Reliability Evaluation Method for the Electricity-gas Coupled System Considering Correlation

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***Abstract*—A reliability evaluation method for the electricity-gas coupled system considering correlation is proposed. Firstly, the point estimate method is applied to the system reliability evaluation process. Since correlated variables could not be used in the point estimate method directly, Nataf transformation is adopted to deal with the correlation. Then, a novel general method to solve the transformed correlation coefficient matrix in Nataf transformation efficiently is researched. Finally, an electricity-gas coupled system consisting of a modified IEEE-RBTS Bus6 NO.4 feeder system and a modified NGS10 node natural gas system is constructed, and results indicate effectiveness and high efficiency of the proposed method.**

***Index Terms*-The electricity-gas coupled system, the reliability evaluation, the point estimate method, Nataf transformation, correlation**

I. Introduction

The utilization of clean energy such as wind power is an inevitable choice to solve the problem of fossil energy shortage and environmental degradation. On the other hand, randomness and intermittence of clean energy also pose challenges to safe operation of the grid. The gas turbine can adjust to outside variation swiftly and compensate the fluctuation of wind turbine outputs, so that the whole system operation becomes relatively stable. Therefore, research on the electricity-gas coupled system has received constant attention.

However, the reliability evaluation of the electricity-gas coupled system appears to need further research efforts. Ref. [1] proposed a Monte Carlo model of combined gas and power network of the Great Britain to determine the reliability of the energy infrastructure. Ref. [2] proposed a reliability model based on two-layer smart agent communication, and calculated the reliability of the integrated energy system which included the electricity-gas coupled system by the Monte Carlo method. Ref. [3] considered the energy hub concept to establish a physical model of the integrated energy microgrid including the electricity-gas coupled system, and the reliability evaluation was carried out by combining Failure Mode and Effects Analysis and the Monte Carlo method. The above references have considered uncertainty of electric loads, gas loads and distributed generators in the electricity-gas coupled system, but do not consider the correlation effect and obtain statistical features of the reliability indices; the adopted Monte Carlo method needs a large number of simulation experiments to attain high precision, so the convergence rate is slow.

This paper proposes a reliability evaluation method for the electricity-gas coupled system considering correlation. First, a mathematical description of the electricity-gas coupled system is proposed. Then, the point estimate method is applied to the system reliability evaluation, which simplifies the reliability evaluation process. Next, Nataf transformation is adopted to deal with the correlated variables, and a novel general method to solve the transformed correlation coefficient matrix more easily and efficiently is studied. Finally, a case study consisting of a modified IEEE-RBTS Bus6 NO.4 feeder system and a modified NGS10 node natural gas system is analyzed to verify the effectiveness of the proposed method.

II. The Point Estimate Method Based on Nataf Transformation

A. The Mathematical Description of the Electricity-gas Coupled System

Most electricity-gas coupled systems have increasing penetration levels of clean energy such as wind energy. Since wind speeds, electric loads and gas loads of the coupled system are all random variables and subject to specific probability distributions, reliability indices of the coupled system are also random variables. The function of reliability indices, wind speeds, electric loads and gas loads can be described by (1).

$$Z = \psi\left(\boldsymbol{X}\right) = \psi\left(X_1, X_2, \cdots, X_n\right) \tag{1}$$

Where Z is the system reliability index; $\boldsymbol{X}$ is the input random vector of wind speeds, electric loads and gas loads; n is the dimensions of $\boldsymbol{X}$; $\psi(\cdot)$ is the function relationship between Z and $\boldsymbol{X}$, which means the reliability evaluation process of the whole system.

In addition, there is a correlation between the loads and the wind speeds in the input random vector $\boldsymbol{X}$. For example, adjacent wind farms often have similar meteorological

The authors gratefully acknowledge the support of National Natural Science Foundation of China (51777078), Science and Technology Project of Guizhou Power Grid Co., Ltd. (066600KK52170055).

conditions, which means the wind speeds of these farms have strong correlations. There is also a similar correlation between the adjacent electric loads and gas loads, as the coupling of power systems and gas systems becomes deeper.

The correlation between the random variables of $\boldsymbol{X}$ is described by the correlation coefficient matrix $\boldsymbol{\rho}$.

$$\boldsymbol{\rho} = \left(\rho_{ij}\right)_{n\times n} = \begin{bmatrix} \rho_{11} & \rho_{12} & \cdots & \rho_{1n} \\ \rho_{21} & \rho_{22} & \cdots & \rho_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{n1} & \rho_{n2} & \cdots & \rho_{nn} \end{bmatrix} \tag{2}$$

Where ρ_{ij} represents the correlation coefficient between two arbitrary random variables X_i and X_j of $\boldsymbol{X}$.

Z is a function of random variables and also a random variable itself. Therefore, Z should be analyzed with probabilistic perspective. The lth order origin moment of Z is calculated by (3).

$$E\left[Z^l\right] = E\left[\left(\psi(\boldsymbol{X})\right)^l\right] = \int_{\Omega}\left(\psi(\boldsymbol{x})\right)^l f_X(\boldsymbol{x})\mathrm{d}\boldsymbol{x} \tag{3}$$

Where $f_X(x)$ is the joint probability density function of the random variables constituting the random vector $\boldsymbol{X}$; Ω is the value space of $\boldsymbol{X}$.

It can be known from (3) that calculating the statistical features of $\boldsymbol{Z}$ directly is highly complicated, because the calculation process involves complex multiple integrals, and both $f_X(x)$ and $\psi(\cdot)$ are difficult to obtain. Therefore, it is essential to research a simplified and effective method to calculate reliability indices of the electricity-gas coupled system.

The point estimate method based on Nataf transformation can be adopted to the above problem. The method can estimate the statistical features of $\boldsymbol{Z}$ by marginal probability density functions and the correlation coefficient matrix of the random variables (X_i) of $\boldsymbol{X}$.

B. *Brief Description of the Point Estimate Method*

The point estimate method utilizes the probability distributions of the input random variables to calculate the origin moments of the output function. The method simplifies the calculation process and obtains estimation results.

$E[Z^l]$ can be estimated by (4).

$$E\left[Z^l\right] \approx \sum_{i=1}^{n}\sum_{k=1}^{2} w_{i,k}\left[\psi\left(\mu_1,\mu_2,\cdots,x_{i,k},\cdots\mu_n\right)\right]^l + w_0\left[\psi\left(\mu_1,\mu_2,\cdots,\mu_n\right)\right]^l \tag{4}$$

Where $w_{i,k}$ and w_0 represent probability concentrations; $X_{i,k}$ denotes the kth estimated value of the input random variable $X_i(i=1,2,\ldots,n)$; μ_i is the expectation value of X_i; $\psi(\mu_1, \mu_2,\ldots,\mu_n)$ denotes the value of Z when all input variables are located at their respective expectation values.

Comprehensive proof process of the point estimate method can be found in [4].

C. *Nataf Transformation*

The mathematical basis of the point estimate method indicates that input variables should be strictly independent from each other. Nataf transformation can be adopted to consider the dependencies of the input variables (X_1, X_2, ..., X_n).

A vector of standard normal variables $\boldsymbol{Y}=[Y_1,Y_2,\ldots,Y_n]$ can be obtained by marginal transformations of the original input random vector $\boldsymbol{X}=[X_1,X_2,\ldots,X_n]$. The relationship between $\boldsymbol{X}$ and $\boldsymbol{Y}$ is described as (5).

$$\begin{cases} y_i = \Phi^{-1}\left[F_i\left(x_i\right)\right] \\ x_i = F_i^{-1}\left[\Phi\left(y_i\right)\right] \end{cases}, \quad i=1,2,\cdots,n \tag{5}$$

Where $F_i(x_i)$ is the marginal probability density function of X_i; $F_i^{-1}(\cdot)$ is the inverse function of $F_i(\cdot)$; $\Phi(\cdot)$ and $\Phi^{-1}(\cdot)$ denote probability density function and its inverse function of the standard normal distribution, respectively.

The correlation coefficient matrices of $\boldsymbol{X}$ and $\boldsymbol{Y}$ are denoted by $\boldsymbol{\rho}$ and $\boldsymbol{\rho}_0$ respectively. The mapping relationship between the components of $\boldsymbol{\rho}$ and $\boldsymbol{\rho}_0$ can be described as (6) .

$$\begin{aligned} \rho_{ij} &= \frac{E\left[\left(X_i-\mu_i\right)\left(X_j-\mu_j\right)\right]}{\sigma_i\sigma_j} = \\ & \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\left(\frac{x_i-\mu_i}{\sigma_i}\right)\left(\frac{x_j-\mu_j}{\sigma_j}\right) f_{X_iX_j}\left(x_i,x_j\right)\mathrm{d}x_i\mathrm{d}x_j = \\ & \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\left(\frac{F_i^{-1}\left(\Phi\left(y_i\right)\right)-\mu_i}{\sigma_i}\right)\left(\frac{F_j^{-1}\left(\Phi\left(y_j\right)\right)-\mu_j}{\sigma_j}\right)\cdot \\ & \varphi_2\left(y_i,y_j,\rho_{0ij}\right)\mathrm{d}y_i\mathrm{d}y_j \end{aligned} \tag{6}$$

Where σ_i denotes the standard deviation of X_i; ρ_{0ij} denotes the correlation coefficient between two arbitrary random variables Y_i and Y_j of $\boldsymbol{Y}$; $\varphi_2(\cdot)$ denotes the two-dimensional joint probability density function of the standard normal distribution; $f_{X_iX_j}\left(x_i,x_j\right)$ denotes the joint probability density function of X_i and X_j.

When $\boldsymbol{\rho}$ is known, $\boldsymbol{\rho}_0$ can be calculated by solving the nonlinear equation (6).

As shown in (7), Cholesky decomposition can be adopted:

$$\boldsymbol{\rho}_0 = \boldsymbol{L}_0\boldsymbol{L}_0^{\mathrm{T}} \tag{7}$$

Where $\boldsymbol{L}_0$ is the lower triangular matrix calculated by (7).

Then (8) can be obtained as below:

$$\boldsymbol{S} = \boldsymbol{L}_0^{-1}\boldsymbol{Y} \tag{8}$$

Where $\boldsymbol{S}=[S_1,S_2,\ldots,S_n]$ is a vector of independent standard normal variables and can be obtained by the point estimate method [4].

Therefore, Nataf transformation can be described as

$$\boldsymbol{X}=\boldsymbol{F}^{-1}\left[\Phi\left(\boldsymbol{L}_0\boldsymbol{S}\right)\right] \tag{9}$$

$\boldsymbol{X}$ can be calculated by (9) and there are $2n+1$ values of $\boldsymbol{X}$ [4]. Applying those values to (4) can estimate $E[Z^l]$.

D. A Novel Method to Calculate ρ_{0ij} from ρ_{ij}

Equation (6) is a highly complex nonlinear equation with integral. Reference [4] adopted semi-empirical formulae to calculate ρ_{ij}, but those formulae are only valid for some specific distributions of $\boldsymbol{X}$. In this paper, a novel general method is proposed and it can calculate ρ_{ij} whatever the distributions of $\boldsymbol{X}$ are.

$\varphi_2(y_i,y_j,\rho_{0ij})$ can be expanded by Hermite polynomials, as shown in (10).

$$\varphi_2\left(y_i,y_j,\rho_{0ij}\right)=\varphi\left(y_i\right)\varphi\left(y_j\right)\sum_{r=0}^{\infty}\frac{\left(\rho_{0ij}\right)^r}{r!}\cdot H_r\left(y_i\right)H_r\left(y_j\right) \tag{10}$$

Where $\varphi(\cdot)$ denotes the one-dimensional probability density function of the standard normal distribution; $H_r(\cdot)$ denotes the rth order probability Hermite polynomial.

By applying (10) to (6), a simplified equation can be obtained, as shown in (11).

$$\rho_{ij}=\sum_{r=1}^{\infty}\frac{1}{r!}I_{i,r}I_{j,r}\left(\rho_{0ij}\right)^r \tag{11}$$

Where $I_{i,r}$ is:

$$I_{i,r}=\int_{-\infty}^{+\infty}\frac{F_i^{-1}\left(\Phi\left(y_i\right)\right)}{\sigma_i}H_r\left(y_i\right)\varphi\left(y_i\right)dy_i \tag{12}$$

$I_{i,r}$ can be calculated by the Gauss-Hermite quadrature formula [5]. The approximate equation of (11) can be described as (13).

$$\rho_{ij}\approx\sum_{r=1}^{Q}\frac{1}{r!}I_{i,r}I_{j,r}\left(\rho_{0ij}\right)^r \tag{13}$$

Equation (13) is a one-dimensional polynomial equation which is much simplified compared to (6). General solutions for polynomial equations, such as the Newton iterative method, can be adopted to calculate ρ_{0ij}.

III. THE RELIABILITY EVALUATION OF THE ELECTRICITY-GAS COUPLED SYSTEM CONSIDERING CORRELATION

A. Reliability Evaluation of the Electricity-gas Coupled System Based on the Minimal Path Method

The minimal path method is widely used for reliability evaluation of distribution networks, and can be adopted to electricity-gas coupled systems [6].

For the minimal path method, minimal path between the loads and power sources should be determined first. Then components in the system can be classified into two groups for a load: in the minimal path and out of the minimal path. Breakdown of components in the minimal path of the load can result in outage of the load, and fault parameters of these components should be directly involved in reliability calculation of the load. However, influence on the load caused by breakdown of components that are out of the minimal path should be equivalent according to different situations, such as the power and locations of distributed sources [6].

For a certain system state ($\boldsymbol{X}$), the reliability evaluation of the electricity-gas coupled system is processed by the minimum path method. According to [4], there are $2n+1$ values of $\boldsymbol{X}$, so origin moments of the reliability index Z can be calculated by (4).

B. The Reliability Evaluation Process of the Electricity-gas Coupled System Considering Correlation

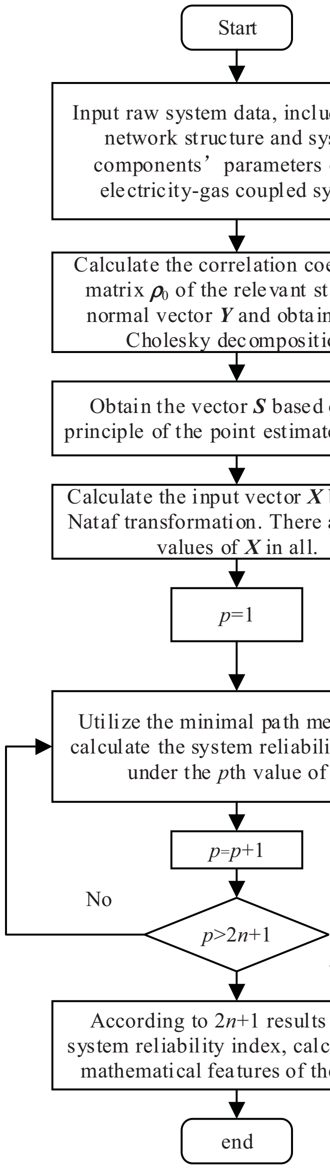

Figure 1. The flow chart of the reliability evaluation process

Fig. 1 shows the reliability evaluation process of the electricity-gas coupled system. Based on the point estimate method and Nataf transformation, which are described in section II, the correlation of input variables can be considered, and mathematical features (e.g. the expectation value, the standard deviation, et al.) of the system reliability index can be calculated.

IV. CASE STUDY

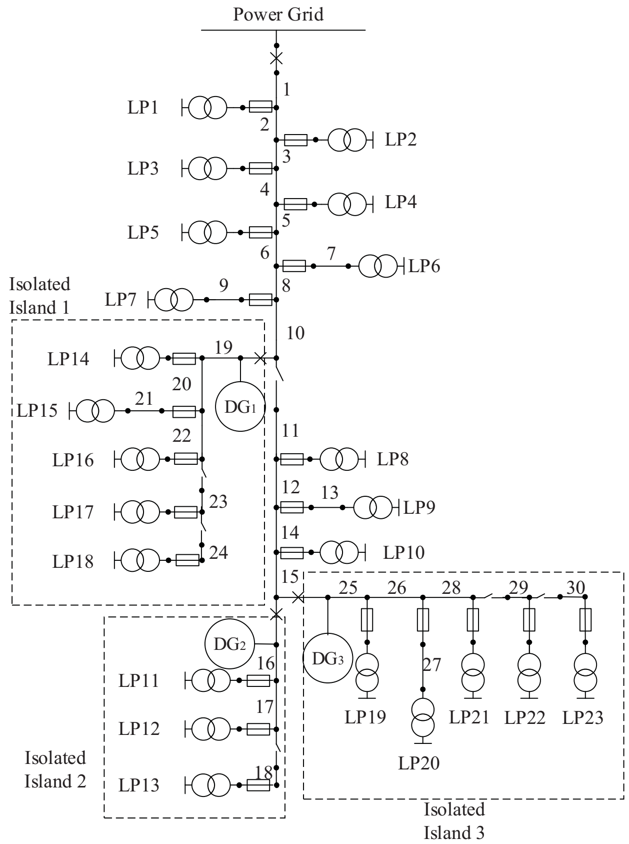

Figure 2. The structure diagram of the modified IEEE RBTS-Bus6 NO.4 feeder system

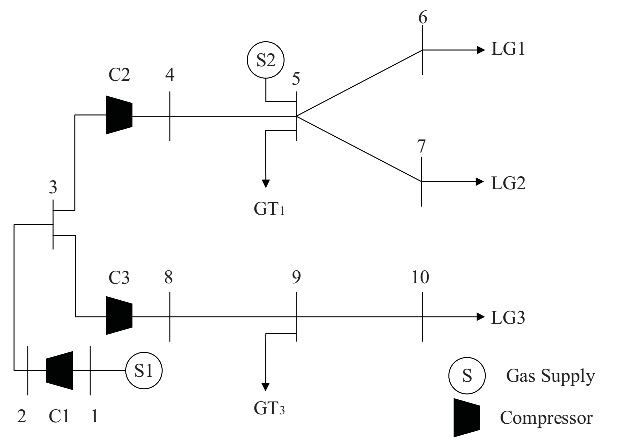

Figure 3. The structure diagram of the modified NGS10 natural gas node system

Here, the electricity-gas coupled system is used as a case study, which consists of a modified IEEE RBTS-Bus6 NO.4 feeder system[7] and a modified NGS10 natural gas system[8].

The IEEE RBTS-Bus6 NO.4 feeder system is modified by adding 3 distributed generators (DG) in Line 16, Line 19 and Line 25 respectively, as shown in Fig. 2. DG_1 contains a 1 MW wind farm and a 0.5 MW gas turbine (GT_1). DG_2 contains a 1 MW wind farm. DG_3 contains a 2 MW wind farm and a 0.5 MW gas turbine (GT_3). Real wind speed data from 3 Dutch wind farms are utilized into above DGs to obtain best fitting distributions, parameters and correlation coefficients. For the wind farms, the cut-in speed is 3m/s, cut-out speed is 20m/s and rated speed is 9m/s.

As shown in Fig. 3, the NGS10 natural gas system is modified by adding a gas supply in Node 5, and 2 above gas turbines in Node 5 (GT_1) and Node 9 (GT_3) respectively.

The electric loads LP1-LP23 use the annual sequential load data of the IEEE-RTS79 reliability test system, and the probability density distribution is obtained by the kernel density estimation method. The gas loads LG1-LG3 are set to the normal distribution (the expectation value is 2 MW; the standard deviation is 0.2 MW). The correlation coefficient between the electric loads is 0.9; the correlation coefficient between the gas loads is 0.9; the correlation coefficient between the electric and gas loads is 0.6.

A. Results and Analysis of the Reliability Indices

The reliability of the case system is evaluated by the proposed method and the Monte Carlo method respectively. The results of the Monte Carlo method (with a sample size of 50000) are the reference. TABLE I shows the expectation values of the system reliability indices calculated by the above two methods respectively. TABLE II shows the standard deviations.

It can be learnt from TABLE I that for the electric loads, the relative errors of the expectation of each index are small, and the maximum error does not exceed 5%. The relative errors of the standard deviation of each index are greater than those of the expectation, because the estimation error of the point estimate method rises with the order increase of the output origin moments. However, the maximum error does not exceed 10%.

It can be learnt from TABLE II that for the gas loads, the relative errors of the expectation of each index are still small, and the maximum error does not exceed 2%. The standard deviations of SAIFI, SAIDI and ASAI tend to zero under the point estimate method, while the results under the Monte Carlo method do not. This is because the gas network does not have isolated island modes [6], which exist in the electrical network. For a gas load, the failure rate and the annual average outage time are completely determined by the component parameters and topology of the gas network, so the failure rate and the annual average outage time are highly deterministic variables, and their standard deviations should tend to zero in theory. As a result, SAIFI, SAIDI and ASAI should be highly deterministic variables and their standard deviations should also tend to zero. The results obtained by the point estimate method are consistent with the theory. On the other hand, the Monte Carlo method has randomness error due to the nature of this method, so the calculated standard deviations do not tend to zero. Besides, for the standard deviation of the ENS, the relative error is still not more than 10%.

The results of the case show that the proposed method has high accuracy to evaluate the reliability of the electricity-gas coupled system.

TABLE I. NUMERICAL CHARACTERISTICS OF THE ELECTRICAL NETWORK RELIABILITY INDICES

Reliability Indices	SAIFI/[interruptions/customer/yr]		SAIDI/[(h/customer/yr)]		ASAI/%		ENS/(MW·h/yr)	
	Expectation	Standard Deviation	Expectation	Standard Deviation	Expectation	Standard Deviation	Expectation	Standard Deviation
Proposed Method	1.1875	0.1413	7.1837	0.9813	0.9992	0.0001	50.2668	19.3685
Monte Carlo Method	1.1640	0.1496	6.9360	1.0765	0.9992	0.0001	52.6555	17.9005
Relative Error/%	1.98	5.87	3.45	9.70	0.00	0.00	4.54	7.58

TABLE II. NUMERICAL CHARACTERISTICS OF THE NATURAL GAS NETWORK RELIABILITY INDICES

Reliability Indices	SAIFI/[interruptions/customer/yr]		SAIDI/[(h/customer/yr)]		ASAI/%		ENS/(MW·h/yr)	
	Expectation	Standard Deviation	Expectation	Standard Deviation	Expectation	Standard Deviation	Expectation	Standard Deviation
Proposed Method	0.2683	0.0000	1.3667	0.0000	0.9998	0.0000	8.2000	0.7945
Monte Carlo Method	0.2697	0.0061	1.3624	0.0116	0.9998	0.0001	8.0715	0.8733
Relative Error/%	0.52	/	0.31	/	0.00	/	1.57	9.92

B. A Comparison of the Simulation Time

In order to analyze the computational efficiency of the proposed method, TABLE III shows a comparison of the simulation time of the proposed method and the Monte Carlo method. The comparison shows that the Monte Carlo method spends remarkably more time than the proposed method, which means the proposed method has a high computing efficiency. If the system reliability index is added to the constraints, the calculation efficiency of the proposed method can satisfy the requirements of real-time optimal dispatch of the electricity-gas coupled system which considers high system reliability. Then, optimal dispatch strategies achieving both high reliability and economy may be obtained.

TABLE III. A COMPARISON OF THE SIMULATION TIME BETWEEN TWO METHODS

Method	Simulation time/s
Proposed Method	9.22
Monte Carlo Method	12772.9

V. CONCLUSION

A reliability evaluation method of the electricity-gas coupled system considering correlation is proposed. The point estimate method is applied to the reliability evaluation process to obtain statistical features. Since the point estimate method could not be applied to correlated variables, those variables are processed by the Nataf transformation. Then a novel general method to calculate ρ_0 is researched. Finally, the proposed method is adopted to a case study. Compared to the Monte Carlo method, the proposed method can provide statistical features of reliability indices of the electricity-gas coupled system swiftly and take correlation effect into account. The calculation efficiency is remarkably higher than that of the Monte Carlo method, while accuracy levels are basically similar. Besides, the proposed method satisfies the requirements of real-time optimal dispatch of the electricity-gas coupled system considering high reliability, and may provide some assistance for the grid dispatchers.

In the future, multiple energy coupling forms in the electricity-gas coupled system should be considered (such as Power to Gas devices). Besides, the method may be adopted to the reliability evaluation of the integrated energy system.

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