Online Supplementary Material for “A Network-Constrained Hydrothermal Unit Commitment Model in the Day-Ahead Market”

Published in the

Journal of Control, Automation and Electrical Systems

DOI: 10.1007/s40313-025-01205-1

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August 2025

A Mathematical formulation

A.1 Notation

Following this, a summary of sets, indices, parameters, and decision variables is enlisted for quick reference.

Sets and indices of the power system:

$\mathcal{BR}$	Set of electric tie-lines; $br \in \mathcal{BR}$
$\mathcal{D}$	Set of loads; $d \in \mathcal{D}$
$\mathcal{D}^E$	Set of elastic loads; $d \in \mathcal{D}^E$
$\mathcal{G}$	Set of all generators in the system; $g \in \mathcal{G} = \mathcal{G}^{\mathrm{TE}} \cup \mathcal{G}^{\mathrm{HI}} \cup \mathcal{G}^{\mathrm{RE}}$
$\mathcal{G}^{\mathrm{HI}}$	Set of hydroelectric generators; $g \in \mathcal{G}^{\mathrm{HI}}$
$\mathcal{G}^{\mathrm{RE}}$	Set of renewable generators in the system; $g \in \mathcal{G}^{\mathrm{RE}}$
$\mathcal{G}^{\mathrm{TE}}$	Set of thermal generators; $g \in \mathcal{G}^{\mathrm{TE}}$
$\mathcal{N}$	Set for electric nodes in the system; $n \in \mathcal{N}$
$\mathcal{R}$	Set of reserve zones; $r \in \mathcal{R}$
$\mathcal{RO}$	Set of operative ranges of generators with prohibit operative zone (POZ); $ro \in \mathcal{RO}$, only for the $\mathcal{G}^{\mathrm{TE}}$ and $\mathcal{G}^{\mathrm{HI}}$
$\mathcal{T}$	Set of time periods in the planning horizon; $t, t' \in \mathcal{T}$

Sets and indices of the hydraulic system:

$\mathcal{E}$	Set of reservoirs; $e \in \mathcal{E}$
$\mathcal{HI}_e$	Set of hydro generators located in reservoir $e \in \mathcal{E}$; $g \in \mathcal{HI}_e$
$\mathcal{HI}_\nu$	Set of hydro generators that discharge water over a river $\nu \in \mathcal{V}_e^{\mathrm{d}}$; $g \in \mathcal{HI}_v$
$\mathcal{V}_e^{\mathrm{c}}$	Set of rivers converging to reservoir $e \in \mathcal{E}$; $\nu \in \mathcal{V}_e^{\mathrm{c}}$
$\mathcal{V}_e^{\mathrm{d}}$	Set of rivers diverging from reservoir $e \in \mathcal{E}$; $\nu \in \mathcal{V}_e^{\mathrm{d}}$

Sets and indices the cost-related aspects:

$\mathcal{B}_d^{\mathrm{e.p.b.}}$	Set of segments of the energy purchase bid for an elastic load $d \in \mathcal{D}^E$; $b \in \mathcal{B}_d^{\mathrm{e.p.b.}}$
$\mathcal{B}_g^{\mathrm{e.s.o.}}$	Set of segments of the energy sale offer for a generator $g \in \mathcal{G}$; $b \in \mathcal{B}_g^{\mathrm{e.s.o.}}$
$\mathcal{B}^{\mathrm{r.r.}}$	Set of segments of regulation reserve requirement; $b \in \mathcal{B}^{\mathrm{r.r.}}$
$\mathcal{B}^{\mathrm{r.10}}$	Set of segments of 10 minutes reserve requirement; $b \in \mathcal{B}^{\mathrm{r.10}}$
$\mathcal{B}^{\mathrm{s.r.10}}$	Set of segments of 10 minutes spinning reserve requirement; $b \in \mathcal{B}^{\mathrm{s.r.10}}$
$\mathcal{B}^{\mathrm{r.30}}$	Set of segments of 30 minutes reserve requirement; $b \in \mathcal{B}^{\mathrm{r.30}}$
$\mathcal{B}^{\mathrm{o.r.d.c}}$	The ORDC is comprised by $\mathcal{B}^{\mathrm{r.r.}} \bigcup \mathcal{B}^{\mathrm{s.r.10}} \bigcup \mathcal{B}^{\mathrm{r.10}} \bigcup \mathcal{B}^{\mathrm{r.30}}$; $b \in \mathcal{B}^{\mathrm{o.r.d.c}}$
$\mathcal{S}_g$	Set of start-up cost curve segments for a generator $g \in \mathcal{G}^{\mathrm{TE}}$; $s \in \mathcal{S}_g$

Parameters:

$a_{1,g}, a_{2,g}, a_{3,g}$	Coefficients that are part of the constant term of the HPF (S40) for a generator $g \in \mathcal{G}^{\mathrm{HI}}$; in MW

$b_{1,g}, b_{2,g}, b_{3,g}$	Linear coefficients that are part of the constant term of the HPF (S40) for a generator $g \in \mathcal{G}^{\mathrm{HI}}$; in MW/m
$c_{1,g}, c_{2,g}, c_{3,g}$	Quadratic coefficient that is part of the constant term of the HPF (S40) for a generator $g \in \mathcal{G}^{\mathrm{HI}}$; in MW s/m^2
$C^{\mathrm{e.p.b.}}_{b,d,t}$	Price of energy purchase bid in a segment $b \in \mathcal{B}^{\mathrm{e.p.b.}}_g$ for load $d \in \mathcal{D}^E$ in a period $t \in \mathcal{T}$; in \$/MWh
$C^{\mathrm{e.s.o.}}_{b,g,t}$	Price of energy sale offers in a segment $b \in \mathcal{B}^{\mathrm{e.s.o.}}_d$ for a generator $g \in \mathcal{G}^{\mathrm{TE}}$ in a period $t \in \mathcal{T}$; in \$/MWh
$C^{m.g.c}_g$	Minimum operating cost of a thermal generator $g \in \mathcal{G}^{\mathrm{TE}}$ that works at least at minimum power $\underline{P}_g$; in \$
$C^{\mathrm{n.s.10}}_{g,t}$,$C^{\mathrm{n.s.30}}_{g,t}$	Price of 10 and 30 minutes non-spinning reserve offer of a generator $g \in \mathcal{G}^{\mathrm{TE}} \cup \mathcal{G}^{\mathrm{HI}}$ in a period $t \in \mathcal{T}$; in \$/MWh
$C^{\mathrm{o.c}}_{g,t}$	Opportunity cost of a hydroelectric generator $g \in \mathcal{G}^{\mathrm{HI}}$ in a period $t \in \mathcal{T}$; in \$/MWh.
$C^{\mathrm{o.r.d.c}}_{b,t}$	Revenue or profit gained by operating reserve demand curve on $b \in \mathcal{B}^{\mathrm{o.r.d.c}}$ in a period $t \in \mathcal{T}$; in \$/MWh
$C^{\mathrm{r.r}}_{g,t}$	Price of regulation reserve offer of a generator $g \in \mathcal{G}^{\mathrm{TE}} \cup \mathcal{G}^{\mathrm{HI}}$ in a period $t \in \mathcal{T}$; in \$/MWh
$C^S_{g,s}$	Start-up offer in a generator $g \in \mathcal{G}^{\mathrm{TE}}$ with a set of segment time $s \in \mathcal{S}_g$; it determines the starting cost by locating a cost in a segment time s in the intervals $[\underline{T}_{g,s}, \overline{T}_{g,s})$; in \$/h
$C^{\mathrm{s.10}}_{g,t}$, $C^{\mathrm{s.30}}_{g,t}$	Price of 10 and 30 minutes spinning reserve offer of a generator $g \in \mathcal{G}^{\mathrm{TE}} \cup \mathcal{G}^{\mathrm{HI}}$ in a period $t \in \mathcal{T}$; in \$/MWh
$DB_{b,d,t}$	Price-sensitive demand bid in a segment $b \in \mathcal{B}^{\mathrm{e.p.b.}}_g$ for a load $d \in \mathcal{D}^E$ in a period $t \in \mathcal{T}$; in \$/MW
$DF_{d,t}$	Fixed demand in a segment $d \in \mathcal{D}$ in a period $t \in \mathcal{T}$; in MW
$\overline{fn}_{br,t}$,$\overline{fp}_{br,t}$	Maximum value for the power counterflow and flow on tie-line $br \in \mathcal{BR}$ in a period $t \in \mathcal{T}$, respectively; in MW.
$LSF_{n,t}$	Sensitivity transmission losses in node $n \in \mathcal{N}$ with regard to changes in power injections in node $n \in \mathcal{N}$ in a period $t \in \mathcal{T}$; dimensionless
$\overline{NP}_g$	Maximum number of stoppages allowed for a generator $g \in \mathcal{G}^{\mathrm{TE}} \cup \mathcal{G}^{\mathrm{HI}}$ during the planning horizon; dimensionless
$ORDC_{b,t}$	Operating reserve demand curve bid on $b \in \mathcal{B}^{\mathrm{o.r.d.c}}$ in a period $t \in \mathcal{T}$; in \$/MWh
P^{sync}_g	Power generation of a generator $g \in \mathcal{G}$ injects into the system immediately after connecting it; in MW
$Pb_{b,g,t}$	Offer quantity of power $b \in \mathcal{B}^{\mathrm{e.p.b.}}_g$ for $g \in \mathcal{G}^{\mathrm{TE}} \cup \mathcal{G}^{\mathrm{RE}}$ in a period $t \in \mathcal{T}$; in MW
$\overline{P}_{g,t}$,$\underline{P}_{g,t}$	Maximum and minimum generation value of a generator $g \in \mathcal{G}$ in a period $t \in \mathcal{T}$, respectively; in MW

$\overline{P}^{RE}_{g,ro,t}$,$\underline{P}^{RE}_{g,ro,t}$	Maximum and minimum regulation limit for a operative zone $ro \in \mathcal{RO}$ of a generator $g \in \mathcal{G}^{\mathrm{TE}} \cup \mathcal{G}^{\mathrm{HI}}$ in a period $t \in \mathcal{T}$, respectively; in MW
$\overline{P}^{RO}_{g,ro}, \underline{P}^{RO}_{g,ro}$	Maximum and minimum operation limit for an operative zone $ro \in \mathcal{RO}$ of a generator $g \in \mathcal{G}^{\mathrm{TE}} \cup \mathcal{G}^{\mathrm{HI}}$ respectively; in MW
$PTDF_{br,n,t}$	Power transfer distribution factors are the incremental changes or sensitivity power flow in tie-lines $br \in \mathcal{BR}$ concerning power injection at any node $n \in \mathcal{N}$ in a period t
$\overline{q}_{g,t}, \underline{q}_{g,t}$	Maximum and minimum water flow passing through the turbines of a generator $g \in \mathcal{G}^{\mathrm{HI}}$ in a period $t \in \mathcal{T}$, respectively; in m^3/s
$\overline{Q}_{\nu,t}, \underline{Q}_{\nu,t}$	Maximum and minimum water discharge limits over a river $\nu \in \mathcal{V}^{\mathrm{d}}_e$ in a reservoir $e \in \mathcal{E}$ in a period $t \in \mathcal{T}$, respectively; in m^3/s
R_{br}	Electric resistance of a tie-line $br \in \mathcal{BR}$; dimensionless
RB_g	Ramp-down rate is the capacity of a generator $g \in \mathcal{G}^{\mathrm{TE}} \cup \mathcal{G}^{\mathrm{HI}}$ to decrease power between two consecutive periods; in MW/h
$RE^{10}_{g,t}$,$RE^{30}_{g,t}$	Emergency ramp rate for 10 and 30 minutes spinning reserve of a generator $g \in \mathcal{G}^{\mathrm{TE}} \cup \mathcal{G}^{\mathrm{HI}}$ in a period $t \in \mathcal{T}$; in MW
$RR_{g,t}$	Regulation ramp rate of a generator $g \in \mathcal{G}^{\mathrm{TE}} \cup \mathcal{G}^{\mathrm{HI}}$ in a period $t \in \mathcal{T}$; in MW
RS_g	Ramp-up rate of a generator $g \in \mathcal{G}^{\mathrm{TE}} \cup \mathcal{G}^{\mathrm{HI}}$ to increase power between two consecutive periods; in MW/h
RS^{sync}_g	Ramp-up rate of a generator $g \in \mathcal{G}^{\mathrm{TE}}$ to increase power when the generator is starting; in MW/h
$RRo^{10}_{g,t}$,$RRo^{30}_{g,t}$	10 and 30 minutes spinning reserve bid of a generator $g \in \mathcal{G}^{\mathrm{TE}} \cup \mathcal{G}^{\mathrm{HI}}$ in a period $t \in \mathcal{T}$; in \$/MWh
$RNR^{10}_{g,t}$,$RNR^{30}_{g,t}$	10 and 30 minutes non-spinning reserve bid of an offline generator $g \in \mathcal{G}^{\mathrm{TE}} \cup \mathcal{G}^{\mathrm{HI}}$ in a period $t \in \mathcal{T}$; in \$/MWh
$RRE_{g,t}$	Regulation reserve bid of a generator $g \in \mathcal{G}^{\mathrm{TE}} \cup \mathcal{G}^{\mathrm{HI}}$ in a period $t \in \mathcal{T}$; in \$/MWh
$\overline{RNR}^{10}_{g,t}$,$\overline{RNR}^{30}_{g,t}$	Maximum capacity limit for 10 and 30 minutes non-spinning reserve of a generator $g \in \mathcal{G}^{\mathrm{TE}} \cup \mathcal{G}^{\mathrm{HI}}$ in a period $t \in \mathcal{T}$; in MW
$\overline{RRo}^{10}_{g,t}$,$\overline{RRo}^{30}_{g,t}$	Maximum capacity limit for 10 and 30 minutes spinning reserve of a generator $g \in \mathcal{G}^{\mathrm{TE}} \cup \mathcal{G}^{\mathrm{HI}}$ in a period $t \in \mathcal{T}$; in MW
$\overline{RRe}_{g,ro,t}$	Maximum capacity limit for regulation reserve of an generator $g \in \mathcal{G}^{\mathrm{TE}} \cup \mathcal{G}^{\mathrm{HI}}$ in a operative zone ro in a period $t \in \mathcal{T}$; in MW
SU^D_g	Start-up time for a generator $g \in \mathcal{G}^{\mathrm{TE}}$; in h
$\overline{T}_{g,s}, \underline{T}_{g,s}$	Start and end of start-up cost segment $s \in \mathcal{S}$, respectively; in h
UT_g,DT_g	minimum up/down time for a generator $g \in \mathcal{G}^{\mathrm{TE}} \cup \mathcal{G}^{\mathrm{HI}}$; in h
$\epsilon_{e,t}$	The amount of water by natural inflows in a reservoir $e \in \mathcal{E}$ in a period $t \in \mathcal{T}$; in m^3
$\kappa_{\nu,t}$	Discharge spillage over a river $\nu \in \mathcal{V}^{\mathrm{d}}_e$ in a period $t \in \mathcal{T}$; in m^3

$\rho_{e,t}$	Water outflow of a reservoir $e \in \mathcal{E}$ in a period $t \in \mathcal{T}$ used for different purposes unrelated to energy generation; in m^3
τ_ν	Water travel time delay of a river $\nu \in \mathcal{V}_e^{\mathrm{c}}$; in h
$\overline{\omega}_e, \underline{\omega}_e$	Maximum and minimum water storage limits on a reservoir $e \in \mathcal{E}$; in m^3

Binary variables:

$u_{g,t}$	Equal to 1 if generator $g \in \mathcal{G}^{\mathrm{TE}} \cup \mathcal{G}^{\mathrm{HI}}$ is online in a period $t \in \mathcal{T}$, and 0 otherwise
$u_{g,t}^{S}$	Equal to 1 if generator $g \in \mathcal{G}^{\mathrm{TE}}$ is starting-up in a period $t \in \mathcal{T}$, and 0 otherwise
$u_{g,ro,t}^{RO}$	Equal to 1 if generator $g \in \mathcal{G}^{\mathrm{TE}} \cup \mathcal{G}^{\mathrm{HI}}$ is in operating range $ro \in \mathcal{RO}$ in a period $t \in \mathcal{T}$, and 0 otherwise
$u_{g,ro,t}^{RE}$	Equal to 1 if generator $g \in \mathcal{G}^{\mathrm{TE}} \cup \mathcal{G}^{\mathrm{HI}}$ is in regulation reserve re in a period $t \in \mathcal{T}$, and 0 otherwise
$u_{g,t}^{rnr10}$	Equal to 1 if generator $g \in \mathcal{G}^{\mathrm{TE}} \cup \mathcal{G}^{\mathrm{HI}}$ is committed to supplying 10 minutes of non-spinning reserve in a period $t \in \mathcal{T}$, and 0 otherwise
$u_{g,t}^{rnr30}$	Equal to 1 if generator $g \in \mathcal{G}^{\mathrm{TE}} \cup \mathcal{G}^{\mathrm{HI}}$ is committed to supplying 30 minutes of non-spinning reserve in a period $t \in \mathcal{T}$, and 0 otherwise
$v_{g,t}$	Equal to 1 if generator $g \in \mathcal{G}^{\mathrm{TE}} \cup \mathcal{G}^{\mathrm{HI}}$ starts up at the beginning of period $t \in \mathcal{T}$, and 0 otherwise
$w_{g,t}$	Equal to 1 if generator $g \in \mathcal{G}^{\mathrm{TE}} \cup \mathcal{G}^{\mathrm{HI}}$ is shut-down at the beginning of period $t \in \mathcal{T}$, and 0 otherwise
$\delta_{g,t,s}$	Equal to 1 if generator $g \in \mathcal{G}^{\mathrm{TE}}$ have a start-up type $s \in \mathcal{S}$ in period $t \in \mathcal{T}$, and 0 otherwise

Real variables:

$DB_{b,d,t}$	Amount of demand price-response of segment b for demand $d \in \mathcal{D}^E$ in a period $t \in \mathcal{T}$, in MW
$f_{br,t}$	Power flow on br in i, in a period $t \in \mathcal{T}$; in MW.
$h_{v,t}$	Net hydraulic head of a river ν in a period $t \in \mathcal{T}$; in m
$iny_{n,t}$	Amount of power input at node $n \in \mathcal{N}$ in a period $t \in \mathcal{T}$, in MW
$Loss_t^{SP}$	Amount of exact transmission losses in a period $t \in \mathcal{T}$, in MW
$Loss_t^{MP}$	Amount of approximate transmission losses in a period $t \in \mathcal{T}$, in MW
$p_{g,t}$	Amount of power a generator $g \in \mathcal{G}$ produces in a period $t \in \mathcal{T}$, in MW
$pb_{b,g,t}$	Amount of power a generator $g \in \mathcal{G}^{\mathrm{TE}} \cup \mathcal{G}^{\mathrm{HI}}$ produces in segment b of period $t \in \mathcal{T}$, in MW
$ps_{g,t}$	Amount of power a generator $g \in \mathcal{G}$ produces during the start-up time in a period $t \in \mathcal{T}$, in MW
$q_{g,t}$	Water discharge of generator $g \in \mathcal{G}$ in a period $t \in \mathcal{T}$; in m^3/s
$rco_{b,t}$	Committed amount of reserve commitment of the OCDR on segment b in a period $t \in \mathcal{T}$; in MW

$rnr_{g,t}^{10}$	Committed amount of 10 minutes non-spinning reserve of a generator $g \in \mathcal{G}$ in a period $t \in \mathcal{T}$, in MW
$rnr_{g,t}^{30}$	Committed amount of 30 minutes non-spinning reserve of a generator $g \in \mathcal{G}$ in a period $t \in \mathcal{T}$, in MW
$rre_{g,t}$	Committed amount of regulation reserve of generator $g \in \mathcal{G}$ in a period $t \in \mathcal{T}$; in MW
$\underline{rre}_{g,t}$	Minimum regulation reserve that can be committed to a generator $g \in \mathcal{G}^{\text{TE}} \cup \mathcal{G}^{\text{HI}}$ in a period $t \in \mathcal{T}$; in MW
$rro_{g,t}^{10}$	Committed amount of 10 minutes spinning reserve of a generator $g \in \mathcal{G}$ in a period $t \in \mathcal{T}$, in MW
$rro_{g,t}^{30}$	Committed amount of 30 minutes spinning reserve of a generator $g \in \mathcal{G}$ in a period $t \in \mathcal{T}$, in MW
$\omega_{e,t}$	Water volume in the reservoir e in a period $t \in \mathcal{T}$; in m^3

A.2 Assumptions

Key assumptions to bear in mind: The limits of generator output power, reserves, offers, and bids may change during the planning horizon. Also, the purchase bids and the energy generation in each period represent the average power demand between the beginning and the end of a period.

A.3 Objective function

The objective function is cast in the form of maximizing the net profit obtained by the difference between the revenue obtained from the selling of energy from the elastic loads and selling of reserve requirements minus all operation costs including start-up costs, minimum generation cost, and cost of the power and reserves supplied by generators.

$$\begin{aligned}
\text{maximize} \quad & \sum_{t\in\mathcal{T}} \sum_{b\in\mathcal{B}^{\text{o.r.d.c}}} C_{b,t}^{o.r.d.c} rco_{b,t} \\
& - \sum_{t\in\mathcal{T}} \Bigg\{ \sum_{g\in\mathcal{G}^{\text{TE}}} \Bigg[C_{g,s}^{S} \delta_{g,t,s} + u_{g,t} C_{g}^{m.g.c} + \sum_{b\in\mathcal{B}_g^{\text{e.s.o}}} C_{b,g,t}^{e.s.o} pb_{b,g,t} \Bigg] \\
& + \sum_{g\in\mathcal{G}^{\text{HI}}} C_{g,t}^{o.c} p_{g,t} + \sum_{g\in\mathcal{G}^{\text{RE}}} \sum_{b\in\mathcal{B}_g^{\text{e.s.o}}} C_{b,g,t}^{e.s.o} pb_{b,g,t} - \sum_{d\in\mathcal{D}^{E}} \Bigg\{ \sum_{b\in\mathcal{B}_d^{\text{e.p.b}}} C_{b,d,t}^{e.p.b} db_{b,d,t} \Bigg\} \\
& + \sum_{g\in\mathcal{G}^{\text{TE}} \bigcup \mathcal{G}^{\text{HI}}} \Bigg[C_{g,t}^{s.10} rro_{g,t}^{10} + C_{g,t}^{n.s.10} rnr_{g,t}^{10} + C_{g,t}^{s.30} rro_{g,t}^{30} \\
& + C_{g,t}^{n.s.30} rnr_{g,t}^{30} + C_{g,t}^{r.r} rre_{g,t} \Bigg] \Bigg\}. \qquad \text{(S1)}
\end{aligned}$$

The constraints of the problem are detailed below.

A.4 Constraints

A.4.1 Power balance

The balance equation per node is outlined in (S2), where the power injections in a node n in a period t are unpacked in (S3):

$$\sum_{n\in\mathcal{N}} iny_{n,t} - Loss_t = 0, \qquad t \in \mathcal{T}, \tag{S2}$$

$$iny_{n,t} = \sum_{g\in\mathcal{G}_n} p_{g,t} - \sum_{d\in\mathcal{D}_n} (DF_{d,t} + \sum_{b\in\mathcal{B}_d^{\text{e.p.b}}} db_{b,d,t}), \qquad n \in \mathcal{N},\ t \in \mathcal{T}. \tag{S3}$$

A.4.2 Loads

The lower and upper limits of the elastic loads are shown as follows:

$$0 \leq db_{b,d,t} \leq DB_{b,d,t}, \qquad b \in \mathcal{B}_d^{\text{e.p.b}},\ d \in \mathcal{D}^E,\ t \in \mathcal{T}. \tag{S4}$$

A.4.3 Offers reserve and requirements reserve

RRE, RRR, RRS, and RRS requirements are met by the generators' committed reserves (regulation, spinning, and non-spinning of 10 and 30 minutes) by constraints (S5)–(S8). Also, the limits of reserve requirements are expressed in (S9):

$$\sum_{g\in\mathcal{G}^{\text{TE}}\cup\mathcal{G}^{\text{HI}}} rre_{g,t} \geq \sum_{b\in\mathcal{B}^{\text{r.r.}}} rco_{b,t}, \qquad t \in \mathcal{T}, \tag{S5}$$

$$\sum_{g\in\mathcal{G}^{\text{TE}}\cup\mathcal{G}^{\text{HI}}} (rre_{g,t} + rro_{g,t}^{10}) \geq \sum_{b\in\mathcal{B}^{\text{r.r.}}\cup\mathcal{B}^{\text{s.r.10}}} rco_{b,t}, \qquad t \in \mathcal{T}, \tag{S6}$$

$$\sum_{g\in\mathcal{G}^{\text{TE}}\cup\mathcal{G}^{\text{HI}}} (rre_{g,t} + rro_{g,t}^{10} + rnr_{g,t}^{10}) \geq \sum_{b\in\mathcal{B}^{\text{r.r.}}\cup\mathcal{B}^{\text{s.r.10}}\cup\mathcal{B}^{\text{r.10}}} rco_{b,t}, \qquad t \in \mathcal{T}, \tag{S7}$$

$$\sum_{g\in\mathcal{G}^{\text{TE}}\cup\mathcal{G}^{\text{HI}}} (rre_{g,t} + rro_{g,t}^{10} + rnr_{g,t}^{10} + rro_{g,t}^{30} + rnr_{g,t}^{30}) \geq \sum_{b\in\mathcal{B}^{\text{o.r.d.c}}} rco_{b,t}, \qquad t \in \mathcal{T}, \tag{S8}$$

$$0 \leq rco_{b,t} \leq ORDC_{b,t}, \qquad b \in \mathcal{B}^{\text{o.r.d.c}},\ t \in \mathcal{T}. \tag{S9}$$

A.4.4 Generation limits

Constraints (S10) establish that, for thermal generators, the power level of a generator is expressed as the sum of the power produced during the start-up process, the minimal operative limit of generator, and the sum of the segment power values associated with each offer segment. For renewable generators, the power level of a generator amounts to the sum of the segment powers associated with each offer segment, as indicated in constraints (S11). For both renewable and thermal generators, the power level amounts of the individual segments offered is limited by constraints (S12).

$$p_{g,t} = ps_{g,t} + u_{g,t}\underline{P}_{g,t} + \sum_{b \in \mathcal{B}_g^{\text{e.s.o}}} pb_{b,g,t}, \qquad g \in \mathcal{G}^{\text{TE}},\ t \in \mathcal{T}, \tag{S10}$$

$$p_{g,t} = \sum_{b \in \mathcal{B}_g^{\text{e.s.o}}} pb_{b,g,t}, \qquad g \in \mathcal{G}^{\text{RE}},\ t \in \mathcal{T}, \tag{S11}$$

$$0 \leq pb_{b,g,t} \leq u_{g,t} Pb_{b,g,t}, \qquad b \in \mathcal{B}_g^{\text{e.s.o}},\ g \in \mathcal{G}^{\text{TE}} \cup \mathcal{G}^{\text{RE}}, \qquad t \in \mathcal{T}. \tag{S12}$$

A.4.5 Generation limits and reserves

For renewable generators, the power limits (maximum and minimum) are modeled in (S13). Concerning the thermal generator in operation, the sum should not exceed its minimal operative power if it is in the start-up process, constraints (S14). Regarding hydro generators, the maximum power level should be greater or equal than its output power, plus the regulation committed, constraints (S15).

$$u_{g,t}\underline{P}_{g,t} \leq p_{g,t} \leq u_{g,t}\overline{P}_{g,t}, \qquad g \in \mathcal{G}^{\text{RE}},\ t \in \mathcal{T}, \tag{S13}$$

$$p_{g,t} + rre_{g,t} + rro^{10}_{g,t} + rro^{30}_{g,t} \leq u_{g,t}\overline{P}_{g,t} + u^{S}_{g,t}\underline{P}_{g,t}, \qquad g \in \mathcal{G}^{\text{TE}},\ t \in \mathcal{T}, \tag{S14}$$

$$p_{g,t} + rre_{g,t} + rro^{10}_{g,t} + rro^{30}_{g,t} \leq u_{g,t}\overline{P}_{g,t}, \qquad g \in \mathcal{G}^{\text{HI}},\ t \in \mathcal{T}. \tag{S15}$$

For both, hydro and thermal generators, their power level should not exceed the maximum level of the operating range selected ro. If the generator does not sell regulation reserve, its power level should not exceed the operating range selected. In case of selling regulation reserve ($u^{RE} = 1$) its power level plus its reserve regulation should not exceed the maximum limit of reserve regulation for the operating range selected, constraints (S16).

$$p_{g,t} + rre_{g,t} \leq \sum_{ro \in \mathcal{RO}_g} \{u^{RO}_{g,ro,t}\overline{P}^{RO}_{g,ro} + u^{RE}_{g,ro,t}(\overline{P}^{RE}_{g,ro,t} - \overline{P}^{RO}_{g,ro})\}, \qquad g \in \mathcal{G}^{\text{TE}} \cup \mathcal{G}^{\text{HI}},\ t \in \mathcal{T}. \tag{S16}$$

Concerning hydro generators, their power level should be gretaer or equal to the minimum level of the operating range selected ro. If the generator does not sell regulation reserve, its power level should be greater or equal to the minimum level of the operating range selected. In case of selling regulation reserve ($u^{RE} = 1$) its power level minus its reserve regulation should be greater or equal to the minimum limit of reserve regulation for the operating range selected, constraints (S17).

$$p_{g,t} - rre_{g,t} \geq \sum_{ro \in \mathcal{RO}_g} \{u^{RO}_{g,ro,t}\underline{P}^{RO}_{g,ro} + u^{RE}_{g,ro,t}(\underline{P}^{RE}_{g,ro,t} - \underline{P}^{RO}_{g,ro})\}, \qquad g \in \mathcal{G}^{\text{HI}},\ t \in \mathcal{T}. \tag{S17}$$

With regard to thermal generators, their power level should be greater or equal to the minimum level of the operating range selected ro. If the generator does not sell regulation reserve, its power level should be greater or equal to the minimum level of the operating range selected plus the synchronous power (start-up power of a generator). In the case of commitment of selling regulation reserve ($u^{RE} = 1$), its power level minus its reserve regulation should be greater or equal to the minimum limit of reserve regulation for the operating range selected plus the synchronizing power. Synchronizing power is the generator's power level once connected and has the same system frequency. The generator is not considered for selling regulation reserve during its start-up time. This is accomplished by these constraints:

$$p_{g,t} - rre_{g,t} \geq \sum_{ro \in \mathcal{RO}_g} \{u^{RO}_{g,ro,t}\underline{P}^{RO}_{g,ro} + u^{RE}_{g,ro,t}(\underline{P}^{RE}_{g,ro,t} - \underline{P}^{RO}_{g,ro})\} + u^{S}_{g,t}P^{sync}_{g,t}, \quad g \in \mathcal{G}^{\mathrm{TE}},\ t \in \mathcal{T}. \quad \text{(S18)}$$

To commit a generator for supplying regulation reserve, a generation operative range $\mathcal{RO}$ should be selected with constraints (S17), avoiding the POZ:

$$u^{RE}_{g,ro,t} \leq u^{RO}_{g,ro,t}, \qquad ro \in \mathcal{RO}_g; g \in \mathcal{G}^{\mathrm{TE}} \cup \mathcal{G}^{\mathrm{HI}},\ t \in \mathcal{T}. \quad \text{(S19)}$$

A.4.6 Reserve limits and ramps

The following constraints establish that the 10- and 30-minute spinning reserves committed should be within the operative limits, respectively:

$$0 \leq rro^{10}_{g,t} \leq u_{g,t}\overline{RRo}^{10}_{g,t}, \qquad g \in \mathcal{G}^{\mathrm{TE}} \cup \mathcal{G}^{\mathrm{HI}},\ t \in \mathcal{T}. \quad \text{(S20)}$$

$$0 \leq rro^{30}_{g,t} \leq u_{g,t}\overline{RRo}^{30}_{g,t}, \qquad g \in \mathcal{G}^{\mathrm{TE}} \cup \mathcal{G}^{\mathrm{HI}},\ t \in \mathcal{T}. \quad \text{(S21)}$$

The constraintys below indicate that the regulation reserve committed should be within the operative limits:

$$\sum_{ro \in \mathcal{RO}_g} u^{RE}_{g,ro,t}\underline{rre}_{g,t} \leq rre_{g,t} \leq \sum_{ro \in \mathcal{RO}_g} u^{RE}_{g,ro,t}\overline{RRe}_{g,ro,t}, \qquad g \in \mathcal{G}^{\mathrm{TE}} \cup \mathcal{G}^{\mathrm{HI}},\ t \in \mathcal{T}. \quad \text{(S22)}$$

The following constraints establish that the 10- and 30-minute non-spinning reserves committed should be within the limits:

$$0 \leq rnr^{10}_{g,t} \leq (1 - u^{s}_{g,t} - u_{g,t})RNR^{10}_{g,t}, \qquad g \in \mathcal{G}^{\mathrm{TE}},\ t \in \mathcal{T}. \quad \text{(S23)}$$

$$0 \leq rnr^{10}_{g,t} \leq (1 - u_{g,t})RNR^{10}_{g,t}, \qquad g \in \mathcal{G}^{\mathrm{HI}},\ t \in \mathcal{T}. \quad \text{(S24)}$$

$$0 \leq rnr^{30}_{g,t} \leq (1 - u^{s}_{g,t} - u_{g,t})RNR^{30}_{g,t}, \qquad g \in \mathcal{G}^{\mathrm{TE}},\ t \in \mathcal{T}. \quad \text{(S25)}$$

$$0 \leq rnr^{30}_{g,t} \leq (1 - u_{g,t})RNR^{30}_{g,t}, \qquad g \in \mathcal{G}^{\mathrm{HI}},\ t \in \mathcal{T}. \quad \text{(S26)}$$

A.4.7 Ramp up/down

The power increase in generators between two consecutive time intervals or ramp-up rate in thermal and hydro generators is limited by the following constraints:

$$p_{g,t} - p_{g,t-1} \leq RS_g, \qquad g \in \mathcal{G}^{\text{TE}} \cup \mathcal{G}^{\text{HI}},\ t \in \mathcal{T}. \tag{S27}$$

The power reduction in generators between two consecutive time intervals or ramp-down rate in thermal and hydro generators is limited by the following constraints:

$$p_{g,t-1} - p_{g,t} \leq RB_g + w_{g,t}(\overline{P}_{g,t-1} - RB_g) \qquad g \in \mathcal{G}^{\text{TE}} \cup \mathcal{G}^{\text{HI}},\ t \in \mathcal{T}. \tag{S28}$$

The power increase in generators in start-up processes between two consecutive time intervals or starting-up ramp in hydro generators are modeled as follows:

$$ps_{g,t} - ps_{g,t-1} \leq P_g^{sync}(u_{g,t}^S - u_{g,t-1}^S) + RS_g^{sync} u_{g,t-1}^S, \qquad g \in \mathcal{G}^{\text{TE}},\ t \in \mathcal{T}. \tag{S29}$$

$$ps_{g,t} - ps_{g,t-1} \geq RS_g^{sync} u_{g,t-1}^S - \underline{P}_{g,t} u_{g,t}, \qquad g \in \mathcal{G}^{\text{TE}},\ t \in \mathcal{T}. \tag{S30}$$

A.4.8 Minimum up and down time

Once a generator is turned on, it is kept on at least a minimum number of time intervals before turning it off. This is modeled in constraints (S31). Likewise, once a generator is turned off, it is kept off at least a minimum number of time intervals before turning it on, as indicated by constraints (S32):

$$\sum_{t'=t+SU_g^D}^{t+SU_g^D+UT_g} u_{g,t'} - UT_g v_{g,t} \geq 0, \qquad g \in \mathcal{G}^{\text{TE}},\ t, t' \in \mathcal{T}. \tag{S31}$$

$$\sum_{t'=t}^{t+DT_g-1} u_{g,t'} + DT_g w_{g,t} \leq DT_g, \qquad g \in \mathcal{G}^{\text{TE}},\ t, t' \in \mathcal{T}. \tag{S32}$$

A.4.9 Variable startup costs

The following constraints are based on the work by Morales-España et al. [2]. The relationship between the start-up segments and the start-up variables is modeled by constraints (S33). Constraints (S34) establish the relationship between the number of hours that the generator has been disconnected and its corresponding segment in the startup costs function for the current interval. These constraints assure that the number of hours that the generator will be disconnected are within the interval of the startup segment delimited between $[\underline{T}_{g,s}, \overline{T}_{g,s})$.

$$v_{g,t} \leq \sum_{s\in\mathcal{S}} \delta_{g,t,s}, \qquad g \in \mathcal{G}^{\mathrm{TE}},\ t \in \mathcal{T}, \tag{S33}$$

$$\delta_{g,t,s} \leq \sum_{t'\in\mathcal{T}'_t} w_{g,t'}, \qquad g \in \mathcal{G}^{\mathrm{TE}},\ t \in \mathcal{T},\ s \in \mathcal{S} - \{|\mathcal{S}|\}, \tag{S34}$$

where

$$\mathcal{T}'_t = \{t' | t - \overline{T}_{g,s} + 1 \leq t' \leq t - \underline{T}_{g,s} 0 < t' < t\}. \tag{S35}$$

A.4.10 Hydraulic balance

The water balance in a reservoir is modeled by constraints (S36). The water inputs to the reservoir are represented by the natural inflow minus other uses, plus the water comes from the reservoir upstream. The water output from the reservoir to the river is the sum of the discharge of water that passes through the turbines plus the spillage. The spillage in the reservoir is a parameter that takes into account the water travel times. The volume of reservoir water should be kept within the minimum and maximum storage operative limits as indicated by (S37).

$$\begin{aligned} \omega_{e,t} = \omega_{e,t-1} &+ \sum_{\nu\in\mathcal{V}^{\mathrm{c}}_e} \sum_{g\in\mathcal{HI}_\nu} q_{g,t-\tau_v} - \sum_{\nu\in\mathcal{V}^{\mathrm{d}}_e} \sum_{g\in\mathcal{HI}_\nu} q_{g,t} \\ &+ \epsilon_{e,t} - \rho_{e,t} - \sum_{\nu\in\mathcal{V}^{\mathrm{d}}_e} \kappa_{\nu,t} + \sum_{\nu\in\mathcal{V}^{\mathrm{c}}_e} \kappa_{\nu,t-\tau_\nu}, \qquad e \in \mathcal{E},\ t \in \mathcal{T} \end{aligned} \tag{S36}$$

$$\underline{\omega}_e \leq \omega_{e,t} \leq \overline{\omega}_e, \qquad e \in \mathcal{E},\ t \in \mathcal{T}. \tag{S37}$$

For each hydro generator, the water discharge that passes through the turbine in the reservoirs should meet minimum and maximum limits, as indicated by constraints (S38). In addition, the sum of the water discharge that passes through turbine generators connected to a reservoir should meet the minimum and maximum limits in the divergent river channel, as set in constraints (S39).

$$\underline{Q}_{\nu,t} \leq \sum_{g\in\mathcal{HI}_\nu} q_{g,t} \leq \overline{Q}_{\nu,t}, \qquad \nu \in \mathcal{V}^{\mathrm{d}}_e, e \in \mathcal{E}, \tag{S38}$$

$$u_{g,t}\underline{q}_{g,t} \leq q_{g,t} \leq u_{g,t}\overline{q}_{g,t}, \qquad g \in \mathcal{G}^{\mathrm{HI}},\ t \in \mathcal{T}. \tag{S39}$$

A.4.11 Hydraulic generation

Hydro generation power depends on the turbine water discharge rate and the reservoir head, both quadratically. The function used here is known as Glimn-Kirchmayer [1]. The equation below is known as the hydropower function (HPF). Its parameters $\{a_{1,g}, ..., c_{3,g}\}$ depend on the reservoir design, turbine and generator features.

$$p_{g,t} = u_{g,t}\big((a_{1,g} + b_{1,g}h_{\nu,t} + c_{1,g}h^2_{\nu,t}) + (a_{2,g} + b_{2,g}h_{\nu,t} + c_{2,g}h^2_{\nu,t})q_{g,t} + (a_{3,g} + b_{3,g}h_{\nu,t} + c_{3,g}h^2_{\nu,t})q^2_{g,t}\big), \qquad g \in \mathcal{HI}_{\nu},\ \nu \in \mathcal{V}^{\mathrm{d}}_e,\ e \in \mathcal{E},\ t \in \mathcal{T}. \tag{S40}$$

The non-linear feature of the HPF constraint requires an approximation method to be successfully integrated into a MILP. The method used in our work is the approximation to Taylor polynomial. This method generates an alternative constraint to replace (S40).

A.4.12 Power flow limits

The power flow in a transmission tie-line is modeled in constraints (S41), where $f_{br,t}$ represents the power flow in a transmission tie-line br which depends on parameters $PTDF_{br,n,t}$ and the power injection $iny_{n,t}$ for each bus n for each period t. A detailed description of the PTDF's calculation is explained by Tejada-Arango et al. [4]. Also, constraints (S42) fixes the maximum limits of flow and counterflow power in tie-line br.

$$f_{br,t} = \sum_{n \in \mathcal{N}} PTDF_{br,n,t}\, iny_{n,t}, \qquad br \in \mathcal{BR},\ t \in \mathcal{T}, \tag{S41}$$

$$\overline{fn}_{br,t} \le f_{br,t} \le \overline{fp}_{br,t}, \qquad t \in \mathcal{T}. \tag{S42}$$

A.4.13 Transmission losses

The losses in the system are calculated using the constraints below. The non-linear feature of these constraints requires an approximation method to be successfully integrated into a MILP solver. The method used in our work is tangent planes and provides a set of alternative constraints to replace these constraints.

$$Loss_t = \sum_{br \in \mathcal{BR}} R_{br}(f_{br,t})^2, \qquad t \in \mathcal{T}. \tag{S43}$$

A.4.14 Logical constraints

A generator cannot start-up and shut-down at the same time, as indicated by constraints (S44). A hydro generator cannot start-up and shut-down and operate (to provide energy in stable fashion within its operative limits) at the same time, as set by constraints (S45).

$$v_{g,t} + w_{g,t} \le 1, \qquad g \in \mathcal{G}^{\mathrm{TE}} \cup \mathcal{G}^{\mathrm{HI}},\ t \in \mathcal{T}, \tag{S44}$$

$$u_{g,t} - u_{g,t-1} - v_{g,t} + w_{g,t} = 0, \qquad g \in \mathcal{G}^{\mathrm{HI}},\ t \in \mathcal{T}. \tag{S45}$$

A thermal generator cannot synchronize, start-up, shut-down, and operate (to provide energy in stable fashion within its operative limits) at the same time, as indicated below:

$$u_{g,t} - u_{g,t-1} - v_{g,t} + w_{g,t} + u^{S}_{g,t} - u^{S}_{g,t-1} = 0, \qquad g \in \mathcal{G}^{\mathrm{TE}},\ t \in \mathcal{T}. \tag{S46}$$

To activate a generator operating range, the unit should be committed for operation [3]:

$$sum_{ro \in \mathcal{RO}_g} u^{RO}_{g,ro,t} = u_{g,t}, \qquad g \in \mathcal{G}^{\mathrm{TE}} \cup \mathcal{G}^{\mathrm{HI}},\ t \in \mathcal{T}. \tag{S47}$$

A generator can start-up only if it was off in the previous time interval, as expressed below:

$$u_{g,t-1} + u^{S}_{g,t} \leq 1, \qquad g \in \mathcal{G}^{\mathrm{TE}},\ t \in \mathcal{T}. \tag{S48}$$

A generator is not committed for operation during the starting-up time intervals, as indicated below:

$$\sum_{t'=t}^{t+SU^D_g-1} u_{g,t'} + SU^D_g v_{g,t} \leq SU^D_g, \qquad g \in \mathcal{G}^{\mathrm{TE}},\ t \in \mathcal{T}. \tag{S49}$$

A generator is committed for the starting-up process during the starting-up time intervals, as established below:

$$\sum_{t'=t}^{t+SU^D_g-1} u^{S}_{g,t'} - SU^D_g v_{g,t} \geq 0, \qquad g \in \mathcal{G}^{\mathrm{TE}},\ t \in \mathcal{T}. \tag{S50}$$

Generator power during the starting-up process should be kept within a minimum limit given by its synchronizing power and a maximum limit given by its minimum operative power, as indicated below:

$$u^{S}_{g,t} P^{sync}_{g} \leq ps_{g,t} \leq u^{S}_{g,t} \underline{P}_{g,t}, \qquad g \in \mathcal{G}^{\mathrm{TE}},\ t \in \mathcal{T}. \tag{S51}$$

The number of shutdowns for a generator during the day cannot exceed an established limit, as set below:

$$\sum_{t \in \mathcal{T}} w_{g,t} \leq \overline{NP}_{g}, \qquad g \in \mathcal{G}^{\mathrm{TE}} \cup \mathcal{G}^{\mathrm{HI}}. \tag{S52}$$

References

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