Online Supplementary Material for A Network-Constrained Hydrothermal Unit Commitment Model in the Day-Ahead Market

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A Mathematical formulation

A.1 Notation

Following this, a summary of sets, indices, parameters, and decision variables is enlisted for quick reference.

Sets and indices of the power system:

$\mathcal{BR}$	Set of electric tie-lines; $br \in \mathcal{BR}$
$\mathcal{D}$	Set of loads; $d \in \mathcal{D}$
$\mathcal{D}^E$	Set of elastic loads; $d \in \mathcal{D}^E$
$\mathcal{G}$	Set of all generators in the system; $g \in \mathcal{G} = \mathcal{G}^{\text{TE}} \cup \mathcal{G}^{\text{HI}} \cup \mathcal{G}^{\text{RE}}$
$\mathcal{G}^{\text{HI}}$	Set of hydroelectric generators; $g \in \mathcal{G}^{\text{HI}}$
$\mathcal{G}^{\text{RE}}$	Set of renewable generators in the system; $g \in \mathcal{G}^{\text{RE}}$
$\mathcal{G}^{\text{TE}}$	Set of thermal generators; $g \in \mathcal{G}^{\text{TE}}$
$\mathcal{N}$	Set for electric nodes in the system; $n \in \mathcal{N}$
$\mathcal{R}$	Set of reserve zones; $r \in \mathcal{R}$
$\mathcal{RO}$	Set of operative ranges of generators with prohibit operative zone (POZ); $ro \in \mathcal{RO}$, only for the $\mathcal{G}^{\text{TE}}$ and $\mathcal{G}^{\text{HI}}$
$\mathcal{T}$	Set of time periods in the planning horizon; $t, t' \in \mathcal{T}$

Sets and indices of the hydraulic system:

$\mathcal{E}$	Set of reservoirs; $e \in \mathcal{E}$
$\mathcal{HI}_e$	Set of hydro generators located in reservoir $e \in \mathcal{E}$; $g \in \mathcal{HI}_e$
$\mathcal{HI}_\nu$	Set of hydro generators that discharge water over a river $\nu \in \mathcal{V}_e^{\text{d}}$; $g \in \mathcal{HI}_v$
$\mathcal{V}_e^{\text{c}}$	Set of rivers converging to reservoir $e \in \mathcal{E}$; $\nu \in \mathcal{V}_e^{\text{c}}$
$\mathcal{V}_e^{\text{d}}$	Set of rivers diverging from reservoir $e \in \mathcal{E}$; $\nu \in \mathcal{V}_e^{\text{d}}$

Sets and indices the cost-related aspects:

$\mathcal{B}_d^{\text{e.p.b.}}$	Set of segments of the energy purchase bid for an elastic load $d \in \mathcal{D}^E$; $b \in \mathcal{B}_d^{\text{e.p.b.}}$
$\mathcal{B}_g^{\text{e.s.o.}}$	Set of segments of the energy sale offer for a generator $g \in \mathcal{G}$; $b \in \mathcal{B}_g^{\text{e.s.o.}}$
$\mathcal{B}^{\text{r.r.}}$	Set of segments of regulation reserve requirement; $b \in \mathcal{B}^{\text{r.r.}}$
$\mathcal{B}^{\text{r.10}}$	Set of segments of 10 minutes reserve requirement; $b \in \mathcal{B}^{\text{r.10}}$
$\mathcal{B}^{\text{s.r.10}}$	Set of segments of 10 minutes spinning reserve requirement; $b \in \mathcal{B}^{\text{s.r.10}}$
$\mathcal{B}^{\text{r.30}}$	Set of segments of 30 minutes reserve requirement; $b \in \mathcal{B}^{\text{r.30}}$
$\mathcal{B}^{\text{o.r.d.c}}$	The ORDC is comprised by $\mathcal{B}^{\text{r.r.}} \bigcup \mathcal{B}^{\text{s.r.10}} \bigcup \mathcal{B}^{\text{r.10}} \bigcup \mathcal{B}^{\text{r.30}}$; $b \in \mathcal{B}^{\text{o.r.d.c}}$
$\mathcal{S}_g$	Set of start-up cost curve segments for a generator $g \in \mathcal{G}^{\text{TE}}$; $s \in \mathcal{S}_g$

Parameters:

$a_{1,g}, a_{2,g}, a_{3,g}$	Coefficients that are part of the constant term of the HPF (40) for a generator $g \in \mathcal{G}^{\text{HI}}$; in MW

$b_{1,g}, b_{2,g}, b_{3,g}$ Linear coefficients that are part of the constant term of the HPF (40) for a generator $g \in \mathcal{G}^{\text{HI}}$; in MW/m

$c_{1,g}, c_{2,g}, c_{3,g}$ Quadratic coefficient that is part of the constant term of the HPF (40) for a generator $g \in \mathcal{G}^{\text{HI}}$; in MW s/m^2

$C^{\text{e.p.b.}}_{b,d,t}$ Price of energy purchase bid in a segment $b \in \mathcal{B}^{\text{e.p.b.}}_g$ for load $d \in \mathcal{D}^E$ in a period $t \in \mathcal{T}$; in \$/MWh

$C^{\text{e.s.o.}}_{b,g,t}$ Price of energy sale offers in a segment $b \in \mathcal{B}^{\text{e.s.o.}}_d$ for a generator $g \in \mathcal{G}^{\text{TE}}$ in a period $t \in \mathcal{T}$; in \$/MWh

$C^{m.g.c}_g$ Minimum operating cost of a thermal generator $g \in \mathcal{G}^{\text{TE}}$ that works at least at minimum power $\underline{P}_g$; in \$

$C^{\text{n.s.10}}_{g,t}$,$C^{\text{n.s.30}}_{g,t}$ Price of 10 and 30 minutes non-spinning reserve offer of a generator $g \in \mathcal{G}^{\text{TE}} \cup \mathcal{G}^{\text{HI}}$ in a period $t \in \mathcal{T}$; in \$/MWh

$C^{\text{o.c}}_{g,t}$ Opportunity cost of a hydroelectric generator $g \in \mathcal{G}^{\text{HI}}$ in a period $t \in \mathcal{T}$; in \$/MWh.

$C^{\text{o.r.d.c}}_{b,t}$ Price of operating reserve demand curve on $b \in \mathcal{B}^{\text{o.r.d.c}}$ in a period $t \in \mathcal{T}$; in \$/MWh

$C^{\text{r.r}}_{g,t}$ Price of regulation reserve offer of a generator $g \in \mathcal{G}^{\text{TE}} \cup \mathcal{G}^{\text{HI}}$ in a period $t \in \mathcal{T}$; in \$/MWh

$C^S_{g,s}$ Start-up offer in a generator $g \in \mathcal{G}^{\text{TE}}$ with a set of segment time $s \in \mathcal{S}_g$; it determines the starting cost by locating a cost in a segment time s in the intervals $[\underline{T}_{g,s}, \overline{T}_{g,s})$; in \$/h

$C^{\text{s.10}}_{g,t}$, $C^{\text{s.30}}_{g,t}$ Price of 10 and 30 minutes spinning reserve offer of a generator $g \in \mathcal{G}^{\text{TE}} \cup \mathcal{G}^{\text{HI}}$ in a period $t \in \mathcal{T}$; in \$/MWh

$DB_{b,d,t}$ Price-sensitive demand bid in a segment $b \in \mathcal{B}^{\text{e.p.b.}}_g$ for a load $d \in \mathcal{D}^E$ in a period $t \in \mathcal{T}$; in \$/MW

$DF_{d,t}$ Fixed demand in a segment $d \in \mathcal{D}$ in a period $t \in \mathcal{T}$; in MW

$\overline{fn}_{br,t}$,$\overline{fp}_{br,t}$ Maximum value for the power counterflow and flow on tie-line $br \in \mathcal{BR}$ in a period $t \in \mathcal{T}$, respectively; in MW.

$LSF_{n,t}$ Sensitivity transmission losses in node $n \in \mathcal{N}$ with regard to changes in power injections in node $n \in \mathcal{N}$ in a period $t \in \mathcal{T}$; dimensionless

$\overline{NP}_g$ Maximum number of stoppages allowed for a generator $g \in \mathcal{G}^{\text{TE}} \cup \mathcal{G}^{\text{HI}}$ during the planning horizon; dimensionless

$ORDC_{b,t}$ Operating reserve demand curve bid on $b \in \mathcal{B}^{\text{o.r.d.c}}$ in a period $t \in \mathcal{T}$; in \$/MWh

P^{sync}_g Power generation of a generator $g \in \mathcal{G}$ injects into the system immediately after connecting it; in MW

$Pb_{b,g,t}$ Offer quantity of power $b \in \mathcal{B}^{\text{e.p.b.}}_g$ for $g \in \mathcal{G}^{\text{TE}} \cup \mathcal{G}^{\text{RE}}$ in a period $t \in \mathcal{T}$; in MW

$\overline{P}_{g,t}$,$\underline{P}_{g,t}$ Maximum and minimum generation value of a generator $g \in \mathcal{G}$ in a period $t \in \mathcal{T}$, respectively; in MW

$\overline{P}^{RE}_{g,ro,t}$,$\underline{P}^{RE}_{g,ro,t}$ Maximum and minimum regulation limit for a operative zone $ro \in \mathcal{RO}$ of a generator $g \in \mathcal{G}^{\text{TE}} \cup \mathcal{G}^{\text{HI}}$ in a period $t \in \mathcal{T}$, respectively; in MW

$\overline{P}_{g,ro}^{RO}, \underline{P}_{g,ro}^{RO}$	Maximum and minimum operation limit for an operative zone $ro \in \mathcal{RO}$ of a generator $g \in \mathcal{G}^{\mathrm{TE}} \cup \mathcal{G}^{\mathrm{HI}}$ respectively; in MW
$PTDF_{br,n,t}$	Power transfer distribution factors are the incremental changes or sensitivity power flow in tie-lines $br \in \mathcal{BR}$ concerning power injection at any node $n \in \mathcal{N}$ in a period t
$\overline{q}_{g,t}, \underline{q}_{g,t}$	Maximum and minimum water flow passing through the turbines of a generator $g \in \mathcal{G}^{\mathrm{HI}}$ in a period $t \in \mathcal{T}$, respectively; in m^3/s
$\overline{Q}_{\nu,t}, \underline{Q}_{\nu,t}$	Maximum and minimum water discharge limits over a river $\nu \in \mathcal{V}_e^{\mathrm{d}}$ in a reservoir $e \in \mathcal{E}$ in a period $t \in \mathcal{T}$, respectively; in m^3/s
R_{br}	Electric resistance of a tie-line $br \in \mathcal{BR}$; dimensionless
RB_g	Ramp-down rate is the capacity of a generator $g \in \mathcal{G}^{\mathrm{TE}} \cup \mathcal{G}^{\mathrm{HI}}$ to decrease power between two consecutive periods; in MW/h
$RE_{g,t}^{10}$,$RE_{g,t}^{30}$	Emergency ramp rate for 10 and 30 minutes spinning reserve of a generator $g \in \mathcal{G}^{\mathrm{TE}} \cup \mathcal{G}^{\mathrm{HI}}$ in a period $t \in \mathcal{T}$; in MW
$RR_{g,t}$	Regulation ramp rate of a generator $g \in \mathcal{G}^{\mathrm{TE}} \cup \mathcal{G}^{\mathrm{HI}}$ in a period $t \in \mathcal{T}$; in MW
RS_g	Ramp-up rate of a generator $g \in \mathcal{G}^{\mathrm{TE}} \cup \mathcal{G}^{\mathrm{HI}}$ to increase power between two consecutive periods; in MW/h
RS_g^{sync}	Ramp-up rate of a generator $g \in \mathcal{G}^{\mathrm{TE}}$ to increase power when the generator is starting; in MW/h
$RRo_{g,t}^{10}$,$RRo_{g,t}^{30}$	10 and 30 minutes spinning reserve bid of a generator $g \in \mathcal{G}^{\mathrm{TE}} \cup \mathcal{G}^{\mathrm{HI}}$ in a period $t \in \mathcal{T}$; in \$/MWh
$RNR_{g,t}^{10}$,$RNR_{g,t}^{30}$	10 and 30 minutes non-spinning reserve bid of an offline generator $g \in \mathcal{G}^{\mathrm{TE}} \cup \mathcal{G}^{\mathrm{HI}}$ in a period $t \in \mathcal{T}$; in \$/MWh
$RRE_{g,t}$	Regulation reserve bid of a generator $g \in \mathcal{G}^{\mathrm{TE}} \cup \mathcal{G}^{\mathrm{HI}}$ in a period $t \in \mathcal{T}$; in \$/MWh
$\overline{RNR}_{g,t}^{10}$,$\overline{RNR}_{g,t}^{30}$	Maximum capacity limit for 10 and 30 minutes non-spinning reserve of a generator $g \in \mathcal{G}^{\mathrm{TE}} \cup \mathcal{G}^{\mathrm{HI}}$ in a period $t \in \mathcal{T}$; in MW
$\overline{RRo}_{g,t}^{10}$,$\overline{RRo}_{g,t}^{30}$	Maximum capacity limit for 10 and 30 minutes spinning reserve of a generator $g \in \mathcal{G}^{\mathrm{TE}} \cup \mathcal{G}^{\mathrm{HI}}$ in a period $t \in \mathcal{T}$; in MW
$\overline{RRe}_{g,ro,t}$	Maximum capacity limit for regulation reserve of an generator $g \in \mathcal{G}^{\mathrm{TE}} \cup \mathcal{G}^{\mathrm{HI}}$ in a operative zone ro in a period $t \in \mathcal{T}$; in MW
SU_g^D	Start-up time for a generator $g \in \mathcal{G}^{\mathrm{TE}}$; in h
$\overline{T}_{g,s}, \underline{T}_{g,s}$	Start and end of start-up cost segment $s \in \mathcal{S}$, respectively; in h
UT_g,DT_g	minimum up/down time for a generator $g \in \mathcal{G}^{\mathrm{TE}} \cup \mathcal{G}^{\mathrm{HI}}$; in h
$\epsilon_{e,t}$	The amount of water by natural inflows in a reservoir $e \in \mathcal{E}$ in a period $t \in \mathcal{T}$; in m^3
$\kappa_{\nu,t}$	Discharge spillage over a river $\nu \in \mathcal{V}_e^{\mathrm{d}}$ in a period $t \in \mathcal{T}$; in m^3
$\rho_{e,t}$	Water outflow of a reservoir $e \in \mathcal{E}$ in a period $t \in \mathcal{T}$ used for different purposes unrelated to energy generation; in m^3
τ_ν	Water travel time delay of a river $\nu \in \mathcal{V}_e^{\mathrm{c}}$; in h
$\overline{\omega}_e, \underline{\omega}_e$	Maximum and minimum water storage limits on a reservoir $e \in \mathcal{E}$; in m^3

Binary variables:

$u_{g,t}$ Equal to 1 if generator $g \in \mathcal{G}^{\text{TE}} \cup \mathcal{G}^{\text{HI}}$ is online in a period $t \in \mathcal{T}$, and 0 otherwise

$u^{S}_{g,t}$ Equal to 1 if generator $g \in \mathcal{G}^{\text{TE}}$ is starting-up in a period $t \in \mathcal{T}$, and 0 otherwise

$u^{RO}_{g,ro,t}$ Equal to 1 if generator $g \in \mathcal{G}^{\text{TE}} \cup \mathcal{G}^{\text{HI}}$ is in operating range $ro \in \mathcal{RO}$ in a period $t \in \mathcal{T}$, and 0 otherwise

$u^{RE}_{g,ro,t}$ Equal to 1 if generator $g \in \mathcal{G}^{\text{TE}} \cup \mathcal{G}^{\text{HI}}$ is in regulation reserve re in a period $t \in \mathcal{T}$, and 0 otherwise

$u^{rnr10}_{g,t}$,$u^{rnr30}_{g,t}$ Equal to 1 if generator $g \in \mathcal{G}^{\text{TE}} \cup \mathcal{G}^{\text{HI}}$ is committed to supplying 10 or 30 minutes of non-spinning reserve in a period $t \in \mathcal{T}$, and 0 otherwise

$v_{g,t}$ Equal to 1 if generator $g \in \mathcal{G}^{\text{TE}} \cup \mathcal{G}^{\text{HI}}$ starts up at the beginning of period $t \in \mathcal{T}$, and 0 otherwise

$w_{g,t}$ Equal to 1 if generator $g \in \mathcal{G}^{\text{TE}} \cup \mathcal{G}^{\text{HI}}$ is shut-down at the beginning of period $t \in \mathcal{T}$, and 0 otherwise

$\delta_{g,t,s}$ Equal to 1 if generator $g \in \mathcal{G}^{\text{TE}}$ have a start-up type $s \in \mathcal{S}$ in period $t \in \mathcal{T}$, and 0 otherwise

Real variables:

$DB_{b,d,t}$ Amount of demand price-response of segment b for demand $d \in \mathcal{D}^{E}$ in a period $t \in \mathcal{T}$, in MW

$f_{br,t}$ Power flow on br in i, in a period $t \in \mathcal{T}$; in MW.

$h_{v,t}$ Net hydraulic head of a river ν in a period $t \in \mathcal{T}$; in m

$iny_{n,t}$ Amount of power input at node $n \in \mathcal{N}$ in a period $t \in \mathcal{T}$, in MW

$Loss^{SP}_{t}$ Amount of exact transmission losses in a period $t \in \mathcal{T}$, in MW

$Loss^{MP}_{t}$ Amount of approximate transmission losses in a period $t \in \mathcal{T}$, in MW

$p_{g,t}$ Amount of power a generator $g \in \mathcal{G}$ produces in a period $t \in \mathcal{T}$, in MW

$pb_{b,g,t}$ Amount of power a generator $g \in \mathcal{G}^{\text{TE}} \cup \mathcal{G}^{\text{HI}}$ produces in segment b of period $t \in \mathcal{T}$, in MW

$ps_{g,t}$ Amount of power a generator $g \in \mathcal{G}$ produces during the start-up time in a period $t \in \mathcal{T}$, in MW

$q_{g,t}$ Water discharge of generator $g \in \mathcal{G}$ in a period $t \in \mathcal{T}$; in m^3/s

$rco_{b,t}$ Committed amount of reserve commitment of the OCDR on segment b in a period $t \in \mathcal{T}$; in MW

$rnr^{10}_{g,t}$,$rnr^{30}_{g,t}$ Committed amount of 10 and 30 minutes non-spinning reserve of a generator $g \in \mathcal{G}$ in a period $t \in \mathcal{T}$, in MW

$rre_{g,t}$ Committed amount of regulation reserve of generator $g \in \mathcal{G}$ in a period $t \in \mathcal{T}$; in MW

$\underline{rre}_{g,t}$ Minimum regulation reserve that can be committed to a generator $g \in \mathcal{G}^{\text{TE}} \cup \mathcal{G}^{\text{HI}}$ in a period $t \in \mathcal{T}$; in MW

$rro^{10}_{g,t}$,$rro^{30}_{g,t}$ Committed amount of 10 and 30 minutes spinning reserve of a generator $g \in \mathcal{G}$ in a period $t \in \mathcal{T}$, in MW

$\omega_{e,t}$ Water volume in the reservoir e in a period $t \in \mathcal{T}$; in m^3

A.2 Assumptions

Key assumptions to bear in mind: The limits of generator output power, reserves, offers, and bids may change during the planning horizon. Also, the purchase bids and the energy generation in each period represent the average power demand between the beginning and the end of a period.

A.3 Objective function

Our goal is to minimize an objective function consisting of start-up costs, minimum generation cost, and selling of the power and reserves supplied by generators minus the revenue obtained by purchasing energy and reserve requirements.

$$\begin{aligned}
\min \sum_{t\in\mathcal{T}} \Bigg\{ & \sum_{g\in\mathcal{G}^{\mathrm{TE}}} \Bigg[C^{S}_{g,s}\delta_{g,t,s} + u_{g,t}C^{m.g.c}_{g} + \sum_{b\in\mathcal{B}^{\mathrm{e.s.o}}_{g}} C^{e.s.o}_{b,g,t} pb_{b,g,t} \Bigg] \\
& + \sum_{g\in\mathcal{G}^{\mathrm{HI}}} C^{o.c}_{g,t} p_{g,t} + \sum_{g\in\mathcal{G}^{\mathrm{RE}}} \sum_{b\in\mathcal{B}^{\mathrm{e.s.o}}_{g}} C^{e.s.o}_{b,g,t} pb_{b,g,t} - \sum_{d\in\mathcal{D}^{E}} \Bigg\{ \sum_{b\in\mathcal{B}^{\mathrm{e.p.b}}_{d}} C^{e.p.b}_{b,d,t} db_{b,d,t} \Bigg\} \\
& + \sum_{g\in\mathcal{G}^{\mathrm{TE}}\bigcup\mathcal{G}^{\mathrm{HI}}} \Bigg[C^{s.10}_{g,t} rro^{10}_{g,t} + C^{n.s.10}_{g,t} rnr^{10}_{g,t} + C^{s.30}_{g,t} rro^{30}_{g,t} \\
& + C^{n.s.30}_{g,t} rnr^{30}_{g,t} + C^{r.r}_{g,t} rre_{g,t} \Bigg] - \sum_{b\in\mathcal{B}^{\mathrm{o.r.d.c}}} C^{o.r.d.c}_{b,t} rco_{b,t} \Bigg\}.
\end{aligned} \tag{1}$$

The constraints of the problem are detailed below.

A.4 Constraints

A.4.1 Power balance

The balance equation per node is outlined in 2. Where the power injections in a node n in a period t are unpacked in 3:

$$\sum_{n\in\mathcal{N}} iny_{n,t} - Loss_t = 0, \qquad t \in \mathcal{T}, \tag{2}$$

$$iny_{n,t} = \sum_{g\in\mathcal{G}_n} p_{g,t} - \sum_{d\in\mathcal{D}_n} (DF_{d,t} + \sum_{b\in\mathcal{B}^{\mathrm{e.p.b}}_{d}} db_{b,d,t}), \qquad n \in \mathcal{N},\ t \in \mathcal{T}. \tag{3}$$

A.4.2 Loads

The lower and upper limits of the elastic loads are shown as follows:

$$0 \le db_{b,d,t} \le DB_{b,d,t}, \qquad b \in \mathcal{B}^{\mathrm{e.p.b}}_{d},\ d \in \mathcal{D}^{E},\ t \in \mathcal{T}. \tag{4}$$

A.4.3 Offers reserve and requirements reserve

RRE, RRR, RRS, and RRS requirements are met by the generators' committed reserves (regulation, spinning, and non-spinning of 10 and 30 minutes), constraint 5, 6, 7, and 8. Also, the limits of reserve requirements are expressed in 9:

$$\sum_{g\in\mathcal{G}^{\mathrm{TE}}\cup\mathcal{G}^{\mathrm{HI}}} rre_{g,t} \geq \sum_{b\in\mathcal{B}^{\mathrm{r.r.}}} rco_{b,t}, \qquad t\in\mathcal{T}, \tag{5}$$

$$\sum_{g\in\mathcal{G}^{\mathrm{TE}}\cup\mathcal{G}^{\mathrm{HI}}} (rre_{g,t} + rro^{10}_{g,t}) \geq \sum_{b\in\mathcal{B}^{\mathrm{r.r.}}\cup\mathcal{B}^{\mathrm{s.r.10}}} rco_{b,t}, \qquad t\in\mathcal{T}, \tag{6}$$

$$\sum_{g\in\mathcal{G}^{\mathrm{TE}}\cup\mathcal{G}^{\mathrm{HI}}} (rre_{g,t} + rro^{10}_{g,t} + rnr^{10}_{g,t}) \geq \sum_{b\in\mathcal{B}^{\mathrm{r.r.}}\cup\mathcal{B}^{\mathrm{s.r.10}}\cup\mathcal{B}^{\mathrm{r.10}}} rco_{b,t}, \qquad t\in\mathcal{T}, \tag{7}$$

$$\sum_{g\in\mathcal{G}^{\mathrm{TE}}\cup\mathcal{G}^{\mathrm{HI}}} (rre_{g,t} + rro^{10}_{g,t} + rnr^{10}_{g,t} + rro^{30}_{g,t} + rnr^{30}_{g,t}) \geq \sum_{b\in\mathcal{B}^{\mathrm{o.r.d.c}}} rco_{b,t}, \qquad t\in\mathcal{T}, \tag{8}$$

$$0 \leq rco_{b,t} \leq ORDC_{b,t}, \qquad b\in\mathcal{B}^{\mathrm{o.r.d.c}},\ t\in\mathcal{T}. \tag{9}$$

The reader can find a comprehensive explanation of the relationship between reserve offers and the ISO's requirements in the supplemental material section A.

A.4.4 Generation limits

For thermal generators the power level of a generator amounts to power produced during the start-up process; plus, the minimal operative limit of generator; plus, the sum of the segment powers associated with each offer segment, constraint 10. For renewable generators, the power level of a generator amounts to the sum of the segment powers associated with each offer segment, constraint 11. For both renewable and thermal generators, the power level amounts to the sum of the segments offered is limited by constraint 12.

$$p_{g,t} = ps_{g,t} + u_{g,t}\underline{P}_{g,t} + \sum_{b\in\mathcal{B}^{\mathrm{e.s.o}}_g} pb_{b,g,t}, \qquad g\in\mathcal{G}^{\mathrm{TE}},\ t\in\mathcal{T}, \tag{10}$$

$$p_{g,t} = \sum_{b\in\mathcal{B}^{\mathrm{e.s.o}}_g} pb_{b,g,t}, \qquad g\in\mathcal{G}^{\mathrm{RE}},\ t\in\mathcal{T}, \tag{11}$$

$$0 \leq pb_{b,g,t} \leq u_{g,t}Pb_{b,g,t}, \qquad b\in\mathcal{B}^{\mathrm{e.s.o}}_g,\ g\in\mathcal{G}^{\mathrm{TE}}\cup\mathcal{G}^{\mathrm{RE}}, \qquad t\in\mathcal{T}. \tag{12}$$

A.4.5 Generation limits and reserves

For renewable generators, the power limits (maximum and minimum) are modeled in 13. Concerning the thermal generator in operation, the sum should equal or less to its minimal operative power if it is in the start-up process, constraint 14. Regarding hydro generators, the maximum power level should be equal or major to its output power, plus the regulation committed, constraint 15.

$$u_{g,t}\underline{P}_{g,t} \leq p_{g,t} \leq u_{g,t}\overline{P}_{g,t}, \qquad g \in \mathcal{G}^{\mathrm{RE}},\ t \in \mathcal{T}, \tag{13}$$

$$p_{g,t} + rre_{g,t} + rro^{10}_{g,t} + rro^{30}_{g,t} \leq u_{g,t}\overline{P}_{g,t} + u^{S}_{g,t}\underline{P}_{g,t}, \qquad g \in \mathcal{G}^{\mathrm{TE}},\ t \in \mathcal{T}, \tag{14}$$

$$p_{g,t} + rre_{g,t} + rro^{10}_{g,t} + rro^{30}_{g,t} \leq u_{g,t}\overline{P}_{g,t}, \qquad g \in \mathcal{G}^{\mathrm{HI}},\ t \in \mathcal{T}. \tag{15}$$

For both, hydro and thermal generators, their power level should be equal or less to the maximum level of the operating range selected ro. If the generator does not sell regulation reserve, its power level should be equal or less to the operating range selected. In case of selling regulation reserve ($u^{RE} = 1$) its power level plus its reserve regulation should be equal or less to the maximum limit of reserve regulation for the operating range selected, constraint 16.

$$p_{g,t} + rre_{g,t} \leq \sum_{ro \in \mathcal{RO}_g} \{u^{RO}_{g,ro,t}\overline{P}^{RO}_{g,ro} + u^{RE}_{g,ro,t}(\overline{P}^{RE}_{g,ro,t} - \overline{P}^{RO}_{g,ro})\}, \qquad g \in \mathcal{G}^{\mathrm{TE}} \cup \mathcal{G}^{\mathrm{HI}},\ t \in \mathcal{T}. \tag{16}$$

Concerning hydro generators, their power level should be equal or major to the minimum level of the operating range selected ro. If the generator does not sell regulation reserve, its power level should be equal or major to the minimum level of the operating range selected. In case of selling regulation reserve ($u^{RE} = 1$) its power level minus its reserve regulation should be equal or major to the minimum limit of reserve regulation for the operating range selected, constraint 17.

$$p_{g,t} - rre_{g,t} \geq \sum_{ro \in \mathcal{RO}_g} \{u^{RO}_{g,ro,t}\underline{P}^{RO}_{g,ro} + u^{RE}_{g,ro,t}(\underline{P}^{RE}_{g,ro,t} - \underline{P}^{RO}_{g,ro})\}, \qquad g \in \mathcal{G}^{\mathrm{HI}},\ t \in \mathcal{T}. \tag{17}$$

With regard to thermal generators, their power level should be equal or major to the minimum level of the operating range selected ro. If the generator does not sell regulation reserve, its power level should be equal or major to the minimum level of the operating range selected plus the synchronous power (start-up power of a generator). In the case of commitment of selling regulation reserve ($u^{RE} = 1$), its power level minus its reserve regulation should be equal or major to the minimum limit of reserve regulation for the operating range selected plus the synchronizing power. Synchronizing power is the generator's power level once connected and has the same system frequency. The generator is not considered for selling regulation reserve during its start-up time.

$$p_{g,t} - rre_{g,t} \geq \sum_{ro \in \mathcal{RO}_g} \{u^{RO}_{g,ro,t}\underline{P}^{RO}_{g,ro} + u^{RE}_{g,ro,t}(\underline{P}^{RE}_{g,ro,t} - \underline{P}^{RO}_{g,ro})\} + u^{S}_{g,t}P^{sync}_{g,t}, \qquad g \in \mathcal{G}^{\mathrm{TE}},\ t \in \mathcal{T}. \tag{18}$$

To commit a generator for supplying regulation reserve, a generation operative range $\mathcal{RO}$ should be selected with constraint 17, avoiding the POZ

$$u^{RE}_{g,ro,t} \leq u^{RO}_{g,ro,t}, \qquad ro \in \mathcal{RO}_g; g \in \mathcal{G}^{\mathrm{TE}} \cup \mathcal{G}^{\mathrm{HI}},\ t \in \mathcal{T}. \tag{19}$$

A.4.6 Reserve limits and ramps

The 10-minute and 30-minute spinning reserves committed should be within the operative limits.

$$0 \leq rro_{g,t}^{10} \leq u_{g,t}\overline{RRo}_{g,t}^{10}, \qquad g \in \mathcal{G}^{\mathrm{TE}} \cup \mathcal{G}^{\mathrm{HI}},\ t \in \mathcal{T}. \tag{20}$$

$$0 \leq rro_{g,t}^{30} \leq u_{g,t}\overline{RRo}_{g,t}^{30}, \qquad g \in \mathcal{G}^{\mathrm{TE}} \cup \mathcal{G}^{\mathrm{HI}},\ t \in \mathcal{T}. \tag{21}$$

The regulation reserve committed should be within the operative limits.

$$\sum_{ro \in \mathcal{RO}_g} u_{g,ro,t}^{RE}\underline{rre}_{g,t} \leq rre_{g,t} \leq \sum_{ro \in \mathcal{RO}_g} u_{g,ro,t}^{RE}\overline{RRe}_{g,ro,t}, \qquad g \in \mathcal{G}^{\mathrm{TE}} \cup \mathcal{G}^{\mathrm{HI}},\ t \in \mathcal{T}. \tag{22}$$

The 10-minute and 30-minute non-spinning reserves committed should be within the limits.

$$0 \leq rnr_{g,t}^{10} \leq (1 - u_{g,t}^{s} - u_{g,t})RNR_{g,t}^{10}, \qquad g \in \mathcal{G}^{\mathrm{TE}},\ t \in \mathcal{T}. \tag{23}$$

$$0 \leq rnr_{g,t}^{10} \leq (1 - u_{g,t})RNR_{g,t}^{10}, \qquad g \in \mathcal{G}^{\mathrm{HI}},\ t \in \mathcal{T}. \tag{24}$$

$$0 \leq rnr_{g,t}^{30} \leq (1 - u_{g,t}^{s} - u_{g,t})RNR_{g,t}^{30}, \qquad g \in \mathcal{G}^{\mathrm{TE}},\ t \in \mathcal{T}. \tag{25}$$

$$0 \leq rnr_{g,t}^{30} \leq (1 - u_{g,t})RNR_{g,t}^{30}, \qquad g \in \mathcal{G}^{\mathrm{HI}},\ t \in \mathcal{T}. \tag{26}$$

A.4.7 Ramp up/down

The power increase in generators between two consecutive time intervals or ramp-up rate in thermal and hydro generators.

$$p_{g,t} - p_{g,t-1} \leq RS_g, \qquad g \in \mathcal{G}^{\mathrm{TE}} \cup \mathcal{G}^{\mathrm{HI}},\ t \in \mathcal{T}. \tag{27}$$

The power reduction in generators between two consecutive time intervals or ramp-down rate in thermal and hydro generators.

$$p_{g,t-1} - p_{g,t} \leq RB_g + w_{g,t}(\overline{P}_{g,t-1} - RB_g) \qquad g \in \mathcal{G}^{\mathrm{TE}} \cup \mathcal{G}^{\mathrm{HI}},\ t \in \mathcal{T}. \tag{28}$$

The power increase in generators in start-up processes between two consecutive time intervals or starting-up ramp in hydro generators are modeled as follows.

$$ps_{g,t} - ps_{g,t-1} \leq P_g^{sync}(u_{g,t}^{S} - u_{g,t-1}^{S}) + RS_g^{sync}u_{g,t-1}^{S}, \qquad g \in \mathcal{G}^{\mathrm{TE}},\ t \in \mathcal{T}. \tag{29}$$

$$ps_{g,t} - ps_{g,t-1} \geq RS_g^{sync}u_{g,t-1}^{S} - \underline{P}_{g,t}u_{g,t}, \qquad g \in \mathcal{G}^{\mathrm{TE}},\ t \in \mathcal{T}. \tag{30}$$

A.4.8 Minimum up and down time

Once a generator is turned on, it will be kept on at least a minimum number of time intervals before turning it off is modeled in constraint 31 . Likewise, once a generator is turned off, it will be kept off at least a minimum number of time intervals before turning it on, in constraint 32

$$\sum_{t'=t+SU_g^D}^{t+SU_g^D+UT_g} u_{g,t'} - UT_g v_{g,t} \geq 0, \qquad g \in \mathcal{G}^{\mathrm{TE}},\ t, t' \in \mathcal{T}. \tag{31}$$

$$\sum_{t'=t}^{t+DT_g-1} u_{g,t'} + DT_g w_{g,t} \leq DT_g, \qquad g \in \mathcal{G}^{\mathrm{TE}},\ t, t' \in \mathcal{T}. \tag{32}$$

A.4.9 Variable startup costs

The following constraints are based on reference Morales-España et al. [2]. The relationship between the start-up segments and the start-up variables is modeled as follows in constraint 33. Constraints (34) and (35) establish the relationship between the number of hours that the generator has been disconnected and its corresponding segment in the startup costs function for the current interval. These constraints assure that the number of hours that the generator will be disconnected are within the interval of the startup segment delimited between $[\underline{T}_{g,s}, \overline{T}_{g,s})$.

$$v_{g,t} \leq \sum_{s \in \mathcal{S}} \delta_{g,t,s}, \qquad g \in \mathcal{G}^{\mathrm{TE}},\ t \in \mathcal{T}, \tag{33}$$

$$\delta_{g,t,s} \leq \sum_{t' \in \mathcal{T}'_t} w_{g,t'}, \qquad g \in \mathcal{G}^{\mathrm{TE}},\ t \in \mathcal{T},\ s \in \mathcal{S} - \{|\mathcal{S}|\}, \tag{34}$$

$$\mathcal{T}'_t = \{t' | t - \overline{T}_{g,s} + 1 \leq t' \leq t - \underline{T}_{g,s} 0 < t' < t\}. \tag{35}$$

A.4.10 Hydraulic balance

The water balance in a reservoir is modeled as follows in constraint 36. The water inputs to the reservoir are represented by the natural inflow minus other uses, plus the water comes from the reservoir upstream. The water output from the reservoir to the river is the sum of the discharge of water that passes through turbines plus the spillage. The spillage in the reservoir is a parameter that takes into account water travel times. Reservoirs water volume should be kept within the minimum and maximum storage operative limits 37.

$$\begin{aligned} \omega_{e,t} = \omega_{e,t-1} + \sum_{\nu \in \mathcal{V}_e^{\mathrm{c}}} \sum_{g \in \mathcal{HI}_\nu} q_{g,t-\tau_v} - \sum_{\nu \in \mathcal{V}_e^{\mathrm{d}}} \sum_{g \in \mathcal{HI}_\nu} q_{g,t} \\ + \epsilon_{e,t} - \rho_{e,t} - \sum_{\nu \in \mathcal{V}_e^{\mathrm{d}}} \kappa_{\nu,t} + \sum_{\nu \in \mathcal{V}_e^{\mathrm{c}}} \kappa_{\nu,t-\tau_\nu}, \qquad e \in \mathcal{E},\ t \in \mathcal{T} \end{aligned} \tag{36}$$

$$\underline{\omega}_e \leq \omega_{e,t} \leq \overline{\omega}_e, \qquad e \in \mathcal{E},\ t \in \mathcal{T}. \tag{37}$$

For each hydro generator, the water discharge passing trough turbine in reservoirs should meet minimum and maximum limits, constraint 38. Also, the sum of water discharge passing through turbine generators linked to a reservoir should meet the minimum and maximum limits in the divergent river channel, constraint 39.

$$\underline{Q}_{\nu,t} \leq \sum_{g \in \mathcal{HI}_\nu} q_{g,t} \leq \overline{Q}_{\nu,t}, \qquad \nu \in \mathcal{V}_e^{\mathrm{d}}\ ,\ e \in \mathcal{E}, \tag{38}$$

$$u_{g,t}\underline{q}_{g,t} \leq q_{g,t} \leq u_{g,t}\overline{q}_{g,t}, \qquad g \in \mathcal{G}^{\mathrm{HI}},\ t \in \mathcal{T}. \tag{39}$$

A.4.11 Hydraulic generation

Hydro generation power depends on the turbine water discharge rate and the reservoir head, both quadratically. The function used here is known as Glimn-Kirchmayer [1]. The constraint below is known as a hydropower function (HPF). Its parameters $\{a_{1,g}, ..., c_{3,g}\}$ depend on the reservoir design, turbine and generator features.

$$\begin{aligned} p_{g,t} =& u_{g,t}\big((a_{1,g} + b_{1,g}h_{\nu,t} + c_{1,g}h_{\nu,t}^2) + (a_{2,g} + b_{2,g}h_{\nu,t} + c_{2,g}h_{\nu,t}^2)q_{g,t} \\ &+ (a_{3,g} + b_{3,g}h_{\nu,t} + c_{3,g}h_{\nu,t}^2)q_{g,t}^2\big), \qquad g \in \mathcal{HI}_\nu,\ \nu \in \mathcal{V}_e^{\mathrm{d}},\ e \in \mathcal{E},\ t \in \mathcal{T}. \end{aligned} \tag{40}$$

The non-linear feature of the HPF constraint requires an approximation method to be successfully integrated into a MILP. The method used in our work is the approximation to Taylor polynomial. This method generates an alternative constraint to replace the former (40).

A.4.12 Power flow limits

The power flow in a transmission tie-line is modeled as follows in constraint41, where $f_{br,t}$ represents the power flow in a transmission tie-line br which depends on parameters $PTDF_{br,n,t}$ and the power injection $iny_{n,t}$ for each bus n for each period t. A detailed description of the PTDF's calculation is explained by Tejada-Arango et al. [4]. Also, constraint 42 fixes the maximum limits of flow and counterflow power in tie-line br.

$$f_{br,t} = \sum_{n \in \mathcal{N}} PTDF_{br,n,t}\, iny_{n,t}, \qquad br \in \mathcal{BR},\ t \in \mathcal{T}, \tag{41}$$

$$\overline{fn}_{br,t} \leq f_{br,t} \leq \overline{fp}_{br,t}, \qquad t \in \mathcal{T}. \tag{42}$$

A.4.13 Transmission losses

The losses in the system are calculated using the constraint 43. The non-linear feature of this constraint requires an approximation method to be successfully integrated into a MILP. The method used in our work is tangent planes and provides a set of alternative constraints to replace this constraint.

$$Loss_t = \sum_{br \in \mathcal{BR}} R_{br}(f_{br,t})^2, \qquad t \in \mathcal{T}. \tag{43}$$

A.4.14 Logical constraints

A generator cannot start-up and shut-down at the same time, constraint 44. A hydro generator cannot start-up and shut-down and operate (to provide energy in stable fashion within its operative limits) at the same time, constraint 45.

$$v_{g,t} + w_{g,t} \leq 1, \qquad g \in \mathcal{G}^{\mathrm{TE}} \cup \mathcal{G}^{\mathrm{HI}},\ t \in \mathcal{T}, \tag{44}$$

$$u_{g,t} - u_{g,t-1} - v_{g,t} + w_{g,t} = 0, \qquad g \in \mathcal{G}^{\mathrm{HI}},\ t \in \mathcal{T}. \tag{45}$$

A thermal generator cannot synchronize, start-up, shut-down and operate (to provide energy in stable fashion within its operative limits) at the same time.

$$u_{g,t} - u_{g,t-1} - v_{g,t} + w_{g,t} + u^S_{g,t} - u^S_{g,t-1} = 0, \qquad g \in \mathcal{G}^{\mathrm{TE}},\ t \in \mathcal{T}. \tag{46}$$

To activate a generator operative range, the unit should be committed for operation [3].

$$sum_{ro \in \mathcal{RO}_g} u^{RO}_{g,ro,t} = u_{g,t}, \qquad g \in \mathcal{G}^{\mathrm{TE}} \cup \mathcal{G}^{\mathrm{HI}},\ t \in \mathcal{T}. \tag{47}$$

A generator can start-up only if it was off in the previous time interval.

$$u_{g,t-1} + u^S_{g,t} \leq 1, \qquad g \in \mathcal{G}^{\mathrm{TE}},\ t \in \mathcal{T}. \tag{48}$$

A generator is not committed for operation during the starting-up time intervals.

$$\sum_{t'=t}^{t+SU^D_g-1} u_{g,t'} + SU^D_g v_{g,t} \leq SU^D_g, \qquad g \in \mathcal{G}^{\mathrm{TE}},\ t \in \mathcal{T}. \tag{49}$$

A generator is committed for the starting-up process during the starting-up time intervals.

$$\sum_{t'=t}^{t+SU^D_g-1} u^S_{g,t'} - SU^D_g v_{g,t} \geq 0, \qquad g \in \mathcal{G}^{\mathrm{TE}},\ t \in \mathcal{T}. \tag{50}$$

Generators power during the starting-up process should be kept within a minimum limit given by its synchronizing power and a maximum limit given by its minimum operative power.

$$u^S_{g,t} P^{sync}_g \leq ps_{g,t} \leq u^S_{g,t} \underline{P}_{g,t}, \qquad g \in \mathcal{G}^{\mathrm{TE}},\ t \in \mathcal{T}. \tag{51}$$

The number of shutdowns for a generator during the day cannot be more than an established limit.

$$\sum_{t \in \mathcal{T}} w_{g,t} \leq \overline{NP}_g, \qquad g \in \mathcal{G}^{\mathrm{TE}} \cup \mathcal{G}^{\mathrm{HI}}. \tag{52}$$

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