Graphical Models and Belief Propagation Hierarchy for Physics-Constrained Network Flows

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Abstract

We review new ideas and the first results from the application of the graphical models approach, which originated from statistical physics, information theory, computer science, and machine learning, to optimization problems of network flow type with additional constraints related to the physics of the flow. We illustrate the general concepts on a number of enabling examples from power system and natural gas transmission (continental scale) and distribution (district scale) systems.

Notes

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