

# Hybrid system modeling and operation schedule optimization for gas transportation network based on combined method of DE, GA and Hybrid Petri net

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**Abstract—** We present an integrated Differential Evolution (DE), Genetic Algorithm (GA) and Hybrid Petri net (HPN) based approach for scheduling of a hybrid dynamical transport network operation. The main advantage of the proposed approach is a reduction of complexity in the optimization problem formulation for a system with non-linear constraints and hybrid discrete-continuous parameters and the impact of discontinuities of the search space ensued by autonomous switching of the network components. In particular the scheduling problem for a natural gas compressor station with parallel pumps is addressed in the case-study.)

**Keywords—**Differential Evolution; Genetic Algorithm; Petri Net; Hybrid Systems, Gas Transport Networks

## I. INTRODUCTION

In this research we consider the scheduling problem for hybrid dynamical transport networks, i.e. networks where a transported product is a subject to non-linear dynamics according to physical laws, while some components of the transport network can be in different discrete states.

Many engineering problems such as scheduling of batch-continuous manufacturing, traffic problems, scheduling of compressor stations in gas networks and pump stations in water distribution networks can be formulated as hybrid system scheduling.

Mathematical model of such scheduling problem includes a set of non-linear and non-convex constraints, which can't be handled by conventional continuous and mixed integer programming solvers [6] Moreover, classical algorithms for non-linear problems are not able to handle discrete variables and constraints in an efficient way. Recent studies propose hybrid approaches mainly based on dynamic programming or sequential linear programming, which yet withdrawn some non-linear aspects of the original problem by introducing various approximations to the objective function and the domain of constraints. [3]. The limitation of using these methods to real cases is mainly due to the complexity of the equations' resolution to ensure the hydraulic balance and the difficulty of

generalizing such methods to different configurations of a transport network.

The remarkable use of evolutionary algorithms in this research topic in recent years is mainly due to heuristic methods provide a great flexibility in exploring the search space and allows an easy link to simulation models. [2, 8] However, in contrast evolutionary algorithms disclose their disadvantages of the speed of the modelling and convergence to optimal solution when applied to complex procedures with many constraints. [8]

Simulation tools for modeling, analysis and control of hybrid systems are currently attracting much attention and many works were devoted to these topics; which were tackled from two different angles. On the one hand, tools conceived for modeling and analysis of continuous systems were adapted to be able to deal with switched systems. This approach consists to integrate the event aspect within a continuous formalism. On the other hand, discrete event systems tools were extended for the modeling and analysis of hybrid systems. [9]

In this paper we propose a combination of two heuristic techniques for scheduling of compressor stations in gas transportation systems. The main algorithm of the proposal is Differential Evolution (DE) that deals with continuous variables (speed of water pumps). Binary Genetic Algorithm (GA) that is integrated into the algorithm of DE, deals with the discrete aspects of the problem (number of active pumps for each station) and guides solution to the best discrete configuration. Thereby we achieve a reduction of the impact of discontinuities of the search space ensued by autonomous switching of the network components. [11, 12]

The substantial reduction in the complexity of the problem formulation for a system with non-linear constraints and hybrid discrete-continuous parameters is achieved by introduction of an Extended Hybrid Petri net (HPN) simulation of a gas transport system into the algorithm. Though there are special-purpose simulation tools designed for the hydraulic analysis of gas transportation networks, the HPN is selected due to the applicability of petri-net based simulations for modeling and

analysis of other cases of dynamical transport networks such as water networks and other types of hybrid systems in general.

The HPN model includes a continuous part, corresponding to continuous gas flow in pipes and a discrete part that simulates transitions between distinct operational states of a compressor station (e.g. number of compressors working) and pipes (in/out of service). The embedded HPN simulation deals with evaluation of the fitness of a candidate solution, i.e. evaluation of the objective function and the degree of constraints violation [4].

## II. PROBLEM FORMULATION

First, Natural gas distribution systems is a transport of compressed fluid with a specified flow rate, quality and pressure to consumers via a network of pipelines and pump stations. In this paper we address the problem of optimization of fuel consumption in natural gas transmission networks. The energy loss is incurred by work of compressors which are installed to support the required pressure in the network. Compressors usually consume up to 5% of the gas flow that makes optimization of their operation aiming minimization of this energy loss be an essential topic for researchers. [3]. A general scheme of the investigated system is introduced in Fig. 1.

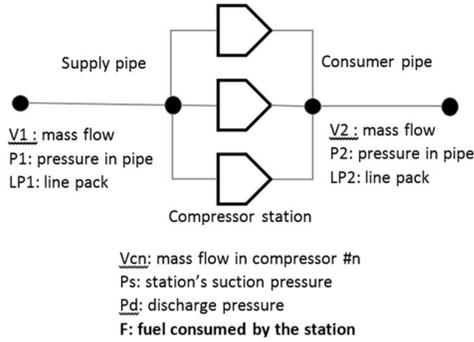


Fig. 1. Compressor station unit

The system is composed of the compressor station unit, which state in time  $t$  is characterized by a number of working compressors, mass flow distribution and a discharge pressure. The compressor station unit is placed between a supply and consumer pipelines. Pipelines are characterized by inlet and outlet flows, pressures and a line-pack in time  $t$ . The line-pack introduces a pipeline's storage capacity. When the demand is less than the steady-state flow rate, gas accumulates in the pipe, which leads to an increase in the pressure. In the opposite case, a complementary amount of gas is drawn from the storage and the average pressure decreases. The goal of using the pipeline storage capacity for the optimization of the compressor station work is to ensure that the pressure along the pipeline will not violate the defined technical limits and service requirements. [1] In our experiment a pipeline can't be fed if the current pressure is above defined upper limit and respectively can't be drawn if the pressure is below the lower limit. Pressure is calculated from the mass of gas in pipe (line-pack) by a formula of the following form.

$$P = LP \frac{ZRT}{\pi L \sqrt{a/z}} \quad (1)$$

, where  $P$  is the pressure (psi),  $LP$  is the line-pack (MMCF),  $Z$  is the gas compressibility (constant),  $R$  is the ideal gas constant,  $T$  is the temperature (Rankine),  $L$  is the length of the pipeline segment (miles) and  $d$  is the diameter of the pipe (ft).

In general, the work of the station can be assessed by the total amount of fuel consumed by compressors. The amount of fuel ( $f$ , MMCF) consumed by one compressor in time  $t$  is defined by an equation of the following form.

$$f_n = \alpha \frac{1}{\eta} v_n \frac{ZRT}{m} \left[ \left( \frac{p_d}{p_s} \right)^m - 1 \right], (v_n, p_s, p_d) \in D_n \quad (2)$$

, where  $v_n$  is the mass flow (MMCFH) through the compressor  $n$ ,  $p_d$  is the discharge pressure (psi) and  $p_s$  is the suction pressure of the station.  $D_n$  represents the non-convex set of feasible values for inlet and outlet pressures of compressors. With the variable flow  $v$ , the feasible set is a three dimensional space that is described by non-linear inequalities. [3]

$$p_i \leq \frac{v_{ij} ZRT}{Q^L}; \quad (3)$$

$$p_i \geq \frac{v_{ij} ZRT}{Q^U}; \quad (4)$$

$$p_j \geq p_i \left( 1 + H^L \frac{m}{ZRT} \right)^{\frac{1}{m}} \quad (5)$$

$$p_j \leq p_i \left( 1 + H^U \frac{m}{ZRT} \right)^{\frac{1}{m}} \quad (6)$$

, where  $p_i, p_j$  are the suction and discharge pressures of a compressor,  $Q^L, Q^U$  are the lower and upper limits for the volumetric flow through a compressor  $H^L, H^U$  are the lower and upper limits of a compressor head. The parameters are defined by technical features of compressors and physical properties of a pumped fluid. [1, 3, 5]

## III. INTEGRATED DE-GA-HPN METHOD

### A. Structure of the Proposed Method

The proposed approach uses combined DE-GA algorithm for computing optimal values of number of working compressors (discrete population) and the flow of gas through each compressor of a pump station (continuous variables) for the scheduling period. HPN module evaluates the schedule based on the population data provided by DE-GA. The achieved objective function is saved as a result of the fitness function in the DE-GA module, then DE crossover and mutation operators are applied to the selected individuals of the continuous population. GA operators intervene into DE iterations when DE search stagnates. The output of GA operators is an adjustment of the schedule of discrete states. The algorithm is repeated for the specific number of generations and the best schedules with respect to the objective function are obtained. The time horizon is divided into hourly periods, that allows changes of the flow rate of each continuous unit and the discrete number of active units for each station every hour.

Fig. 2 illustrates the main structure of the proposed algorithm in which the HPN module is embedded within the DE-GA iterations.

### B. Encoding of DE-GA Chromosomes

The DE-GA chromosome used in this algorithm consists of two distinct parts, representing the flowrates for compressors of each station (DE chromosome) and the number of active compressors for a station (GA chromosome). Since we consider an equal load sharing (equal flow rates) for active compressors within a station and since the settings can change every hour in the time horizon of 24 hours, 24 genes are considered. The combination of a DE gene and GA gene describes the discrete continuous state of a compressor station for one hour. Fig. 3 describes the encoding of an individual chromosome.

We use standard DE mechanism to deal with evaluation of constraints violation and multi-objectiveness embedded into the original code of the algorithm published by authors at the homepage of DE [10]. As soon as most of the constraints related to the transient behavior of the system are included into the HPN model, the evaluation of constraints violation is anticipated by monitoring of the number of undesired discrete transitions (failures) during a simulation. The cost function (total energy consumptions) can be computed after a simulation run using the equations described above (2 – 6).

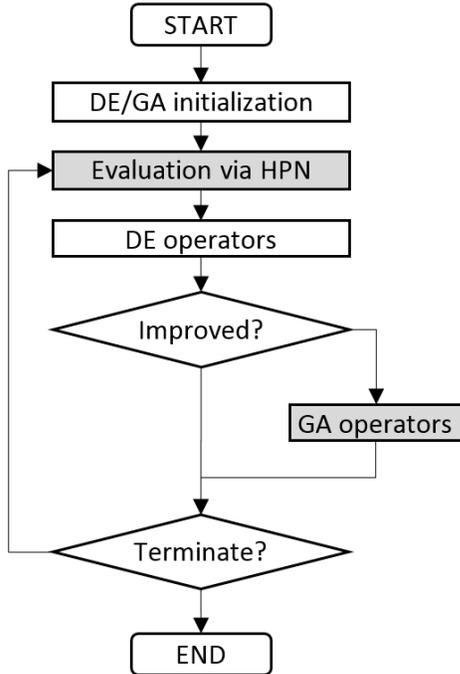


Fig. 2. Overall structure of the proposed algorithm

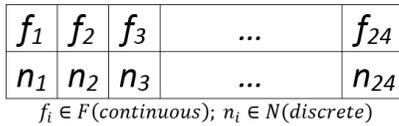


Fig. 3. Encoding of DE-GA chromosome for the compressor station scheduling

### C. Handling of Constraints

An extended hybrid PN modeling of the compressor station unit is shown in Fig. 4. The model of a compressor station is

composed of a continuous part, corresponding to continuous gas flow in pipes and a discrete part corresponding to the distinct states of a compressor station (number of compressors working) and pipes (in/out of operation). The continuous and discrete parts of the model are connected with each other via read arcs, which allow to specify positive side conditions. Continuous transition T2 represents the flow through the compressor station. It is enabled only if there are two tokens in P6, that is when the supply pipe is in service (pressure is above minimum) and the customer pipe is filled below its maximum capacity (pressure is below maximum). Places LP1 and LP2 correspond to the amount of mass of gas (line-pack) in pipes. These places are connected to discrete immediate transitions by read arcs, which allow the transitions when the line-pack reaches values corresponding to maximum allowable pressures, thereby switching the discrete states of the model and allow or disallow ‘feed’ (T1), ‘draw’ (T3) or ‘compression’ (T2) transitions. LP\_U1 and LP\_U2 represent buffer limits of pipelines and behave inverse to LP1 and LP2.

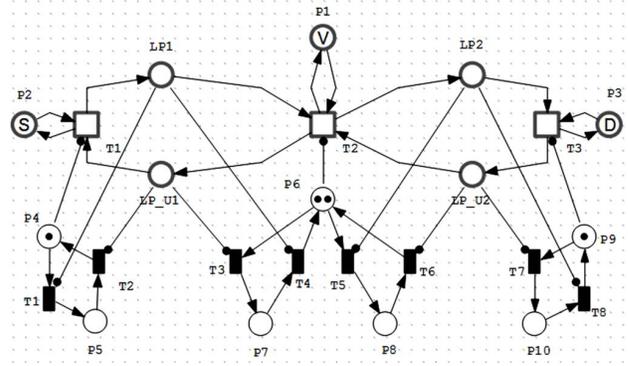


Fig. 4. Extended HPN model of the compressor station unit

## IV. CASE STUDY

The proposed solution have been released in Matlab R2014b using the original code of DE algorithm published by authors of DE [10], a binary version of GA published by Yarpiz project [12] and the free hybrid petri net simulation tool – HYPENS [7]. As other heuristic techniques, DE and GA requires definition of several control parameters. The following configuration of the required settings is selected after a number of experiments.

TABLE I. CONTROL PARAMETERS OF DE AND GA

Differential Evolution		GA	
Population	120	Population	120
Iterations	100	Iterations	-
Crossover probability	1	Crossover percentage	0.6
Mutation step	0.1	Mutation percentage	0.4
DE strategy	DE/rand/1	Mutation rate	0.2

Input parameters (initial state) of the model are: line-packs in the supply (LP1) and demand (LP2) pipe equals to 30000 MMCF; supply (S) is 600 MMCFH. Both pipes are of equal length (50 miles) and diameter (3 ft.). The interval of operational pressure limits for both pipes is defined between 600 and 800 psi. The lower and upper flow rate limits for a single compressor unit are 150 MMCFH and 250 MMCFH. The maximum number of active units in a station is 3. Other constants required for the calculations related to the gas flow in compressors and pipes are

compressibility factor  $Z=0.95$ , gas constant  $R=85.2$ , temperature  $T=520$ , friction factor  $f=0.0085$  and  $K=4.104$ . The discharge from LP2 varies in time according to the demand pattern described in Table 2.

TABLE II. VARIABLE DEMAND PATTERN (MMCF)

<b>Time (hours, h)</b>	1	2	3	4	5	6
<b>Demand (MMCF)</b>	300	300	300	300	400	500
<b>Time (hours, h)</b>	7	8	9	10	11	12
<b>Demand (MMCF)</b>	600	600	400	400	500	600
<b>Time (hours, h)</b>	13	14	15	16	17	18
<b>Demand (MMCF)</b>	600	600	600	700	700	700
<b>Time (hours, h)</b>	19	20	21	22	23	24
<b>Demand (MMCF)</b>	800	800	800	800	600	300

## V. RESULT

The schedule obtained by the algorithm is presented in the following Table 3. Total fuel consumption is 382.32 MMCF that accounts for around 3.28 % of the total flow through the compressor station in 24 hours and assessed as below average consumption (5%). The schedule supports stable operation of the transport system without interruptions due to low pressure in pipelines. The final line-pack in the pipelines is 32747.81 MMCF and 28452.19 MMCF correspondingly for LP1 and LP2 that is within the defined limits of the operational state of the system.

TABLE III. SCHEDULE OF COMPRESSORS WORK

<b>Time (hours)</b>	1	2	3	4	5
<b>Flow (MMCF)</b>	0	563.52	0	667.5	348.04
<b>Number of units</b>	0	3	0	3	2
<b>Time (hours)</b>	6	7	8	9	10
<b>Flow (MMCF)</b>	426.96	589.11	462.44	638.4	672.78
<b>Number of units</b>	2	3	2	3	3
<b>Time (hours)</b>	11	12	13	14	15
<b>Flow (MMCF)</b>	667.17	686.25	598.74	725.46	679.17
<b>Number of units</b>	3	3	3	3	3
<b>Time (hours)</b>	16	17	18	19	20
<b>Flow (MMCF)</b>	205	422.1	677.88	627.57	555.03
<b>Number of units</b>	1	2	3	3	3
<b>Time (hours)</b>	21	22	23	24	
<b>Flow (MMCF)</b>	643.89	465.3	329.88	0	
<b>Number of units</b>	3	3	2	0	
<i>Total flow through the station</i>	11652.19 MMCF				
<i>Total fuel consumption</i>	382.32 MMCF (3.28%)				

The algorithm converges to a feasible solution (a schedule without breaks of operation due to low pressure in pipes or incorrect load sharing of compressors) in around 20 iterations and proceed to converge to a near optimal solution in around 70 iterations. The procedure is terminated when the maximum number of DE iterations is reached. GA operators are executed 74 times within 100 iterations of DE.

## VI. CONCLUSION

The purpose of this work is to demonstrate how the integration of Differential Evolution and Genetic Algorithm can be applied to the optimization of hybrid systems with both discrete and continuous behavior. It was shown that the implementation of Extended Hybrid Petri Net results in a substantial reduction of the complexity of the problem formulation for a system with non-linear constraints and hybrid

discrete-continuous parameters. In the proposed algorithm, HPN simulation is embedded into DE-GA algorithm for the purposes of evaluation of the fitness of candidate solutions. Though there are special-purpose simulation tools designed for the hydraulic analysis of gas transportation networks, the HPN is selected due to the applicability of petri-net based simulations for modeling and analysis of other cases of dynamical transport networks such as water networks and other types of hybrid systems. The potential of the algorithm was illustrated through a natural gas compressor station case study, which is an example of a hybrid system. The considered topics for the direction of future research include: algorithm's capabilities to cope with uncertain parameters of a fluid transportation network to obtain a robust solution; smoothness of the solution from the compressor station control point of view; computation time of an iterative algorithm with an embedded simulation model; other potential areas of application of the proposed methodology.

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