum_{l\in L}\sum_{k\in K} y_{lki}^s - \sum_{l\in L}\sum_{k\in K} c_{li} y_{lki}^s\right) \qquad i \in I^s, s \in S \tag{3.10}$$

$$\sum_{l\in L}\sum_{k\in K} y_{lki}^s + z_i^s \geq 1 \qquad i \in I^s, s \in S \tag{3.11}$$

$$f_i^s + g_i^s + h_i^s + w_i^s + z_i^s = 1 \qquad i \in I^s, s \in S \tag{3.12}$$

$$x_{lk} \in \mathbb{Z}^+, y_{lki}^s \in \{0,1\} \qquad l \in L, k \in K, i \in I^s, s \in S \tag{3.13}$$

$$f_i^s, g_i^s, h_i^s, w_i^s, z_i^s \in \{0,1\} \qquad i \in I^s, s \in S. \tag{3.14}$$

The objective function (3.1) maximizes the expected value of the weighted coverage of emergencies. The parameters $\alpha_1, \alpha_2, \alpha_3$ and α_4 are normalized weights that ponder the coverage type, and ϕ is the penalty for the null coverage. We assume that every scenario is equally probable since each $s \in S$ represents a sample of the high-demand period in which we are interested.

Constraints (3.2) establish the available number of ambulances per type. Constraints (3.3) establish the relationship between the first and second-stage variables, meaning no ambulances can be dispatched from a potential site if no ambulances are located there. The *total* coverage of an emergency is defined by constraints (3.4). In fact, if the time response of the location of the ambulances to the emergency is less than τ, then all $c_{li} = 1$ and total coverage variables f_i^s can be equal to one, for $l \in L, i \in I^s, s \in S$. The *total-late* coverage is defined by constraints (3.5) and (3.6). Constraints (3.5) allow the total-late coverage variables g_i^s to be one when dispatching variables are active. Meanwhile, constraints (3.6) track the demand points where the response time is between $(\tau, \tau_{\max})$ when the difference in the right-hand side of the equation is positive, that is, when there is a value $c_{lj} < 1$ associated with a dispatched ambulance for $l \in L, i \in I^s, s \in S$. Note that this difference may be decimal, so we include a big M value. The *partial* coverage is defined by constraints (3.7) and (3.8). Recall that in this case, not all needed ambulances are dispatched to emergencies, but those dispatched have an ideal response time. Thus, constraints (3.7) activate variables h_i^s if the number of dispatched ambulances is less than the required ones. Quadratic constraints (3.8) ensure that ambulances dispatched arrive within the ideal response time, that is, their corresponding value $c_{li} = 1$, for $l \in L, i \in I^s, s \in S$. Constraints (3.9) and (3.10) define the *partial-late* coverage. Constraints (3.9) activate the w_i^s variables when the number of required ambulances exceeds the number of dispatched ones. Similarly to the total-late coverage, constraints (3.10) track the ambulances with a response time larger than the ideal one and must be multiplied by a big M. The *null* coverage is activated by constraints (3.11). All coverage constraints are related to the constraint (3.12), which ensures only one type of coverage for each emergency. Finally, (3.13) and (3.14) establish the nature of the decision variables.

The novelty of the MEC model is the stochastic total/partial coverage per emergency by two types of ambulances. However, the related number of variables and constraints is usually large. In addition, constraints (3.8) are quadratic. An integer linear stochastic model could easily be formulated with a classical linearization method. Still, previous experiments showed similar times between the linearized and the quadratically constrained models when solved with integer programming solvers, so we keep the quadratic one for the scenario-based feedback methodology presented in the next section.

3.4 Surrogate-based feedback method for the EVCP problem

The EVCP problem is $\mathcal{NP}$-hard since the classical $\mathcal{NP}$-hard facility location problem [41] could be polynomially reduced to it. The MEC model is experimentally challenging to solve, even for medium-sized instances, as shown in Section 5. Thus, we propose a surrogate-based feedback method (SBFM) to obtain approximate solutions to the EVCP problem based on an auxiliary disaggregated model, named *Surrogate Ambulance-Based Coverage* (SABC) model, which is faster to solve.

The SABC model's essential characteristic is that its objective function does not rely on emergency coverage, as in the MEC model; it only counts the number of ambulances sent on time, late, or null to emergency demand points. Moreover, its resolution time is extremely fast since it requires fewer variables and constraints than the MEC model. However, disaggregating an emergency situation into the number of ambulances needed does not capture emergency coverage, which is crucial for an EMS system.

In addition to the location variables x_{li}, the SABC model requires the following ambulance dispatching binary variables for $k \in K, l \in L, i \in I^s, s \in S$:

$$u^s_{lki} = \begin{cases} 1 & \text{if ambulance of type } k \text{ is dispatched from site } l \text{ to point } i \\ & \text{with response time less than } \tau, \\ 0 & \text{otherwise,} \end{cases}$$

$$v^s_{lki} = \begin{cases} 1 & \text{if ambulance of type } k \text{ is dispatched from site } l \text{ to } i \\ & \text{with response time in } (\tau, \tau_{\max}), \\ 0 & \text{otherwise.} \end{cases}$$

Variables u^s_{lki} indicate the ambulances with an ideal response time dispatched from the location sites corresponding to a decay function value $c_{li} = 1$. While variables v^s_{lki} indicate the ones with a larger than τ response time which have a value $c_{li} < 1$. The number of required ambulances k in an emergency demand point i that are not dispatched are counted by integer variable ζ^s_{ki}, for $k \in K, i \in I^s, s \in S$. The SABC is as follows.

$$\max_x \ \mathbb{E}_s[\mathcal{G}^s(x)], \tag{3.15}$$

$$\text{where } \mathcal{G}^s(x) = \left[\sum_{l \in L} \sum_{k \in K} \sum_{i \in I^s} (\beta_1 u^s_{lki} + \beta_2 v^s_{lki}) - \sum_{k \in K} \sum_{i \in I^s} \phi \zeta^s_{ki} \right] \tag{3.16}$$

$$\text{s.t.} \quad \sum_{l \in L} x_{lk} \leq \eta_k \qquad k \in K \tag{3.17}$$

$$\sum_{i \in I^s} (u^s_{lki} + v^s_{lki}) \leq x_{lk} \qquad l \in L, k \in K, s \in S \tag{3.18}$$

$$u^s_{lki} \leq c_{li} \qquad l \in L, i \in I^s, k \in K, s \in S \tag{3.19}$$

$$u^s_{lki} + v^s_{lki} \leq 1 \qquad l \in L, i \in I^s, k \in K, s \in S \tag{3.20}$$

$$a^s_{1i} + a^s_{2i} = \sum_{l \in L} \sum_{k \in K} (u^s_{lki} + v^s_{lki} + \zeta^s_{ki}) \qquad i \in I^s, s \in S \tag{3.21}$$

$$a^s_{2i} \leq \sum_{l \in L} (u^s_{l2i} + v^s_{l2i} + \zeta^s_{2i}) \qquad i \in I^s, s \in S \tag{3.22}$$

$$x_{lk}, \zeta^s_{ki} \in \mathbb{Z}^+, u^s_{lki}, v^s_{lki} \in \{0,1\} \qquad l \in L, k \in K, i \in I^s, s \in S$$

The objective function (3.15) maximizes the expected value of the on-time and late dispatched ambulances minus a penalty ϕ for the required ambulances that could not be dispatched in less than $\tau_{\max}$ time response. The weights $\beta_1 > \beta_2$ are normalized parameters that prioritize the ambulances dispatched with a response time less than τ. As in the previous model, no more than the available ambulances can be located on the sites, corresponding to constraints (3.17). The number of ambulances dispatched on time or late is less than the number of ambulances located, as indicated by constraints (3.18). Constraints (3.19) define the ambulances dispatched with an ideal response time of less than τ. Thus, if $c_{li} = 1$, then the ambulance will have an ideal response time, while constraints (3.20) activate the late variables for which their response time is between $(\tau, \tau_{\max})$. With constraints (3.21) and (3.22), the non-covered emergencies, ζ^s_{ki} variables are defined for $i \in I^s, s \in S$. Recall that advanced ambulances can be dispatched instead of basic ones. Finally, the nature of the variables is stated.

The surrogate-based feedback method: Under the SBFM, the SABC stochastic model is solved first. From its optimal solution, we obtain the location of the ambulances of the first stage corresponding to the value of x_{lk} variables, for $l \in L$, $k \in K$. Let the solution vector of these values be called x^{SABC}. Then, in the allocation stage, we solve MEC taking x^{SABC} as input. We call this model MEC(x^{SABC}), or simply MEC(SABC), implying that it is the solution of the MEC model with the location variables fixed with the solution of the surrogate model SABC. Since the first stage variables are fixed, the MEC(SABC) model becomes easier to solve and yields high-quality solutions. We could implement a local search neighborhood based on the location variables x_{lj} to diversify the solution yield by variables x^{SABC}. However, experimental results show that the quality of the SBFM solutions is exceptionally high with a single feedback.

As mentioned, the SABC auxiliary model is a surrogate for the MEC formulation. Thus, the solutions obtained by the MEC and SABC are not equivalent. However, the solutions of the SABC model can be mapped into solutions for the EVCP problem, as shown in Algorithm 1. In this manner, we can compare both models in terms of emergency coverage, even if the SABC model is short-sighted regarding this objective. Step 3 activates the total coverage when all the required ambulances arrive in less than τ response time. Step 4 verifies if a dispatched ambulance has a response time in $(\tau, \tau_{\max})$, corresponding to the total-late coverage. Step 6 checks that not all the required ambulances are dispatched but they arrive between the ideal time, while Step 8 verifies that the dispatched ambulances are not all the required ones and at least one of them has a response time in $(\tau, \tau_{\max})$. Finally, Step 10 activates the null variable.

Algorithm 1 Transformation of a SABC solution into a MEC solution

1: **require** solution of the SABC model $(\bar{x}, \bar{u}, \bar{v})$
2: **for** $i \in I^s, s \in S$ **do**
3:     **if** $\sum_{l \in L, k \in K} \bar{u}^s_{lki} = a^s_{ki}$ **then** $f^s_i = 1$ ▷ total coverage
4:     **if** $\sum_{l \in L, k \in K} \bar{u}^s_{lki} < a^s_{ki}$ and $\sum_{l \in L, k \in K} \bar{u}^s_{lki} + \bar{v}^s_{lki} = a^s_{ki}$
5:         **then** $g^s_i = 1$ ▷ total-late coverage
6:     **if** $\sum_{l \in L, k \in K} \bar{u}^s_{lki} < a^s_{ki}$ and $\sum_{l \in L, k \in K} \bar{v}^s_{lki} = 0$
7:         **then** $h^s_i = 1$ ▷ partial coverage
8:     **if** $\sum_{l \in L, k \in K} \bar{u}^s_{lki} + \bar{v}^s_{lki} < a^s_{ki}$ and $\sum_{l \in L, k \in K} \bar{v}^s_{lki} > 0$
9:         **then** $w^s_i = 1$ ▷ partial-late coverage
10:     **otherwise** $z^s_i = 1$ ▷ null coverage
11: **return** MEC solution $(\bar{x}, f, g, h, w, z)$

3.5 Matheuristic to improve the MEC model

The Surrogate-based feedback method for the MEC(SABC) model has good results. However, a disadvantage of this method is that we obtain only one solution for the ambulance location from the surrogate model SABC. This is a problem because we disown the optimal solution for the MEC model and we are not sure if the SABC solution is close to that optimality. Trying to improve the solution x^{SABC} we proposed a local search procedure, which is a matheuristic considering four neighborhoods, named as SABC Matheuristic. These four different neighborhoods are as follows:

- Neighborhood 1, (N_1): exchange one active potential site with another active potential site.

- Neighborhood 2, (N_2): pick half of an active potential site and add it to a non-active potential site.
- Neighborhood 3, (N_3): pick half of an active potential site and add it to another active potential site.
- Neighborhood 4, (N_4): exchange one active potential site with one non-active potential site.

N_1 and N_4 change BLS ambulances with BLS ambulances and then ALS ambulances with ALS ambulances. N_2 and N_3 change only BLS ambulances with BLS ambulances due to the small quantity of ALS ambulances in the EMS system.

The algorithm to solve the SABC Matheuristic has the x^{SABC} as an initial solution. First, we obtain the value of the objective function for this initial solution, defined as t^{SABC}, which is the best solution at this point. Then, we construct the first neighborhood from the x^{SABC}. The t^{SABC} objective value is compared with each neighbor's objective value, obtained using the MEC(SABC) methodology. If a neighbor's solution is better than t^{SABC}, we consider this solution as the best one for the Matheuristic and we save the objective value and variables' results. Otherwise, we have the initial solution as the best one when the algorithm is finished. Regardless of whether the new best solution is the initial solution or not, we construct the second neighborhood from x^{SABC}, and each neighbor is compared with the best solution at the moment. We repeat this procedure for the other two neighborhoods and when the comparisons are finished, we obtain the best solution, as we can see in the Algorithm 2.

This procedure calculates each neighbor's objective value as in the MEC(SABC) methodology in the four different neighborhoods, which makes the SABC Matheuristic take so much time to check each neighborhood. This Matheuristic aids in improving x^{SABC} solutions, but not for all instances, as we can see in Chapter 5.

The next section compares the MEC, MEC(SABC), and even the SABC solutions.

Algorithm 2 SABC Matheuristic to improve the MEC model

require solution of the SABC model x^{SABC}
$x^* = x^{SABC}$
$t^* = MEC(x^{SABC}) = t^{SABC}$
while not all neighborhoods N_i, where $i \in \{1, 2, 3, 4\}$, have been visited **do**
    for $x' \in N_i$ **do**
        Evaluate $t' = MEC(x')$
        if $t' > t^*$ **then** $x^* = x', t^* = t'$
return solution (x^*, f, g, h, w, z) and objective value t^*

CHAPTER 4

EXPERIMENTAL ASSESSMENT

This chapter presents an empirical assessment of models and the solution methodology previously described to solve the EVCP problem. We used Gurobi Optimizer 10.0.2 with Python 3.10 to solve the integer programming models MEC, SABC, and MEC(SABC). The experiments were carried out on an Intel Core i7 at 3.1 GHz with 16 GB of RAM under the macOS Catalina 10.15.7 operating system. Each execution of the integer linear programming solvers had a CPU time limit of 10800 seconds. For SABC Matheuristic the solver had a CPU limit of 300 seconds for neighbor's evaluation and 10800 seconds in total.

4.1 INSTANCE GENERATION

The value ranges of our instance generator are based on real-world data taken from Monterrey, Mexico. In the literature, there are no suitable benchmarks for our problem. The databases for the Monterrey case study showed a larger number of possible demand points, $|I| \in \{168, 270, 500, 900, 1500\}$ compared to the one from the literature with $|I| \leq 270$ [55]. The number of possible locations for ambulances in Monterrey is $|L| \in \{16, 50, 100\}$, which is also larger than the one from the literature (≤ 30) since not only hospitals and fire stations can be considered. We consider the whole city of Monterrey, so the number of ambulances $(\eta_1, \eta_2) = (35, 20)$ is also greater than the ones from the literature cases (6 ambulances per type [55]). The number of scenarios is set to be as large as that in the literature $|S| \in \{10, 50, 100, 150, 200\}$. Thus, our benchmark has 15 instances for which five different scenario settings were built.

For each instance, we simulated a two-hour high-demand period. Each scenario $s \in S$ consists of a set of demand values per ambulance type and per demand point

$\{a_{ki}^s\}_{k\in K, i\in I, s\in S}$. Fewer demand points imply a larger city grid and a larger proportion of emergencies per demand point. Therefore, when $|I| = 168$, around 30% of the demand points may have a value different from 0. In contrast, when $|I| = 1500$, only 1% of the demand points will require ambulances. This setting reflects the number of emergencies per hour observed in the case study. Instances are built such that most emergencies require a single ambulance, but as observed in real cases, some of them may require up to three ambulances.

The ideal ambulance response time is $\tau = 10$ minutes, while the maximum response time is $\tau_{\max} = 30$ minutes. For the MEC formulation, we use the following weights in the objective function (3.1): $\alpha_1 = 0.65, \alpha_2 = 0.2, \alpha_3 = 0.1$, and $\alpha_4 = 0.05$. In this manner, the total coverage is the most sought-after, while the partial-late cover has less benefit. Surprisingly, the value of the big M of the model is not the main cause of the execution time of the MEC model. Thus, a simple value $M = 1000$ is set.

For the SABC objective function (3.16) we use $\beta_1 = 0.7$ and $\beta_2 = 0.3$. These values reflect the aim to send primordially the required ambulances with an ideal response time. The penalty for null coverage in the MEC model or when a required ambulance cannot be dispatched to the emergency in less than $\tau_{\max}$ time in the SABC model is set to $\phi = 1/|S| + 0.0005$.

All instances with their related scenarios and detailed solutions are available at `https://doi.org/10.6084/m9.figshare.25928401`.

CHAPTER 5

EXPERIMENTAL WORK

In this chapter, we analyze the parameters of the EVCP problem that impact the performance of the objective values of our stochastic methodologies. Several questions arise. We wish to investigate how sensitive the model is to the number of scenarios in terms of solution quality and solution time. We also want to determine the size of tractable instances.

5.1 OBJECTIVE VALUES FOR THE MEC, MEC(SABC) AND SABC MATHEURISTIC

In this first experiment, we solve the instances using the original MEC model. Figure 5.1 consists of six plots. The three plots in the left column vary the number of demand points (x-axis), comparing each one to the value of the objective function when different scenarios are tested. The three plots in the right-hand side column vary the tested number of scenarios and show the variation in the solution value for each number of demand points. The upper plots consider a number of possible locations for the ambulances of $|L| = 16$, the middle plots of $|L| = 50$, and the lower plots of $|L| = 100$. Straight lines are the best objective values, while dotted ones are the best bounds found.

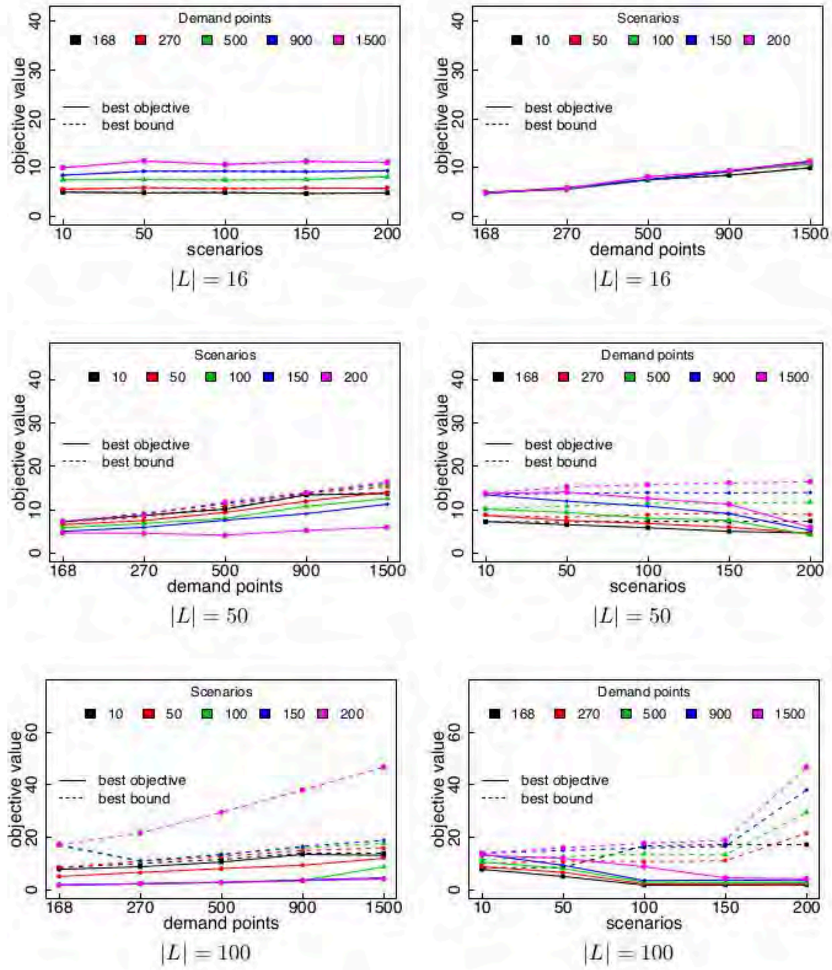


Figure 5.1: Objective values for the MEC problem.

As can be seen from the plots, the difference between the best objective and the best bound (and thus, the relative optimality gaps[1]) are negligible for small instances with 16 potential location sites. Still, the gaps become larger for the instances with 50 and 100 potential sites. The number of demand points where emergencies may occur and the scenarios considered make the instances harder to solve optimally within the time limit. Thus, the deterministic equivalent integer program of MEC can only handle small instances with a few scenarios, demand points (emergency points), and potential ambulance sites. Note that the larger the number of scenarios

[1](best objective - best bound)/best objective.

in the plots on the left-hand side, the better the objective function. This implies that a better sampling of the emergency demand points benefits the quality of the solution related to the ambulance response time. The plots on the right side show that the larger the size of the demand point set, the harder it is to solve the instance.

For the MEC(SABC) methodology, we have the solution represented in Figure 5.2, which has a similar structure to the previous one. For this methodology, the number of scenarios does not affect the results for the objective value due to the MEC(SABC) only considering the ambulances serving the accidents. To this methodology, the optimal is found in an easier and faster way than the MEC, but it is different to the optimal at the MEC, so we need to compare the objective values of both of them.

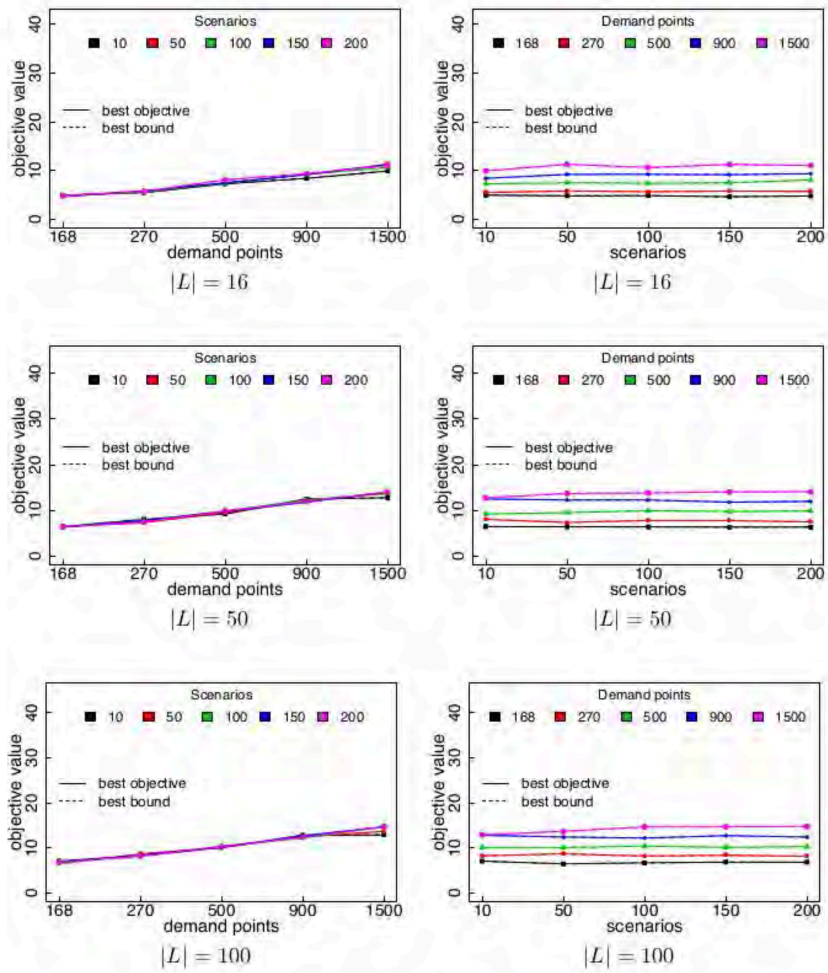


Figure 5.2: Objective values for the MEC(SABC) methodology.

Now, we compare the solution values of the equivalent integer program of MEC with those obtained by MEC(SABC) in Figure 5.3. As can be seen, while the number of scenarios, demand points, and potential sites slightly affects the performance of MEC(SABC), it obtains better objective function values than those obtained by the MEC model for the larger instances that reported positive gaps. Indeed, the optimality gaps of the MEC(SABC) model always equal 0 within the time limit that we established. In addition, the MEC(SABC) model tends to be less dependent on the number of scenarios. Thus, although we cannot guarantee optimality with the MEC(SABC) model, it obtains faster and higher-quality solutions than those obtained by the MEC equivalent model.

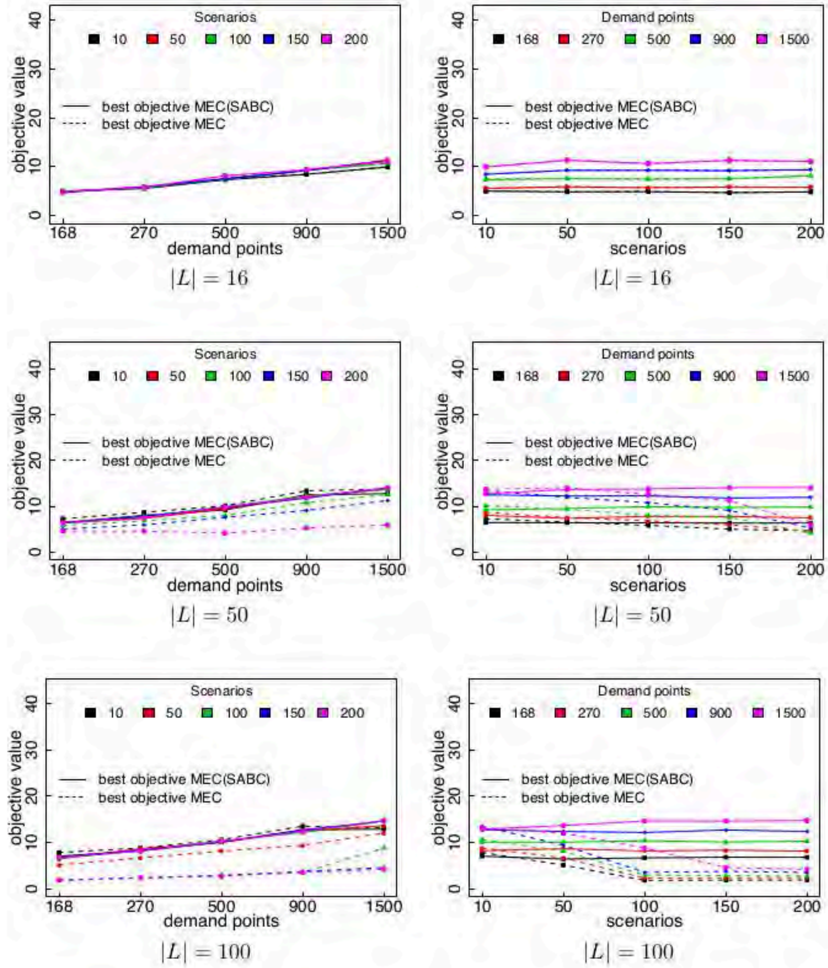


Figure 5.3: Objective values of MEC(SABC).

Once we analyze the MEC(SABC), we have in Figure 5.4 the SABC Matheuristic results. The axis on the left-hand and the right-hand sides of the figure can be described as previous ones. Straight lines indicate the best objective value found throughout the Matheuristic, i.e., the solution for the best neighbor found from the four neighborhoods explored. As we can see, even for the larger scenarios, we can obtain a solution close to the best bound, which is represented by no straight lines. This means that fixing locations facilitates the exploration in the second stage of the MEC and solutions for Matheuristic can be found efficiently. However, we need to compare this solution with the solutions from the MEC and MEC(SABC) to evaluate this mat heuristic.

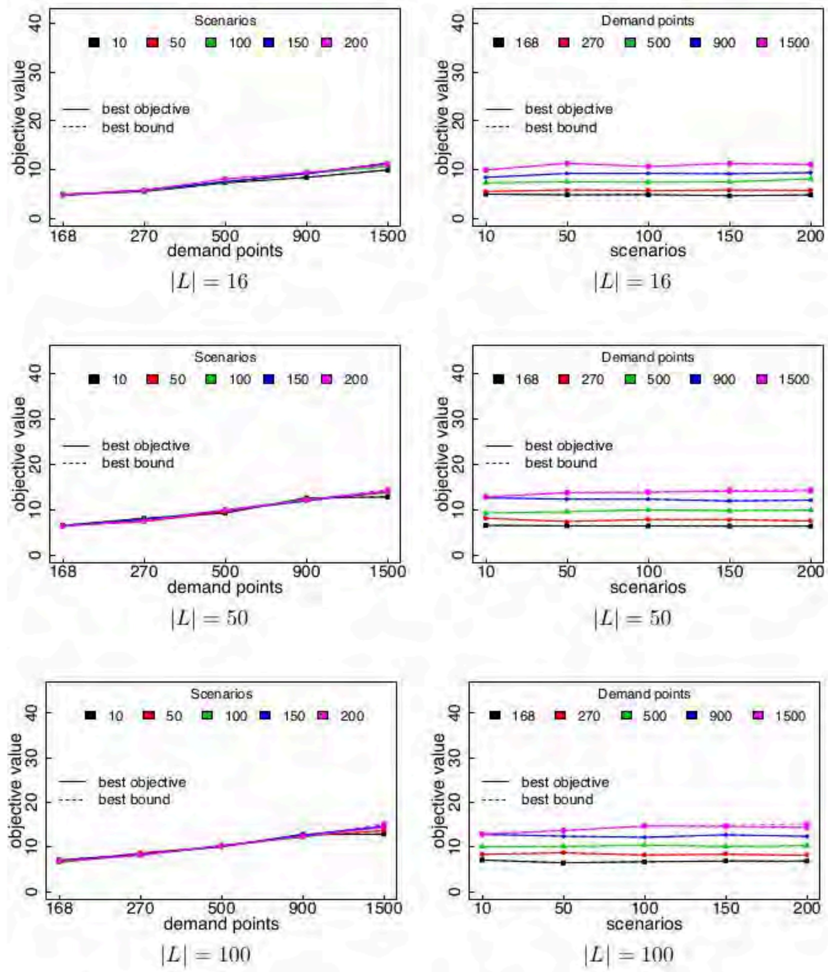


Figure 5.4: Objective values of SABC matheuristic.

To see if the Matheuristic shows better results than MEC and MEC(SABC) methodology, we compare the results between the objective values obtained from three of them. Figure 5.5 shows these results. MEC objective values are represented for the straight lines, and we can see that the other two types of lines, the dotted and the dashed ones, have better solutions than the straight line. Nevertheless, there is no significant difference between MEC(SABC) and SABC Matheuristic. This small improvement may be due to the computational limit time that the solver had to solve each instance. However, we can see the improvement percentage between the MEC and the MEC(SABC) in column five of Table 5.1, and the improvement percentage between the MEC(SABC) and the SABC Matheuristic in column six. Column one defines the instance name, which is given by the number of demand points, potential sites, and scenarios. Columns two, three, and four have the objective values for the MEC, the MEC(SABC), and the SABC MAtheuristic, respectively. Something interesting is that SABC Matheuristic mostly improves MEC solution for those instances where MEC(SABC) does not improve the objective value. Table 5.1 shows only those instances in which we obtain an improvement for the optimal solution.

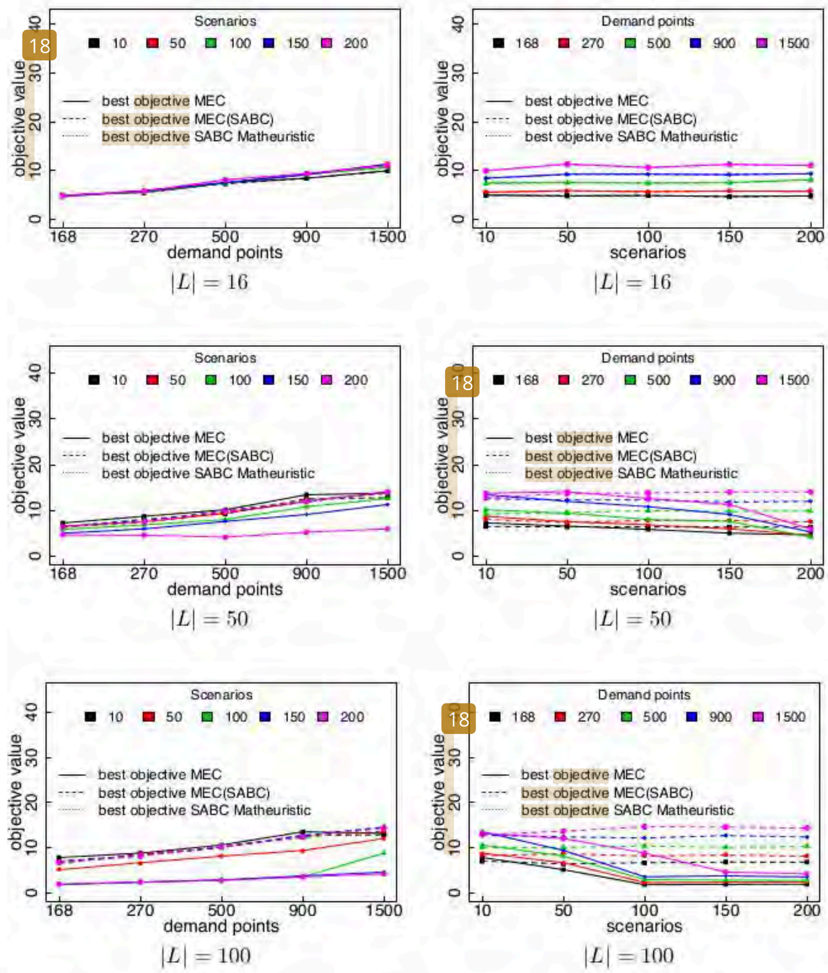


Figure 5.5: Comparissons between the objective values of the MEC, MEC(SABC) and SABC matheuristic.

Table 5.1: Comparissons between a subset of instances for the objective values of the MEC, MEC(SABC) and SABC matheuristic with a limited computational time of 1800 seconds per neighbor.

Objective values

Continued on next page

Table 5.1: Comparissons between a subset of instances for the objective values of the MEC, MEC(SABC) and SABC matheuristic with a limited computational time of 1800 seconds per neighbor. (Continued)

Instance $\|I\|$ $\|L\|$ $\|S\|$	MEC	MEC(SABC)	Matheuristic (Math)	% Improve MEC vs MEC(SABC)	% Improve MEC(SABC) vs Math
1500 16 200	11.0536	11.05635	11.05635	0.024878773	-
168 50 10	7.285	6.485	6.53	-	0.693909021
168 50 100	5.86825	6.4167	6.4167	9.346057172	-
168 50 150	5.04052222	6.375988893	6.375988893	26.4946093	-
168 50 200	4.6735	6.3773	6.3773	36.4566171	-
270 50 10	8.725	8.05	8.085	-	0.434782609
270 50 50	7.535	7.381	7.389	-	0.108386398
270 50 100	6.85685	7.83305	7.83305	14.23685803	-
270 50 150	5.9789	7.80876667	7.80876667	30.60540685	-
270 50 200	4.59045	7.504750002	7.504750002	63.48615064	-
500 50 50	9.4105	9.547000009	9.547000009	1.450507504	-
500 50 100	8.02305	9.9521	9.9521	24.04384866	-
500 50 150	7.61198889	9.7781	9.7781	28.45657216	-
500 50 200	4.17615	9.883800001	9.883800001	136.6725333	-
900 50 10	13.4365	12.535	12.603	-	0.542481053
900 50 50	11.987	12.286	12.301	2.4943689	0.122090184
900 50 100	10.8234	12.29315	12.29565	13.57937432	0.020336529
900 50 150	9.11984444	11.83602222	11.91226667	29.78315908	0.644172873
900 50 200	5.26805	11.98405	12.04855	127.4855022	0.53821538
1500 50 50	14.084	13.709	13.7235	-	0.10576994
1500 50 100	12.6053	13.82855	13.82855	9.704251386	-
1500 50 150	11.2970444	14.03975556	14.03975556	24.27812983	-
1500 50 200	5.9664	14.0559	14.0723	135.5842719	0.116676984
168 100 50	5.156	6.463000009	6.463000009	25.34910801	-
168 100 100	1.8485	6.6976	6.6976	262.3262105	-
168 100 150	1.9081	6.85182223	6.85182223	259.0913594	-
168 100 200	1.93335	6.8329	6.8329	253.4228153	-

Continued on next page

Table 5.1: Comparissons between a subset of instances for the objective values of the MEC, MEC(SABC) and SABC matheuristic with a limited computational time of 1800 seconds per neighbor. (Continued)

270 100 10	8.82500001	8.255000064	8.355	-	1.211386255
270 100 50	6.71	8.6855	8.6855	29.44113264	-
270 100 100	2.32665	8.208650004	8.208650004	252.8098341	-
270 100 150	2.41097778	8.375244449	8.375244449	247.3795788	-
270 100 200	2.4159	8.15665	8.15665	237.6236599	-
500 100 50	8.169	10.101	10.124	23.6503856	0.22770023
500 100 100	2.97905	10.40835	10.4282	249.384871	0.190712265
500 100 150	2.86453333	10.12661111	10.12661111	253.5169584	-
500 100 200	2.78645	10.29585	10.29585	269.4970304	-
900 100 10	13.5535	12.79	12.8315	-	0.324472244
900 100 50	9.42	12.4165	12.4165	31.80997877	-
900 100 100	3.59	12.18375	12.18375	239.3802228	-
900 100 150	3.78992222	12.71902222	12.71902222	235.6011411	-
900 100 200	3.5589	12.4005	12.4005	248.4363146	-
1500 100 10	13.206	12.887	12.889	-	0.015519516
1500 100 50	12.063	13.6875	13.6875	13.4667993	-
1500 100 100	8.8318	14.67245	14.67245	66.13204556	-
1500 100 150	4.62758889	14.60712222	14.60712222	215.6529798	-
1500 100 200	4.2533	14.3678	14.41435	237.8035878	0.323988387

SABC Matheuristic depends on the time we allow for the neighborhoods exploration. As the number of demand points, potential sites and scenarios increases, the number of neighbors in each neighborhood also increases. Considering that each neighbor is verified for the MEC(SABC), the computational time required is very large if we let SABC Matheuristic do a complete procedure. However, we checked some instances increasing the computational time from 1800 seconds to 10800 seconds per neighbor. The results for a selected set are in Table 5.1, whose structure is similar to the previous table. In these results, we can see that improvements are very small, but we were able to get an improvement for instances where we did not get improvements before. However, since these are small instances, the MEC is still better.

Table 5.2: Comparissons between a subset of instances for the objective values of the MEC, MEC(SABC) and SABC matheuristic with a limited computational time of 10800 seconds per neighbor.

Objective values					
Instance |I| |L| |S|	MEC	MEC(SABC)	Matheuristic Math	% Improve MEC vs MEC(SABC)	% Improve MEC(SABC) vs Math
168 16 10	4.995000008	4.995	4.995000016	-	3.16983E-07
168 16 50	4.849	4.848	4.848000001	-	1.43126E-08
168 16 100	4.91145	4.88345	4.887450001	-	0,081909319
168 16 150	4.713244444	4.677400001	4.678066671	-	0,014252984
168 16 200	4.84205	4.8221	4.82355	-	0,030069887
168 50 10	7.285	6.485	6.540000002	-	0,848111051
168 50 50	6.5825	6.441	6.464	-	0,357087409
168 50 100	5.86825	6.4167	6.4623	9.346057172	0,710645659
168 50 150	5.040522223	6.375988893	6.417422223	26.4946093	0,649833788
168 50 200	4.6735	6.3773	6.4149	36.4566171	0,589591206
168 100 10	7.865	7.070000095	7.18	-	1.555868511
168 100 50	5.156	6.463000009	6.513	25.34910801	0,773634398
168 100 100	1.8485	6.6976	6.717600001	262.3262105	0,298614435
168 100 150	1.9081	6.85182223	6.8787	259.0913594	0,392271858
168 100 200	1.93335	6.8329	6.8721	253.4228153	0,573694917
270 16 10	5.580000003	5.525	5.535	-	0,180995475
270 16 50	5.9145	5.8675	5.872500001	-	0,085215177
270 16 100	5.719	5.70245	5.70465	-	0,038579906
270 16 150	5.857844447	5.834588893	5.835666673	-	0,018472238
270 16 200	5.7808	5.77065	5.771	-	0,006065175

5.2 Response time for the MEC, MEC(SABC) and SABC matheuristic

The following experiment compares the running times of the equivalent MEC model with the MEC(SABC) method. Recall that MEC(SABC) attempts to exploit that the surrogate model SABC is very tractable and solved relatively quickly. To this end, Figure 5.6 shows plots of the running time in seconds of the instances with $|L| = 16$, $|L| = 50$ and $|L| = 100$ potential location sites for the equivalent MEC, and Figure 5.7 shows response times for the SBFM with the same potential sites. The x-axis of the plots corresponds to the number of scenarios, and we vary the number of emergency demand points. Recall that the MEC model with $|L| = \{50, 100\}$ reaches the time limit even for ten scenarios and few demand points.

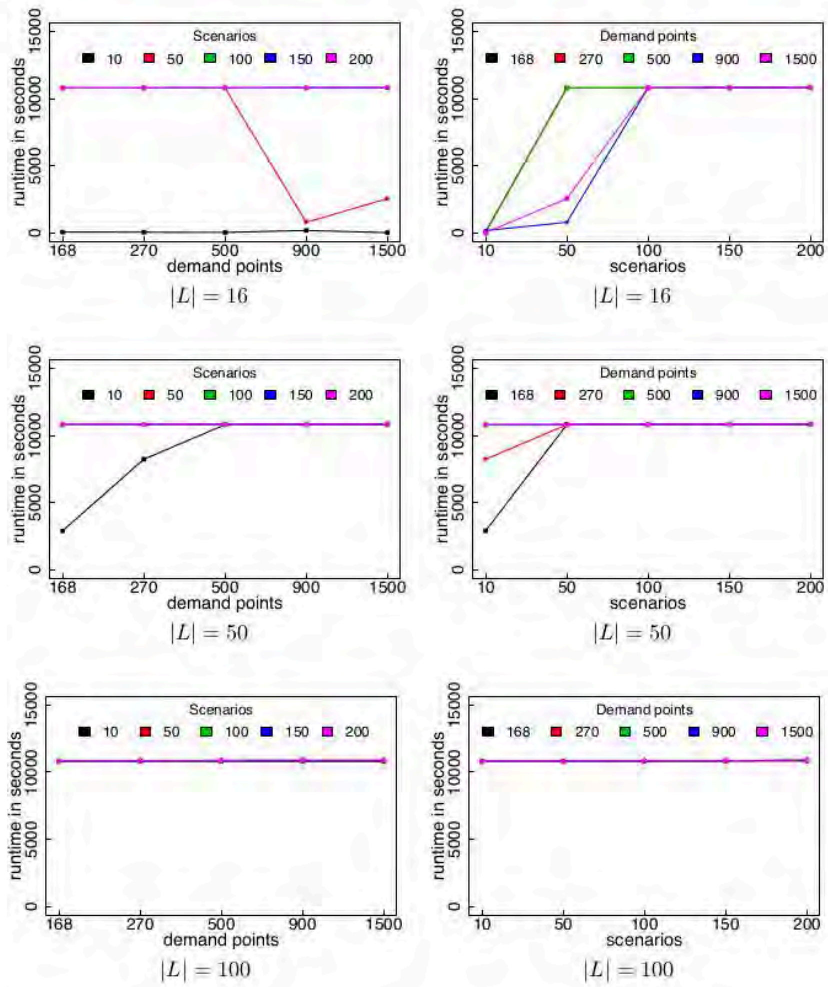


Figure 5.6: Runtime in seconds for the MEC.

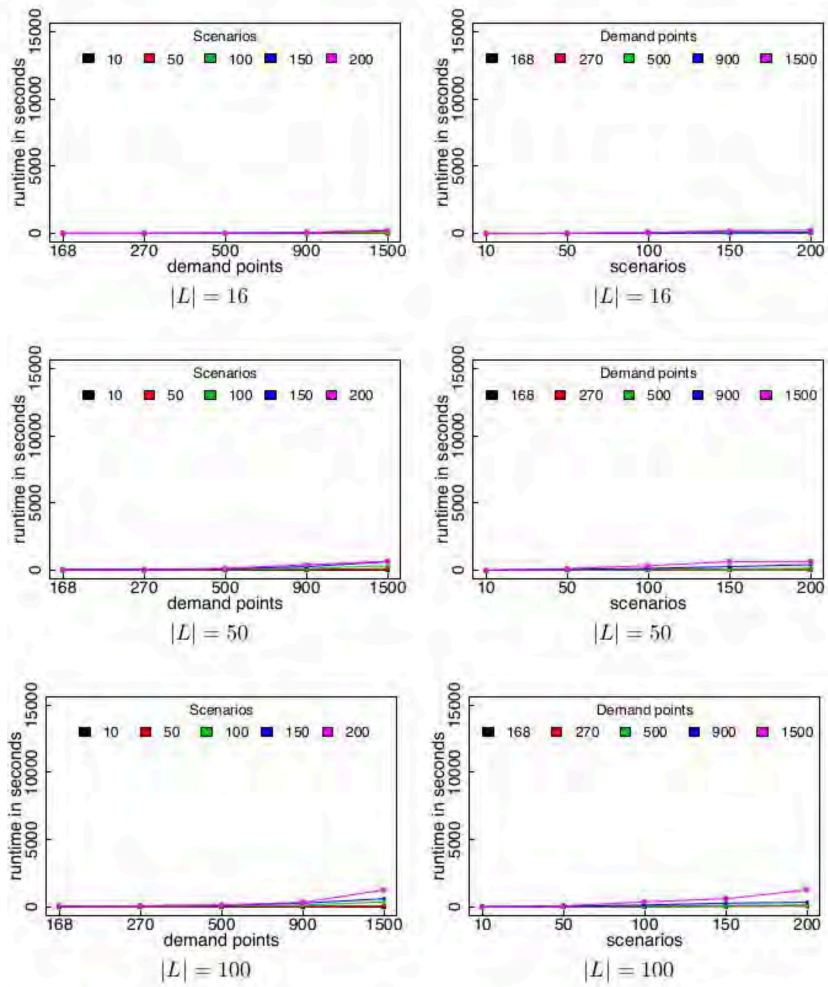


Figure 5.7: Runtime in seconds for the MEC(SABC).

Figure 5.6 shows that the main disadvantage of the MEC model is its computational time, which increases significantly with the number of demand points, potential sites, and scenarios, even for small instances with 16 potential location sites for ambulances. The SABC model is extremely fast, even for large instances, and yields an initial solution to the assignment of ambulance location in a short time to allow the MEC(SABC) model, Figure 5.7, to be solved faster than the MEC model and obtain high-quality solutions. The MEC(SABC) location-allocation strategy inherits not only its fast computational time from the SABC, but also yields coverage per emergency situation, which is the main objective for the EVCP problem. The

MEC(SABC) model is an approximated mathod, but it gives solutions that are as good as the MEC and even better when the MEC instances do not reach optimality and its gaps are large. The SBFM solves most of the instances in less than a minute.

An interesting advantage of the SBFM is that only one iteration is needed. In fact, once the location of the ambulances has been retrieved from the SABC model and fed back to the MEC model, we could perturb the ambulances either randomly or with a local search, the allocation of the ambulances, and iterate again. However, we could not systematically generate a neighborhood around a location solution that yields better solutions with the MEC(SABC) approach. This implies that local maximums are often reached with this first feedback and that complex or more diverse neighborhoods should be built to allow escaping from these solutions. It would probably be interesting to enable local search movements that do not yield immediate benefits.

Comparing the SABC Matheuristic runtime with MEC and MEC(SABC) we show Figure 5.8. MEC still has a very large computational time compared to the other two methodologies. As we can see, MEC(SABC) methodology computational time is always less than or equal to SABC Matheuristic computational time. Even though SABC Matheuristic is a local search procedure, the evaluation in MEC(SABC) for each neighbor increases the computational time when instances are large. Still, it is not a time that should alarm us, which indicates that it is a good procedure.

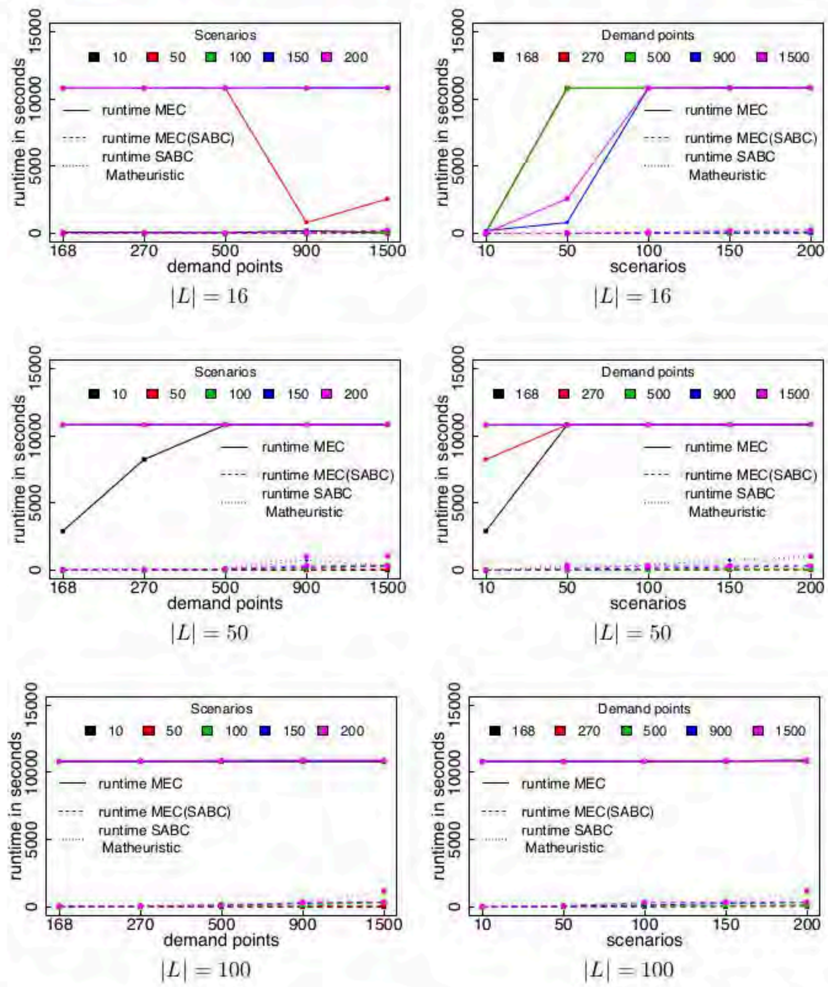


Figure 5.8: Comparissons between runtime in seconds for the MEC, MEC(SABC) and SABC matheuristic.

5.3 Coverage for the MEC, MEC(SABC) and SABC matheuristic

The objective values and running times are crucial for evaluating the models' performance. However, the most critical objective of the EVCP problem is to cover the largest number of demand points within a fixed response time. Thus, a central question arises: Is the emergency coverage quality of the MEC(SABC) as good as the one yielded by the MEC model?

The percentage of emergency coverage for all instances is presented with the equivalent MEC model and the SBFM in Figures 5.9 and 5.10, respectively.. Two columns with three plots each, varying the number of scenarios and the location sites. Each plot shows the type of ambulance percentage coverage obtained: T is for Total coverage (all required ambulances on time), TL is for Total-late coverage (all required ambulances, but at least one arrives late), P is for Partial coverage (at least one required ambulance is not dispatched, but the dispatched ones all arrive in time), PL is for Partial-late coverage (at least one required ambulance is not dispatched, at least one of the dispatched arrives late), and N for Null (no ambulances assigned to the demand point). The upper plots are for $|L| = \{16\}$ potential sites, the middle ones for $|L| = 50$, and the lower ones for $|L| = 100$.

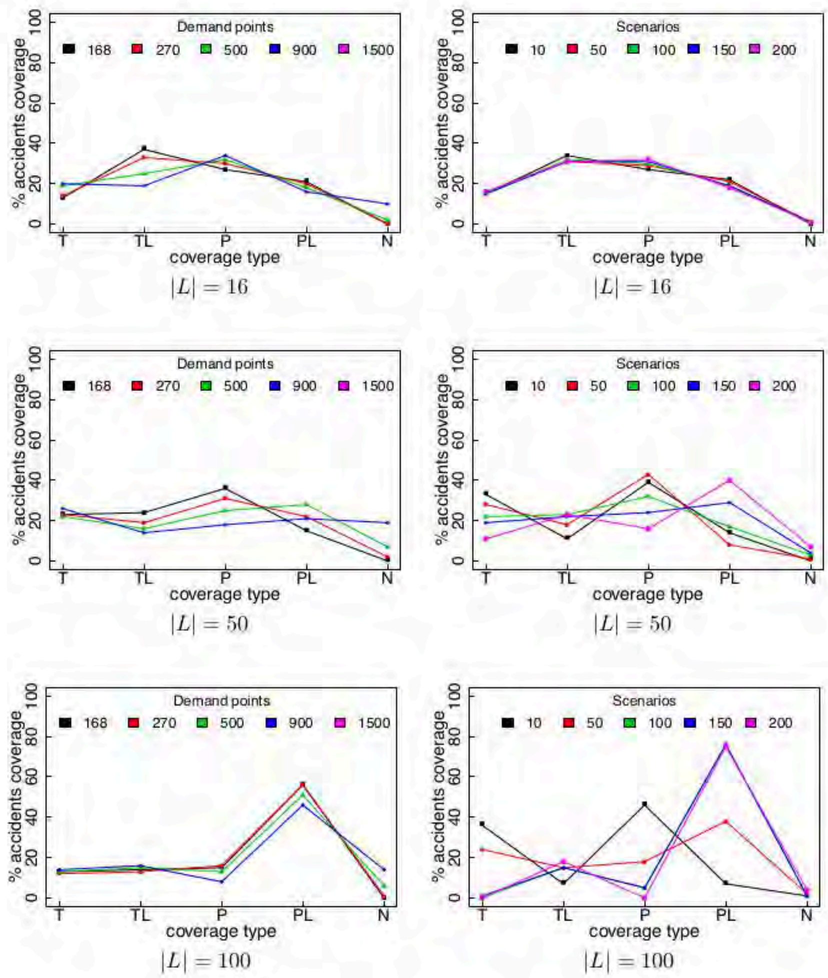


Figure 5.9: Percentage coverage type for the MEC.

Figure 5.9, shows that the MEC model tends to leave very few demand points with null coverage, which is the primary concern of the emergency services in our case study. As the number of potential sites $|L|$ increases, the coverage tends to be partial-late for the MEC model. This behavior is probably related to the large gaps obtained by the MEC model for large instances, but the number of null coverage is still remarkably low. Figure 5.10 shows that the SBFM is robust in terms of the number of scenarios. That is, the demand point coverage is independent of the scenario number. In this way, 100 scenarios are sufficient to handle a high-quality coverage solution. Moreover, the MEC(SABC) model inherits the characteristic of having very few null demand point coverage from the MEC model. Interestingly, partial coverage tends to be larger than partial late coverage, which is mainly desired in real life because it can be translated into first-aid medical care on time, increasing the probability of saving lives.

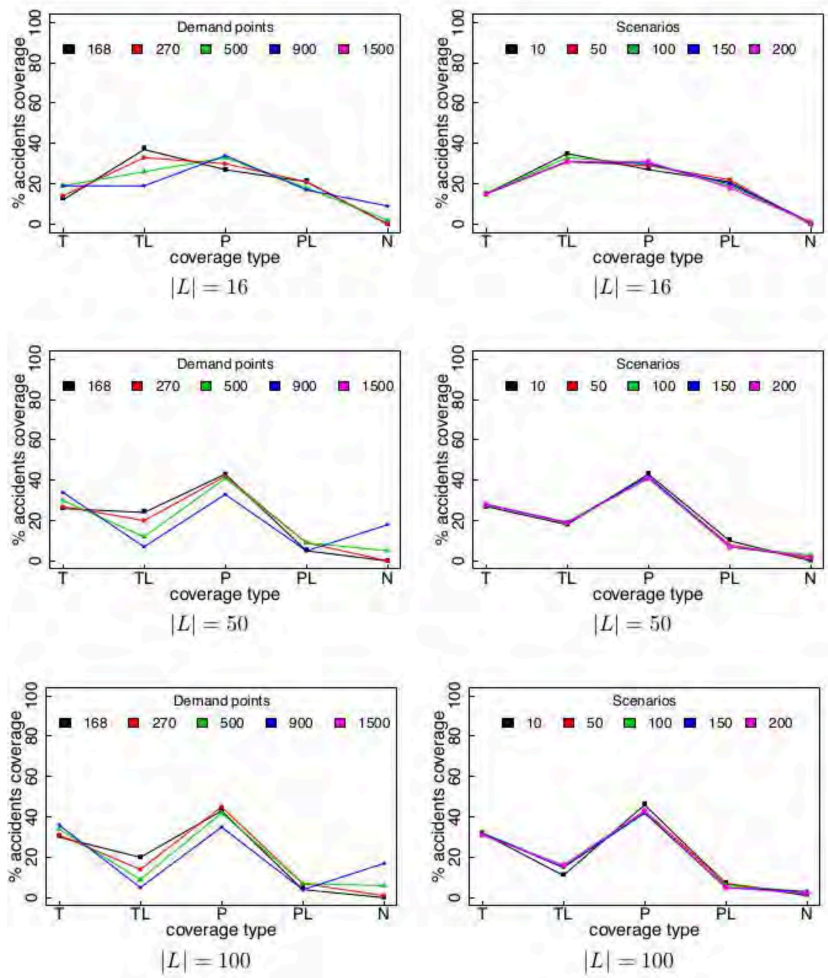


Figure 5.10: Percentage coverage type for the MEC(SABC).

For the SABC matheuristic the coverage results are in Figure 5.11. As we saw in Figure 5.10, SABC Matheuristic tends to have a large partial coverage. The results are for the best neighbor found throughout the matheuristic, which has not a significant difference to the MEC(SABC) results.

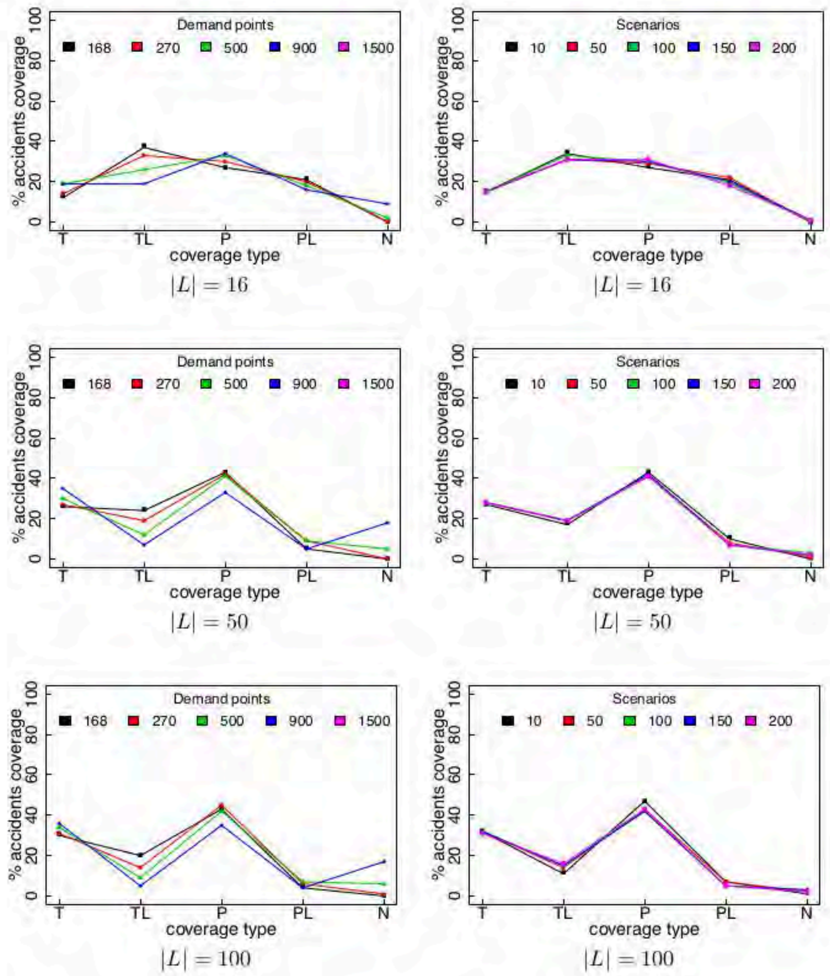
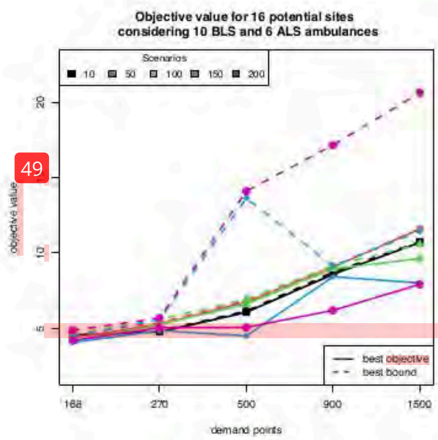


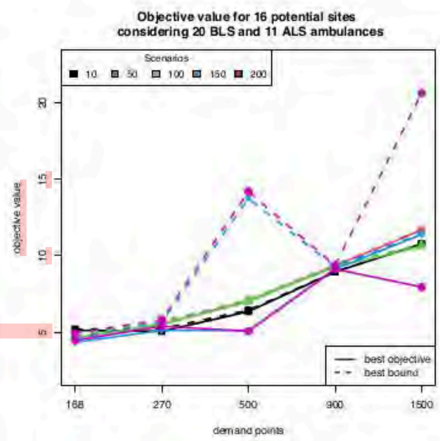
Figure 5.11: Percentage coverage type for the SABC matheuristic.

All the previous experiments were executed with the number of ambulances equal to $(\eta_1, \eta_2) = (35, 20)$. A central feature of the EVCP problem is that an ALS ambulance can be sent instead of the BLS one, which gives a more flexible setting but may induce difficulty when solving the models. Thus, what is the effect of the number of available ambulances in the EVCP problem on the objective function value and the running time?

For a preliminary evaluation, we launched experiments changing the number of ambulances for the MEC problem, considering $|L| = 16$ potential sites for all sizes of demand points and scenarios. Figure 5.12 presents objective values. As we expected, the number of ambulances increases the gap between best objective and best bound found in some cases, mostly when the number of scenarios increases for (η_1, η_2)=(10,6) and (20,11). Although, as the number of ambulances increases, the gap decreases, and we decided to reduce our instances set considering just (η_1, η_2)=(35,20) to obtain more realistic results according to the number of ambulances in our case of the EMS system in Nuevo León. In addition, in Figure 5.13, we can see that there is no significant difference for the runtime varying the number of ambulances, so considering a few number of them does not help the model to use a low computational time.



Considering (η_1, η_2)=(10,6)



Considering (η_1, η_2)=(20,11)

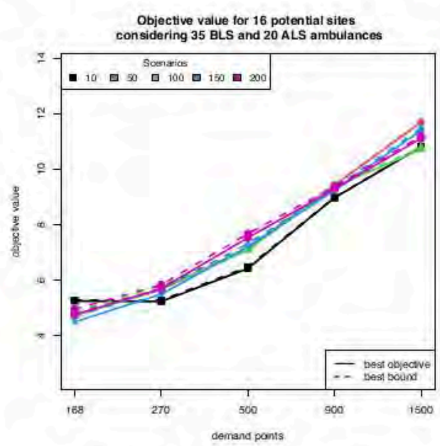


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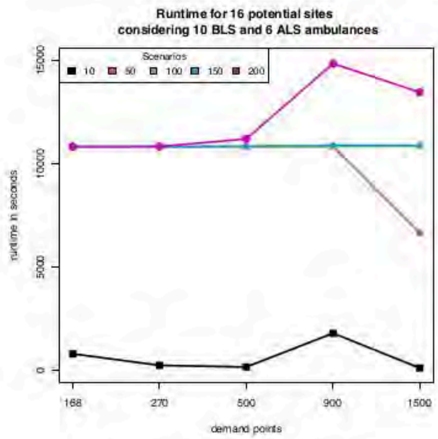
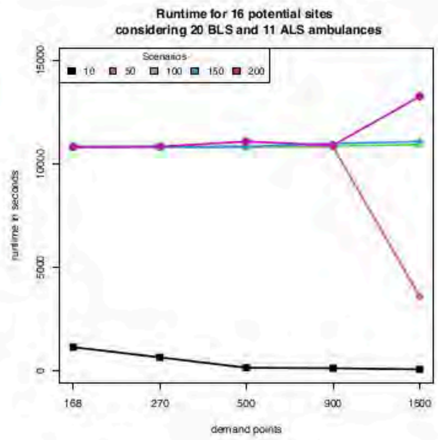
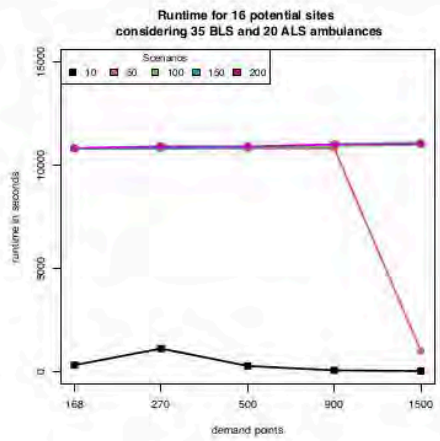
Figure 5.12: Objective values for a preliminar experiments to evaluate different number of ambulances.



Considering (η_1, η_2)=(10,6)



Considering (η_1, η_2)=(20,11)



Considering (η_1, η_2)=(35,20)

Figure 5.13: Runtime in seconds for a preliminar experiments to evaluate different number of ambulances.

We execute all the instances with emergency demand points fixed to 900, 100 scenarios, and 50 ambulance location sites. For this experiment, we vary the number of ambulances. Figure 5.14 shows two columns of two plots each. The objective value (upper plots) and the running time (lower plots) are on the y-axis, while the x-axis varies the number of ambulances: (η_1, η_2)=(10,6), (20,11), and (η_1, η_2)=m,(35,20). The left plots correspond to the MEC stochastic model, while the right ones are for the SBFM.

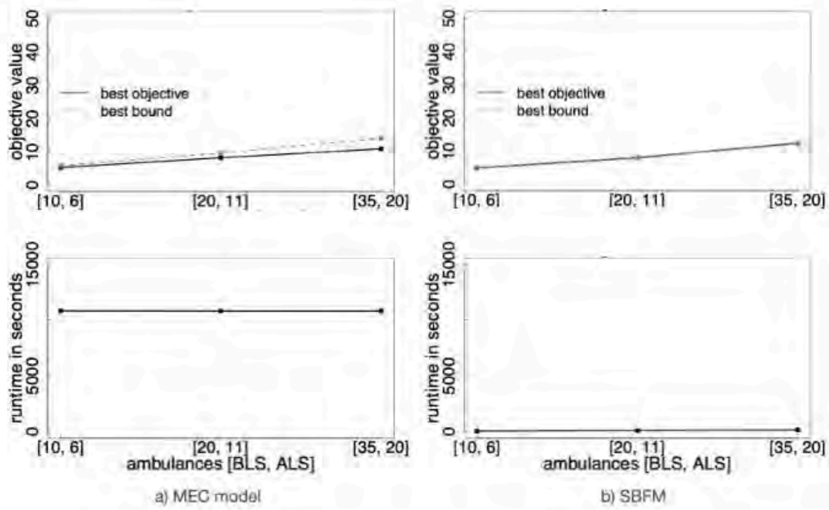


Figure 5.14: Objective value and running time versus the number of ambulances for a) MEC model and b) SBFM.

From Figure 5.14a, we observe that the difference between the best objective and the best bound for the MEC model (left-hand side plots) increases slightly with the number of ambulances. Thus, the larger the number of ambulances, the harder the instances for the MEC model. Furthermore, the time limit is reached for every tested instance of the MEC model. For the SBFM, the relative optimality gaps are equal to 0 for all instances. In addition, the objective values are comparable to the MEC model for all different ambulance settings, which is a prominent characteristic. Furthermore, under the SBFM, all instances are solved in less than one minute, and this time is not affected by the number of ambulances.

Chapter 6

Conclusions

EMS systems in developing countries, such as Mexico, suffer from a shortage of ambulances. Thus, one of the main goals addressed in this work was to investigate and develop tools that allow us to decide whether an emergency can be uncovered, or totally or partially covered.

The *Emergency Vehicle Covering and Planning* (EVCP) problem consists of locating a limited number of two heterogeneous types of ambulances in different city locations and dispatching them to the emergency points so as to maximize the coverage with short medical first aid response time. In the EVCP problem, these two interrelated decisions are simultaneously considered in a novel two-stage stochastic program. The EVCP stochastic model allows for partial coverage of the accidents by the ambulances based on a decay function.

We propose a two-stage stochastic program for the EVCP problem that can be solved by branch-and-bound for small instances with a restrictive number of scenarios with the MEC formulation. We also propose a surrogate-based feedback method, which is essentially a location-allocation procedure that relies on the solution of an auxiliary surrogate model. This method is faster to solve and allows us to obtain high-quality solutions significantly faster than the previous model. The SBFM was tested over a broad set of randomly generated instances based on real-world data from a local system. An important feature of the proposed approach is that it can be implemented by calling any off-the-shelf integer solver without employing complex decomposition techniques.

SABC-matheuristic aims to improve the solutions obtained from the SABC methodology. There were slight improvements in the solutions.

In this manner, we have proposed exact and metaheuristics methodologies to solve the ECVP problem. Moreover, we have tackled a real problem, and now we can test it in practice.

Our stochastic model can solve large-scale instances, but as the sizes of the instances increase, the convergence time becomes very long. This is why one of the main contributions of this thesis is the SAFM. MEC model experiments show that large-scale problems cannot be solved because the solver does not have enough time or memory to obtain an optimal solution. However, small instances results generate feasible solutions for ambulance location and dispatching decisions in different scenarios. Partial rate coverage allows sending ambulances even if there are not enough ambulances to cover an accident totally or if an ambulance or more than one ambulance are out of the desired response time for patient attention. These results help us cover more demand points in the system, allowing us to start giving attention to patients, which can be finished after providing first aid to them at those demand points where not all ambulances were sent.

6.1 Future work

Our future work involves more than one service provider in the system, considering the differences between them and the preferences that public ambulances can have compared to private ambulances. In fact, in Monterrey, there are at least three emergency service providers who are cooperating but also competing with each other: *Cruz Roja, Protección Civil, CRUM.* Thus, game theory or bi-level programming could be used to determine the best policies in such a way that the population is benefited.

Naturally, there are several lines of work that can be investigated further. For example, another interesting aspect we observed is that there are some private EMS services that also dispatch vehicles to accident sites. Some of these are neither regulated nor coordinated by the state. In some cases, this provokes a conflict as too many ambulances arrive at the site, leaving other points unattended. This situation could, of course, benefit if coordinated through decision-making tools as the ones developed here.

With respect to the methodology, to solve the preliminarily model and future models, we will include Benders cuts or another solution method that we are studying. The difficulty is that our problem considers integer and binary variables in the second stage of the stochastic programming, thus, we have to evaluate if considering real variables yields feasible solutions and how much the reduction in the quality is.

Also, we want to include queues at hospitals. During the beginning of the

COVID-19 pandemic, some hospitals only attended to COVID-19 patients, which caused other hospitals to have ambulance queues due to overdemand. This wasted time waiting for attention affects ambulance availability and must be counted in the EMS system.

Another approach is to consider scenarios by clusters as it is done with the work of Hewitt et al. [30]. It could reduce the size of the stochastic program, and maybe it will not be necessary to use the Benders decomposition, and it may be competitive to the heuristic methods.

Finally, we are improving SABC matheuristic by considering other heuristic or matheuristic approaches to obtain better solutions compared to the solutions obtained from the local search procedure that we used in this investigation.

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Resumen autobiográfico

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Nací el 20 de enero de 1995 en el municipio de San Nicolás de los Garza en el estado de Nuevo León. Mis padres Jesús García Gámez y Beatriz Ramos Larralde me han cuidado y educado desde mi nacimiento, al igual que a mi hermana Karina Guadalupe García Ramos. Concluí mis estudios como Licenciado en Matemáticas en junio del año 2017 en la Facultad de Ciencias Físico Matemáticas perteneciente a la Universidad Autónoma de Nuevo León. Obtuve mi grado como Maestro en Ingeniería de Sistemas en noviembre de 2019 en la Facultad de Ingeniería Mecánica y Eléctrica perteneciente a la Universidad Autónoma de Nuevo León, en donde también inicié mis estudios de Doctorado en Ingeniería en Sistemas en agosto del año 2020.

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