US009915399B1

(12) **United States Patent**
Latshaw et al.

(10) **Patent No.:** **US 9,915,399 B1**
(45) **Date of Patent:** **Mar. 13, 2018**

(54) **CONTROL SYSTEM IN A GAS PIPELINE NETWORK TO SATISFY DEMAND CONSTRAINTS**

(71) Applicant: **Air Products and Chemicals, Inc.,** Allentown, PA (US)

(72) Inventors: **Catherine Catino Latshaw,** Fogelsville, PA (US); **Ali Esmaili,** Emmaus, PA (US); **Joshua David Isom,** Allentown, PA (US); **Camilo Mancilla,** Allentown, PA (US)

(73) Assignee: **Air Products and Chemicals, Inc.,** Allentown, PA (US)

(*) Notice: Subject to any disclaimer, the term of this patent is extended or adjusted under 35 U.S.C. 154(b) by 0 days.

(21) Appl. No.: **15/490,344**

(22) Filed: **Apr. 18, 2017**

(51) **Int. Cl.**
G06F 19/00	(2011.01)
F17D 3/01	(2006.01)
F17D 1/04	(2006.01)
F17D 5/00	(2006.01)
G05D 7/06	(2006.01)

(52) **U.S. Cl.**
CPC **F17D 3/01** (2013.01); **F17D 1/04** (2013.01); **F17D 5/005** (2013.01); **G05D 7/0629** (2013.01)

(58) **Field of Classification Search**
CPC G06F 17/10; G06F 17/11; G06F 19/24; G06Q 10/06; G06Q 10/04; G06Q 10/0631; G06Q 50/06; G01F 15/005; G01F 1/36; G05D 16/20; G05D 7/0635; Y10T 137/268
See application file for complete search history.

(56) **References Cited**

U.S. PATENT DOCUMENTS

4,200,911 A	*	4/1980	Matsumoto	E03B 7/02 137/119.06
4,835,687 A	*	5/1989	Martin	F17D 3/00 417/2
6,236,894 B1	*	5/2001	Stoisits	G05B 13/0265 166/52

(Continued)

OTHER PUBLICATIONS

Martin et al., Mixed Integer Models for the Stationary Case of Gas Network Optimization, Mat. Program., Ser. B 105, 563-582 (2006).*

(Continued)

Primary Examiner — Darrin Dunn
(74) *Attorney, Agent, or Firm* — Larry S. Zelson

(57) **ABSTRACT**

Controlling flow of gas in a gas pipeline network, wherein flow of gas within each pipeline segment is associated with a direction (positive or negative). Minimum and maximum delivery rates to each gas receipt facility are determined. Lower and upper flow bounds of gas delivery rate are created by bounding minimum and maximum signed flow rates using minimum and maximum delivery rates, respectively, for each pipe segment. A pressure drop relationship for each pipeline segment within the lower and upper flow bounds is linearized to create a linear pressure drop model for each pipeline segment. A network flow solution is calculated, which includes flow rates for each pipeline segment and pressures for each network nodes to satisfy the lower and upper flow bounds on the gas delivery rate. The network flow solution is associated with control element setpoints used by a controller to control one or more control elements.

7 Claims, 20 Drawing Sheets
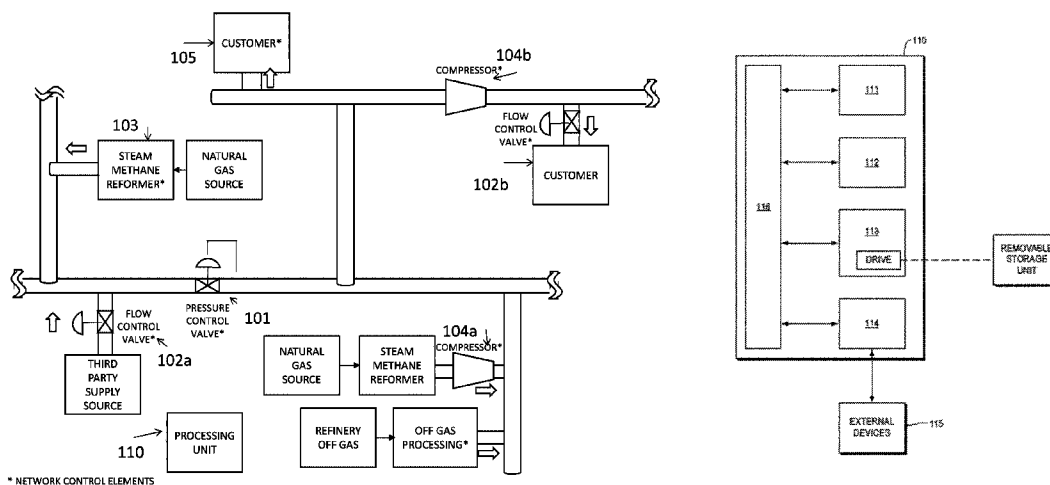
* NETWORK CONTROL ELEMENTS

(56)   **References Cited**

U.S. PATENT DOCUMENTS

6,697,713	B2 *	2/2004	Megan	G05D 16/20
				137/119.03
6,701,223	B1 *	3/2004	Rachford, Jr.	F16K 37/0075
				137/45
6,993,403	B1 *	1/2006	Dadebo	G05B 23/0294
				700/108
7,561,928	B2 *	7/2009	Peureux	G06Q 50/00
				700/28
7,720,575	B2 *	5/2010	Ferber	G05D 7/0647
				700/281
8,762,291	B2	6/2014	Neagu et al.	
9,599,499	B1 *	3/2017	Chaudhary	G01F 22/02
2003/0144766	A1	7/2003	Megan et al.	
2003/0192693	A1	10/2003	Wellington	
2005/0137810	A1	6/2005	Esposito, Jr. et al.	
2005/0273204	A1	12/2005	Hansen et al.	
2007/0204930	A1	9/2007	Phallen et al.	
2007/0260333	A1	11/2007	Peureux et al.	
2009/0265292	A1	10/2009	Harper	
2010/0152900	A1	6/2010	Gurciullo et al.	
2010/0174517	A1	7/2010	Slupphaug et al.	
2010/0222911	A1	9/2010	Castelijns et al.	
2013/0211601	A1	8/2013	Cheng et al.	
2014/0005841	A1	1/2014	Cheng et al.	
2015/0136248	A1 *	5/2015	Nagase	G05D 7/0635
				137/468
2016/0072891	A1	3/2016	Joshi et al.	
2016/0209851	A1	7/2016	Aughton et al.	

OTHER PUBLICATIONS

Martin et al., A Mixed Integer Approach for the Transient Case of Gas Network Optimization, 2006.*

C.M. Correa-Posada, et al, "Gas Network Optimization: A comparison of Piecewise Llinear Models", Chemical Engineering Science, 2014, 1-24.

A. Gopalakrishnan, et al, "Economic Nonlinear Model Predictive Control for periodic optimal operation of gas pipeline networks", Computers and Chemical Engineering, 2013, 52, 90-99.

F. Tabkhi, et al, "Improving the Performance of Natural Gas Pipeline Networks Fuel Consumption Minimization Problems", American Institute of Chemical Engineers, 2010, 56, 946-964.

R.Z. Rios-Mercado, et al, "Optimization problems in natural gas transportation systems: A state-of-the-art review", Applied Energy, 2015, 147, 536-555.

K.G. Murty, et al, "Some NP-Complete Problems in Quadratic and Nonlinear Programming", Mathematical Programming, 1987, 39, 117-129.

R.K. Ahuja, et al, "Network Flows, Chapter 4 Shortest Paths: Label-Setting Algorithms", 1993, 93-132.

Ajinkya A. More, et al, "Analytical solutions for the Colebrook and White equation and for pressure drop in ideal gas flow in pipes", Chemical Engineering Science, 2006, 61, 5515-5519.

Dejan Brkic, "Lambert W Function in Hydraulics Problems", Mathematica Balkanica, 2012, 26, 285-292.

Alexander Martin, et al, "Mixed Integer Models for the Stationary Case of Gas Network Optimization", Math. Program., 2006, 563-582.

A. Martin, et al, "A Mixed Integer Approach for the Transient Case of Gas Network Optimization", 2007.

U.S. Appl. No. 15/490,268, filed Apr. 18, 2017, first named inventor Camilo Mancilla, titled Control System in a Gas Pipeline Network to Satisfy Pressure Constraints.

U.S. Appl. No. 15/490,268, Office Action dated Jul. 7, 2017.

U.S. Appl. No. 15/490,308, filed Apr. 18, 2017, first named inventor Camilo Mancilla, titled Control System in an Industrial Gas Pipeline Network to Satisfy Energy Consumption Constraints at Production Plants.

U.S. Appl. No. 15/490,308, Office Action dated Aug. 11, 2017.

U.S. Appl. No. 15/490,394, filed Apr. 18, 2017, first named inventor Ali Esmaili, titled Control System in a Gas Pipeline Network to Increase Capacity Factor.

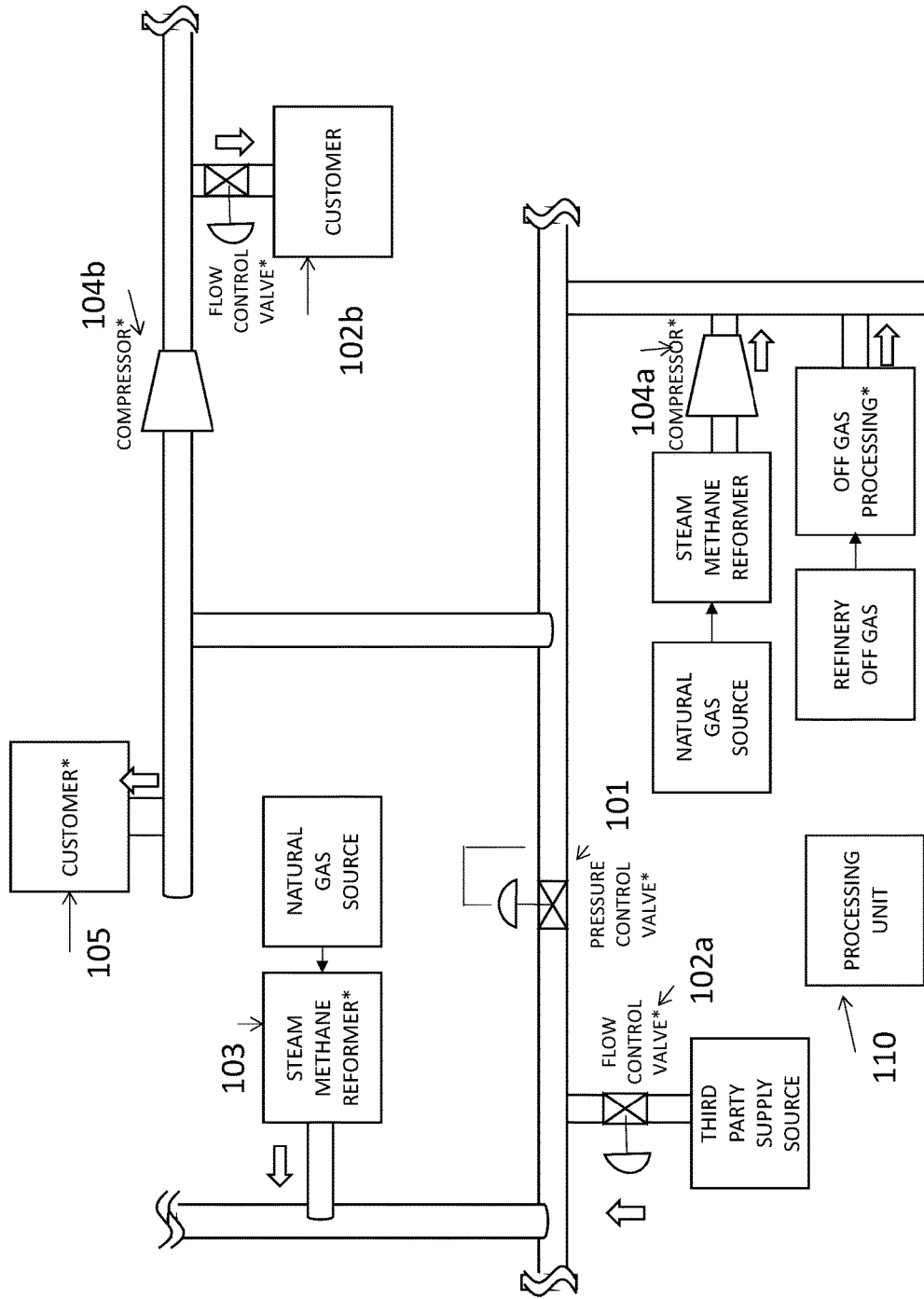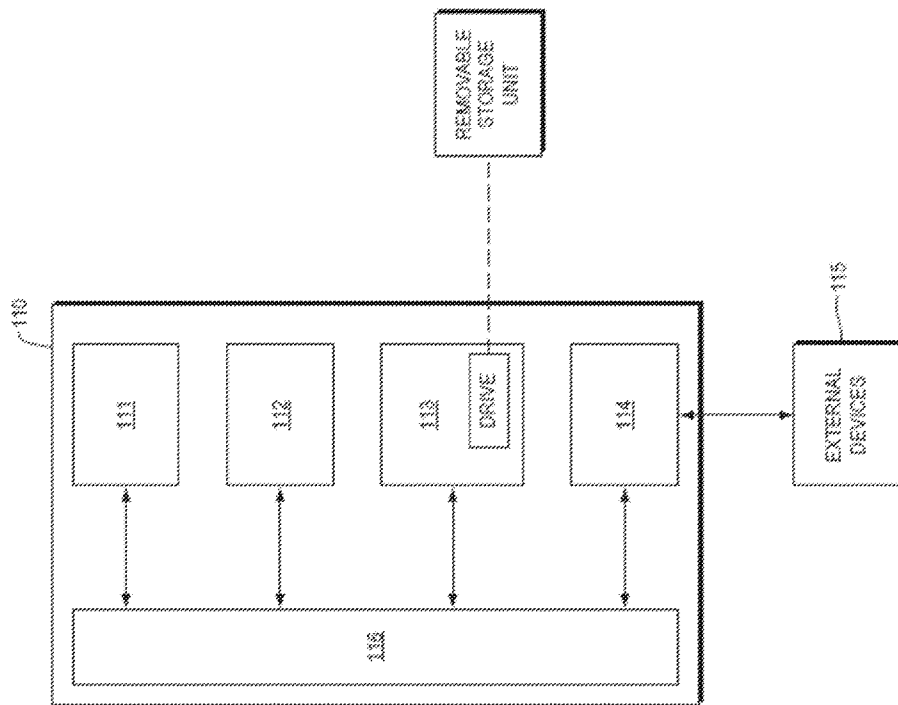U.S. Appl. No. 15/490,394, Office Action dated Jul. 12, 2017.

* cited by examiner

FIG. 1A

FIG. 1B

FIG. 2

FIG. 3

FIG. 4

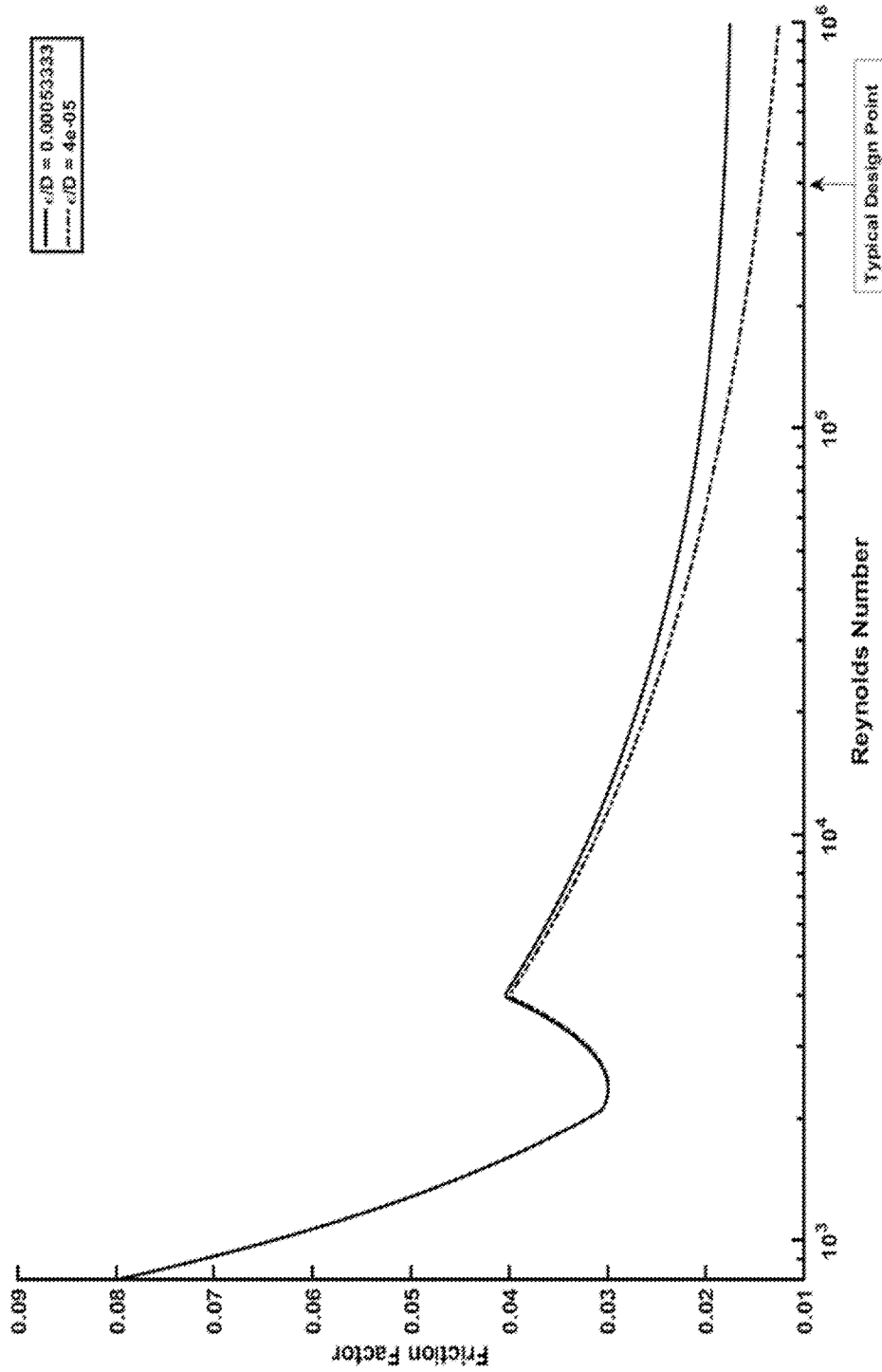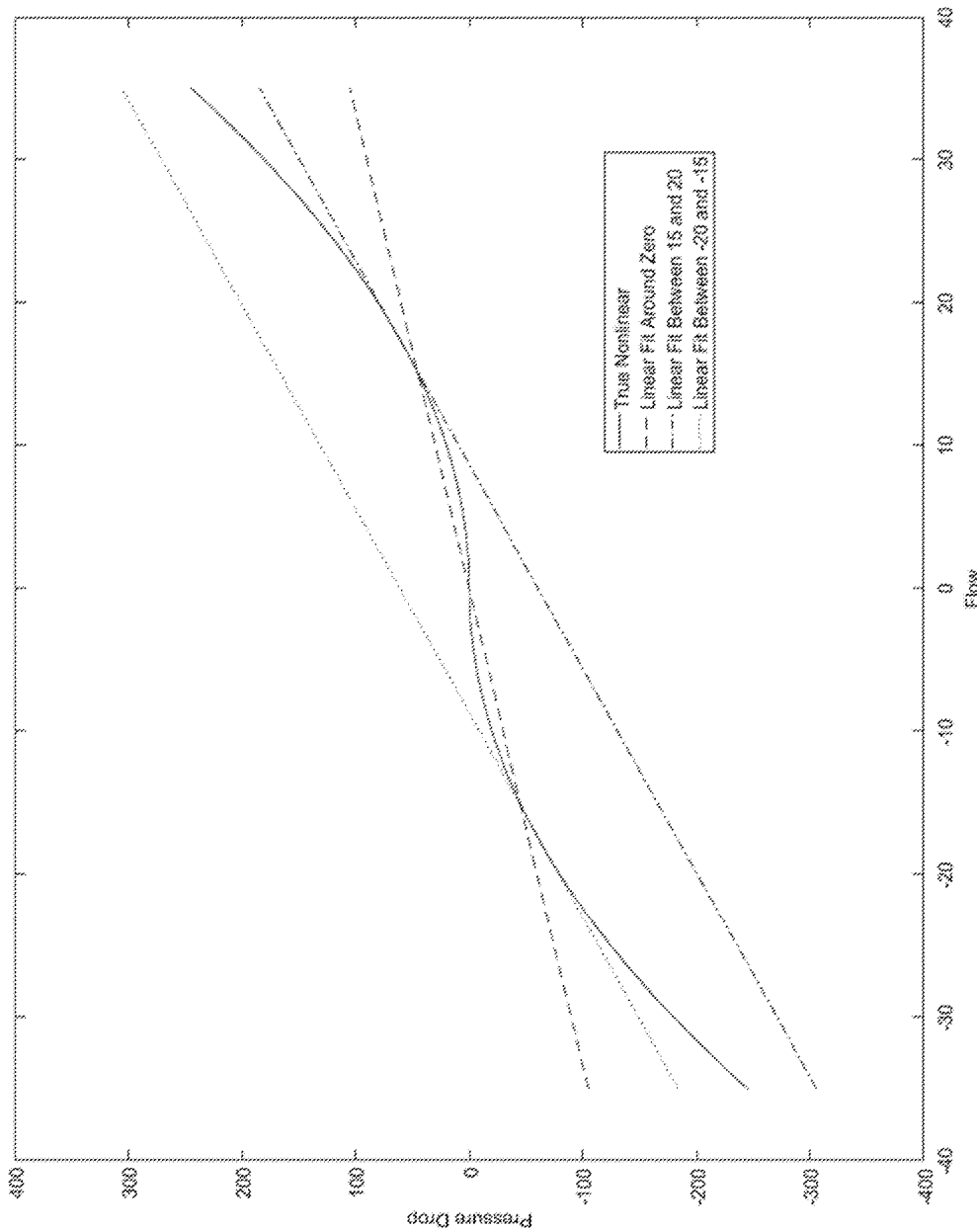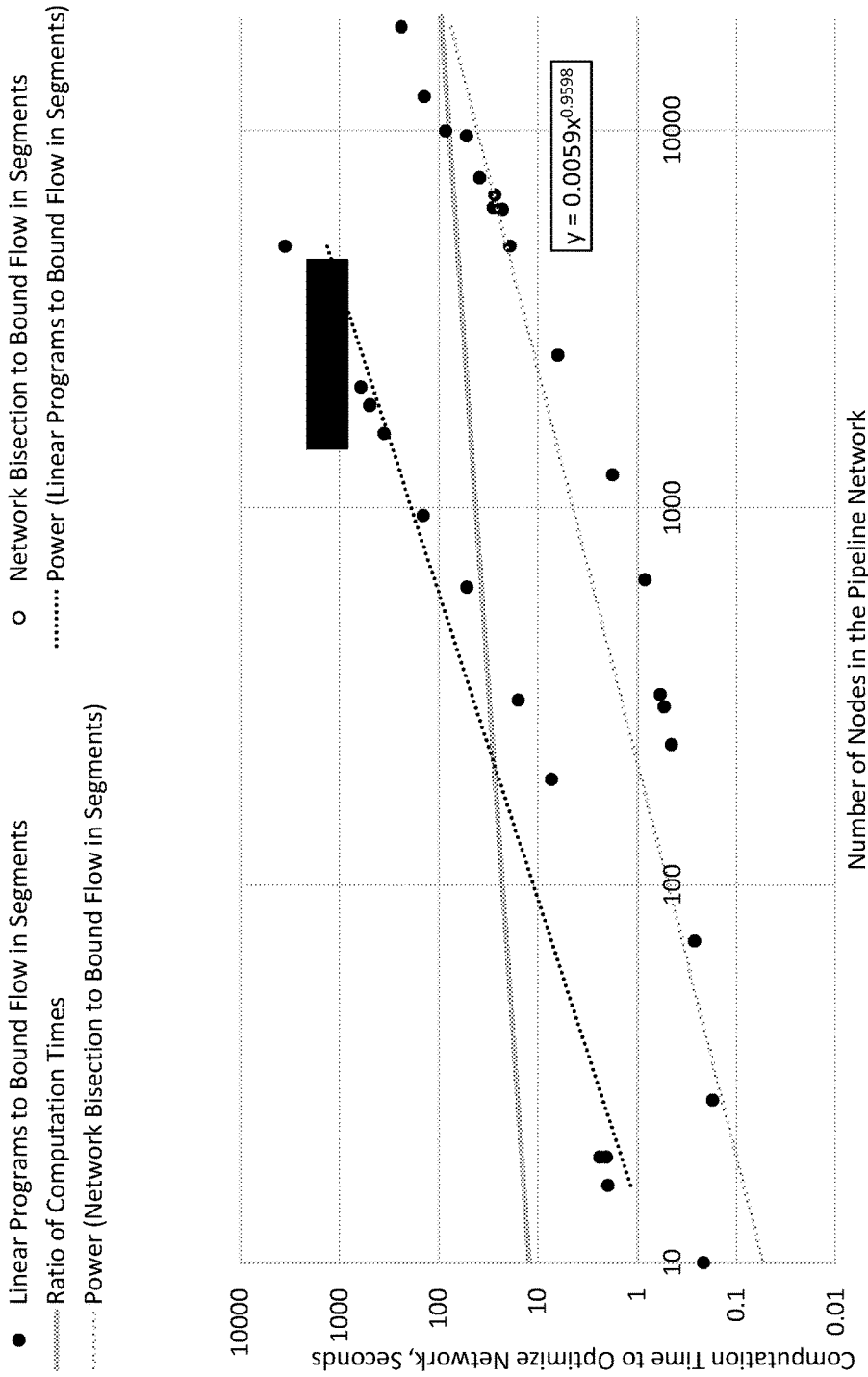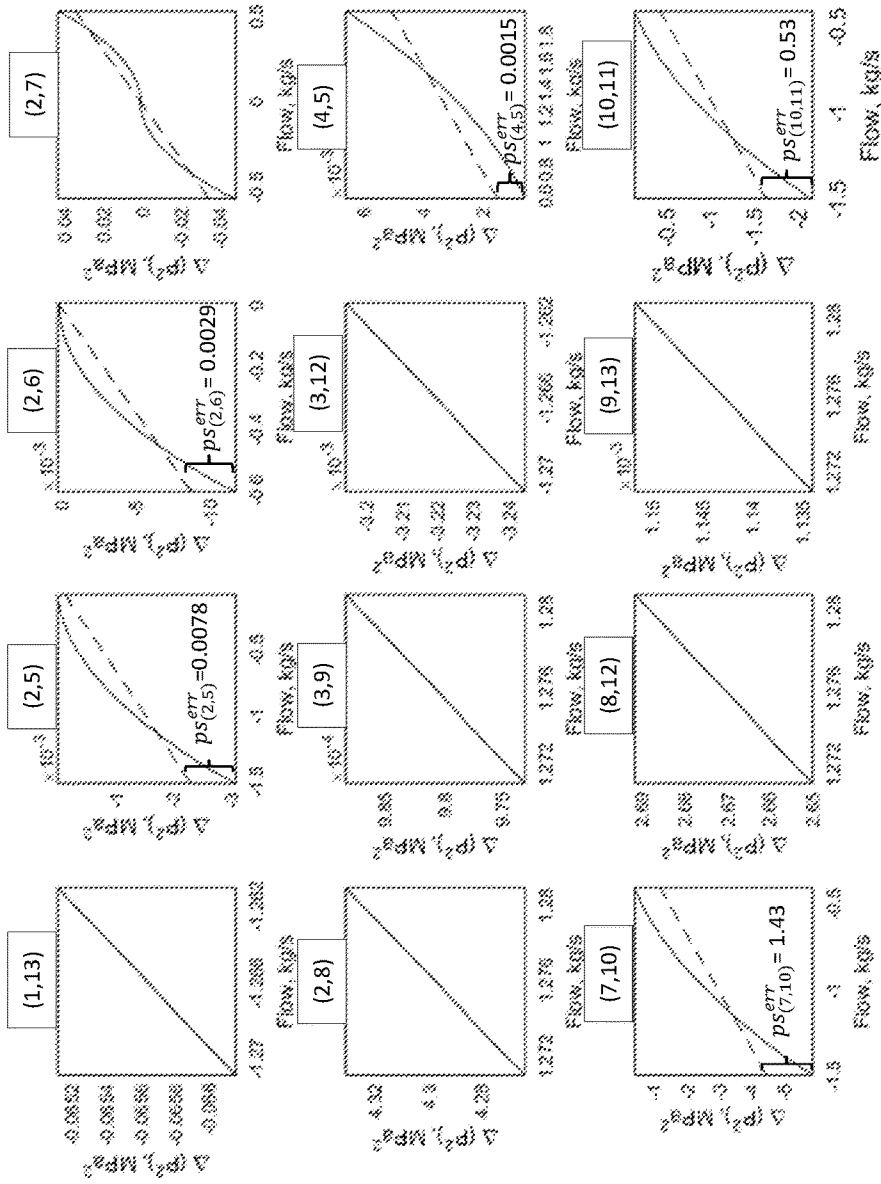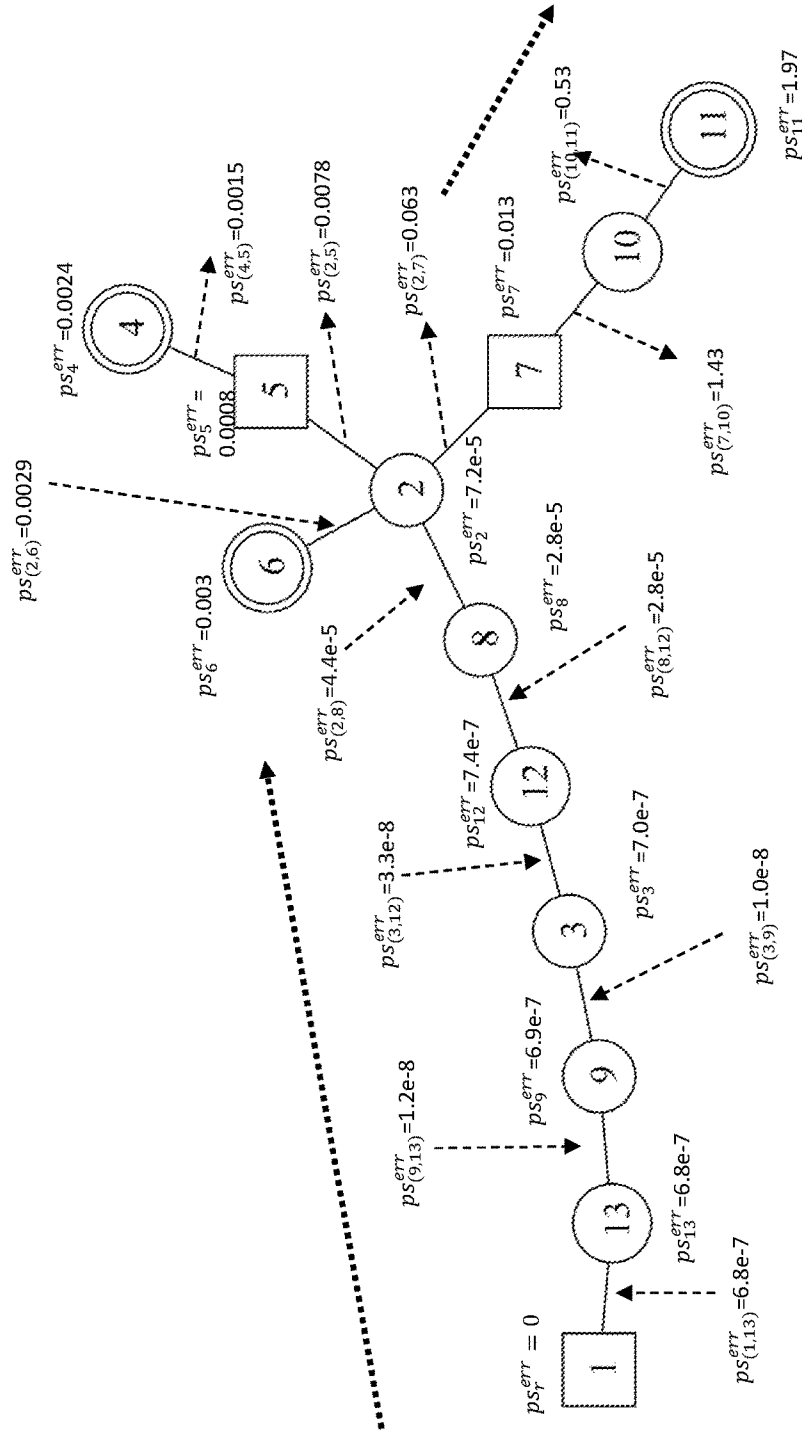$$d_{R_j}^{min} = \left(\sum_{n \in R} d_n^{min}\right) - \left(\sum_{n \in R} s_n^{max}\right) = (8 + 3 + 7) - (15 + 8 + 10 + 12) = -27$$

$$d_R^{max} = \left(\sum_{n \in R} d_n^{max}\right) - \left(\sum_{n \in R} s_n^{min}\right) = (16 + 10 + 12) - (10 + 7 + 3 + 5) = 13$$

$$s_{L_j}^{min} = \left(\sum_{n \in L} s_n^{min}\right) - \left(\sum_{n \in L} d_n^{max}\right) = (0) - (11) = -11$$

$$s_L^{max} = \left(\sum_{n \in L} s_n^{max}\right) - \left(\sum_{n \in L} d_n^{min}\right) = (0) - (6) = -6$$

$$q_j^{min} = \max\left\{s_{L_j}^{min}, d_{R_j}^{min}\right\} = \max\{-11, -27\} = -11$$

$$q_j^{max} = \min\left\{s_{L_j}^{max}, d_{R_j}^{max}\right\} = \min\{-6, 13\} = -6$$

$d_1^{min} = 6$
$d_1^{max} = 11$

$s_{10}^{min} = 7$
$s_{10}^{max} = 12$

$s_{17}^{min} = 5$
$s_{17}^{max} = 10$

$s_{13}^{min} = 3$
$s_{13}^{max} = 8$

$s_2^{min} = 10$
$s_2^{max} = 15$

$d_9^{min} = 8$
$d_9^{max} = 16$

$d_{16}^{min} = 3$
$d_{16}^{max} = 10$

$d_{12}^{min} = 7$
$d_{12}^{max} = 12$

FIG. 5

FIG. 6

$$d_{R_j}^{min} = \left(\sum_{n \in R} d_n^{min} \right) - \left(\sum_{n \in R} s_n^{max} \right) = (8 + 3 + 7 + 6) - (15 + 8 + 10 + 12) = -21$$

$$d_R^{max} = \left(\sum_{n \in R} d_n^{max} \right) - \left(\sum_{n \in R} s_n^{min} \right) = (16 + 10 + 12 + 11) - (10 + 7 + 3 + 5) = 24$$

$$q_j^{min} = \max \left\{ s_{L_j}^{min}, d_{R_j}^{min} \right\} = \max\{7, -21\} = 7$$

$$q_j^{max} = \min \left\{ s_{L_j}^{max}, d_{R_j}^{max} \right\} = \min\{12, 24\} = 12$$

$$s_{L_j}^{min} = \left(\sum_{n \in L} s_n^{min} \right) - \left(\sum_{n \in L} d_n^{max} \right) = (7) - (0) = 7$$

$$s_L^{max} = \left(\sum_{n \in L} s_n^{max} \right) - \left(\sum_{n \in L} d_n^{min} \right) = (12) - (0) = 12$$

$d_1^{min} = 6$
$d_1^{max} = 11$

$s_{10}^{min} = 7$
$s_{10}^{max} = 12$

$s_{17}^{min} = 5$
$s_{17}^{max} = 10$

$s_{13}^{min} = 3$
$s_{13}^{max} = 8$

$s_2^{min} = 10$
$s_2^{max} = 15$

$d_9^{min} = 8$
$d_9^{max} = 16$

$d_{16}^{min} = 3$
$d_{16}^{max} = 10$

$d_{12}^{min} = 7$
$d_{12}^{max} = 12$

$$s_{Lj}^{min} = \left(\sum_{n\in L} s_n^{min}\right) - \left(\sum_{n\in L} d_n^{max}\right) = (3 + 10 + 5) - (16 + 10 + 12) = -20$$

$$s_L^{max} = \left(\sum_{n\in L} s_n^{max}\right) - \left(\sum_{n\in L} d_n^{min}\right) = (15 + 8 + 10) - (8 + 3 + 7) = 15$$

$$d_{Rj}^{min} = \left(\sum_{n\in R} d_n^{min}\right) - \left(\sum_{n\in R} s_n^{max}\right) = (6) - (12) = -6$$

$$d_R^{max} = \left(\sum_{n\in R} d_n^{max}\right) - \left(\sum_{n\in R} s_n^{min}\right) = (11) - (7) = 4$$

$$q_j^{min} = \max\left\{s_{Lj}^{min}, d_{Rj}^{min}\right\} = \max\{-20, -6\} = -6$$

$$q_j^{max} = \min\left\{s_{Lj}^{max}, d_{Rj}^{max}\right\} = \min\{15, 4\} = 4$$

$d_1^{min} = 6$
$d_1^{max} = 11$

$s_{10}^{min} = 7$
$s_{10}^{max} = 12$

$s_{17}^{min} = 5$
$s_{17}^{max} = 10$

$s_2^{min} = 10$
$s_2^{max} = 15$

$s_{13}^{min} = 3$
$s_{13}^{max} = 8$

$d_{16}^{min} = 3$
$d_{16}^{max} = 10$

$d_9^{min} = 8$
$d_9^{max} = 16$

$d_{12}^{min} = 7$
$d_{12}^{max} = 12$

FIG. 7

FIG. 8

FIG. 9

FIG. 10

FIG. 11

FIG. 12
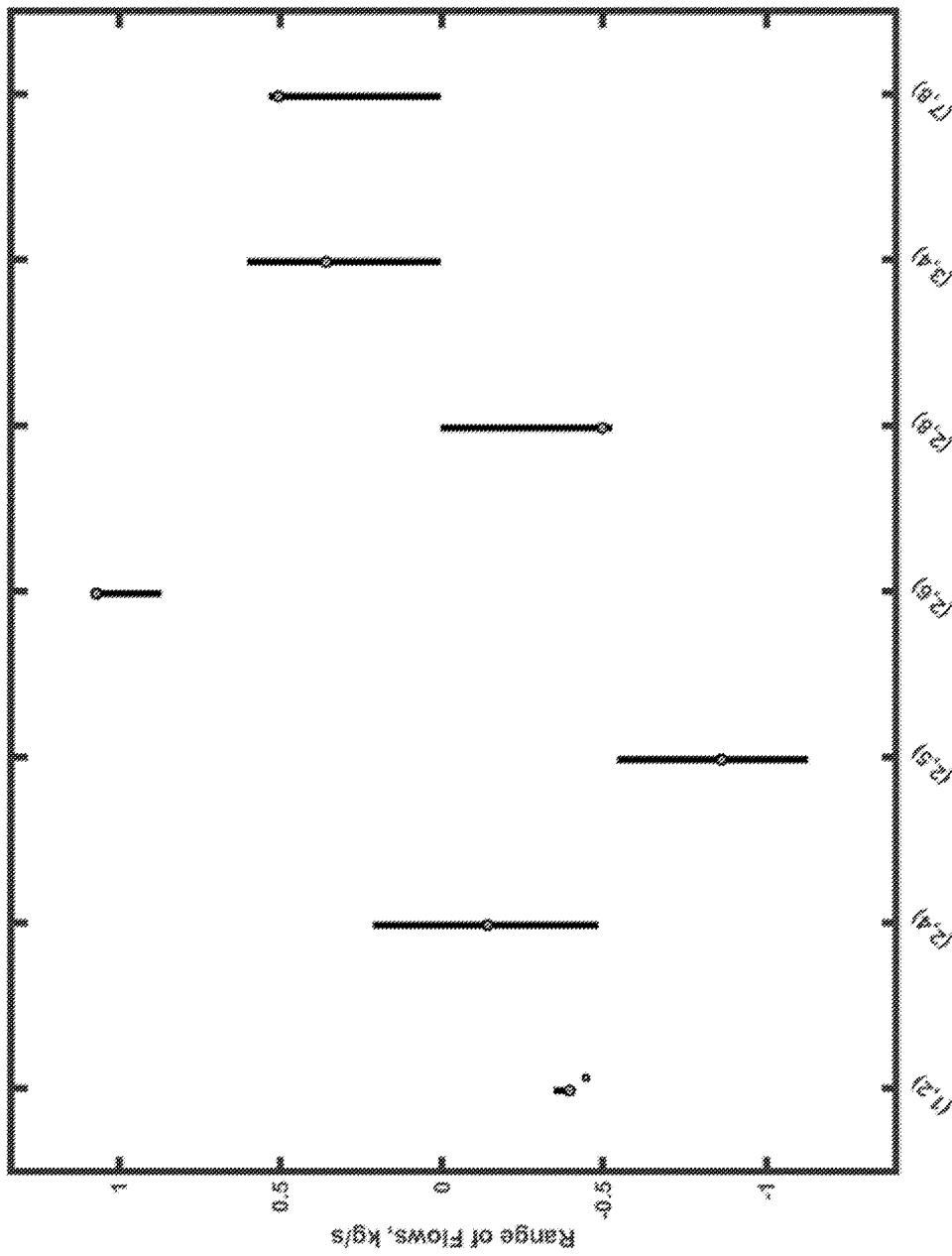
FIG. 13
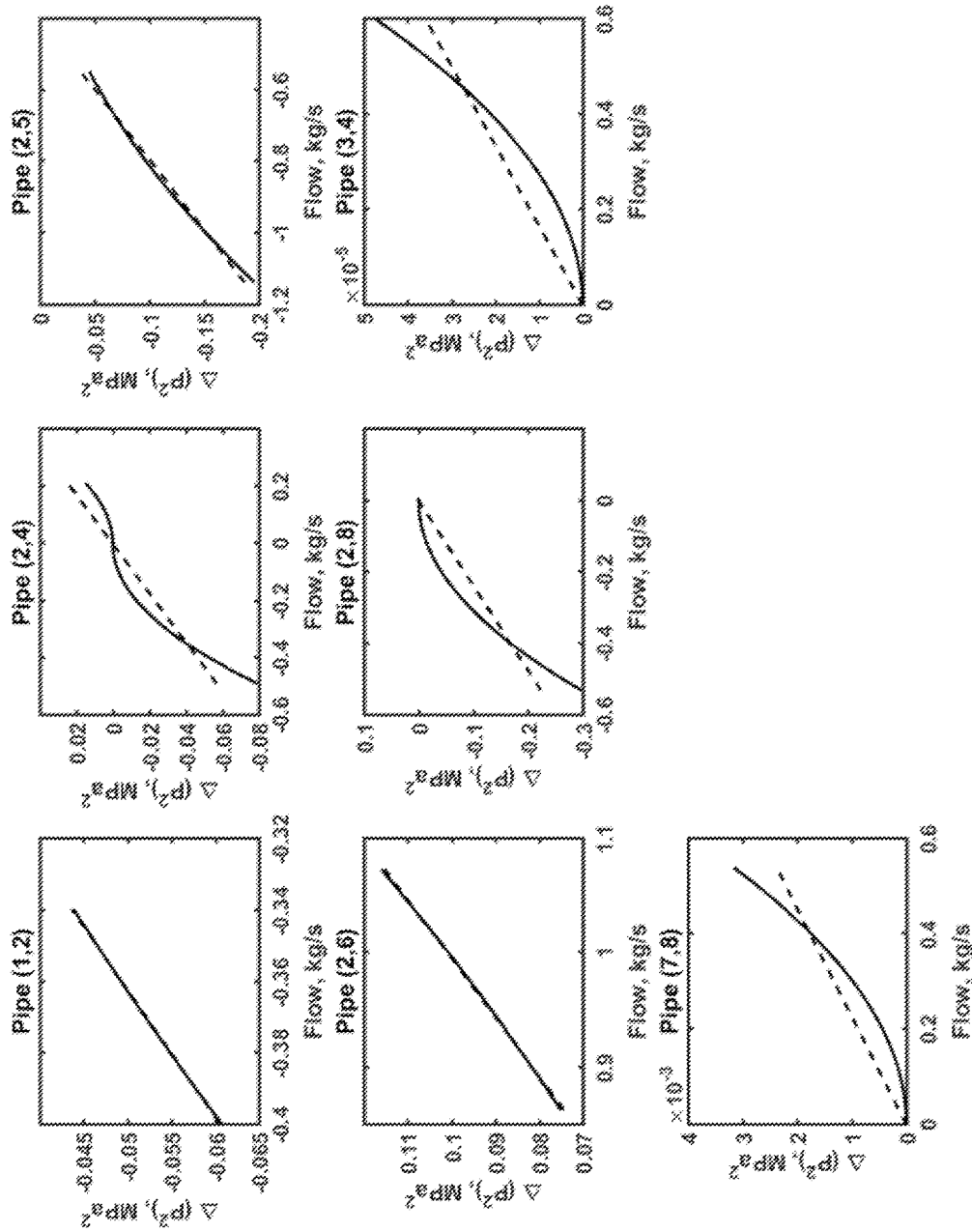
FIG. 14

FIG. 15

FIG. 16

FIG. 17

FIG. 18

1901 - Determine minimum and maximum flow rate for delivery to each customer	→	Minimum rate based on contractual constraints, maximum rate based on most recent request
1902- Calculate minimum and maximum signed flow rate for each pipeline segment	→	Network bisection method
1903 - Calculate linearization of pressure drop relationship for each pipeline segment based on minimum and maximum signed flow rate	→	Least squares linearization
1904 - Bound pressure prediction error for each network node	→	Shortest path for weighted graph using Dijkstra's method
1905 - Use pressure drop linearization and pressure prediction error bounds to compute network flow solution	→	Linear program
1906 - Control elements receive setpoints determined from network flow solution	→	Setpoints received by flow control and pressure control elements

FIG. 19

CONTROL SYSTEM IN A GAS PIPELINE NETWORK TO SATISFY DEMAND CONSTRAINTS

FIELD OF THE INVENTION

The invention relates to control of gas pipeline networks for the production, transmission and distribution of a gas.

BRIEF SUMMARY OF THE INVENTION

The present invention relates to a system and method for controlling flow of gas in a gas pipeline network. The gas pipeline network comprises at least one gas production plant, at least one gas receipt facility of a customer, a plurality of pipeline segments, a plurality of network nodes, and a plurality of control elements. Flow of gas within each of the plurality of pipeline segments is associated with a direction, the direction being associated with a positive sign or a negative sign. The system further includes one or more controllers and one or more processors. A minimum delivery rate and maximum delivery rate of the gas to each of the gas receipt facilities in the gas pipeline network is determined. A lower flow bound and an upper flow bound of delivery rate of gas for each of the plurality of pipeline segments is created by bounding a minimum signed flow rate for each pipe segment using a minimum delivery rate as a lower bound and bounding a maximum signed flow rate for each pipe segment using a maximum delivery rate as an upper bound. A pressure drop relationship for each of the plurality of pipeline segments within the lower flow bound and the upper flow bound is linearized to create a linear pressure drop model for each of the plurality of pipeline segments. A network flow solution is calculated. The network flow solution comprises flow rates for each of the plurality of pipeline segments and pressures for each of the plurality of network nodes to satisfy the lower flow bound and to satisfy the upper flow bound on the delivery rate of the gas. The network flow solution being associated with control element setpoints. The controllers receive data describing the control element setpoints and control at least some of the plurality of control elements based on the data describing the control element setpoints.

In some embodiments, the minimum delivery rate comprises a contractual minimum flow rate to which the customer is entitled and the maximum delivery rate is a rate most recently requested or taken by the customer.

In some embodiments, the processor is further configured to calculate the minimum signed flow rate and the maximum signed flow rate by: bisecting an undirected graph representing the gas pipeline network using at least one of the plurality of pipeline segments to create a left subgraph and right subgraph; calculating a minimum undersupply in the left subgraph by subtracting a sum of demand rates for each of the gas receipt facilities in the left subgraph from a sum of minimum production rates for each of the gas production plants in the left subgraph; calculating a minimum unmet demand in the right subgraph by subtracting a sum of maximum production rates for each of the gas production plants in the right subgraph from a sum of demand rates for each of the gas receipt facilities in the right subgraph; calculating the minimum signed flow rate for at least one of the pipeline segments as a maximum of a minimum undersupply in the left subgraph and a minimum unmet demand in the right subgraph; calculating a maximum oversupply in the left subgraph by subtracting the sum of the demand rates for each of the gas receipt facilities in the left subgraph from

the sum of the maximum production rates for each of the gas production plants in the left subgraph; calculating a maximum unmet demand in the right subgraph by subtracting a sum of the minimum production rates for each of the gas production plants in the right subgraph from the sum of the demand rates for each of the gas receipt facilities in the right subgraph; and calculating the maximum signed flow rate for at least one of the pipeline segments as a minimum of a maximum oversupply in the left subgraph and a maximum unmet demand in the right subgraph.

BACKGROUND

Gas pipeline networks have tremendous economic importance. As of September 2016, there were more than 2,700,000 km of natural gas pipelines and more than 4,500 km of hydrogen pipelines worldwide. In the United States in 2015, natural gas delivered by pipeline networks accounted for 29% of total primary energy consumption in the country. Due to the great importance of gas pipelines worldwide, there have been attempts to develop methods for calculating network flow solutions for gas pipeline networks. Some solutions involve formulating the problem as a nonconvex, nonlinear program. However, such methods cannot effectively scale for large gas pipeline networks. Other approaches involve stipulating in advance the direction of the flow in each pipeline segment. This approach has the advantage of reducing the complexity of the optimization problem. However, not allowing for flow reversals severely restricts the practical application. Still other approaches formulate the solution as a mixed-integer linear program. However, constructing efficient mixed-integer linear program formulations is a significant task as certain attributes can significantly reduce the solver effectiveness.

BRIEF DESCRIPTION OF THE DRAWINGS

The foregoing summary, as well as the following detailed description of embodiments of the invention, will be better understood when read in conjunction with the appended drawings of an exemplary embodiment. It should be understood, however, that the invention is not limited to the precise arrangements and instrumentalities shown.

In the drawings:

FIG. 1A illustrates an exemplary gas pipeline network.

FIG. 1B illustrates an exemplary processing unit in accordance with an exemplary embodiment of the present invention.

FIG. 2 shows the typical range of Reynolds numbers and friction factors for gas pipeline networks.

FIG. 3 shows the non-linear relationship between flow and pressure drop.

FIG. 4 represents an example pipeline network for illustrating method for bounding flow rates in pipe segments.

FIG. 5 is a first example illustrating the bisection method for bounding flows in pipes.

FIG. 6 is a second example of the bisection method for bounding flows in pipes.

FIG. 7 is a third example illustrating the network bisection method.

FIG. 8 shows a comparison of the computation times for two different methods for bounding flow in pipe segments.

FIG. 9 depicts a pipeline network which is used to illustrate how pressure prediction errors are calculated for each network node.

FIG. 10 illustrates identifying the maximum error in predicted pressure drop for each pipe segment.

3

FIG. **11** shows propagating pressure prediction errors from the reference node to all other nodes in the network.

FIG. **12** illustrates a flow network.

FIG. **13** shows bounds on the signed flow rate for each pipeline segment for the flow network of illustrated in FIG. **12**.

FIG. **14** illustrates linearizing the pressure drop relationship between the minimum and maximum signed flow rate for each pipe segment.

FIG. **15** shows the directions of flows for the network flow solution for the flow network illustrated in FIG. **12**.

FIG. **16** shows pressures for each node in the pipeline network, as predicted by the linear and nonlinear model for the network flow solution illustrated in FIG. **12**.

FIG. **17** is a diagram showing that the pressure predictions of the tight linear model agree well with those of the nonlinear model, and that lower bounds on pressure for customer nodes are satisfied.

FIG. **18** shows the pressure predictions from a naive linearization for the flow network illustrated in FIG. **12**.

FIG. **19** is a flowchart illustrating a preferred embodiment of a method of the present invention.

DETAILED DESCRIPTION OF THE EXEMPLARY EMBODIMENTS

The invention relates to the control of gas pipeline networks for the production, transmission, and distribution of a gas. Examples of gas pipeline networks include 1) natural gas gathering, transmission, and distribution pipeline networks; 2) pipeline networks for the production, transmission, and distribution of hydrogen, carbon monoxide, or syngas; 3) pipeline networks for the production, transmission, and distribution of an atmospheric gas.

Gas pipeline networks include customers which receive the gas for an end-use. These customers exhibit a certain demand for consumption of the gas. It is desirable for the operator of a gas pipeline network to meet customer demand for the gas. Meeting customer demand for a gas is not a trivial endeavor, especially during periods of peak demand for the gas. In both industrial gas networks and natural gas networks, there are periods when it is necessary to reduce the supply of the gas to some sets of customers below the amount that they have requested. This situation is sometimes known as curtailment. According to the website of a large natural gas marketing company, "A curtailment is the reduction of gas deliveries due to a shortage of supply or because demand for service exceeds a pipeline's capacity. Usually there is a hierarchy of customers, in which some may be required to partially or totally cut back takes of gas before others. Industrial users, for example, are usually curtailed before service to residential users is reduced."

In periods of curtailment, gas suppliers must provide flows of gas to each customer in some range. A lower bound on the flow that is provided to each customer is typically dictated by a contract. An upper bound on the flow that is provided to each customer is typically the most recent amount that they have requested or taken.

One of the primary reasons that it is difficult to control flows to meet customer demand is that there are constraints on pressures within the gas pipeline network. In gas pipeline networks, flow through the network is driven by pressure gradients wherein gas flows from higher pressure regions to lower pressure regions. As a gas travels through a pipeline network, the pressure decreases due to frictional losses. The greater the flow of gas through a particular pipeline segment, the greater the pressure drop through that segment.

4

Gas pipeline networks have certain constraints on the pressure of the gas within the network. These include lower bounds on the pressure of a gas delivered to a customer, and upper bounds on the pressure of a gas flowing through a pipeline. It is desirable for the operator of a gas pipeline network to meet pressure constraints. If upper limits on pressure are not satisfied, vent valves may open to release gas from the network to the atmosphere. If lower bounds on the pressure of gas supplied to a customer are not met, there may be contractual penalties for the operator of the gas pipeline network.

To meet constraints on flows delivered to customers, and pressures within the network, gas pipeline networks include control elements which are operable to regulate pressure and flow. FIG. **1A** illustrates an exemplary hydrogen gas pipeline network. This exemplary network illustrates at least certain of the physical elements that are controlled in accordance with embodiments of the present invention. Flow control elements are operable to receive setpoints for the flow or pressure of gas at a certain location in the network, and use feedback control to approximately meet the setpoint. Thus, control elements include pressure control elements **101** and flow control elements **102***a*, **102***b*.

Industrial gas production plants associated with a gas pipeline network are control elements, because they are operable to regulate the pressure and flow of gas supplied into the network. Examples of industrial gas production plants include steam methane reformer plants **103** for the production of hydrogen, carbon monoxide, and/or syngas; and air separation units for the production of oxygen, nitrogen, and/or argon. These plants typically are equipped with a distributed control system and/or model predictive controller which is operable to regulate the flow of feedgas into the production plant and the flow and/or pressure of product gas supplied to the gas pipeline network.

Natural gas receipt points are control elements, because they include a system of valves and/or compressors to regulate the flow of natural gas into the natural gas pipeline network.

Natural gas delivery points are control elements, because they include a system of valves and/or compressors to regulate the flow of natural gas out of the natural gas pipeline network.

Natural gas compressor stations **104***a*, **104***b* are control elements, because they are operable to increase the pressure and regulate the flow of natural gas within a natural gas pipeline network.

Industrial gas customer receipt points **105** are control elements, because they are operable to receive a setpoint to regulate the flow and/or pressure of an industrial gas delivered to a customer.

In order to operate a gas pipeline network, it is desirable to provide setpoints to flow control elements in such a fashion that customer demand constraints and pressure constraints are satisfied simultaneously. To ensure that setpoints for flow control elements will result in satisfying demand and pressure constraints, it is necessary to calculate simultaneously the flows for each gas pipeline segment and gas pressures at network nodes. As described herein, in an exemplary embodiment, a network flow solution includes numerical values of flows for each pipeline segment and pressures for each pipeline junction that are: 1) self-consistent (in that laws of mass and momentum are satisfied), 2) satisfy customer demand constraints, and 3) satisfy pressure constraints.

The network flow solution may be determined using processing unit **110**, an example of which is illustrated in

FIG. 1B. Processing unit **110** may be a server, or a series or servers, or form part of a server. Processing unit **110** comprises hardware, as described more fully herein, that is used in connection with executing software/computer programming code (i.e., computer readable instructions) to carry out the steps of the methods described herein. Processing unit **110** includes one or more processors **111**. Processor **111** may be any type of processor, including but not limited to a special purpose or a general-purpose digital signal processor. Processor **111** may be connected to a communication infrastructure **116** (for example, a bus or network). Processing unit **110** also includes one or more memories **112**, **113**. Memory **112** may be random access memory (RAM). Memory **113** may include, for example, a hard disk drive and/or a removable storage drive, such as a floppy disk drive, a magnetic tape drive, or an optical disk drive, by way of example. Removable storage drive reads from and/or writes to a removable storage unit (e.g., a floppy disk, magnetic tape, optical disk, by way of example) as will be known to those skilled in the art. As will be understood by those skilled in the art, removable storage unit includes a computer usable storage medium having stored therein computer software and/or data. In alternative implementations, memory **113** may include other similar means for allowing computer programs or other instructions to be loaded into processing unit **110**. Such means may include, for example, a removable storage unit and an interface. Examples of such means may include a removable memory chip (such as an EPROM, or PROM, or flash memory) and associated socket, and other removable storage units and interfaces which allow software and data to be transferred from removable storage unit to processing unit **110**. Alternatively, the program may be executed and/or the data accessed from the removable storage unit, using the processor **111** of the processing unit **110**. Computer system **111** may also include a communication interface **114**. Communication interface **114** allows software and data to be transferred between processing unit **110** and external device(s) **115**. Examples of communication interface **114** may include a modem, a network interface (such as an Ethernet card), and a communication port, by way of example. Software and data transferred via communication interface **114** are in the form of signals, which may be electronic, electromagnetic, optical, or other signals capable of being received by communication interface **114**. These signals are provided to communication interface **114** via a communication path. Communication path carries signals and may be implemented using wire or cable, fiber optics, a phone line, a wireless link, a cellular phone link, a radio frequency link, or any other suitable communication channel, including a combination of the foregoing exemplary channels. The terms "non-transitory computer readable medium", "computer program medium" and "computer usable medium" are used generally to refer to media such as removable storage drive, a hard disk installed in hard disk drive, and non-transitory signals, as described herein. These computer program products are means for providing software to processing unit **110**. However, these terms may also include signals (such as electrical, optical or electromagnetic signals) that embody the computer program disclosed herein. Computer programs are stored in memory **112** and/or memory **113**. Computer programs may also be received via communication interface **114**. Such computer programs, when executed, enable processing unit **110** to implement the present invention as discussed herein and may comprise, for example, model predictive controller software. Accordingly, such computer programs represent controllers of processing

unit **110**. Where the invention is implemented using software, the software may be stored in a computer program product and loaded into processing unit **110** using removable storage drive, hard disk drive, or communication interface **114**, to provide some examples.

External device(s) **115** may comprise one or more controllers that receive setpoint data from the software and are operable to control the network control elements described with reference to FIG. 1A.

It is difficult to calculate a network flow solution for a gas pipeline network because of a nonlinear equation that relates the decrease in pressure of a gas flowing through a pipeline segment (the "pressure drop") to the flow rate of the gas. This nonlinear relationship between flow and pressure drop requires that a nonconvex nonlinear optimization program be solved to calculate a network flow solution. Nonconvex nonlinear programs are known to be NP-complete (see Murty, K. G., & Kabadi, S. N. (1987). Some NP-complete problems in quadratic and nonlinear programming. Mathematical programming, 39(2), 117-129.). The time required to solve an NP-complete problem increases very quickly as the size of the problem grows. Currently, it is not known whether it is even possible to solve a large NP-complete quickly.

It is difficult and time-consuming to solve a large NP-complete program. Also, the nature of the solution of a nonconvex mathematical program typically depends greatly on the way the mathematical program is initialized. As a result of these difficulties in solving a nonconvex mathematical program, it has not been practical to control flows in a gas pipeline to satisfy pressure constraints using network flow solutions produced by nonconvex mathematical programs.

Because of the difficulty of computing network flow solutions, it is not uncommon to have so-called stranded molecules in a gas pipeline network. Stranded molecules are said to exist when there is unmet demand for a gas simultaneous with unused gas production capacity, due to pressure limitations in the network.

Because of the difficulty of computing network flow solutions, flows of gas pipeline segments, and gas pressures in a gas pipeline network, it is not uncommon to vent an industrial gas to the atmosphere when there are flow disturbances in the network.

There exists a need in the art for a reliable and computationally efficient method of computing a network flow solution which can be used to identify setpoints for control elements in a gas pipeline network and, more particularly, a sufficiently accurate linearization of the relationship between flow and pressure drop in pipeline segments that could be used to quickly calculate network flow solutions which could, in turn, be used to identify setpoints for network flow control elements.

The systems and methods of the present invention use information on customer demand ranges and available plant capacity ranges to bound the minimum and maximum flow rate for each pipeline segment in a pipeline network. In an exemplary embodiment, these bounds are computed using a computationally efficient network bisection method which is based on bounding the demand/supply imbalance on either side of a pipe segment of interest. Embodiments of the systems and methods of the present invention find the best linearization of the relationship between flow rate and pressure drop for each pipe segment, given the true nonlinear relationship between flow rate and pressure drop as well as the computed minimum and maximum flow rates for each segment. Then, a linear program may be used to compute a

network flow solution, given the linearization of the relationship between flow rate and pressure drop for each segment. The linear program incorporates prior bounds on the inaccuracy of the pressure drop linearization to ensure that the network flow solution will meet pressure constraints, given the actual nonlinear pressure drop relationship. Finally, certain setpoints for flow control elements are identified from the network flow solution. The setpoints are received by flow control elements to ensure that network pressure constraints are satisfied while also satisfying customer demand constraints.

The following provides the notation used to describe the preferred embodiments of the invention. The first column in the tables below show the mathematical notation, the second column is a description of the mathematical notation, and the third column indicates the units of measure that may be associated with the quantity.

convention, we use the notation (m, n), rather than the notation {m, n}, and (m, n) and (n, m) are considered to be the same arc. If (m, n) is an arc in an undirected graph, it can be said that (m, n) is incident on nodes m and n. The degree of a node in an undirected graph is the number of arcs incident on it.

If (m, n) is an arc in a graph G=(N, A), it can be said that node m is adjacent to node n. The adjacency relation is symmetric for an undirected graph. If m is adjacent to n in a directed graph, it can be written m n.

A path of length k from a node m to a node m' in a graph G=(N, A) is a sequence $<n_0, n_1, n_2, \ldots, n_k>$ of nodes such that $m=n_0$, $m'=n_k$, and $(n_{i-1}, n_i) \in A$ for i=1, 2, ..., k. The length of the path is the number of arcs in the path. The path contains the nodes $n_0, n_1, n_2, \ldots, n_k$ and the arcs (n_0, n_1), (n_1, n_2), ..., n_k). (There is always a 0-length path from m

Sets		
$n \in N$	Nodes (representing pipeline junctions)	
$j \in A$	Arcs (representing pipe segments and control elements)	
G = (N, A)	Graph representing the layout of the gas pipeline network	
$e \in \{in, out\}$	Arc endpoints	
$(n, j) \in A_{in}$	Inlet of arc j intersects node n	
$(n, j) \in A_{out}$	Outlet of arc j intersects node n	
$n \in D \subset N$	Demand nodes	
$n \in S \subset N$	Supply nodes	
$j \in P \subset A$	Pipe arcs	
$j \in C \subset A$	Control element arcs	
$L_j \in N$	Left subgraph for arc j	
$R_j \in N$	Right subgraph for arc j	
Parameters		
D_j	Diameter of pipe j	[m]
R	Gas constant	[N m kmol^{-1} K^{-2}]
Z	Compressibility factor	[no units]
L_j	Length of pipe j	[m]
M_W	Molecular weight of the gas	[kg kmol^{-1}]
T_{ref}	Reference temperature	[K]
ϵ	Pipe roughness	[m]
α	Nonlinear pressure drop coefficient	[Pa kg^{-1} m^{-1}]
f_j	Friction factor for pipe j	[no units]
μ	Gas viscosity	[Pa s]
Re_j	Reynolds number for flow in pipe j	[no units]
q_j^{min}	Minimum flow rate for flow in pipe j	[kg/s]
q_j^{max}	Maximum flow rate for flow in pipe j	[kg/s]
b_j	Intercept for linear pressure drop model for pipe j	[Pa^2]
m_j	Slope for linear pressure drop model for pipe j	[Pa^2 s/kg]
d_n^{min}	Minimum demand in node n	[kg/s]
d_n^{min}	Maximum demand in node n	[kg/s]
s_n^{min}	Minimum production in node n	[kg/s]
s_n^{min}	Maximum production in node n	[kg/s]
Variables		
d_n	Demand supplied rate in node n	[kg/s]
q_j	Flow rate in pipe j	[kg/s]
s_n	Production rate in node n	[kg/s]
p_n^{node}	Pressure at node n	[Pa]
p_j^e	Pressure at a particular end of a particular pipe	[Pa]
ps_n^{node}	Squared pressure at node n	[Pa^2]
ps_j^e	Squared pressure at a particular end of a particular pipe	[Pa^2]
ps_j^{err}	Maximum absolute squared pressure drop error for pipe j	[Pa^2]
ps_n^{err}	Maximum absolute squared pressure error for node n	[Pa^2]

For the purposes of computing a network flow solution, the layout of the pipeline network is represented by an undirected graph with a set of nodes (representing pipeline junctions) and arcs (representing pipeline segments and certain types of control elements). The following provides some basic terminology associated with undirected graphs.

An undirected graph G=(N, A) is a set of nodes N and arcs A. The arc set A consists of unordered pairs of nodes. That is, an arc is a set {m, n}, where m, n $\in$ N and m≠n. By

to m). If there is path p from m to m', we say that m' is reachable from m via p. A path is simple if all nodes in path are distinct.

A subpath of path $p=<n_0, n_1, n_2, \ldots, n_k>$ is a contiguous subsequence of its nodes. That is, for any 0≤i≤j≤k, the subsequence of nodes $<n_i, n_{i+1}, \ldots, n_j>$ is a subpath of p.

In an undirected graph, a path $<n_0, n_1, n_2, \ldots, n_k>$ forms a cycle if k≥3, $n_0=n_k$, and $n_1, n_2, \ldots, n_k$ are distinct. A graph with no cycles is acyclic.

9

10

An undirected graph is connected if every pair of nodes is connected by a path. The connected components of a graph are the equivalence classes of nodes under the "is reachable from" relation. An undirected graph is connected if it has exactly one connected component, that is, if every node is reachable from every other node.

A graph G'=(N', A') is a subgraph of G=(N, A) if N'⊆N and A'⊆A. Given a set N'⊆N, the subgraph of G induced by N' is the graph G'=(N', A'), where A'={(m, n) ∈A: m, n ∈N'}.

To establish a sign convention for flow in a gas pipeline network represented by an undirected graph, it is necessary to designate one end of each pipe arc as an "inlet" and the other end as an "outlet":

$(n,j) \in A_{in}$ Inlet of arc j intersects node n

$(n,j) \in A_{out}$ Outlet of arc j intersects node n

This assignment can be done arbitrarily, as embodiments of the present invention allows for flow to travel in either direction. By convention, a flow has a positive sign if the gas is flowing from the "inlet" to the "outlet", and the flow has a negative sign if the gas is flowing from the "outlet" to the "inlet".

Some nodes in a network are associated with a supply for the gas and/or a demand for the gas. Nodes associated with the supply of a gas could correspond to steam methane reformers in a hydrogen network; air separation units in an atmospheric gas network; or gas wells or delivery points in a natural gas network. Nodes associated with a demand for the gas could correspond to refineries in a hydrogen network; factories in an atmospheric gas network; or receipt points in a natural gas network.

A set of mathematical equations govern flows and pressures within a gas pipeline network. These equations derive from basic physical principles of the conservation of mass and momentum. The mathematical constraints associated with a network flow solution are described below.

Node Mass Balance

The node mass balance stipulates that the total mass flow leaving a particular node is equal to the total mass flow entering that node.

$$d_n + \sum_{j|(n,j)\in A_{in}} q_j = \sum_{j|(n,j)\in A_{out}} q_j + s_n$$

The left-hand side of the equation represents the flow leaving a node, as d_n is the customer demand associated with the node. The term $\sum_{j|(n,j)\in A_{in}} q_j$ represents the flow associated with pipes whose "inlet" side is connected to the node. If the flow q_j is positive, then it represents a flow leaving the node. The right-hand side of the equation represents the flow entering a node, as s_n is the plant supply associated with the node. The term $\sum_{j|(n,j)\in A_{out}} q_j$ represents the flow associated with pipe segments whose "outlet" side is connected to the node. If the flow term q_j is positive, then it represents a flow entering the node.

Node Pressure Continuity

The node pressure continuity equations require that the pressure at the pipe ends which is connected to a node should be the same as the pressure of the node.

$$p_j^{in} = p_n^{node} \forall (n, j) \in A_{in}$$

$$p_j^{out} = p_n^{node} \forall (n, j) \in A_{out}$$

Pipe Pressure Drop

The relationship between the flow of a gas in the pipe is nonlinear. A commonly used equation representing the nonlinear pressure drop relationship for gas pipelines is presented here. Other nonlinear relationships may be used in connection with alternative embodiments of the present invention.

This nonlinear pressure drop equation for gases in cylindrical pipelines is derived based on two assumptions. First, it is assumed that the gas in the pipeline network is isothermal (the same temperature throughout). This is a reasonable assumption because pipelines are often buried underground and there is excellent heat transfer between the pipeline and the ground. Under the isothermal assumption, an energy balance on the gas in the pipeline yields the following equation:

$$(p_j^{in})^2 - (p_j^{out})^2 = q_j|q_j|\frac{4ZRT}{M_w\pi^2 D_j^4}\left[\frac{4f_j L_j}{D_j} + 2\ln\left(\frac{p_j^{in}}{p_j^{out}}\right)\right]$$

For gas pipelines, because the pipe lengths are large relative to the diameters, the term

$$\frac{4f_j L_j}{D}$$

is so much greater than the term

$$2\ln\left(\frac{p_j^{in}}{p_j^{out}}\right)$$

that the latter term can be neglected. Under this assumption, then the nonlinear pressure drop relationship reduces to:

$$(p_j^{in})^2 - (p_j^{out})^2 = \alpha q_j|q_j|$$

with

$$\alpha = \frac{16ZRf_j T_{ref} L_j}{M_w\pi^2 D_j^5}$$

where Z is the compressibility factor for the gas, which in most pipelines can be assumed to be a constant near 1; R is the universal gas constant; T_{ref} is the reference temperature; L_j is the length of the pipeline segment; and the term f_j^e is a friction factor for a pipe segment, which varies weakly based on the Reynolds number of flow in the pipe, and for most gas pipelines is in the range 0.01-0.08. Below an explicit formula is provided for the friction factor in terms of the Reynold's number. The dimensionless Reynold's number is defined as

$$Re_j = \frac{4|q_j|}{\pi D_j \mu},$$

where μ is the gas viscosity.

If the flow is laminar ($Re_j^e < 2100$) then the friction factor is

$$f_{j,L} = \frac{64}{Re_j}$$

If the flow is turbulent ($Re_j^e > 4000$), then the friction factor may be determined using the implicit Colebrook and White equation:

$$\frac{1}{\sqrt{f_{j,TR}}} = -2\log_{10}\left(\frac{\epsilon}{3.71D} + \frac{2.51}{Re_j\sqrt{f_j}}\right).$$

An explicit expression for the friction factor for turbulent flow that is equivalent to the Colebrook and White equation is

$$f_{j,TR} = \frac{1}{[c[W_0(e^{a/bc}/bc)] - a/b]^2}$$

where

$$a = \frac{\epsilon}{3.71D}, \; b = \frac{2.51}{Re}, \; \text{and} \; c = \frac{2}{\ln(10)} = 0.868589$$

and $W_0(\cdot)$ is the principal Lambert-W function. See (More, A. A. (2006). Analytical solutions for the Colebrook and White equation and for pressure drop in ideal gas flow in pipes. Chemical engineering science, 61(16), 5515-5519) and (Brkic, D. (2009). Lambert W-function in hydraulics problems. In MASSEE International Congress on Mathematics MICOM, Ohrid.)

When the Reynolds number is between 2100 and 4000, the flow is in a transition range between laminar and turbulent flow and the accepted approach in the literature is to interpolate the friction factor between the laminar and the turbulent value, based on the Reynolds number, as follows:

$$f_{j,TS} = f_{j,L|2100}\beta + f_{j,TF|4000}(1 - \beta)$$

with $\beta = (4000 - Re_j)/(4000 - 2100)$.

Typical Design Parameters for Gas Pipeline Networks

Mainline natural transmission pipes are usually between 16 and 48 inches in diameter. Lateral pipelines, which deliver natural gas to or from the mainline, are typically between 6 and 16 inches in diameter. Most major interstate pipelines are between 24 and 36 inches in diameter. The actual pipeline itself, commonly called 'line pipe', consists of a strong carbon steel material, with a typical roughness of 0.00015 feet. Thus, the relative roughness for natural gas transmission pipelines is typically in the range 0.00005 to 0.0003 and the friction factor is in the range 0.01 to 0.05 under turbulent flow conditions.

Hydrogen distribution pipelines typically have a diameter in the range 0.3-1.2 feet, and a typical roughness of 0.00016 feet. Thus, the relative roughness for hydrogen transmission pipelines is typically in the range 0.0001 to 0.0005 and the friction factor is in the range 0.012 to 0.05 under turbulent flow conditions.

For gas pipeline networks, a typical design Reynold's number is 400,000. FIG. 2 shows the typical range of Reynold's numbers and the associated friction factors for gas pipeline networks.

Establishing Bounds on the Flows in Pipe Segments

A key enabler for the efficient computation of network flow solutions is the linearization of the nonlinear pressure drop relationship. To produce an accurate linearization of the pressure drop relationship for pipe segments, it is critical to bound the range of flow rates for each pipe segment. In examples below, a linearization based on tightly bounded flow rates is referred to as a "tight linearization".

FIG. 3 illustrates the nonlinear relationship between pressure drop and flow. The true nonlinear relationship is indicated by the solid line. If one approximates the true nonlinear relationship with a linear fit centered around zero, the linear fit severely underestimates the pressure drop for flow magnitudes exceeding 20. If one does a linear fit of the true pressure drop relationship in the range of flows between 15 and 20, the quality of the pressure drop estimate for negative flows is very poor. If one does a linear fit of the true pressure drop relationship in the range between −20 and −15 MMSCFD, the pressure drop estimate for positive flows is very poor.

Bounds on flow rates can be determined using mass balances and bounds on production for plants and demand for customers, even in the absence of any assumptions about pressure constraints and pressure drop relationships.

One method for bounding flows in pipeline segments based on mass balances is to formulate and solve a number of linear programs. For each pipe segment, one linear program can be used to determine the minimum flow rate in that segment and another linear program can be used to determine the maximum flow rate in that segment.

An exemplary embodiment of the present invention involves a method of bounding the flow rate in pipeline segments that is simple and computationally more efficient than the linear programming method.

For the pipe segment of interest (assumed to not be in a graph cycle), the pipeline network is bisected into two subgraphs at the pipe segment of interest: a "left" subgraph and a "right" subgraph associated with that pipe. Formally, the left subgraph L_j associated with pipe j is the set of nodes and arcs that are connected with the inlet node of pipe j once the arc representing pipe j is removed from the network. Formally, the right subgraph R_j associated with pipe j is the set of nodes and arcs that are connected with the outlet node of pipe j once the arc representing pipe j is removed from the network. Given the bisection of the flow network into a left subgraph and a right subgraph, it is then possible to calculate the minimum and maximum signed flow through pipe segment j, based on potential extremes in supply and demand imbalance in the left subgraph and the right subgraph.

To bound the flow rate in each pipeline segment, some quantities describing the imbalance between supply and demand are defined in the left and right subgraphs. The minimum undersupply in the left subgraph for pipe j is defined as $s_{L_j}^{min} = (\Sigma_{n \in L} s_n^{min}) - (\Sigma_{n \in L} d_n^{max})$. The minimum unmet demand in the right subgraph for pipe j is defined as $d_{R_j}^{min} = (\Sigma_{n \in R} d_n^{min}) - (\Sigma_{n \in R} s_m^{max})$. The maximum oversupply in the left subgraph for pipe j is defined as $s_L^{max} = (\Sigma_{n \in L} s_n^{max}) - (\Sigma_{n \in L} d_n^{min})$. The maximum unmet demand in the right subgraph for pipe j is defined as $d_R^{max} = (\Sigma_{n \in R} d_n^{max}) - (\Sigma_{n \in R} s_n^{min})$.

Given the definitions above, the minimum and maximum feasible signed flow in the pipe segment are given by:

$$q_j^{min} = \max\{s_{L_j}^{min}, d_{R_j}^{min}\},$$

$$q_j^{max} = \min\{s_{L_j}^{max}, d_{R_j}^{max}\}.$$

The equation for q_j^{min} indicates that this minimum (or most negative) rate is the maximum of the minimum undersupply in the left subgraph and the minimum unmet demand in the right subgraph. The equation for q_j^{max} indicates that this maximum (or most positive) rate is the minimum of the maximum oversupply in the left subgraph and the maximum unmet demand in the right subgraph.

The equations in the previous paragraph for calculating q_j^{min} and q_j^{max} can be derived from the node mass balance relationship, as follows. The node mass balance relationship, which was previously introduced, is

$$d_n + \sum_{j|(n,j)\in A_{in}} q_j = \sum_{j|(n,j)\in A_{out}} q_j + s_n.$$

Consider the left subgraph associated with pipe j. The left subgraph contains the node connected to the inlet of The method of claim 1, where the error in pressure prediction for each network node is bounded and the error bounds are used to ensure that the network flow solution produced using the linearized pressure drop models is sufficiently conservative to satisfy pressure constraints when the nonlinear pressure drop model is used.

Consider collapsing the entire left subgraph into the single node connected to the inlet of pipe j. Then,

$$q_j^{in} = \sum_{n \in L_j} s_n - d_n$$

An upper bound for the inlet flow is $q_j^{in} \leq \sum_{n \in L_j} s_n^{max} - d_n^{min}$, and a lower bound for the inlet flow is $q_j^{in} \geq \sum_{n \in L_j} s_n^{min} - d_n^{max}$. Similarly, an upper bound for the outlet flow is $q_j^{out} \leq \sum_{n \in R_j} d_n^{max} - s_n^{min}$ and a lower bound is $q_j^{out} \geq \sum_{n \in R_j} d_n^{min} - s_n^{max}$.

At steady state, the pipe inlet flow equals the outlet flow and

$$\sum_{n \in L_j} s_n^{min} - d_n^{max} \leq \sum_{n \in R_j} d_n^{min} - s_n^{max} \leq q_j^{in} =$$

$$q_j^{out} = q_j \leq \sum_{n \in R_j} d_n^{max} - s_n^{min} \leq \sum_{n \in L_j} s_n^{max} - d_n^{min}.$$

Equivalently,

$$\max\left\{\sum_{n \in L_j} s_n^{min} - d_n^{max}, \sum_{n \in R_j} d_n^{min} - s_n^{max}\right\} \leq q_j^{in} =$$

$$q_j^{out} = q_j \leq \min\left\{\sum_{n \in R_j} d_n^{max} - s_n^{min}, \sum_{n \in L_j} s_n^{max} - d_n^{min}\right\}$$

or

$$q_j^{min} = \max\{s_{L_j}^{min}, d_{R_j}^{min}\} \leq = q_j \leq \min\{s_{L_j}^{max}, d_{R_j}^{max}\} = q_j^{max},$$

which completes the proof.

The bisection method for bounding flow rates in pipe segments is illustrated with an example. An example flow network is depicted in FIG. 4. This flow network has four customer demand nodes (nodes 1, 9, 12, and 16), and four plant supply nodes (nodes 2, 10, 13, and 17).

FIG. 5 illustrates how the bisection method can be used to bound the flow rate in the pipe segment connecting node 1 with node 5. Recall that the sign convention for flow rates is that a flow is negative if it is in the direction going from a lower-numbered node to a higher-numbered node. In this case, the minimum and maximum flow rate are −6 and −11 kg/s, which are consistent with the bounds on the flow rate provided to the customer at node 1.

FIG. 6 shows how the network bisection method is used to bound the flow rate in the pipe segment going from node 10 to node 11. In this case, the range of flows is between 7 and 12 kg/s, which is consistent with flow of the gas from the production plant at node 10 to the rest of the network. This range is consistent with the minimum and maximum production rate of the plant.

While simplistic for illustration purposes, the results of these examples validate the correctness of the network bisection method for bounding the flow rates in pipes. The next example, presented in FIG. 7, is a more complex example of using the network bisection method to bound the flow rate in the pipe leading from node 3 to node 15. In this case, the flow can vary from −6 kg/s (a flow going from node 15 to node 3) to 4 kg/s (a flow going from node 3 to node 15).

FIG. 8, which shows data from computational experiments performed using Matlab on a computer with an Intel Core I 2.80 GHz processor, shows that the network bisection method for bounding the flow in pipeline segments is between 10 and 100 times faster than the linear programming method.

Finding a Linear Pressure-Drop Model

A further step in the method of the exemplary embodiments of the invention involves linearizing the nonlinear pressure drop relationship for each pipe, based on the flow bounds established for each pipe. This can be done analytically (if the bounded flow range is narrow enough that the friction factor can be assumed to be constant over the flow range), or numerically (if the bounded flow range is sufficiently wide that the friction factor varies significantly over the flow range). Below is described how a linearization can be accomplished either analytically or numerically. What is sought is a linear pressure drop model of the form

$$ps_j^{in} - ps_j^{out} = m_j q_j + b_j \forall j \in P.$$

Bounding the flow range is critical to produce a good linear model. Without these bounds, a naïve linear model may be produced which is based on linearizing the nonlinear relationship about zero with a minimum and maximum flow magnitude equal to the total network demand. As will be shown in examples below, this generally does not produce good network flow solutions.

Finding the Least-Squares Linear Pressure-Drop Model Analytically: Slope-Intercept Form

If the bounded flow range is fairly narrow, then the friction factor as well as the nonlinear pressure drop coefficient α will be nearly constant and an analytical solution may be found for the least squares linear fit of the nonlinear pressure drop relationship.

Least squares solution for a linear model with $g=q_j^{min}$ and $h=q_j^{max}$

$$(m_j^*, b_j^*) = \underset{m,b}{\mathrm{argmin}} \int_g^h (\alpha q|q| - mq - b)^2 dq$$

Evaluating the definite integral:

$$\int_g^h (\alpha q|q| - mq - b)^2 dq = b^2 h - b^2 g -$$

$$g^3 \left(\frac{m^2}{3} - \frac{2\alpha b \,\mathrm{sign}(g)}{3} \right) + h^3 \left(\frac{m^2}{3} - \frac{2\alpha b \,\mathrm{sign}(g)}{3} \right) - \frac{\alpha^2 g^5 \mathrm{sign}(g)^2}{5} +$$

$$\frac{\alpha^2 h^5 \mathrm{sign}(h)^2}{5} - bg^2 m + bh^2 m + \frac{\alpha g^4 m \,\mathrm{sign}(g)}{2} - \frac{\alpha h^4 m \,\mathrm{sign}(h)}{2}$$

This quantity is minimized when the partial derivatives with respect to b and m are simultaneously zero. These partial derivatives are

$$\frac{\partial \int_g^h (\alpha q|q| - mq - b)^2 dq}{\partial b} =$$

$$2bh - 2bg - g^2 m + h^2 m + \frac{2ag^3 \mathrm{sign}(g)}{3} - \frac{2ah^3 \mathrm{sign}(h)}{3}$$

$$\frac{\partial \int_g^h (\alpha q|q| - mq - b)^2 dq}{\partial m} =$$

$$bh^2 - bg^2 - \frac{2g^2 m}{3} + \frac{2h^2 m}{3} + \frac{ag^4 \mathrm{sign}(g)}{2} - \frac{ah^4 \mathrm{sign}(h)}{2}$$

Setting the partial derivatives equal to zero, and solving for b and m, the form of the slope-intercept least squares linear model is:

$$b^* = - \frac{\begin{array}{c} (\alpha g^5 \mathrm{sign}(g) - \alpha h^5 \mathrm{sign}(h) - 8\alpha g^3 h^2 \mathrm{sign}(g) + 8\alpha g^2 h^3 \mathrm{sign}(h) + \\ \alpha g^4 h \,\mathrm{sign}(g) - \alpha g h^4 \mathrm{sign}(h)) \end{array}}{(6(g-h)(g^2 - 2gh + h^2))}$$

$$m = \frac{(\alpha g^4 \mathrm{sign}(g) - \alpha h^4 \mathrm{sign}(h) - 2\alpha g^3 h \,\mathrm{sign}(g) + 2\alpha g h^3 \mathrm{sign}(h))}{(g^3 - 3g^2 h + 3gh^2 - h^3)}$$

Finding the Least Squares Model Empirically: Slope-Intercept Model

If the bounded flow range for a pipe segments spans more than a factor of two, then the friction factor may vary significantly over that flow range and there is no analytical expression for the least-squares linear fit of the nonlinear pressure drop relationship. In this case, one exemplary preferred approach for developing a least-squares linear fit of the nonlinear pressure drop is a numerical approach.

This approach entails using numerical linear algebra to calculate the value of the slope and intercept using the formula.

$$\begin{bmatrix} m \\ b \end{bmatrix} = (Q^T Q)^{-1} Q^T y$$

where m is the slope of the line, b is the intercept of the line, Q is a matrix the first column of the matrix Q contains a

vector of flow rates ranging from the minimum signed flow rate for the segment to the maximum signed flow rate for the segment, and the second column is a vector of ones.

$$Q = \begin{bmatrix} q_{min} & 1 \\ \vdots & \vdots \\ q_{max} & 1 \end{bmatrix}$$

The vector y contains the pressure drop as calculated by the nonlinear pressure drop relationship, at flow rates ranging from the minimum signed flow rate to the maximum signed flow rate. Since the friction factor varies over this flow range, a different value of the nonlinear pressure drop relationship a may be associated with each row of the vector.

$$y = \begin{bmatrix} \alpha_{min} q_{min} |q_{min}| \\ \vdots \\ \alpha_{max} q_{max} |q_{max}| \end{bmatrix}$$

As an example, consider the following data from a nonlinear pressure drop model:

Flow, kg/s	Change in squared pressure, Pa^2
2.0	7.7
3.0	12.1
4.0	17.9
5.0	25.3
6.0	34.1
7.0	44.3

Given this data, q_{min}=2.0, q_{max}=7.0,

$$Q = \begin{bmatrix} 2.0 & 1 \\ 3.0 & 1 \\ 4.0 & 1 \\ 5.0 & 1 \\ 6.0 & 1 \\ 7.0 & 1 \end{bmatrix},$$

and

$$y = \begin{bmatrix} 7.7 \\ 12.1 \\ 17.9 \\ 25.3 \\ 34.1 \\ 44.3. \end{bmatrix}.$$

Applying the formula

$$\begin{bmatrix} m \\ b \end{bmatrix} = (Q^T Q)^{-1} Q^T y,$$

we determine that the parameters of the least-squares linear fit are m=7.33 and b=−9.40.

Finding the Least Squares Model Numerically: A Slope Only Model

In some instances, if the flow range includes transition turbulent flow, includes laminar flow, or includes both turbulent and laminar flow regimes, there is no analytical expression for the least-squares linear fit of the nonlinear pressure drop relationship. In this case, the preferred approach for developing a least-squares linear fit of the nonlinear pressure drop is a numerical approach.

This approach involves calculating the value of the

$$m = (q^T q)^{-1} q^T y$$

where m is the slope of the line, q is a vector of flow rate values ranging from the minimum signed flow rate for the segment to the maximum signed flow rate for the segment

$$q = \begin{bmatrix} q_{min} \\ \vdots \\ q_{max} \end{bmatrix}$$

The vector y contains the pressure drop as calculated by the nonlinear pressure drop relationship, at flow rates ranging from the minimum signed flow rate to the maximum signed flow rate. Since the friction factor varies over this flow range, a different value of the nonlinear pressure drop relationship a may be associated with each row of the vector

$$y = \begin{bmatrix} \alpha_{min} q_{min} |q_{min}| \\ \vdots \\ \alpha_{max} q_{max} |q_{max}| \end{bmatrix}.$$

As an example, consider the following data from a nonlinear pressure drop model:

Flow, kg/s	Change in squared pressure, Pa²
−3.0	−24.2
−2.0	−7.5
−1.0	−1.0
0.0	0.0
1.0	1.0
2.0	7.5

Given this data, q_{min}=2.0, q_{max}=7.0,

$$q = \begin{bmatrix} -3.0 \\ -2.0 \\ -1.0 \\ 0.0 \\ 1.0 \\ 2.0 \end{bmatrix},$$

and

$$y = \begin{bmatrix} -24.2 \\ -7.5 \\ -1.0 \\ 0.0 \\ 1.0 \\ 7.5 \end{bmatrix}.$$

Applying the formula $m = (q^T q)^{-1} q^T y$, it is determined that the parameter of the least-squares linear fit is m=5.51.

Choosing the Most Appropriate Linear Model

Above described are several methods for calculating the best linear fit of the nonlinear pressure drop relationship, given the minimum and maximum flow rates. Also, described is how to find the best slope-only linear model, given the minimum and maximum flow rates. An open question is in which situations it is appropriate to use the slope/intercept model, and in which situations it is best to use the slope-only model. A key principle here is that the linear model should always give the correct sign for the pressure drop. In other words, for any linear model exercised over a bounded flow range, the sign of the predicted pressure drop should be consistent with the flow direction. Pressure should decrease in the direction of the flow. Note that the slope-only model has an intercept of zero, and thus the slope-only model will show sign-consistency regardless of the flow range. So, a slope-intercept model should be used unless there is a point in the allowable flow range where there would be a sign inconsistency; if a slope-intercept model would create a sign-inconsistency, then the slope-only model should be used.

Identifying the Nonlinear Pressure Drop Coefficient from Experimental Data

The methods described above for creating a linearization of the nonlinear pressure drop relationship rely on knowledge of the nonlinear pressure drop parameter α.

In some cases, the nonlinear pressure drop coefficient α may be calculated directly using the formula

$$\alpha = \frac{16 Z R f_j T_{ref} L_j}{M_w \pi^2 D_j^5}$$

if the length of the pipe segment, the diameter of the pipe segment, the friction factor, and the gas temperature are known. In other cases, these quantities may not be known with sufficient accuracy. In such situations, α can still be estimated if historical data on flow rates and pressure drops for the pipe are available.

If historical data on flow rates and pressure drops for a pipe are available, with a minimum signed flow rate of q_{min}=g and a maximum signed flow rate of q_{max}=h, then the first step in estimating a is to fit a line to the data $(p_j^{in})^2 - (p_j^{out})^2$ as a function of the flow rate q. The line of best slope is parameterized by a calculated slope m and intercept b.

Given a linear fit for data in slope-intercept form over a given flow range, it is now shown how to recover a least-squares estimate of the nonlinear pressure drop parameter α. The best estimate α^*, given the flow range (g, h), the best slope estimate m, and the best intercept estimate b satisfies the least squares relationship

$$\alpha^* = \underset{\alpha}{\mathrm{argmin}} \int_g^h (\alpha q |q| - mq - b)^2 dq$$

It can be shown that an equivalent expression for α^* is as a function of the flow range (g, h), the best slope estimate m, and the best intercept estimate b is

$$\alpha^* = \frac{\begin{array}{c}20bg^3\text{sign}(g) - 20bh^3\text{sign}(h) + \\ 15g^4 m\text{sign}(g) - 15h^4 m\text{sign}(h)\end{array}}{12g^5\text{sign}(g)^2 - 12h^5\text{sign}(h)^2}$$

which is the formula that can be used to estimate α given historical data of pressure drop over a flow range.

Bounding the Error in the Linearized Pressure Predictions for the Pipeline Network

Above, a method is described for how to linearize the pressure drop relationship for each pipe in the network by first bounding the range of flow rates which will be encountered in each pipe segment. In accordance with exemplary embodiments of the present invention, the linearized pressure drop models are used to calculate a network flow solution. Although the linearized pressure drop models fit the nonlinear models as well as possible, there will still be some error in the pressure estimates in the network flow solution relative to the pressures that would actually exist in the network given the flows from the network flow solution and the true nonlinear pressure drop relationships. To accommodate this error while still ensuring that pressure constraints are satisfied by the network flow solution, it is necessary to bound the error in the linearized pressure prediction at each node in the network.

To bound the error in the pressure prediction at each node in the network, the error in the prediction of the pressure drop for each arc is bound. For pipe arcs, this is done by finding the maximum absolute difference between the linear pressure drop model and the nonlinear pressure drop model in the bounded range of flows for the pipe segment. By definition,

$$ps_j^{err} = \max_{q_j^{min} \le q \le q_j^{max}} |\alpha_j q|q| - m_j^* q - b_j^*| \forall\ j \in P.$$

For control arcs, the maximum error in the prediction of the change in pressure associated with the arc depends on the type of arc. Some control elements, such as valves in parallel with variable speed compressors, have the capability to arbitrarily change the pressure and flow of the fluid within certain ranges, and for these there is no error in the pressure prediction. Other types of control elements, such as nonlinear valves, may be represented by a linear relationship between pressure drop and flow based on the set valve position. For these, there may be a potential linearization error similar to that for pipes. In what follows, it is assumed without loss of generality that $ps_j^{err} = 0 \forall j \in C$.

Next, a known reference node r in the network is identified. This is a node where the pressure is known with some bounded error. Typically, the reference node is a node which is incident from a pressure control element arc. The maximum absolute pressure error for the reference value can be set to zero, or it can be set to some small value associated with the pressure tracking error associated with the pressure control element.

To compute the error associated with nodes in the network other than the reference node, the undirected graph representing the pipeline network is converted to a weighted graph, where the weight associated with each pipeline arc is

the maximum absolute pressure error for the pipe segment. The shortest path is then found, in the weighted graph, between the reference node and any other target node.

In a shortest-path problem, a weighted, directed graph G=(N, A), with weight function w: A→R mapping arcs to real-valued weights is used. The weight of path p= $\langle n_0, n_1, \ldots, n_k \rangle$ is the sum of the weights of its constituent arcs:

$$w(p) = \sum_{i=1}^{k} w(n_{i-1}, n_i).$$

The shortest-path weight from n to m is defined by

$$\delta(m, n) = \begin{cases} \min\{w(p): m \xrightarrow{p} n\} & \text{if there is a path from } m \text{ to } n \\ \infty & \text{otherwise.} \end{cases},$$

A shortest-path from node m to node n is then defined as any path p with weight $w(p)=\delta(m, n)$.

In the weighted graph used here, the weight function is the maximum absolute pressure prediction error associated with the pipe segment connecting the two nodes. To compute the shortest-path weight $\delta(m, n)$, an implementation of Dijkstra's algorithm can be used (see Ahuja, R. K., Magnanti, T. L., & Orlin, J. B. (1993). Network flows: theory, algorithms, and applications.) The maximum pressure error for the target node is the maximum pressure error for the reference node plus the shortest path distance between the reference node and the target node. In mathematical notation,

$$ps_m^{err} = ps_r^{err} + \sigma(r, m)$$

where the weight function for the shortest path is $w_j = ps_j^{err}$.

If a pipeline network has more than one pressure reference node $r_1 \ldots, r_n$, then one calculates the shortest path between each reference node and every other reference node. The pressure error is then bounded by the minimum of the quantity $ps_r^{err} + \delta(r, m)$ over all reference nodes:

$$ps_m^{err} = \min_{r \in \{r_1, \ldots, r_n\}} \{ps_r^{err} + \sigma(r, m)\}.$$

FIG. **9** is an unsigned graph representing a gas pipeline network which is used for the purpose of illustrating how to bound the error associated with linearized pressure drop models. Double circle nodes represent production plants, square nodes represent customers, and single circle nodes represent pipeline junctions. The arcs connecting the nodes are labeled. In this example, the network bisection method is used to bound the flow rate in each pipe segment, and then a least-squares linear model is fitted to the nonlinear pressure drop relationship. The nonlinear pressure drop relationship for each pipe (a solid line), along with the least squares linear fit for each pipe is shown in plots (as FIG. **10**) for each of the pipe segments. FIG. **10** also graphically depicts the maximum squared pressure drop error between the linear and nonlinear relationship.

FIG. **11** shows the results of the application of Dijkstra's method to calculate the maximum pressure prediction error for each of the pipeline nodes, given the bounded error for each of the pipe arcs.

Calculating a Network Flow Solution

Above it is described how to 1) bound the minimum and maximum flow rate for each pipe segment in a computationally efficient fashion; 2) compute an accurate linear approximation of the nonlinear pressure drop relationship given the bounded flow range; 3) bound the pressure prediction error associated with the linear approximation. Next described is how to calculate a network flow solution, that is, to determine values of pressures for pipeline junctions and flows for pipeline segments which 1) satisfy constraints associated with the conservation of mass and momentum; 2) are consistent with bounds on the flow delivered to each customer, 3) satisfy pipeline pressure constraints with appropriate margin to accommodate errors associated with the linearization of the nonlinear pressure drop relationship. The governing equations are summarized here.

Node Mass Balance

The node mass balance stipulates that the total mass flow leaving a particular node is equal to the total mass flow entering that node.

$$d_n + \sum_{j|(n,j)\in A_{in}} q_j = \sum_{j|(n,j)\in A_{out}} q_j + s_n$$

Node Pressure Continuity

The node pressure continuity equations require that the pressure of all pipes connected to a node should be the same as the pressure of the node.

$$p_j^{in} = p_n^{node} \forall\, (n,\, j) \in A_{in}$$
$$p_j^{out} = p_n^{node} \forall\, (n,\, j) \in A_{out}$$

Linearized Pressure Drop Mode

It is shown how to develop a linear pressure drop model of the form

$$ps_j^{in} - ps_j^{out} = m_j q_j + b_j.$$

Pressure Constraints at Nodes

At nodes in the pipeline network, there are minimum and maximum pressure constraints. These constraints must be satisfied with sufficient margin, namely ps_n^{err}, to allow for potential inaccuracy associated with the linearized pressure drop relationships:

$$ps_n^{min} + ps_n^{err} \leq ps_n^{node} \leq ps_n^{max} - ps_n^{err}, \forall n \in N.$$

This ensures that the pressures constraints will be satisfied even when the nonlinear pressure drop model is used to calculate network pressures based on the flow values associated with the network flow solution. Above, it is shown how to compute ps_n^{err} using Dijkstra's algorithm for a certain weighted graph.

Production Constraints

This constraint specifies the minimum and maximum production rate for each of the plants.

$$s_n^{min} < s_n < s_n^{max}$$

Finally, the following linear program can be formulated to find a network flow solution:

Given

d_n $\forall n \in N$	Demand rate in node n
(m_j, b_j) $\forall j \in P$	Linearized pressure drop model for pipe j
ps_n^{err} $\forall\, n \in N$	Maximum squared pressure error for node n, given linearized pressure drop models
$s_n^{min} < s_n < s_n^{max}$	Minimum and maximum production rates at node n

CALCULATE

q_j $\forall\, j \in A$	Flow rate in arcs
s_n $\forall\, n \in S$	Production rate in supply node
d_n $\forall\, n \in D$	Rate supplied to demand node
ps_n^{node} $\forall\, n \in N$	Squared pressure at each node
ps_j^e $\forall\, j \in A$	Squared pressure at the ends of each arc

SUCH THAT

$d_n + \Sigma_{j|(n,j)\in A_{in}}q_j =$ $\Sigma_{j|(n,j)\in A_{out}}q_j + s_n$ $\forall\, n \in N$	Node mass balance
$ps_j^{in} = ps_n^{node}$ $\forall(n, j) \in A_{in}$	Node pressure equality constraints
$ps_j^{out} = ps_n^{node}$ $\forall(n, j) \in A_{out}$	Node pressure equality constraints
$ps_j^{in} - ps_j^{out} = m_j\, q_j + b_j$ $\forall j \in P$	Linearized pressure drop model for pipes
$ps_n^{min} + ps_n^{err} \leq ps_n^{node} \leq$ $ps_n^{max} - ps_n^{err}$, $\forall\, n \in N$	Pressure bounds with margin for error
$s_n^{min} < s_n < s_n^{max}$ $\forall\, n \in S$	Production bounds
$d_n^{min} < d_n < d_n^{max}$ $\forall\, n \in D$	Demand bounds

The above linear program can be quickly solved by a wide variety of linear programming solvers, including those in MATLAB, Gurobi, or CPLEX. Note that additional linear constraints, such as min or max flow rates in certain arcs, can be added to the above linear program. In addition, an objective function can be added such that a single unique flow solution can be identified based on criteria such as economic considerations. This would result in an optimization such as the following:

GIVEN

d_n $\forall\, n \in N$	Demand rate in node n
(m_j, b_j) $\forall\, j \in P$	Linearized pressure drop model for pipe j
ps_n^{err} $\forall\, n \in N$	Maximum squared pressure error for node n, given linearized pressure drop models
$s_n^{min} < s_n < s_n^{max}$	Minimum and maximum production rates at node n

CALCULATE

q_j $\forall\, j \in A$	Flow rate in arcs
s_n $\forall\, n \in S$	Production rate in supply node
d_n $\forall\, n \in D$	Rate supplied to demand node
ps_n^{node} $\forall\, n \in N$	Squared pressure at each node
ps_j^e $\forall\, j \in A$	Squared pressure at the ends of each arc

IN ORDER TO MINIMIZE

$f(s_1, \ldots, s_n)$	A function (such as unit cost) of the production rate at each plant

SUCH THAT

$d_n + \Sigma_{j|(n,j)\in A_{in}}q_j =$ $\Sigma_{j|(n,j)\in A_{out}}q_j + s_n$ $\forall\, n \in N$	Node mass balance
$ps_j^{in} = ps_n^{node}$ $\forall(n, j) \in A_{in}$	Node pressure equality constraints
$ps_j^{out} = ps_n^{node}$ $\forall(n, j) \in A_{out}$	Node pressure equality constraints

-continued

$ps_j^{in} - ps_j^{out} = m_j q_j + b_j \ \forall \ j \in P$	Linearized pressure drop model for pipes
$ps_n^{min} + ps_n^{err} \leq ps_n^{node} \leq$ $ps_n^{max} - ps_n^{err}, \ \forall n \in N$	Pressure bounds with margin for error
$s_n^{min} < s_n < s_n^{max} \ \forall \ n \in S$	Production bounds
$d_n^{min} < d_n < d_n^{max} \ \forall \ n \in D$	Demand bounds

As can be seen in the practical implementation, a customer may get less than their most recent requested or taken flow rate, but the flow rates delivered to them would still satisfy the lower bound (typically associated with contractual constraints). In real time, a customer may be curtailed based on decisions. Customers may go in and out of a curtailment situation, in order that network-level hydraulic, demand, and production constraints may still be satisfied.

Controlling the Gas Pipeline Network Using the Network Flow Solution

Once the network flow solution has been computed, it can be used to control the gas pipeline network. Flow control elements (e.g., such as those illustrated with reference to FIG. 1A) receive setpoints which are identified using the network flow solution. There are two representations of flow control elements in the undirected graph representation of the network. First, nodes associated with supply or demand are control elements, and the network flow solution indicates the supply or demand flow that should be associated with each plant or customer in the network. Second, in some networks there are also control arcs (representing compressors, valves, or a combination of compressors in valves). The network flow solution indicates the flows and pressures that should be accomplished by these control elements.

EXAMPLES

Embodiments of the invention are illustrated in the following examples.

Example 1

This example is small enough that extensive detail can be provided. In this example, there are three customers and three plants. In the network diagram of FIG. 12, customers are represented as squares and plants are represented as double circles.

Parameters for each of the eight nodes in the network are shown in Table 1. For the customer demand nodes, the minimum acceptable pressure is 2 Pa (corresponding to a

squared pressure of 4 Pa2). For the plant supply nodes, the maximum acceptable pressure is 5 Pa (corresponding to a squared pressure of 25 Pa2). The table shows that the lower and upper bounds demand for the customer at node **1** are 0.349 and 0.399 kg/s, respectively; the lower and upper bounds demand for the customer at node **4** are 0.108 and 0.208 kg/s, respectively; and the lower and upper bounds demand for the customer at node **4** are 0.863 and 1.063 kg/s, respectively. The table also shows that the gas production plant located at node **3** can range from 0 to 0.597 kg/s; the gas production plant located at node **5** can produce between 0.546 kg/s and 1.135 kg/s; and the gas production plant located at node **7** can produce between 0 and 0.530 kg/s.

TABLE 1

Parameters for the nodes for Example 1

n	d_n^{min}, kg/s	d_n^{max}, kg/s	s_n^{min}, kg/s	s_n^{max}, kg/s	ps_n^{min}, Pa²	ps_n^{max}, Pa²	ps_n^{err}, Pa²
1	0.3496	0.399	0	0	4	25	0
2	0	0	0	0	4	25	0.00022
3	0	0	0	0.597	4	25	0.02276
4	0.108	0.208	0	0	4	25	0.022748
5	0	0	0.5461	1.135	4	25	0.0093463
6	0.863	1.063	0	0	4	25	0.00096791
7	0	0	0	0.530	4	25	0.075683
8	0	0	0	0	4	25	0.074895

The first step in the exemplary implementation of the invention is to bound the flow rate in each of the pipe segments, using the graph layout shown in FIG. **12**, the information in Table 1, and the network bisection method described in detail above. The results are shown in FIG. **13**, which displays the range of possible flows for each arc in the graph. By convention, the "inlet" for each pipe is at the lower numbered node on which it is incident, and the "outlet" for each pipe is at the higher numbered node on which it is incident. As a result, by convention, flows are indicated as negative if the flow is going from a higher numbered node to a lower numbered node.

The next step in an implementation of the invention is to linearize the nonlinear pressure drop relationship for each pipe segment in the network. The results of the linearization are shown in FIG. **14**. Each subgraph shows a range of flows for a particular pipe segment (on the x-axis), with the corresponding change in squared pressure (on the y-axis). The solid plot line shows the nonlinear pressure drop relationship, and the dashed line shows the least-squares linear fit of the nonlinear pressure drop relationship over the flow range.

Key parameters are results associated with the arcs in the undirected graph are shown in Table 2. The table shows the length and diameter of each pipe segment, as well as the nonlinear pressure drop coefficient α. The table also shows the slope and intercept associated with the linearization of the nonlinear pressure drop relationship. Note that for some arcs, such as (2,4), (2,8), (3,4), and (7,8), there is slope-only line; whereas for the arcs (1,2), (2,5), and (2,6) there is a slope-intercept line.

TABLE 2

Parameters for the arcs for Example 1

ID	L, m	D, m	α	ps_j^{err}, Pa^2	m_j	b_j	q_j, kg/s
(1,2)	3983.2	0.153	0.3801	0.00022809	0.28104	0.051828	0.44964
(2,4)	3983.2	0.157	0.3322	0.02252	0.11619	0	−0.1446
							—
(2,5)	571.2	0.125	0.1521	0.0091182	0.25431	0.10175	0.86775
(2,6)	378.0	0.125	0.1007	0.00073982	0.1949	−0.09397	1.06315
							—
(2,8)	3983.2	0.125	1.0606	0.074667	0.422114	0	0.50037
(3,4)	1.6	0.158	0.0001	1.141e−05	5.99E−05	0	0.35314
(7,8)	499.9	0.206	0.0108	0.00078808	0.004399	0	0.50037

Once flow rates in each pipe segment have been bounded, and the linearized pressure drop model for each pipe has been created, the next step is to bound the potential pressure prediction error associated with the linearization. The maximum absolute pressure drop error for the pipe segments is shown in fifth column of Table 2, and the maximum absolute pressure error for network nodes is shown in the seventh column of Table 1.

Next, a network flow solution is computed using the linear program:

___

GIVEN

d_n $\forall$ n $\in$ N	Demand rate in node n
(m_j, b_j) $\forall$ j $\in$ P	Linearized pressure drop model for pipe j
ps_n^{err} $\forall$ n $\in$ N	Maximum squared pressure error for node n, given linearized pressure drop models
$s_n^{min} < s_n < s_n^{max}$	Minimum and maximum production rates at node n

CALCULATE

q_j $\forall$ j $\in$ A	Flow rate in arcs
s_n $\forall$ n $\in$ S	Production rate in supply node
d_n $\forall$ n $\in$ D	Rate supplied to demand node
ps_n^{node} $\forall$ n $\in$ N	Squared pressure at each node
ps_j^{e} $\forall$ j $\in$ A	Squared pressure at the ends of each arc

SUCH THAT

$d_n + \Sigma_{j\|(n,\,j)\in A_{in}} q_j =$ $\Sigma_{j\|(n,\,j)\in A_{out}} q_j + s_n$ $\forall$ n $\in$ N	Node mass balance
$ps_j^{in} = ps_n^{node}$ $\forall$(n, j) $\in$ A$_{in}$	Node pressure equality constraints
$ps_j^{out} = ps_n^{node}$ $\forall$(n, j) $\in$ A$_{out}$	Node pressure equality constraints
$ps_j^{in} - ps_j^{out} = m_j q_j + b_j$ $\forall$ j $\in$ P	Linearized pressure drop model for pipes
$ps_n^{min} + ps_n^{err} \leq ps_n^{node} \leq$ $ps_n^{max} - ps_n^{err}$, $\forall$ n $\in$ N	Pressure bounds with margin for error
$s_n^{min} < s_n < s_n^{max}$ $\forall$ n $\in$ S	Production bounds
$d_n^{min} < d_n < d_n^{max}$ $\forall$ n $\in$ D	Demand bounds

___

The results of the linear program include a specification of the flow rate in each pipeline arc, the quantity q_j which is shown in the eighth column of Table 2. The results also include a specification of the production rate at each plant which is required to meet network pressure constraints. FIG. **15** shows the direction of flows in the network from the network flow solution.

FIG. **16** and FIG. **17** show that the pressures associated with the linear models in the network flow solution match closely the pressures that would be predicted by the nonlinear models, given the flows from the network flow solution. Furthermore, as shown in FIG. **17**, the prior bounds calculated to bound the error associated with the pressure prediction from the linear model do, indeed, contain the pressure that would be calculated from the nonlinear model. This guarantees that the flow solution from the linear

program will satisfy the pressure constraints, given the true nonlinear relationship between pressure and flow.

Finally, with regard to this example, it can be noted that a more naïve linearization of the pressure drop, such as simply bounding the flow in any pipe based on the total network demand for hydrogen, produces pressure estimates which do not closely match those of the nonlinear model. This result is illustrated in FIG. **18**.

FIG. **19** is a flow chart illustrating an exemplary method of an embodiment of the present invention. The steps of the method are shown, alongside a preferred exemplary means of implementing each step. In step **1901**, the minimum flow rate and maximum flow rate for delivery to each customer is determined. In one embodiment, the minimum rate is based on contractual constraints and the maximum rate is based on the most recent request of the customer. In step **1902**, the minimum and maximum signed flow rates for each pipeline segment are calculated. In one embodiment, this is achieved using a network bisection method. In step **1903**, the linearization of the pressure drop relationship for each pipeline segment is calculated based on the minimum and maximum signed flow rates. In one embodiment, this is accomplished using least squares linearization. In step **1904**, the pressure prediction error for each network node is bound. In some embodiments, this is accomplished using the shortest path for a weighted graph using Dijkstra's method. In step **1905**, the pressure drop linearization and pressure prediction error bounds are used to compute the network flow solution. In some embodiments, this is accomplished using linear programming. In step **1906**, control elements (e.g., flow control and pressure control elements) receive setpoints determined from network flow solution.

It will be appreciated by those skilled in the art that changes could be made to the exemplary embodiments shown and described above without departing from the broad inventive concept thereof. It is understood, therefore, that this invention is not limited to the exemplary embodiments shown and described, but it is intended to cover modifications within the spirit and scope of the present invention as defined by the claims. For example, specific features of the exemplary embodiments may or may not be part of the claimed invention and features of the disclosed embodiments may be combined. Unless specifically set forth herein, the terms "a", "an" and "the" are not limited to one element but instead should be read as meaning "at least one".

It is to be understood that at least some of the figures and descriptions of the invention have been simplified to focus on elements that are relevant for a clear understanding of the invention, while eliminating, for purposes of clarity, other elements that those of ordinary skill in the art will appreciate may also comprise a portion of the invention. However,

27

because such elements are well known in the art, and because they do not necessarily facilitate a better understanding of the invention, a description of such elements is not provided herein.

Further, to the extent that the method does not rely on the particular order of steps set forth herein, the particular order of the steps should not be construed as limitation on the claims. The claims directed to the method of the present invention should not be limited to the performance of their steps in the order written, and one skilled in the art can readily appreciate that the steps may be varied and still remain within the spirit and scope of the present invention.

What is claimed is:

1. A system for controlling flow of gas in a gas pipeline network comprising:

a gas pipeline network comprising one or more gas production plants each having a minimum and maximum production rate, one or more gas receipt facilities of a customer, a plurality of pipeline segments, a plurality of network nodes, and a plurality of control elements,

wherein flow of gas within each of the plurality of pipeline segments is associated with a direction, the direction being associated with a positive sign or a negative sign;

one or more controllers; and

one or more processors configured to:

determine a minimum delivery rate and maximum delivery rate of the gas to each of the gas receipt facilities in the gas pipeline network;

calculate a single minimum signed flow rate and maximum signed flow rate for each pipeline segment as a function of the minimum and maximum production rates of the one or more gas production plants and the minimum and maximum delivery rates of the one or more gas receipt facilities, the minimum signed flow rate constituting a lower bound for flow in each pipeline segment and the maximum signed flow rate constituting an upper bound for flow in each pipeline segment;

linearize a pressure drop relationship for each of the plurality of pipeline segments within the lower flow bound and the upper flow bound to create a linear pressure drop model for each of the plurality of pipeline segments; and

calculate a network flow solution comprising flow rates for each of the plurality of pipeline segments and pressures for each of the plurality of network nodes to satisfy the lower flow bound and to satisfy the upper flow bound on the delivery rate of the gas, the network flow solution being associated with control element setpoints;

at least one of the controllers receiving data describing the control element setpoints and controlling at least some of the plurality of control elements based on the data describing the control element setpoints.

2. The system of claim 1 where the minimum delivery rate comprises a contractual minimum flow rate to which the customer is entitled.

3. The system of claim 1, where the maximum delivery rate is a rate last requested or taken by the customer.

4. The system of claim 1, wherein the processor is further configured to calculate the minimum signed flow rate and the maximum signed flow rate by:

28

bisecting an undirected graph representing the gas pipeline network using at least one of the plurality of pipeline segments to create a left subgraph and right subgraph;

calculating a minimum undersupply in the left subgraph by subtracting a sum of maximum delivery rates for each of the gas receipt facilities in the left subgraph from a sum of minimum production rates for each of the gas production plants in the left subgraph;

calculating a minimum unmet demand in the right subgraph by subtracting a sum of maximum production rates for each of the gas production plants in the right subgraph from a sum of minimum delivery rates for each of the gas receipt facilities in the right subgraph;

calculating the minimum signed flow rate for at least one of the pipeline segments as a maximum of the minimum undersupply in the left subgraph and the minimum unmet demand in the right subgraph;

calculating a maximum oversupply in the left subgraph by subtracting the sum of the minimum delivery rates for each of the gas receipt facilities in the left subgraph from the sum of the maximum production rates for each of the gas production plants in the left subgraph;

calculating a maximum unmet demand in the right subgraph by subtracting a sum of the minimum production rates for each of the gas production plants in the right subgraph from the sum of the maximum delivery rates for each of the gas receipt facilities in the right subgraph; and

calculating the maximum signed flow rate for at least one of the pipeline segments as a minimum of the maximum oversupply in the left subgraph and the maximum unmet demand in the right subgraph.

5. A system for controlling flow of gas in a gas pipeline network comprising:

a gas pipeline network comprising at least one gas production plant, at least one gas receipt facility of a customer, a plurality of pipeline segments, a plurality of network nodes, and a plurality of control elements,

wherein flow of gas within each of the plurality of pipeline segments is associated with a direction, the direction being associated with a positive sign or a negative sign;

one or more controllers; and

one or more processors configured to:

determine a minimum delivery rate and maximum delivery rate of the gas to each of the gas receipt facilities in the gas pipeline network;

create a lower flow bound and an upper flow bound of delivery rate of gas for each of the plurality of pipeline segments by bounding a minimum signed flow rate for each pipe segment using a minimum delivery rate as a lower bound and bounding a maximum signed flow rate for each pipe segment using a maximum delivery rate as an upper bound;

linearize a pressure drop relationship for each of the plurality of pipeline segments within the lower flow bound and the upper flow bound to create a linear pressure drop model for each of the plurality of pipeline segments; and

calculate a network flow solution comprising flow rates for each of the plurality of pipeline segments and pressures for each of the plurality of network nodes to satisfy the lower flow bound and to satisfy the upper flow bound on the delivery rate of the gas, the network flow solution being associated with control element setpoints;

at least one of the controllers receiving data describing the
control element setpoints and controlling at least some
of the plurality of control elements based on the data
describing the control element setpoints,
wherein the minimum signed flow rate and the maximum
signed flow rate are calculated by:
  bisecting an undirected graph representing the gas
    pipeline network using at least one of the plurality of
    pipeline segments to create a left subgraph and right
    subgraph;
  calculating a minimum undersupply in the left sub-
    graph by subtracting a sum of demand rates for each
    of the gas receipt facilities in the left subgraph from
    a sum of minimum production rates for each of the
    gas production plants in the left subgraph;
  calculating a minimum unmet demand in the right
    subgraph by subtracting a sum of maximum produc-
    tion rates for each of the gas production plants in the
    right subgraph from a sum of demand rates for each
    of the gas receipt facilities in the right subgraph;
  calculating the minimum signed flow rate for at least
    one of the pipeline segments as a maximum of the

minimum undersupply in the left subgraph and the
minimum unmet demand in the right subgraph;
calculating a maximum oversupply in the left subgraph
by subtracting the sum of the demand rates for each
of the gas receipt facilities in the left subgraph from
the sum of the maximum production rates for each of
the gas production plants in the left subgraph;
calculating a maximum unmet demand in the right
subgraph by subtracting a sum of the minimum
production rates for each of the gas production plants
in the right subgraph from the sum of the demand
rates for each of the gas receipt facilities in the right
subgraph; and
calculating the maximum signed flow rate for at least
one of the pipeline segments as a minimum of the
maximum oversupply in the left subgraph and the
maximum unmet demand in the right subgraph.

6. The system of claim 5 where the minimum delivery rate
comprises a contractual minimum flow rate to which the
customer is entitled.

7. The system of claim 5, where the maximum delivery
rate is a rate last requested or taken by the customer.

*   *   *   *   *