Contribution to modeling and optimization of home healthcare

Bushra Bashir

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Bushra BASHIR

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Contribution to Modeling and Optimization of Home Healthcare

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ARTIBA Hakim, examinateur
CAUX Christophe, directeur de thèse
CHABROL Michelle, co-directeur de thèse
GUINET Alain, rapporteur
LAMOTHE Jacques, rapporteur
QUILLIOT Alain, examinateur

I dedicate this work to my beloved daughter Suha.

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GENERAL INTRODUCTION

A healthcare network or health system consists of all organizations, actions and people who participate to promote, restore or maintain people's health. The health care systems in many developed countries are facing increasing costs. The major reason is the changing age distribution of the population with more elderly people in need of support. Increasing healthcare costs has created new alternatives to traditional hospitalization in which one is Home Health Care (HHC). Home health care or domiciliary care is the provision of health care and assistance to people in their own homes, according to a formal assessment of their needs. HHC has attained a specific place in healthcare network. HHC programs have now been successfully implemented in many countries. The purpose of HHC is to provide the care and support needed to assist patients to live independently in their own homes. HHC is primarily performed by means of personal visitations of healthcare workers to patients in their homes, where they provide care assistance according to patients' needs.

In this thesis we have considered different aspects of planning problems for home health care services. The efficient use of resources is necessary in continuous healthcare services. To meet the increased demand of HHC, operation research specialist can play an important role by solving the various combinatorial optimization problems arising in HHC. These problems can be tactical, strategic or operational with respect to planning horizon. Strategic problems are those which help in attaining long term goals or objectives, e.g. higher level of quality for HHC patients and efficient use of resources. These strategic objectives can be achieved through tactical i.e. medium term panning and operational planning i.e. short term planning. The main purpose of our thesis is to identify these potential optimization problems and solve them via recent metaheuristics.

HHC is an alternative to traditional hospitalization and has got a significant share in the organization of healthcare in developed countries. The change in aging demographics, recent development in technology and the increase in the demand of healthcare services are major reasons for this rapid growth. Some studies show HHC as a tool to reduce costs of care, which

is a major preoccupation in developed countries. Some others reveal that it leads to the improvement of patients' satisfaction without increasing the resources.

Home health care, i.e. visiting and nursing patients in their homes, is a flourishing realm in the medical industry. The number of companies has grown largely both in public and private sectors. The staffing needs for HHC companies have been expanded as well. Also they face the problem of assigning geographically dispersed patients to home healthcare workers and preparing daily schedules for these workers. The challenge of this problem is to combine aspects of vehicle routing and staff rostering. Both of them are well known NP- hard combinatorial optimization problems [106, 107, 108] it means the amount of computational time required to find solution increases exponentially with problem size.

Home healthcare workers scheduling problem is difficult to solve optimally due to presence of large number of constraints. These are two types of constraints: hard constraints and soft constraints. The hard constraints are the restrictions to be fulfilled for the schedules to be applicable and soft constraints are preferences to improve the quality of these schedules.

Additionally, these planning problems are idiosyncratic in nature, for example different HHC companies have their own set of constraints, their own definition of good schedules, specific geographical constraints, routing and travelling constraints and their own choice about the use of constraint type (as a soft or hard constraint). So the solution of these problems requires their explicit definition. Therefore we have given assumptions, sets, notations, parameters and mathematical formulation for each considered problem.

Two types of algorithms are used in literature to solve planning problems, exact algorithms and approximation algorithms. Exact algorithms are able to find optimal solution, however in HHC planning problems exact algorithms constitute brute-force style procedures. And because these problems have the exponential growth rates of the search spaces, thus these kinds of algorithms can be applied only for small size problems. On the other side, approximation algorithms may construct optimal solution or not but they can produce good practically useable solutions. Thus due to these factors we have proposed approximation algorithms to solve planning problem and districting problem.
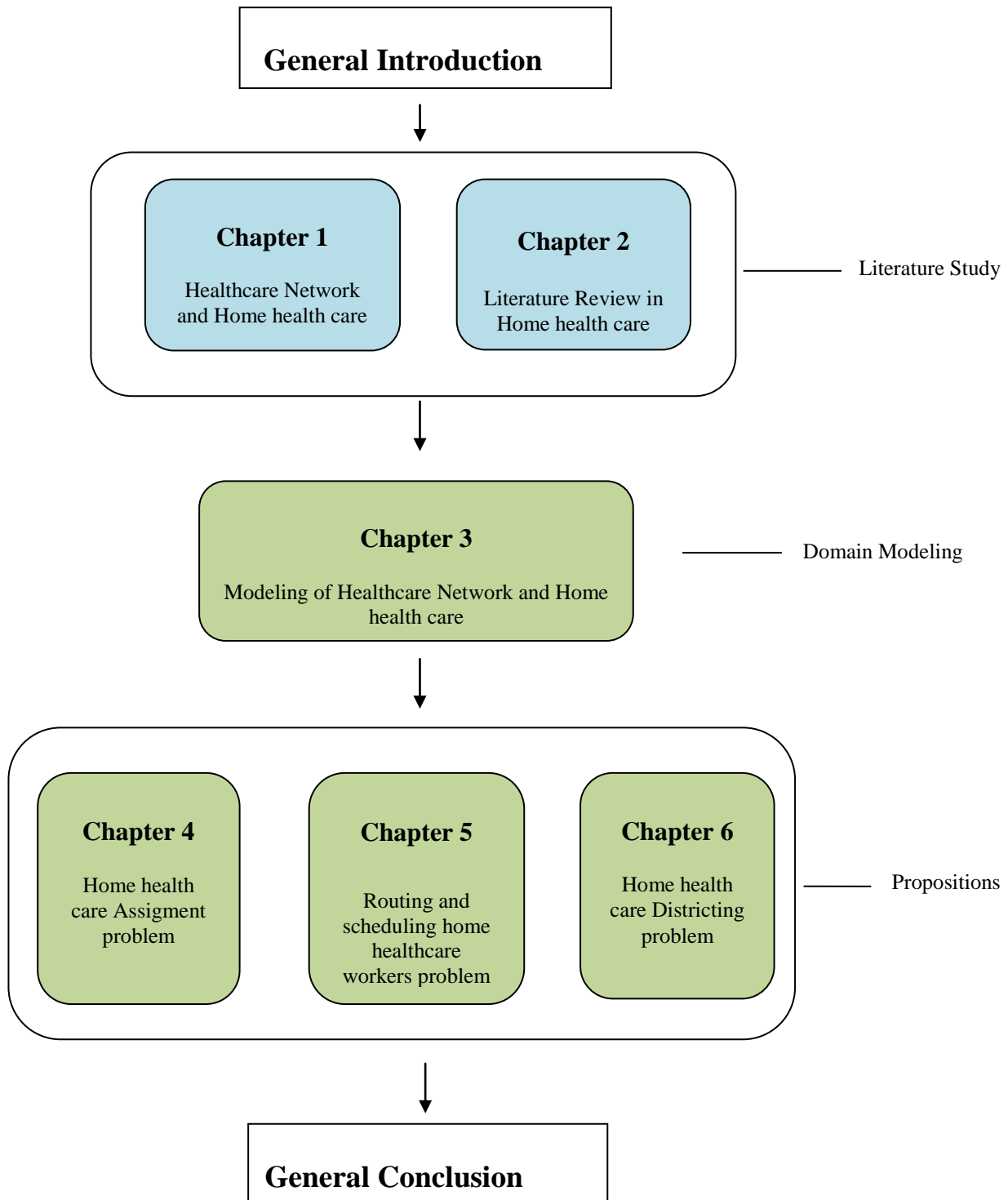
Flow chart represents Thesis Structure. Here green colour is used to show our contribution part and blue colour for literature study.

This thesis studies the problem of improving home healthcare workers productivity through the development of improved solution methodologies for the tactical, strategic and operational logistics planning problems encountered in the home health industry. At the tactical level, the home health districting problem (i.e. allocation of home healthcare workers to geographic service regions) is addressed. The strategic decision is the assignment of patients to home healthcare workers and at the operational level, the problem of developing daily visit schedules for home healthcare workers is studied.

We propose metaheuristics to solve considered problems, thus we mostly discuss metaheuristics based algorithms such as variable neighborhood search, simulated annealing and genetic algorithm. To construct these algorithms, solution representation, neighbourhood moves, genetic operators and local search procedures are described in detail for each considered problem.

Our thesis is structured as follows.

In Chapter 1 we give an introduction to the healthcare network and specifically home health care. We give the terminology, features and constraints associated to home health care. Then we identify the various problems in HHC which can be solved using operation research techniques. We analyze the application of HHC practices in different countries and present the pros and cons of HHC in comparison with traditional hospitalization.

In Chapter 2, the literature on HHC related optimization problems is reviewed. This review identifies two potential problems of HHC which we study in detail. These are planning of HHC human resources and division of a geographical region to smaller units for HHC service employment. We also give a review of different solution techniques used to solve this kind of problems, concentrating mainly on metaheuristics.

In Chapter 3, we give class diagrams for home health care and health care network. Their purpose is to better understand the system, its various elements and their relationship with one another. We propose a unified approach to integrate HHC and health care network. Then we show how a location allocation problem can be simplified to a home healthcare workers assignment problem. This problem is explained in detail in Chapter 4.

In Chapter 4, we describe home healthcare workers assignment problem in detail. We define different sets, sub sets and parameters to formulate a binary integer mathematical model for this problem.

Most of the works in literature perform these two decisions simultaneously. In doing so the assignment of health care workers is done with respect to skill level requirements and patient preferences for a particular home healthcare worker. But to our knowledge, none of them considers the geographical positions of patients and home healthcare workers. The feature that we add to this problem is to consider their geographical positions along with other constraints of skill level and preferences. The objective is to assign patients to home healthcare worker taking into account the geographical positions of patients and travelling costs. We use the data from literature and edit it for adapting to our problem. This is solved by solver LINGO and detailed results for different scenarios are presented.

The primary contributions of Chapter 5 are following. We integrate the routing constraints into home healthcare workers assignment problem discussed earlier in chapter 4. The objective of adding these constraints is to prepare daily schedules for home healthcare workers. These schedule will subject to some restrictions and soft constraints. We reformulate this modified problem. For this we describe assumptions, parameters and variables considered for the problem in detail.

In this chapter, we present a new 0-1 linear integer programming formulation for routing and scheduling home healthcare workers problem. RSHCW problem deals two decisions (i.e. assignment of patients to home healthcare workers and preparing routing plan for each home healthcare worker to visit assigned patients). We have formulated mathematical model in a generalized way by gathering many constraints from different organizational environments in a single formulation. For this model the objective function is also generalized and contains many criteria that can be adopted for specific problems by choosing the values of weighted constants.

The main purpose of this chapter is to adopt the existing metaheuristics that can find good solutions for routing and planning home healthcare workers problem. We propose neighbourhood based metaheuristics and population based metaheuristics. These

metaheuristics are described in detail by giving definitions of neighbourhood moves and pseudo code for each process applied. To compare the performance of these metaheuristics, a random data is generated for our computational experiments.

In Chapter 6 we describe the home health care districting problem. The districting problem in HHC is to study how to partition a comparatively large zone into many minor districts for the employment of home healthcare workers. This problem is important in real world, because the districting plan directly influences the outcome of HHC service. We give the considered assumptions and the parameters. Then we formulate it mathematically via binary integer programming. The objective function and constraints are explained. This chapter concerns mainly an application of the metaheuristics techniques to a graph–theoretic formulation of the districting problem in home healthcare context. As HHCD problem is computationally very hard to solve; thus it makes sense to look at metaheuristics, in order to find good feasible solutions with a modest computational effort. Our objective is the comparison of different metaheuristics in terms of quality of the solution found and computation time. We apply four metaheuristics to solve this problem. Their pseudo codes are given in detail. At the end of this chapter, we discuss the results obtained by solving HHC districting problem by means of the four proposed metaheuristics.

Finally, we state the conclusion of our work and present ideas for further work.

Chapter 1

1 HEALTH CARE NETWORK AND HOME HEALTH CARE

Health care is a major issue for a country's welfare status in all levels, either economically and socially, or demographically. We live in an age that the health care sector demands reducing costs and simultaneously improving its quality and access to all [109]. Here we use the term health care network to describe different types of health care facilities, organizations and trained personnel engaged in providing health care within a geographical area. Sometimes it is called health care system. Health care systems are organizations or policies in practice that are designed to plan and provide medical care for people. Hospitals, clinics and community health centers are examples of healthcare systems. Providers of health care insurance are also examples of healthcare systems.

Home health care has developed to respond the changes of healthcare system such as in the hospital sector (bed closures, increase in ambulatory care clinics, and day surgery) and in the long term care facilities sector (waiting lists for beds, limited availability). As a result, home health care has become an essential component of health care system. Home health care also works as a bridge between the various settings of care, including acute care hospitals, emergency rooms, supportive living, long term care homes, and the physician's clinic.

This chapter is organized as follows. Section 1.1 describes home health care (HHC) and its evolution in context of France. Also this section provides the purpose of literature study, the terminology used in health care or home health care, features of HHC and set of constraints which need to be taken into account. In section 1.2 we identify the various problems relevant to the new development of HHC structure. Section 1.3 presents role of HHC studies in some developed countries and developing countries. Some pros and cons of HHC are given in section 1.4. In the end, concluding remarks are given in section 1.5.

1.1   Home health care

Home health care is a generic term which includes all forms of structures (i.e. public, private or community) that receive patients requiring long and regular care. This care is provided outside the hospital by a team of medical professionals, allied health, rehabilitation, social workers and / or volunteers and it is relied on various services and facilities as a resource.

The demand for home healthcare (HHC) services is growing as population ages and the number of people living with chronic illnesses increases. In France, 100,100 frail people needed HHC services and 3 901 637days have been devoted in 2011 and the number of HHC organizations in France has been doubled between 2005 and 2009 [164]. The aging population has created an increase in the number of people with chronic degenerative diseases that result in functional disabilities and handicaps. Patients who undergo a treatment for chronic progressive or palliative care, they want a support that removes them as little as possible from their family environment for reasons of personal comfort. Moreover, these new needs do not require the care that mobilizes a high-level technical apparatus and thus making heavy load on the hospital. These are the reasons that the structures of health care for patients have been developed outside hospital walls in recent years in patient's homes. So there is need to adapt the patient's home environment for taking medical, paramedical, social and psychological care comparable to that offered at hospital. In the treatment of kidney deficiencies, the realization of dialysis at home requires the care equipment not complicated, but the establishing of an environment for surveillance and monitoring is the important task.

Furthermore, health care sector is confronted to economic and capacity constraints, and this alternative structure of care (HHC), to the traditional hospitalization, consists in reducing medical costs and solving the problem of hospitals' capacity saturation. The HHC structures are facing an important manpower deployment and staff organisation problem.

In 2011, nearly 12 000 patients were treated each day by HHC, which is still far from the symbolic objective of the 15 000 places. This objective should have been achieved in 2010 [164].

1.1.1 Overview

The purpose of this work is to do a detailed state of art of the problems as well as scientific knowledge in the domain of home health care. We review problems related to three levels of planning (strategic, tactical and operational). This allows us to better identify the variation of the objectives of HHC structure. The problems of high value are selected. The study allows diagnostics and modelling of existing good practices. The bibliographic knowledge finds the potential contributions of operations research, industrial engineering, management science and combinatorial optimization theory. An initial search of the literature revealed that the study of problem of HHC is in its infancy. We have found a limited number of publications in engineering sciences and medical sciences that concern the operational management of HHC facilities. In this work, we have analyzed the various types of problems that can arise within this type of care structure. A systematic literature search was performed in a number of international databases. This review presents the existing problems treated in HHC context and also points out some problems which could not get much attention of the researchers.

1.1.2 Terminology

We noted that the exact definitions of the terms in health care system are variable from author to author, from country to country, from company to company, from state to state, and from policy form to policy form, even between forms of the same company. Different authors use same term for different purpose. So there is need to describe these terms explicitly in order to avoid any confusion.

Homecare - Homecare describes any form of care given within the home. This can range from care provided by a home health aide, home health nurse, companion or caregiver and includes intermittent care, respite care and home therapies. The term homecare covers both medical and non-medical forms of care.

Home health care - The term home health care refers to medical-related care while homecare encompasses all medical and non-medical homecare services.

The term home health care and homecare are often interchanged. Some authors use the term home care for the care provided by healthcare professional to a patient in his home [1, 2, 4, 9, 11]. On the other hand, we have seen in literature that the term home health care has also been used to describe the same activity [7, 17, 49].We think the term home health care is more explicit. So we retain the term home health care throughout this work. For clarity, we also describe here the other terms used

Home healthcare worker A person licensed or certified to provide home health care services is called home healthcare worker.

1.1.3   The features of home health care process

Home healthcare processes are very complex. They involve clinical and administrative tasks. The features of home healthcare process are given in the following; some of them are common to both traditional healthcare and home healthcare. However, we noticed several differences in the application.

We are interested to work in the research problems related to coordination, network of the home health care structures and delivery of health care regarding cost control and quality of health care.

1.1.3.1 Coordination

Coordination is the act of organizing, making different people or things work together for a goal or effect to fulfil desired goals in an organization. Coordination is a managerial function in which different activities of the business are properly adjusted and interlinked.

HHC structures can treat patients whose healthcare needs are difficult and comprehensive in terms of treatment plan (i.e. medical, paramedical, psychological support, social support etc.). It requires them to implement extremely fine coordination among these structures and various professionals involved in this healthcare process. Their objective is to monitor quality of health care (as perceived by the patient and professionals), to ensure efficiency (minimizing delays, cancellations, redundancy) and to control costs.

1.1.3.2 The network

A network is a group of legally independent companies or subsidiary business units that use various methods of coordinating and controlling their interaction in order to appear like a larger entity. The HHC network consists of a large number of actors (home healthcare workers, patients, patients' families, etc.) which need to coordinate among one another to provide health care to ensure good quality of health care services. Here other than healthcare workers the patients are decision makers as well with their own different objectives. So it is a network of many different actors and it is required to control, manage and drive across this complex network in order to provide the well coordinated health care services.

1.1.3.3 Heterogeneity

The important feature of HHC organizations is its heterogeneity as they provide wide range of services through different kinds of actors. They involve private or public organisations: healthcare organisations, social organisations, home-care materials and technologies suppliers, chemists, patients' transportation. HHC involves the heterogeneous activities such as contracting new patients, managing the logistical aspects of the care activities and human resources, managing materials suppliers (beds, drugs, telesurveillance tools, sensors, etc.), providing (or helping families to do so) patients' meals, house work, etc and managing the network of the different actors involved in the care process: medical, paramedical, social, etc. These activities may take place at different levels of granularity. Granularity is the extent to which a system is broken down into small parts, either the system itself or its description or observation. It is the extent to which a larger entity is subdivided. For example, a yard broken into inches has finer granularity than a yard broken into feet.

1.1.3.4 Delivery of care

Home health care organizations are facing many organisational and logistical challenges. Operation research specialists can play an important role to address some of these issues among which are the management of complex HHC organizations and an efficient and effective healthcare delivery to an ageing and ever more demanding population, with limited

budgets, advancing medical technology [103] and increased expectations. Here we identify the optimization problems faced in the delivery of care. Our aim is therefore to analyze the optimization problems that arise in context of the delivery of health care in HHC structure. HHC structucre is acomplex network involving many decision makers and many resources through the various constraints.

1.1.3.5 Constraints

The delivery of health care in HHC is a complex process. It is highly constrained process. For example the constraints on the coordination of structures are:

- human resource constraints such as constraints related to decision makers and healthcare workers due to their own objectives; constraints related to availability and qualification level, etc.
- constraints related to patients' choice, lack of availability of these patients i.e. time of visit, finite number of day care activities, etc.;
- constraints related to the performed activities i.e. relationship dependence among the visits, timing of visits, excluding visits , pre allocated visits, etc.;
- constraints of resource availability (i.e. equipment, drugs, sterile medical devices) and constraints related to the expiration of certain resources (i.e. preparation of chemotherapy);

In all these cases home health care system is noticed due to its various human dimensions, the need to consider uncertainty and incompleteness on variables. These factors h increase the complexity of the network related to the decision makers and resources.

1.2 Identification of various types of problems in Home health care

We are interested in the history of the home health care. We then study the various works that have addressed this issue both in the publications of engineering and in the medical world. After conducting a literature review of various works related to HHC, we try to analyze HHC

structures to study how they work and identify various points where we could make improvements.

For the efficient organization of HHC in terms of human and material resource management, we need to analyze the various types of problems that we have to face within this type of care structure. The problems which we identify are given in the following:

- Guide and information exchange among different actors of Care
- Defining the criteria of quality to meet the necessity to guarantee a satisfactory service
- Modeling of health care system
- Dimensioning the resources
- Management of human and material resources
- Cost saving study (cost comparison of HHC)
- Resource allocation
- Coordination of HHC with other health care network
- Planning and scheduling of nurses and doctors
- Assignment of human resource to patients
- Districting problem (i.e. the problem of partitioning a territory into districts)
- Routing Problem for health care personnel and medicines
- Taking into account of uncertainties and emergencies
- Workload balance among health care personnel and satisfaction of their preferences
- Efficient use of technological support in HHC service delivery

Many problems solved by operational research in industry can be applied to problems in HHC. In fact, the two fields treat the same problems and thus, the industrial problems can be adapted to HHC logistic. But HHC services have a large number of new constraints and human dimension so this adaptation is not an easy and straightforward task. It requires deep understanding of the given situation.

1.3   Home health care in developed countries

Now we describe the state of HHC in various countries, how it works, what is the chosen system which handles it, private or public. We perform this analysis from articles and reports published in this domain.

1.3.1 Europe

R. Tarricone and A. D. Tsouros [27] analyzed the different factors in detail that can influence the supply of HHC and demand for HHC in Europe. These factors include: policy priorities and choices such as deinstitutionalization, community-based solutions and constraints on public expenditure; demographic shifts such as ageing population and changing dependency ratios; social changes such as small family units, female labour market participation and mobility across countries; changes in epidemiology such as rise in non-communicable diseases; science and technical innovation such as medical science advances, medical and non-medical technology advances; changes in attitudes and expectations such as increased choice and individualized care.

According to FNEHAD Report [28], HHC sector had known an important growth since its creation sixty years ago and especially over this last decade in France. Indeed, the number of HHC structures increased by 87.7% between 2005 and 2008. During the same period, the number of days during which patients were followed up by home healthcare workers had risen steadily from 1 505 814 to 3 298 104 which corresponded to an increase of 119.02%. Similarly, the number of patients had increased by 147.52% between 2005 and 2008 in France.

A variety of provision models was found, including monopolist agencies providing comprehensive services in an area; agencies for specific services, such as nursing or domestic care (e.g. in Sweden B. Malmberg *et al.*[29]; competing commercial and non-commercial private providers and public providers. Private provision (including non-profit) was growing in several countries, such as Ireland V. Timonen and M. Doyle [30], Finland T. Kroger [31], Sweden G. Sundstrom *et al*. [32] and England A. Netten *et al.* [33], either replacing public

provision or compensating for its absence. The introduction of market mechanisms in some countries appeared to have weakened co-governance between the third sector (voluntary sector) and the public sector I. Bode [34]. The for-profit private providers may have been better adapted to the new market forces than the voluntary organizations, as was the case in the UK, where managers of voluntary organizations were more likely to have greater problems with negotiating contracts than private provider managers J. Kendall *et al.* [35].

B. L. Bihan and C. Martin [36] is comparative studies of European social policies towards frail elderly people typically focus on the systems and their implementation. The study was conducted in 2001 in six European countries (Germany, Spain, Italy, France, the United Kingdom and Sweden). Their aim was comparing the rights of the individuals within the different health care systems. The methodology used was a case study approach, which drew on a series of situations of dependent elderly people. Therefore, the analysis focused on the public authorities' responses the care packages, which determined the type of care required and the financial contribution of the user in each of the six countries, in relation to the concrete situations of frail elderly people. As local variations were important, in all the countries studied, local authorities had been chosen in each of the countries. This approach gave interesting concrete elements on the services and financial help which could be given to frail elderly people. This study also enables us to understand precisely the national care systems organized in the different countries and the main difficulties encountered by public authorities in facing this problem of frail elderly people.

N. Genet *et al.* [37] presented a review of HHC in Europe. This review included publications provided information on 18 countries. No information was found on Bulgaria, Croatia, Cyprus, Estonia, Greece, Hungary, Iceland, Latvia, Lithuania, Luxembourg, Malta, Romania and Slovakia. Single country studies described features of HHC in 15 countries. Country comparisons were made in eight publications and these contained information on three countries for which no single country studies were found (Germany, Czech Republic and Austria). The countries addressed in the largest number of publications were Sweden and the UK. The review yielded very little information on HHC in the countries situated in Central and Eastern Europe. They concluded that HHC systems appeared to differ among different

countries and also within the same country. The papers included by these authors, however, provided only a limited picture of home health care.

Many studies only focused on one aspect of HHC system and international comparative studies were rare. Furthermore, little information emerged on HHC financing and on HHC in general in Eastern Europe. This review clearly showed the need for more scientific publications on HHC, especially studies comparing countries. A comprehensive and more complete insight into the state of HHC in Europe requires the gathering of information using a uniform framework and methodology.

1.3.2 USA

J. D. Goldberg *et al.* [38] discussed the characteristics and organization of HHC structures in the United States. They also evaluated the costs associated with providing appropriate access to care to low-income beneficiaries, those residing in medically underserved areas and those with varying severity levels illness. They proposed changes that can ensure increased access to HHC for these beneficiaries. The literature addressed varying definitions of vulnerable populations and their associated barriers to care as well as potentially uncompensated costs.

R. B. Greene [39] described that HHC monitoring might have an important role as a strategy to provide effective and cost efficient health care for heart failure patients. The use of new improved technology to monitor patients along with the support of a health care provider significantly improves heart failure management while reducing the cost of health care.

K. Buhler-Wilkerson [40] described that the problems of caring for patients with disabling illnesses who neither get well nor die are not new in US. Such patients have always required assistance at home from family, benevolent volunteers, or paid healthcare workers. Despite two centuries of experimentation, however, no agreement exists concerning the balance between the public and private resources to be allocated through state funding, private insurance and family contributions for the daily and routine care at home for chronically ill persons of all ages. This article examined these issues and the unavoidable tensions between fiscal reality and legitimate need. It also used historical and policy analyses to explain why HHC had never become the cornerstone for caring for the chronically ill.

1.3.3 Canada

In 2006, more than 2.5 % (875,000 individuals) of the Canadian population reported receiving HHC and home support; about 60 % of this group received HHC only according to CIHI report [41]. In 2006, about 160,000 nurses and personal carers worked in the long-term care (LTC) sector on a full-time basis and close to 70,000 on a part-time basis OECD Health Data, 2010 based on Census 2006 [42]. P.C. Coyte [43] presented an overview of Canadian HHC services; it highlighted health policy assumptions that had resulted in an increasing reliance on in-home services; and it assessed roles for the private and public sectors in the financing of HHC services.

J.P. Hirdes et *al.* [44] developed a methodology for prioritizing access to community and facility based services for HHC patients. They analyzed Canadian and international data to find predictors for nursing home placement, healthcare worker distress and for being rated as requiring alternative placement. They showed that the method for assigning priority levels (MAPLe) algorithm was a strong predictor of all three outcomes in the derivation sample. They presented validation of this algorithm with additional data from five other countries, three other provinces, and an Ontario sample. They concluded that the MAPLe algorithm could provide a psychometrically sound decision-support tool that might be helpful to make informed choices related to allocation of HHC resources and prioritization of patients needing home or facility based services.

1.3.4 Home health care in developing countries

The developing countries face great difficulties in planning and production medical care of good quality. Taking into account their political, demographic problems, socio-economic and resource constraints in these countries, access to health care becomes an activity of great complexity. The risk of nosocomial infections, propagation of bacteria, problems of transportation, electricity, and lack of a hospital information system are all factors that stimulate the need of implementing new techniques in order to improve the quality of health care. In these countries, it is necessary to find the alternatives that afford to take care of some

patients with certain diseases and that prevent them from making long distances between home and hospital.

The implementation of this structure is already experienced in some third world countries. For example in Mozambique, with the participation of several NGOs (Organization Non-Governmental) national and international, it was established a day hospital and a HHC structure for HIV patients and AIDS patients. This structure responded to an agreement signed between the Ministry of Health of Mozambique and the French Cooperation in 1995 [45].

The objective in this approach was to centralize operations and monitoring of patients infected with HIV. Countries like Cuba, French Guiana, Guadeloupe and Martinique were experiencing a huge success in the field of production of health care, since the implementation of this structure of HHC in this territory according to a report called Health in the Americas [46], J. M. Spiegel and A. Yassi [47]. With this method of HHC, the physician can remotely diagnose certain diseases in the patient at his home. It can also monitor the patient's blood pressure. This avoids unnecessary hospitalizations; beds and facilities are well used in a more rational way. The Cuban experience in setting up a structure of HHC is a tangible evidence of the application of this system in developing countries. The objective of the work N. Germani *et al*. [48] was to identify and analyze constraints to the problems of developing a HHC structure in Haiti.

1.4   Pros and cons of Home health care

The home health care has largely evolved during this last decade thanks to its human, medical and economical advantages. The pros and cons of HHC discussed by E. Benzarti [49] are given in the following.

L. Com-Ruelle and E. Lebrun [50] showed that the main advantage of HHC was to be treated in their familial environment according to 93% of the patients included in their survey. This would allow, according to N. R. Pihan [51], the free time organization. It would also reduce the risk of appearance or worsening of a dependence state linked to a hospital stay. Furthermore, HHC avoids the de-socialization generally caused by the traditional

hospitalization which is psychologically harmful for the patients. If the patient is being cared for in his or her own home, this comes with a feeling of security, comfort and privacy. Moreover, HHC is also advantageous for the family members as it supports them psychologically and avoids them to go to the hospital every day. HHC also guarantees the continuity of care by collaborating with home healthcare workers and coordinating the care delivered by the different home healthcare workers.

Despite this, HHC is also advantageous for the practitioners as it enables them to take care of a reduced number of patients and thus to be less stressed, more autonomous and closer to their patients. HHC also presents a medical advantage that consists in reducing considerably the risks of hospital-acquired infections by approximately three times E. Benzarti [49]. Finally, HHC represents an economical advantage by avoiding the hospital capacity saturation which would lead to the containment of the whole health system's costs. Indeed, HHC reduces the average duration of stay within hospitals and accelerates the turnover of the hospital beds and consequently allows keeping highly specialized human and material resources to the patients suffering from acute diseases. HHC is thus less costly than the traditional hospitalization according to A. Aligon *et al.* [52]. HHC is associated with patient satisfaction and sometimes it leads to decreased hospitalizations and emergency department visits [101].

However, despite all its advantages, HHC also presents some disadvantages. The most important disadvantage of HHC is the absence of a permanent medical supervision (guaranteed 24 hours per day). This type of hospitalization necessitates thus the permanent presence and availability of the family members who feel themselves stressed and overloaded by the domestic work according to N. R. Pihan [51]. Additionally, due to the intensity and frequency of care to deliver and severity of the pathologies treated, HHC is perceived by the general practitioners who can prescribe it since October 1992 as a badly defined responsibility that is heavy to take for. Private homes are not equipped with the emergency response systems usually in place at health care facilities.

Finally, even if HHC is economically interesting for the health system as a whole, it can paradoxically cost much for the patients and their families due to the fact that a part of the costs is shifted to the patients and their families such as lighting, hot water, acquisition of medical and/or paramedical equipments (wheelchair, specialized beds, etc.), etc.

1.5   Conclusion

In this chapter, we present the main issues, constraints and various types of problems faced in the home health care. We give an introduction to the healthcare network and specifically home health care. Although the concept of home health care is very old, it lacks the complete description of the process and uniform terminology. We give the terminology used in our work, features and constraints associated to home health care. Then we identify the various problems in HHC which can be solved using operation research techniques. We analyze the application of HHC practices in different countries and present the pros and cons of HHC in comparison with traditional hospitalization.

The potential opportunities for research can be the work on the methodological aspects of the home health care process, the organization of the resources, the design of healthcare network, the integration of home health care in the health care network and presentation of models and methods who can take into account of uncertainties of the health care process.

Chapter 2

2 LITERATURE REVIEW IN HOME HEALTH CARE

Due to the important growth of the home health care (HHC) in the last decades, it has got interest of many researchers. We survey the studies in HHC literature. The main issues are delivery problem, scheduling and assignment problem, HHC modeling, resource dimensioning problem, districting problem, HHC in developing world and cost comparison of HHC and hospitalization. We have also done a survey of solution methodologies applied to these problems. This guides us to choose appropriate solution methods to solve the chosen HHC problems.

This chapter is organized as follows. We have divided this chapter into two main parts: first part presents the literature review about the problems in HHC and second part reviews the different types of solution methods applied to solve these problems in the literature. In section 2.1.1 we describe work done about location-allocation (LA) problem. We have implemented location–allocation model in HHC to present a simplified form which is discussed in Chapter 3. Section 2.1.2 reviews the main optimization problems in HHC. In section 2.1.3 we give a cost comparison studies for HHC.

Section 2.2 is concerned to the solution methods used in literature. These are of three types: exact methods, heuristics and metaheuristics. Section 2.2.1 reviews exact methods and Section 2.2.2 is about heuristics. In section 2.2.3 contains metaheuristics. Due to efficient performance of metaheuristics in real life problems, we concentrate on the recent metaheuristics. We provide the classic basic algorithms for neighbourhood based metaheuristics and population based metaheuristics.

Finally, in Section 2.3 we present the conclusions drawn from this literature review of HHC related problems and their solution methodologies.

2.1 Literature Review for Problems in Home health care

Now we review the work done in literature for the problems identified in the domain of home health care. This section provides a state-of-the-art on the resource dimensioning, modeling approaches in healthcare network and home health care (HHC) systems, scheduling, assignment, delivery and districting problems.

2.1.1 Location-Allocation Problem

The Location-Allocation (LA) problem determines the optimal facility locations and the assignment of customers to the selected facilities. The objective is to maximize the population accessibility and to minimize the costs. Given the locations of n customers and their demands, the LA problem is concerned with locating m facilities in the Euclidean plane and allocating them to the customers in order to satisfy their demand at minimum total cost. The objectives that are usually considered in location problems can be different. Some of them are the minimizing total setup cost or fixed cost, minimizing the longest distance from the existing facilities, minimizing total annual operating cost, maximizing service, minimizing average time or distance traveled, minimizing maximum time or distance traveled, minimizing the number of located facilities, balance of workload of health care facilities and maximizing responsiveness etc.

The research literature on LA problem in health care facility location is abundant. M. J. Côté and S.S. Syam [18] solved the cost minimization problem of the location and allocation of specialized health care services with service proportion requirements. D. G. Kim and Y. D. Kim [19] worked on a long-term care facility location problem with the objective of balancing the loads of the facilities, developed a heuristic algorithm and found a better performance as compared to the commercial solver Cplex. R. L. Church [20] proposed a new model formulation for the p-median problem that contained both exact and approximate features. This new p-median formulation is called Both Exact and Approximate Model Representation (BEAMR).Their computational results showed that the BEAMR model is particularly efficient for large number of facilities. J. C. Teixeira and A. P. Antunes [21] presented a hierarchical version of LA problem which considered different levels of demands and several

types of facilities. They also added capacity constraints and user-to-facility assignment constraints to improve public confidence and acceptance.

In analyzing the efficacy of health services provision, the ability of patients to access facilities, in terms of acceptable distances and adequate transport provision, is the key factor enabling effective treatment. These patients include mothers in all stages of natal care, children receiving immunizations and routine checkups, persons requiring emergency treatment of injury and illness and persons receiving routine care. The problem of health services accessibility is particularly pronounced in rural regions of lesser developed countries; this has been well documented by studies in a number of countries including Nigeria [22], Bolivia [23] and Ghana [24]. Another study [25] examined the effects of rainy season on in third word Location-Allocation Application. For a comprehensive review of LA models applied to health facility location in developing areas, see [26].

2.1.2 Resource dimensioning and resource allocation problem

We are also interested in resource dimensioning problem. This problem is the one dealing with the determination of number of healthcare workers with particular skills and quantity of material resources of different types, necessary to meet predicted care demand with the satisfactory service quality and minimum cost.

This problem is treated by C. R. Busby and M. W. Carter [13] in which they created a decision tool for the Simcoe County Community Care Access Center (SCCCAC) in Ontario. The tool enables the SCCCAC to quantitatively assess the trade-offs among three key factors: cost, quality and waiting time of their HHC patients. This information can then be used to negotiate reasonable funding levels with the Ontario government and to appropriately allocate this funding among the various patient groups at the SCCCAC. This work can be expanded to other health care organizations that use prioritized waiting lists.

D. Boldy and N. Howell [132] have done a case-study for the allocation of a certain amount of home help resources to four geographical districts within the Devon Social Services Department. Their approach is divided into three main parts: the assessment process (it consists in evaluating the nature and level of the services required), the allocation procedure

(how to distribute equitably the service units between the districts according to the average level of the service required for each type of client and the number of clients of each type within each district) and the survey information (information related to the patients i.e. age, disability, charge paid, housing conditions, etc.; provision of related services already available within the same territory; amount of home help actually received and ideal amount of home help to provide). This approach can be considered as a decision making tool which does not produce one solution but proposes different possible allocations to the decisions-makers who would choose the best one according to the assumptions that they consider to be the most important ones.

V. De Angelis [133] has also addressed this problem for HHC structures delivering services to AIDS patients (local problem) and the problem of evaluating the suitability of the budgets assigned to the HHC structures by public-health authorities (global problem) in the city of Rome, Italy. The author has developed a stochastic linear programming model which is linked to an epidemiological model and has integrated the uncertainty in terms of patients' number and level of care required by each patient's class (the patients are classified into classes of dependency between which transition rates are defined). This model aims at maximizing the number of new patients admitted based on constraints related to resources' availability, minimum standard of service, variability of the demand, transition rates among classes and fixed budget.

2.1.3 Home health care Modeling

C. V. Campen and I. B. Woittiez [1] estimated the volume and composition of referrals to HHC on the basis of applicant characteristics. The relationships between the background and care needs of applicant groups on the one hand and the referral of HHC packages on the other hand, were studied by means of a multinomial logit model. The model was estimated on the basis of more than 7000 requests for HHC in the northern part of the Netherlands. In the modeling of HHC, emphasis had been placed on the differentiation of clients and products. They found for instance that elderly chronically ill applicants had a greater chance of being referred for domestic help only, while applicants with psychosocial disorders were more liable to be offered packages that include social support.

Patients discharged from hospital had a greater chance of a referral to domestic help only when they were slightly disabled, and were more likely to be offered packages including physical care when they were more disabled. The model had a range of policy applications in assessing the impact of changes in the health care system on the volume and structure of the demand for HHC services. Examples were presented of the consequences of the ageing population and earlier discharge from hospitals on demand for HHC packages.

B. Thomé *et al*. [2] study was a review of the empirical literature for the description of HHC as a phenomenon and as a concept, especially with regard to who were the care recipients, what actions and assessments were performed and what effects were achieved for the care recipient in terms of functional health status and quality of life.

C. Clémentz *et al*. [3] presented the application of concepts, methods and tools of systems engineering to production systems of HHC.

J. Bajo *et al*. [4] described a case study in HHC scenarios applying an abstract architecture and a computational model for large scale open multi-agent systems based on a service oriented approach. The architecture used was THOMAS (MeTHods, Techniques and Tools for Open Multi Agent Systems), which specifically addressed the design of HHC systems. A case study example was employed as an illustration of the usage of THOMAS components and services.

2.1.4 Delivery Problem

O. Bräysy *et al*. [5] discussed the communal home meal delivery problem. The problem was modelled as a multiple travelling salesman problem with time windows that was closely related to the well-studied vehicle routing problem with time windows. Experimental results were reported for a real-life case study from Central Finland over several alternative scenarios using the SPIDER commercial solver. They compared their results with the practice used at that time and revealed that a significant savings potential could be obtained using off-the shelf optimization tools. As such, the potential for supporting real-life communal routing problems could be considered to be important for vehicle routing problem practitioners.

R. B. Bachouch *et al*. [6] dealt with a drug delivery problem in a French homecare. The carriers were assigned to specific areas. The duty of carriers was to visit each patient once a day while minimizing the total travelled distance. They proposed to explore four strategies of delivery: (i) starting deliveries when a specified number of deliveries was achieved, (ii) starting deliveries if a specified distance was reached regarding to the planned deliveries, (iii) starting deliveries on a fixed number of deliveries per carrier and, (vi) starting deliveries on fixed hours. They used a mixed linear programming which took into account various constraints and minimize the total travelled distance. The results obtained for each strategy were compared in order to identify which one was the most efficient to solve the drug delivery problem at the homecare.

S. Bertels and T. Fahle [7] presented the core optimization components of the PARPAP software. In the optimization kernel, a combination of linear programming, constraint programming and (meta-) heuristics for HHC problem was used. They showed how to apply these different heuristics efficiently to solve HHC problems. The overall concept was to adapt to various changes in the constraint structure, thus providing the flexibility needed for real-world settings.

C. Salma *et al*. [8] presented a brief description of the supply chain of anti-cancer drugs and the problem of coupling production - distribution of these drugs detailing associated assumptions, the approach for its resolution and analysis of results.

2.1.5 Assignment and scheduling Problem

C. Akjiratikarl e*t al.* [9] gave an application of a Particle Swarm Optimization (PSO) - based scheduling algorithm to home healthcare worker scheduling. This algorithm combined local improvement techniques to schedule home healthcare workers effectively and efficiently. The objective was the minimization of the total distance travelled by all healthcare workers, while satisfying the capacity and delivery time window constraints. The algorithm utilized the population-based evolutionary searching characteristic of PSO to explore the solution space globally, and also exploited the local search ability of local improvement procedures (swap and insertion) to fine-tune the neighbourhood area more thoroughly. A parameter study had been performed using Taguchi design of experiments in order to find the best combination of

parameter values for this specific application. PSO-based methodology had been tested on real demand data and the results compared with those obtained with the existing manual approach and those obtained by the AiMES Centre at the University of Liverpool using ILOG in order to assess the solution qualities and computational performance. The PSO-based algorithm produced significantly and consistently better results across all the test problems. This work contributed to the development of an efficient methodology to improve the scheduling of healthcare workers and also introduced the application of PSO-based algorithm to solve this type of problem and all classes of similar problems.

The home health care service is rapidly growing in the France and in other countries around the world. HHC can be provided directly by the state or an independent provider with the aim of achieving best value, in terms of quality and cost. The drive to maximize quality and minimize costs creates a need for home healthcare worker scheduling algorithms to support the planning process by reducing costs, improving customer service and reducing the cost of planning, etc. P. Eveborn *et al.* [12] focused on a staff planning problem arising in Sweden where people receive HHC from the local authorities. The objective was to develop visiting schedules for healthcare workers that could incorporate some restrictions and soft objectives. They described the development of a decision support system LAPS CARE to aid the planners. The system consisted of a number of components including information data bases, maps, optimization routines and report possibilities. They formulated the problem using a set partitioning model. For a solution method, they made use of a repeated matching algorithm. This time saving by itself was about 7% of the total working time. The savings in travel time corresponded to about 20%.

A. Guinet *et al.* [10] described an exact method based on developing a mathematical model for mixed linear programming to assign each nurse a set of patients to visit during his tour. This model was solved by two solvers LINDO SYSTEMS of LINGO and ILOG OPL-CPLEX ILOG STUDIO. Their comparison of results showed almost similar results for the two solvers but CPLEX gave better arrival times for nurses.

L. Ettor and M. Andrea [11] dealt with the problem of assigning a HHC patient to a health professional among a set of possible ones, under the constraint of continuity of care. Starting from a previously proposed assignment policy, they first compared the policy with that

implemented in real organizations and then they derived some rules to choose the health professional that would be in charge to deliver the care service to a new admitted patient. The proposed assignment rules took into account the variability of the patient needs, expressed in terms of the number of visits requested along the time.

Portugal. O. Bräysy *et al*. [5] had studied a case from the Finnish city of Jyvaskyla. The route optimization problem was modelled as a multiple Travelling Salesman Problem with time windows, with minimization of vehicles and total distance as objective components. To illustrate the potential of widely available routing software, a commercial, heuristic route optimization tool called SPIDER Designer was used to implement the model and to automatically create optimized routes. Experimental results were presented for several alternative scenarios and compared against the current manual solution. The results showed a significant savings potential, up to 50% in both distance and number of vehicles, offering quantitative decision support to communal decision makers for renewing outsourcing contracts.

2.1.6 Districting problem

The districting problem consists in grouping small geographic areas into larger clusters called "districts" in a way that these latter are "good" according to relevant criteria, each district being under the responsibility of a multidisciplinary team.

The districting problem has been treated in the operations research literature in a broad range of applications: the design of political districts [102]; the definition of districts for salesmen; the establishment of districts for schools, salt spreading and police command; the construction of turfs for telecommunication workers and also the definition of districts for the home healthcare workers. Hence, the political and the sales domains are the two most important applications in terms of number of publications. The application of districting approach in HHC is not well studied in literature and we found a few articles for districting problem in HHC.

The districting approach had been studied by M. Blais *et al* [14] for the case of the Côtes-des-Neiges local community health clinic in Montreal, Canada. For partitioning this community

into six districts, the authors had proposed a multi-criteria approach similar to the one proposed by B. Bozkaya *et al.* [15] for the political districting problem, where the criteria related to the visiting personnel mobility and the workload equilibrium were combined into a single objective function whereas the criteria related to the indivisibility of the basic units, the respect of borough boundaries and the connectivity were considered as hard constraints. The problem was solved by means of a Tabu search technique.

After that, N. Lahrichi *et al.* [16] had reviewed the optimality of the method proposed by M. Blais *et al* [14] by analyzing the historical data of the years 1998-1999 and 2002-2003 related to the total number of visits and the distribution of these visits among districts. This analysis had proved that the territorial approach presents two main shortcomings. First, this approach could lead to workload imbalance among the healthcare workers due to the fact that it cannot forecast the fluctuation of the demand in each district. This imbalance could conduct to inequities in terms of the service quality among the districts. Second, this approach was not flexible enough in terms of the assignment of the home healthcare workers to the districts which did not encourage the collaboration between the different home healthcare workers.

In order to alleviate these shortcomings, N. Lahrichi *et al.* [16] had proposed two solutions. The first one was a dynamic approach which consisted in assigning the patients to the home healthcare workers according to the home healthcare workers' workload and the patients' caseload instead of the geographical location of patients. The second solution consisted in combining the approach proposed by M. Blais *et al* [14] with the dynamic approach. To do this, the home healthcare workers were split into two groups: the first one represented home healthcare workers assigned to a fixed district while the second one grouped home healthcare workers that could work in all or a part of the territory.

Once the territory was divided into districts, the different resources must be equitably assigned to the designed districts so that the workload of the home healthcare workers and the quality of the services delivered to the patients were roughly the same.

In E. Benzarti *et al.* [17], they presented a linear integer programming for districting problem applied in HHC. Their objective functions were the workload balance, the minimizing the travel time and the weighted sum of the two criteria. In the mathematical formulation, they

considered the profile of the patient which largely affected the workload balance among care workers. For this purpose, they had defined two parts of profiles, one with minimization of maximum deviation of workload and the second with tolerance interval for workload. They had also added the time dimension in their integer linear program by replacing the decision variables. The shortcoming of this approach according to N. Lahrichi *et al.* [16] is that it did not take into account the fluctuations of the demand which lead to imbalance of workload. These fluctuations could be caused due to various changes i.e. change in home location of patient, increase or decrease in number of patients.

2.1.7 Cost comparison of Home health care

In this section an economic analysis is presented in detail. Reducing costs by avoiding admission to hospital and decreasing hospital length of stay are often presented as central goals of HHC. Different researchers have thus been interested in the economical evaluation of home health care.

M. Hensher *et al.* [53] studied three HHC schemes in West London for early discharge orthopaedic patients, comparing their costs with those of hospital care and showed that HHC might substitute for hospitalization but it would not necessarily save the resources. R. Remonnay *et al.* compared the costs of anti-cancer chemotherapy in home versus hospital care in a French area. The results showed that the interest of developing HHC in chemotherapy was questionable as regards costs [54]. F. W. J. M. Smeenk *et al.* [55] showed that HHC program significantly reduced drug and re-hospitalization costs while increasing standard community nursing care and home help costs, when compared to the standard care available in the Netherlands. However, on the whole there was no significant difference in the total sum of health care costs. In view of this F. W. J. M. Smeenk *et al.* [56], and that of the previous study had shown that the intervention contributed significantly to a better quality of life of patients and their direct healthcare workers, the implementation of a HHC program for terminal cancer patients was recommended for all hospitals with a large multidisciplinary oncology unit in F. W. J. M. Smeenk *et al.* [57].

J. Jones *et al.* [58] concluded that HHC structure could deliver care with similar or lower costs than the traditional hospitalization for an equivalent admission. O'Brien and Nelson [59]

also conducted a comparison between the traditional hospitalization costs and HHC costs for elderly people who needed acute care. Their conclusion was that HHC was less expensive than the traditional hospitalization as it allowed the saving of 30 billion dollars per year. After that, A. Aligon *et al.* [52] had compared the average costs of nursing care within HHC context and the traditional hospitalization context between 2005 and 2007. The results of this study clearly proved that HHC was less expensive than traditional hospitalization. Another economic analysis had been developed by A. Vergnenègre *et al.* [60] in which the authors had compared the costs of the chemotherapy delivered to patients suffering from bronchipulmonary cancers at home and in hospital. The results of this study proved that HHC allowed the saving of 16.15% of the chemotherapy costs per treatment's cycle compared to the traditional hospitalization.

C. R. Verjan *et al.* [61] showed to have at least an equivalent biomedical impact and quality of life for some pathology in HHC, but this point had to be secure for each pathology and treatment. When bio-medical impact was equivalent, patient's satisfaction was higher at home.

The economic factors that considerably increase costs must be analyzed in order to compare the costs of both types of hospitalization as pertinently as possible. L. O. Brien and C. W. Nelson [59] had enumerated these factors. On one hand, the four factors that explained the increase of the traditional hospitalization's costs had been presented: medical errors (annual additional costs of 200 billion dollars), hospital acquired infections (the annual costs related to the infections were estimated to 4 billion dollars), decline of patients' autonomy (the hospital stay of 75% of the patients aged of more than 75 years old was extended by 12.3 days that corresponded to 4.233 dollars per patient and per day) and death rate.

On the other hand, the authors had explained that HHC costs could increase due to the risks that patients made errors for taking the corresponding drugs, for using a medical equipment, etc. during the absence of the home healthcare workers. HHC costs could also increase due to the additional costs related to the home fitting; home support; transportation services; acquisition of non-medical equipment such as special chairs, ramps into the house, adapted toilets, showers, baths, special beds, etc. given by R. (Tarricone and A. D. Tsouros [27] and A. Aligon *et al.* [52]).

In terms of costs even if a lot of studies showed that HHC was cheaper than traditional hospitals, costs studies were very sensitive to the point of view of costs and to the kind of costs included in the research. A lot of assumptions must be made and it could not be made sure that the conclusions would be unchanged when these assumptions were not valid anymore (N. Germani *et al.* [48]).

Like it was pointed by C. D. Amstrong *et al.* [63], one of the difficulties of a HHC was the lack of pressure to discharge patients from the hospitals. This was very important since the exams were often made in the hospitals, the patients arrived at the hospitals and a lot of doctors worked at hospitals. It was very probable that patients stayed there and it was very difficult to pressure doctors to give their patient to another institution or structure where they might not be implicated.

The cost-effectiveness of HHC is debated. This is partly because HHC is not a homogeneous entity. And economic analyses had focused on different service models, conditions and treatments, ranging from acute to post-acute and rehabilitation. The results of economic analyses were therefore not necessarily comparable between studies. Also, HHC is a diverse entity and varies in definition from state to state and country to country. As such, it is difficult to apply the findings of any particular study to HHC as a general form of care.

From our literature study, we find that the choice of care structure for a patient is not simple and depends on many criteria and constraints.

2.2   Literature Review for Solution Methodologies

Now we see which solution methods can be used solving the problems arisen in health care network and more specifically in homecare.

- Exact Methods
- Heuristics
- Metaheuristics

2.2.1 Exact Methods

The exact methods are used to find optimal solutions for a combinatorial optimization problem and these prove the optimality of the solution or show that there is no optimal solution for the considered problem. It is guaranteed that solution obtained by exact method is optimal solution and one can prove its optimality for every finite size instance of a combinatorial optimization problem within an instance-dependent, finite run-time, or one can prove that considered problem has not a feasible solution [65].

Integer Programming (IP) is a linear programming, where all decision variables are integers. These are exact methods which use the integer characteristic of the decision variables. Some well known exact methods are implicit total enumeration, Divide and Conquer, Tree search-space, Branch-and-Bound, Constraint programming, Branch-and-Price, Branch-and-Cut, Dynamic Programming and Lagrangean Relaxation [67].

There is a significant improvement in performance of exact methods for some combinatorial problems (see for example [68] for the Travelling Salesman Problem). One can get these advantages from IP exact methods (i) if the algorithm gets solution, one can get proven optimal solutions (ii) if the algorithm does not succeed, one can obtain valuable information on the upper or lower bounds on the optimal solution. IP methods can be used as approximation methods if one defines a stopping criterion to stop algorithm before solving the problem or before its completion (iii) IP methods can guide to eliminate the parts of the search space where optimal solutions are not located.

There are also many disadvantages of exact methods, despite the known successes. Firstly, the exact methods are practical for problems with instances of limited size. If one increases size of instance, the computational time increases strongly. Secondly, the exact algorithms use very large memory and may go towards the early abortion or termination of a program. Thirdly, many good exact algorithms for combinatorial optimization problems are problem specific and experts in linear programming and algorithms can only develop these algorithms. Finally, it is often difficult to extend the good performance exact algorithms of one problem to another new problem, if some details of the problem formulation are changed. It is evident

by literature review that an exact method can solve huge problem instances very fast for some problems, while for some other problems it could not solve even small size instances [65].

Another advantage of exact methods is that good research code like Minto [69], or powerful, general-purpose commercial tools like CPLEX [70] are available, which can often find solutions for some combinatorial problems. Many general, advanced solvers and modeling languages are available (XPRESS, COIN-OR, AMPL, EXCEL, MPL, GAMS, etc.) for discrete optimization. But the actual implementations of these solvers require knowledge in both modeling and solution methods.

2.2.2 Heuristics

Heuristic is used to find a good enough solution quickly for the considered problem. This may not necessarily construct best solution of all the actual solutions to this problem. This gives simply an approximate to the exact solution, but it is still worthwhile because finding it does not require a long computational time. The major benefit of using heuristics is its simplicity and the relatively less computational time. Therefore good initial solutions or seed values can be found with heuristics. These can be used in combination with other optimization algorithms to improve their efficiency as well.

Many complex optimization problems are NP-hard [71]. One needs to get solutions of the real-world applications of these problems. Thus the use of heuristics can be a good option.

Decision of using a heuristic for solving a considered problem includes a compromise on certain criteria which are given in the following [72]:

- *Optimality:* When there are several solutions for a considered problem, does the heuristic guarantee that the best solution is found? Is there actually need of the best one?
- *Completeness:* When there are several solutions for a considered problem, does the heuristic find them all? Is there actually need to find all solutions? Many heuristics are meant to find only one solution.
- *Accuracy and precision:* Does the heuristic provide a confidence interval for the solution? Is the error bar on the purported solution unreasonably large?

- *Execution time*: Is the purposed heuristic best known heuristic for solving this type of problem? Some heuristics convergence is faster than others. Some heuristics are only marginally quicker than classic methods.

In some cases, it is difficult to apply heuristics, because a heuristic may provide a good solution but it does not theoretically elaborate it. So, one is not assured about the quality of a solution found by the heuristics.

2.2.3 Metaheuristics

A metaheuristic is a set of concepts which can be applied to define heuristic methods that can be used to a wide set of different problems. It means that, a metaheuristic can be considered as a general algorithmic framework that can be applied to different optimization problems with relatively few modifications to make them adapted to a specific problem. [73].

Christian Blum and Andrea Roli [74] presented and compared most important metaheuristic methods and outlined fundamental properties that characterize metaheuristics:

- Metaheuristics are strategies that guide the search process.
- The goal is to efficiently explore the search space in order to find (near) optimal solutions.
- Techniques which constitute metaheuristic algorithms range from simple local search procedures to complex learning processes.
- Metaheuristic algorithms are approximate and usually non-deterministic.
- They may incorporate mechanisms to avoid getting trapped in confined areas of the search space.
- The basic concepts of metaheuristics permit an abstract level description.
- Metaheuristics are not problem-specific.
- Metaheuristics may make use of domain-specific knowledge in the form of heuristics that are controlled by the upper level strategy.
- Today's more advanced metaheuristics use search experience (embodied in some form of memory) to guide the search.

Different metaheuristics search strategies are highly dependent on the philosophy of the metaheuristic itself. Several different philosophies exist in the existing metaheuristics. Some of them can be considered as extensions of local search algorithms. The main purpose of this type of metaheuristic is to get away from local minima in order to proceed in the exploration of the search space and to move on to search other hopefully better local minima. This can be seen in the case of Iterated Local Search, Variable Neighbourhood Search and Simulated Annealing. These metaheuristics use one or several neighbourhood structure(s) imposed on the solutions of the search space. One can find a different philosophy in algorithms like Evolutionary Computation and Ant Colony Optimization, because a learning component is incorporated in the sense that they implicitly or explicitly attempt to learn correlations among decision variables to identify high quality areas in the search space. They perform, in a sense, a biased sampling of the search space, for example, this is achieved by recombination of solutions in Evolutionary Computation.

Now we give a detailed description of the most important used metaheuristics.

- Variable Neighbourhood Search
- Simulated Annealing
- Evolutionary Algorithm

2.2.3.1 Neighbourhood system

Definition

If X is a topological space and p is a point in X, a neighbourhood of p is a subset V of X, which includes an open set U containing p, $p \in U \subseteq V$. This is shown in Figure 2.1.

The set of all neighbourhoods of a point is called the neighbourhood system at the point. If S is a subset of X then a neighbourhood of S is a set V which includes an open set U containing S (as given in definition). It means that a set V is a neighbourhood of S if and only if it is a neighbourhood of all the points in S. Furthermore, it follows that V is a neighbourhood of S if and only if S is a subset of the interior of V [120].
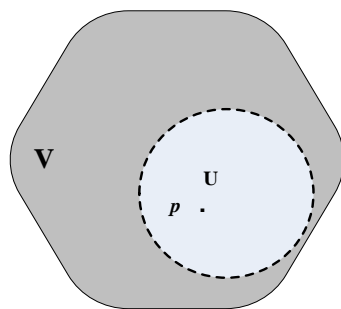
Figure 2.1: A set V in the plane is a neighbourhood of a point *p* if a small disk around *p* is contained in V.

neighbourhood for x_n
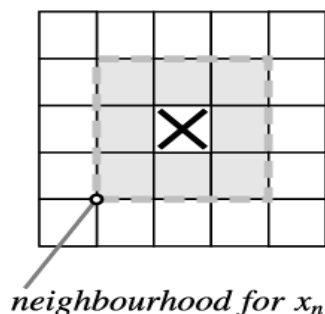
Figure 2.2: Representation of neighbourhood of a solution

2.2.3.2 A locally minimal solution or local minimum

Definition: A local minimum of a function *f* is a value *f(c)* of *f* where $f(x) \geq f(c)$ for all *x* in some neighbourhood of *c;* if *f(c)* is a local minimum, *f* is said to have a local minimum at *c*.

2.2.3.3 Variable Neighbourhood Search

Metaheuristics are general framework to build heuristics for combinatorial and global optimization problems. Variable Neighbourhood Search [75, 76, 77, 78, 92] is a valuable metaheuristic. Variable Neighbourhood Search (VNS) explores different neighbourhoods of the current incumbent solution to find new improved solution. It exploits systematically the idea of neighbourhood change, both in descent to local minima and in escape from the valleys which contain them. One applies Local search method repeatedly to get from solutions in neighbourhood to local optima. VNS is designed for finding solutions of continuous and discrete optimization problems. According to these, it is aimed for solving linear

programming problems, integer programming problems, nonlinear programming problems and mixed integer programming problems.

VNS strategy is to iteratively explore larger and larger neighbourhoods for a given local optima until an improvement is found. According to N. Mladenovic [80], VNS is built upon the following perceptions:

FACT 1. A local minimum with respect to one neighbourhood structure is no necessarily so for another.

FACT 2. A global minimum is a local minimum with respect to all possible neighbourhood structures.

FACT 3. For many problems local minima with respect to one or several neighbourhoods are relatively close to each other.

This last observation is empirical, implies that a local optimum often gives some information about the global optimum. For example the both optima may have several variables with the same value. However, it is usually not known which ones are such. An organized study of the neighbourhood of this local optimum is therefore in order, until a better one is found.

Y. Chen *et al*. [81] presented a very good illustration of concept of VNS algorithm graphically. This is presented in Figure.2.3. This shows the traditional concept of VNS. In an example of minimizing the makespan during job production sequence decisions, VNS uses shaking approach to generate the x variable. VNS is followed with a localized search, beginning at the first search area ($N_1 (x)$) and constantly spreading. VNS demonstrates its ability to avoid the regional local makespan solution.

2.2.3.3.1   *Basic Algorithm*

P. Hansen and N. Mladenovic [82] had given the basic algorithm for VNS shown in Figure 2.4. According to this basic algorithm, firstly a series of neighbourhood structures are chosen. These define different neighbouring points in the solution space. Then the local search is applied which leads to a local minimum say x. By the definition of the first neighbourhood of x, a point x' is selected at random and a local search with gradient is applied. This gives a new

local minimum x". Now there are three possible outcomes: (i) x" = x, i.e. one is again at the bottom of the same valley; in this case the procedure is iterated using the next neighbourhood (ii) x'' is different than x but having equal or greater objective function value i.e. another local optimum has been found, which is not better than the previous best solution (or incumbent); in this case too, the procedure is iterated using the next neighbourhood; (iii) x" is different than x but with lesser objective function value i.e. another local optimum, better than the incumbent has been found; in this case the search is continued around x" and begins again with the first neighbourhood. This procedure is iterated until a stopping condition, for example a maximum time or maximum number of iterations or maximum number of iterations since the last improvement, is satisfied.

2.2.3.3.2   *Variants of VNS*

- The variable neighbourhood descent

The variable neighbourhood descent (VND) [83] method can be obtained by changing neighbourhoods in a deterministic way. In the explanations of their algorithms, they considered that an initial solution x is already given. Few neighbourhoods are used in most of the local search heuristics in their descent phase. The final solution is a local minimum with respect to all neighbourhoods; hence to get a global one have larger chances when using VND in comparison with a single neighbourhood structure.

- The reduced VNS

The reduced VNS (RVNS) [84] method is obtained if random points are selected from $N_k(x)$ and no descent is made. The values of these new points are compared with the incumbent points and one makes an update in case of improvement. A stopping condition can be chosen like maximum number of iterations between two improvements or the maximum CPU time allowed.

To simplify the description of the algorithms it is used t_max below. Therefore, RVNS uses two parameters: t_max and k_max. RVNS is useful in very large instances, for which local search is costly. It has been observed that the best value for the parameter k_max is often 2.
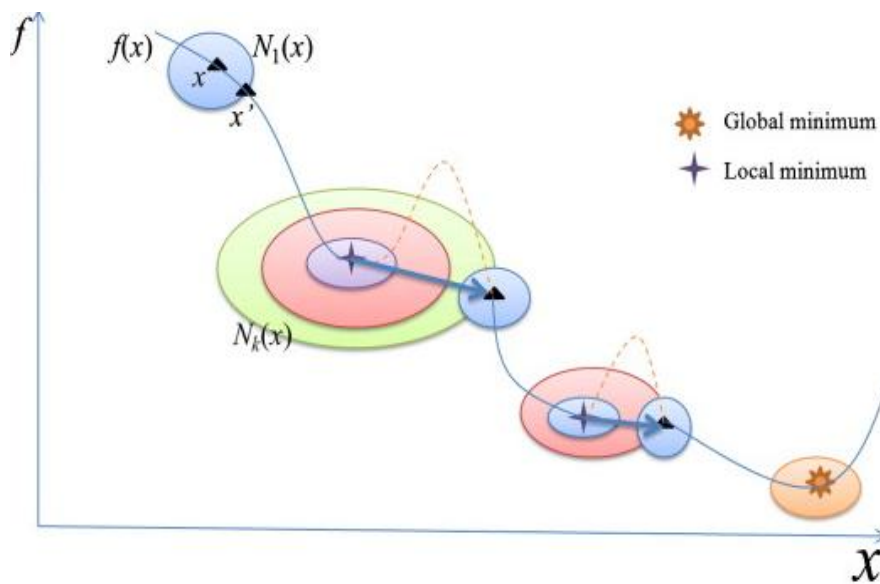
Figure 2.3: Basic VNS concept Diagram [81]

Initialization. Select the set of neighbourhood structures $N_k, \forall k = 1,...,k_{max}$ that will be used in the search; find an initial solution *x;* choose a stopping condition;

Repeat the following sequence until no improvement is obtained:

1) Set *k=1*;
2) *Repeat* the following steps until *k= kmax*
   a) *Shaking.* Generate a point *x'*at random from the *kth* neighbourhood of *x (x' in Nk(x))*;
   b) *Local search.* Apply some local search method with *x'* as initial solution; denote with *x''* the so obtained local optimum;
   c) *Move or not.* If the local optimum *x''* is better than the incumbent *x,* move there *(x = x''),* and continue the search with N_1 *(k= 1)*; otherwise, set *k= k+1.*

Figure 2.4: Steps of the basic VNS [82]

In addition, the maximum number of iterations between two improvements is usually used as a stopping condition. RVNS is akin to a Monte-Carlo method, but is more systematic.

- Skewed VNS

The skewed VNS (SVNS) method in P. Hansen *et al.* [85] addresses the problem of exploring valleys far from the incumbent solution. Indeed, the purpose of the algorithm is to go some way else to obtain an improved solution, once the best one in a large region has been found. Solutions drawn at random in distant neighbourhoods may differ substantially from the incumbent and VNS can then degenerate, to some extent, into the Multistart heuristic (in which descents are made iteratively from solutions generated at random, a heuristic which is known not to be very efficient). Consequently, some compensation for distance from the incumbent must be made.

- Variable Neighbourhood Decomposition Search

The variable neighbourhood decomposition search (VNDS) method in P. Hansen *et al*. [86] extends the basic VNS into a two-level VNS scheme based upon decomposition of the problem. For ease of presentation, but without loss of generality, it is assumed that the solution *x* represents the set of some elements.

- Parallel VNS

Several ways of parallelizing VNS have recently been proposed for solving the p-Median problem. In F. Garcia-Lopez *et al*. [87] three of them are tested: (i) parallelize local search; (ii) augment the number of solutions drawn from the current neighbourhood and make a local search in parallel from each of them and (iii) do the same as (ii) but update the information about the best solution found. Three Parallel VNS strategies are also suggested for solving the travelling purchaser problem in L. S. Ochi *et al*. [88].

- Primal-dual VNS

For most modern heuristics, the difference in value between the optimal solution and the obtained one is completely unknown. Guaranteed performance of the primal heuristic may be determined if a lower bound on the objective function value is known. To this end, the

standard approach is to relax the integrality condition on the primal variables, based on a mathematical programming formulation of the problem.

However, when the dimension of the problem is large, even the relaxed problem may be impossible to solve exactly by standard commercial solvers. Therefore, it seems a good idea to solve dual relaxed problems heuristically as well. It was obtained guaranteed bounds on the primal heuristics performance. In Primal-dual VNS (P. Hansen *et al.* [89]), one possible general way to attain both the guaranteed bounds and the exact solution was proposed.

- Variable Neighbourhood Branching. [90]

The mixed integer linear programming (MILP) problem consists of maximizing or minimizing a linear function, subject to equality or inequality constraints, and integrality restrictions on some of the variables.

- Variable Neighbourhood Formulation Space Search [91].

Variable Neighbourhood Formulation Space Search (FSS) is a method which is very useful because, one problem can be defined in additional formulations and moving through formulations is legitimate. It is proved that local search works within formulations, implying a final solution when started from some initial solution in first formulation. Local search systematically alternates among different formulations and it is investigated for circle packing problem where stationary point for a nonlinear programming formulation of this problem in Cartesian coordinates is not strictly a stationary point in polar coordinates.

2.2.3.3.3   Applications of VNS

Applications of VNS, or of varieties of VNS are very abundant and numerous. We mention some fields where it can be found collections of scientific papers: knapsack and packing problems, mixed integer problem, time tabling, scheduling, vehicle routing problems, arc routing and waste collection, fleet sheet problems, extended vehicle routing problems, problems in biosciences and chemistry, continuous optimization and other optimization problems.

2.2.3.3.4   Features of VNS

VNS implies several features which are presented in P. Hansen and N. Mladenovic [92] and we are presenting some here:

(i) Simplicity: VNS is a simple and clear which is universally applicable;

(ii) Precision: VNS is formulated in precise mathematical definitions;

(iii) Coherence: all actions of the heuristics for solving problems follow from the VNS principles;

(iv) Effectiveness: VNS supplies optimal or near-optimal solutions for all or at least most realistic instances;

(v) Efficiency: VNS takes a moderate computing time to generate optimal or near-optimal solutions;

(vi) Robustness: the functioning of the VNS is coherent over a variety of instances;

(vii) User friendliness: VNS has no parameters, so it is easy for understanding, expressing and using;

(viii) Innovation: VNS is generating new types of application.

(ix) Generality: VNS is inducing to good results for a wide variety of problems;

(x) Interactivity: VNS allows the user to incorporate his knowledge to improve the resolution process;

(xi) Multiplicity: VNS is able to produce a certain near-optimal solutions from which the user can choose;

2.2.3.4 Genetic Algorithm

Genetic Algorithms are search algorithms that are based on concepts of natural selection and natural evolution. The idea of genetic algorithm is to simulate the process observed in natural

evolution. This process operates on chromosomes (organic devices for encoding the structure of living being). The genetic algorithm differs from other search methods in that it searches among a population of points and works with a coding of parameter set, rather than the parameter values themselves. It also employs the information about objective function without any gradient information. The transition scheme of the genetic algorithm is probabilistic, whereas traditional methods use gradient information. Because of these features of genetic algorithm, they are used as general purpose optimization algorithm. They also provide means to search irregular space and hence are applied to a variety of function optimization, parameter estimation and machine learning applications [105].

This is the most famous type of Evolutionary Algorithm (EA). The solution of a problem is found in the form of strings of numbers (traditionally binary, although the best representations are usually those that reflect characteristics of the problem being solved), by applying operators like crossover and mutation. This type of EA is often used in optimization problems.

First a representation is chosen to encode solutions of the problem being solved such as binary encoding as strings of 0s and 1s [93], but other encodings can also be used. A genetic algorithm begins with a population of candidate solutions (called individuals, creatures, or phenotypes). This population is evolved toward better solutions in each generation. Each candidate solution possesses some properties which can be exchanged or altered via application of genetic operators.

The first generation is usually generated randomly. In each generation, the fitness of every individual in the population is evaluated, the better individuals are stochastically selected for modifying their genome (during recombination and mutation) to form a new population. The procedure is iterated to form a new generation during each iteration of the algorithm. The algorithm terminates usually on achieving some stopping criteria which can be either a maximum number of generations or a satisfactory fitness level for the population.

The basic requirements of a typical genetic algorithm are:

  i.   a genetic representation of the solution,
 ii.   a fitness function to evaluate the solution

A candidate solution can be represented as an array of bits [93]. The benefit of this standard representation is its property that their parts are easily aligned due to their fixed size, which makes it suitable for simple crossover operation. The length of representations may also be variable but crossover operation becomes more complex in this case. The other used representations are tree-like representations in genetic programming and graph-form representations in evolutionary programming; a mix of both linear chromosomes and trees is explored in gene expression programming.

The definition of fitness function is dependent on the problem being solved. After choosing the genetic representation and the fitness function, a genetic algorithm begins with initializing a population of solutions and then to improve it through implementation of the mutation, crossover and selection operators in an iterative process.

2.2.3.4.1 *Initialization*

Genetic algorithms are used generally with an initial population. The population size is chosen according to the problem in such a way that we can get abundant possible solutions. The initial population sometimes is generated randomly. The purpose of random initialization is to allow a wide range in possible solutions. Sometimes these initial solutions may be repaired by using special techniques [62, 66, 100, 110]. Another possibility is the solutions may be chosen from an area of solution space where optimal solutions are likely to be found.

2.2.3.4.2 *Selection*

After generating of initial population in beginning or during each following generation, two or more individuals are chosen to breed or recombine. The criteria of selection are usually based on fitness or objective function value of the individual solutions. The most used methods are elitism, roulette wheel selection, random selection and tournament selection. Elitism rates the fitness of each solution and selects the best solutions [111]. Roulette wheel selection is a probability based method. In this method, preference is given to good solutions but the bad solutions have chances to be selected as well [113]. In tournament selection, a specific number of chromosomes are selected from the population, and then their fitness values are compared and one having best fitness value will go for the next generation [112, 114]. Some

authors used only a random sample of the population, as the former processes may be time-taking.

The definition of fitness function is over the genetic representation of the problem being solved and shows the quality of the represented solution. The fitness function is always problem dependent. Sometimes it is hard to define a fitness function, so it can be replaced by a simulation to estimate the fitness function value of a phenotype.

2.2.3.4.3 *Genetic operators*

These selected parents breed and/ or mutate to generate a next generation of population of solutions. This process contains applying the genetic operators: crossover (also called recombination or reproduction), and/or mutation. In crossover, a child solution is created which inherits a part of its characteristics by each of its parents. The breeding process continues until a desired size of population is attained. For each crossover, new parents are selected for each new child. Traditionally two parents are selected to breed in genetic algorithm but some authors [94, 95] described that the selection of more than two parents for crossover might produce higher quality child solutions.

In classic genetic algorithm, mutation is applied on child solutions before putting them into population. This is a random change in the characteristics of the child solution. The purpose of mutation is to produce new solutions having characteristics different from the parent solutions. The importance of crossover versus mutation is not well decided. The modified genetic algorithms can be found which support the use of only mutation or only crossover [115]. Although crossover and mutation are the main genetic operators, other operators such as regrouping, colonization-extinction, or migration [96] may also be applied in genetic algorithm. These genetic operators create the next generation of population different from the initial generation. In general, the genetic algorithm results in the population with the increased average fitness value.

2.2.3.4.4 *Termination*

This generational process is repeated until a termination condition has been reached. Common terminating conditions are: maximum number of generations, maximum computational time

limit, specific number of successive iterations with no improvement in fitness function, finding the solution that satisfies minimum criteria, manual inspection or combination of these conditions.

2.2.3.5 Simulated Annealing

The simulated annealing algorithm is generally applied to problems for which the set of possible solutions is large. This is analogous to the process of physical annealing of solids in which a solid is heated and then allowed to cool slowly to achieve its minimum internal energy state. Simulated annealing copies this type of thermodynamic behaviour to find global minima for a discrete optimization problem. In simulated annealing, a path is generated through the solution space, from one solution to its neighbouring solution, reaching finally to the optimum solution. This path contains solutions chosen from the locality of the previously found solution by a probabilistic function of the improvement gained by this move. So a step may result in a better or worse solution, but each step has a certain probability of moving to improvement. In the beginning, all steps have equal likeliness. We accept all moves either resulting in better solution or worse solution. But as the algorithm proceeds, the tolerance for not improving solutions decreases, and in the end of algorithm only improving solutions are accepted. In this way the algorithm can find the optimum solution without becoming trapped in local optima [97]. For simulated annealing algorithm, the important factors are choice of cooling schedule, the acceptance probability function and choice of neighbourhood structure.

2.2.3.5.1 The Structure of a Simulated Annealing Algorithm

Simulated annealing process has several distinct steps during which the control parameter called temperature is reduced and randomness is applied to the input values. Figure 2.5 displays a flowchart of this process. There are two major processes that take place in the simulated annealing algorithm [116]. First, for each temperature, the simulated annealing algorithm goes through a number of cycles. The number of cycles is predefined parameter. In each cycle, we apply the randomness to the input values. For example in the travelling salesman problem, the input can be a permutation or list of the cities that the travelling salesman visits. The orders of cities i.e. solutions which improve the desired objective function are saved.

After completing the specified number of training cycles, the temperature is lowered. Once the temperature is lowered, it is compared with the lowest temperature allowed. If the temperature is not lower than the lowest temperature allowed, then the temperature is lowered and another cycle of randomizations will take place. Otherwise the simulated annealing algorithm terminates. This randomization takes place through an acceptance probability function and neighbourhoods moves. The acceptance probability function is directly proportional to the objective function value of a solution and it is inversely proportional to the number of iterations of simulated annealing algorithm.

This randomization process always depends on the problem being solved. So it is a difficult task to find an appropriate parameter values for a simulated annealing application. The following Table 2.1 represents analogy between annealing in physics and simulated annealing algorithm [104].

Original term	Analogous term	Original term	Analogous term
Physical System	Optimization Problem	Metastable state	Local optimum
System state	Solution	Rapid quenching	Local search
Molecular positions	Decision variables	Temperature	Control parameter T
Energy	Objective function	Careful annealing	Simulated annealing
Ground state	Global optimal solution		

Table 2.1: Analogy between the Physical System and the Optimization Problem [104]
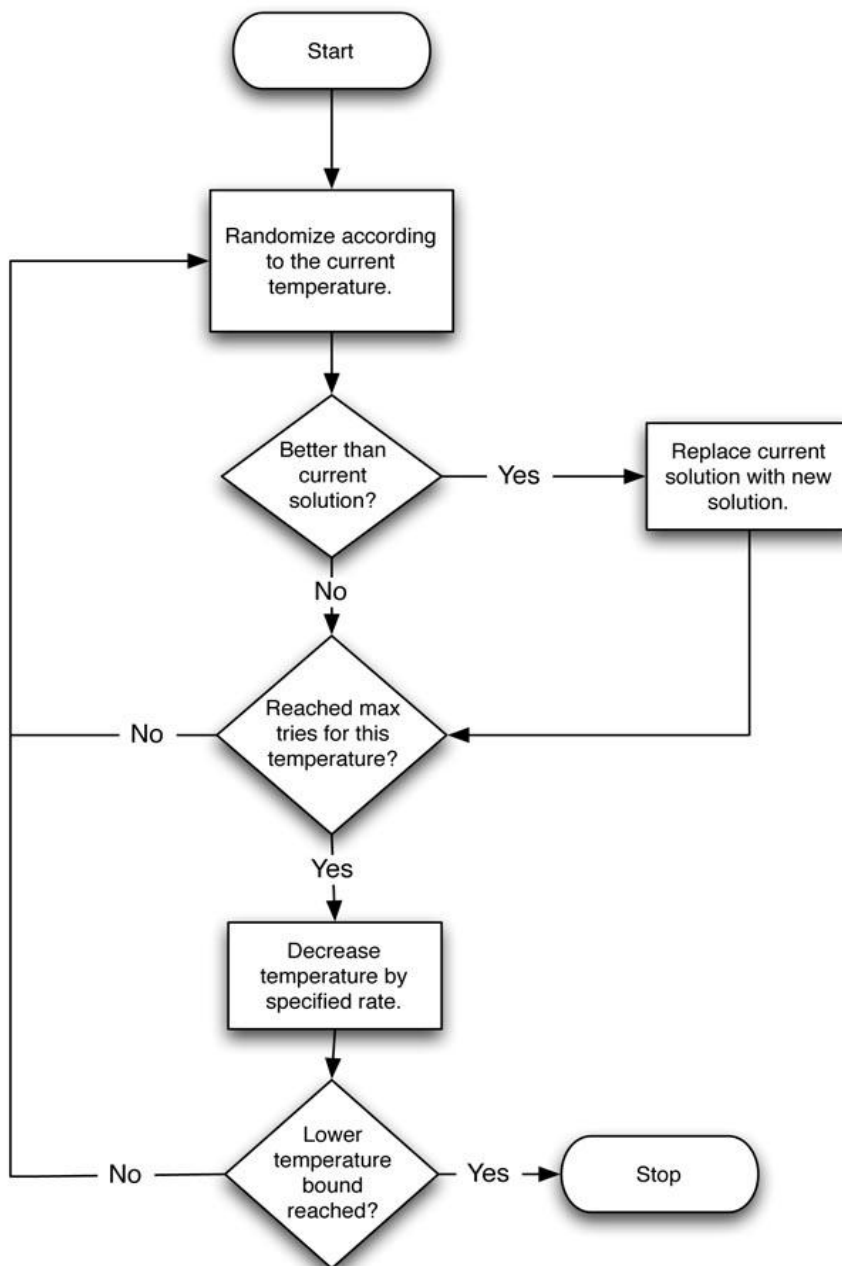
Figure 2.5: Overview of the simulated annealing process [116]

2.3 Conclusion

We conclude that majority of the literature work is about the assignment and routing problems. But the issues like resource dimensioning, home health care modeling and districting problem are less treated.

We have addressed the modeling of home health care issue in Chapter 3. Since home health care districting problem is related to daily planning and scheduling issues of home healthcare workers, we start our research work by the problem of assigning home healthcare workers to patients. This problem is studied in detail in Chapter 4. Then we have integrated the routing constraints in this assignment problem and proposed routing and scheduling problem in Chapter 5. And home health care districting is studied in the Chapter 6.

After choosing the potential problems for study, the next step is to apply appropriate solution methods for the sake of getting good solutions. On the basis of our literature survey, we think that neighbourhood based metaheuristics are better option for the home health care related optimization problems. The simplicity and robustness of these metaheuristics can help to find good solutions to these highly constrained problems. We also implement an evolutionary algorithm for the sake of comparison.

Chapter 3

3 MODELIING OF HEALTHCARE NETWORK AND HOME HEALTH CARE

Class diagram has wide variety of applications including both conceptual or domain modeling and detailed design modeling. It represents the static view of an application. Class diagram is used for visualizing, describing and documenting different aspects of a system. We have proposed class diagrams to model healthcare network, home health care (HHC) individually and then for a unified approach for integrating home health care in the traditional healthcare network. This unified model describes the decision making process of healthcare system. It starts from when a person needs healthcare service. He first seeks treatment by a general physician doctor for whom we use the term family physician in the model given later. If this person needs formal healthcare, his family physician along with help of evaluations center and patient's personal criteria choose an appropriate type of healthcare structure for the patient. It comprises a number of successive decisions with alternative options. This study of modeling the decisions of home health care referral may play an important role in examining the determinants of demand for healthcare services.

This chapter is organized as follows. In Section 3.1 we give model of home health care and healthcare network. Section 3.2 proposes a unified approach for health care system and discusses the criteria and benefits of the proposed approach in real-world setting. In section 3.3, we give how operations research can be applied in home health care context. Section 3.4 provides a simplified form of Location-Allocation (LA) problem. Lastly, the concluding remarks and the future research directions are given in Section 3.5.

Most developed countries have been trying to integrate different components of healthcare as a solution to some problems of the health care systems [121, 122]. We propose a unified approach for the optimization of health care system and introduce a simplified version of location allocation problem called patient allocation problem. We discuss the criteria for

applying this new approach and conclude its effects on the health care system. Our problem is to find a system which can deal these two problems simultaneously.

3.1 Model of Home health care and healthcare network

Home health care or Hospital at home is a form of health care service which is provided at patient's home. Patients can receive home health care services whether they live in their own homes, with or without family members, or in an assisted living facility. The purpose of home health care is to promote, maintain, or restore a patient's health and reduce the effects of disease or disability. We used class diagrams to represent healthcare network and home health care. The purpose is to understand all the elements of each system and its specific role. A class diagram of home health care is given in Figure 3.1. We can see the different entities of home health care and their relationship with each other.
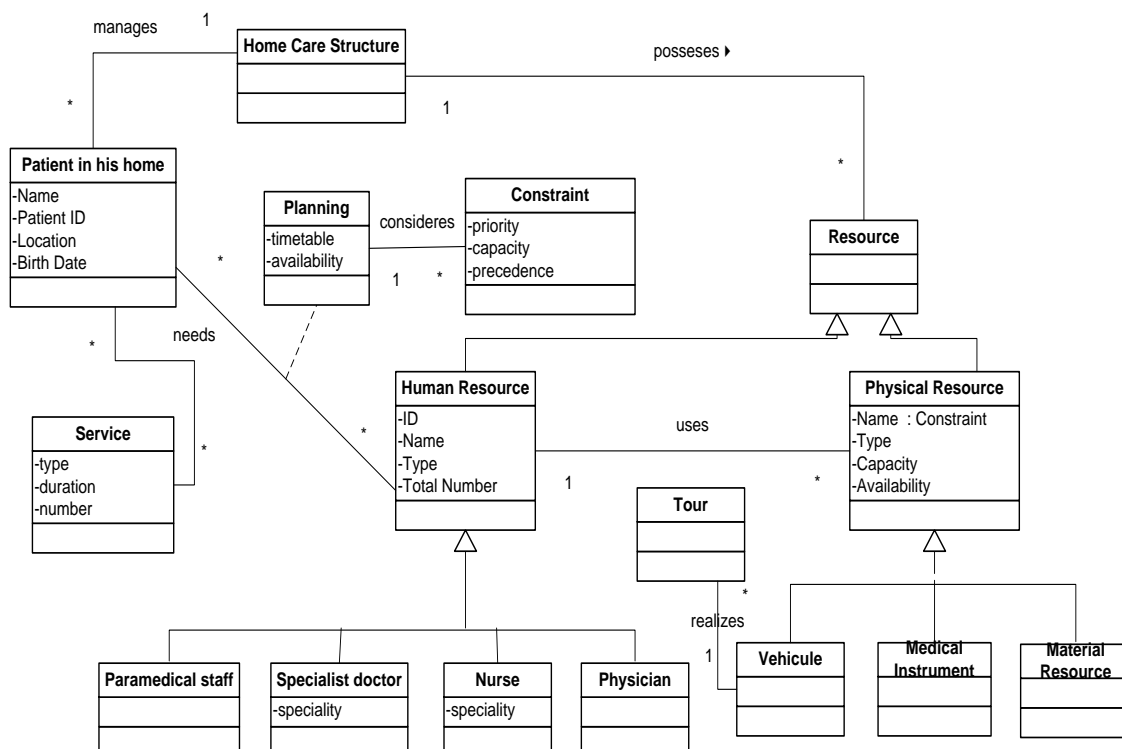
Figure 3.1: Class diagram of Home health care

Health Care Network means complex of facilities, organizations, and trained personnel engaged in providing health care within a geographical area. Here we have combined different healthcare structures such as clinics, community hospitals, small healthcare centers, university medical hospitals and regional hospitals into a hospital network.

We have modelled the similar human resources and physical resources for our models of home health care and healthcare network. The major difference between them is the place of treatment of patient. A class diagram of Health Care Network is given in Figure 3.2.
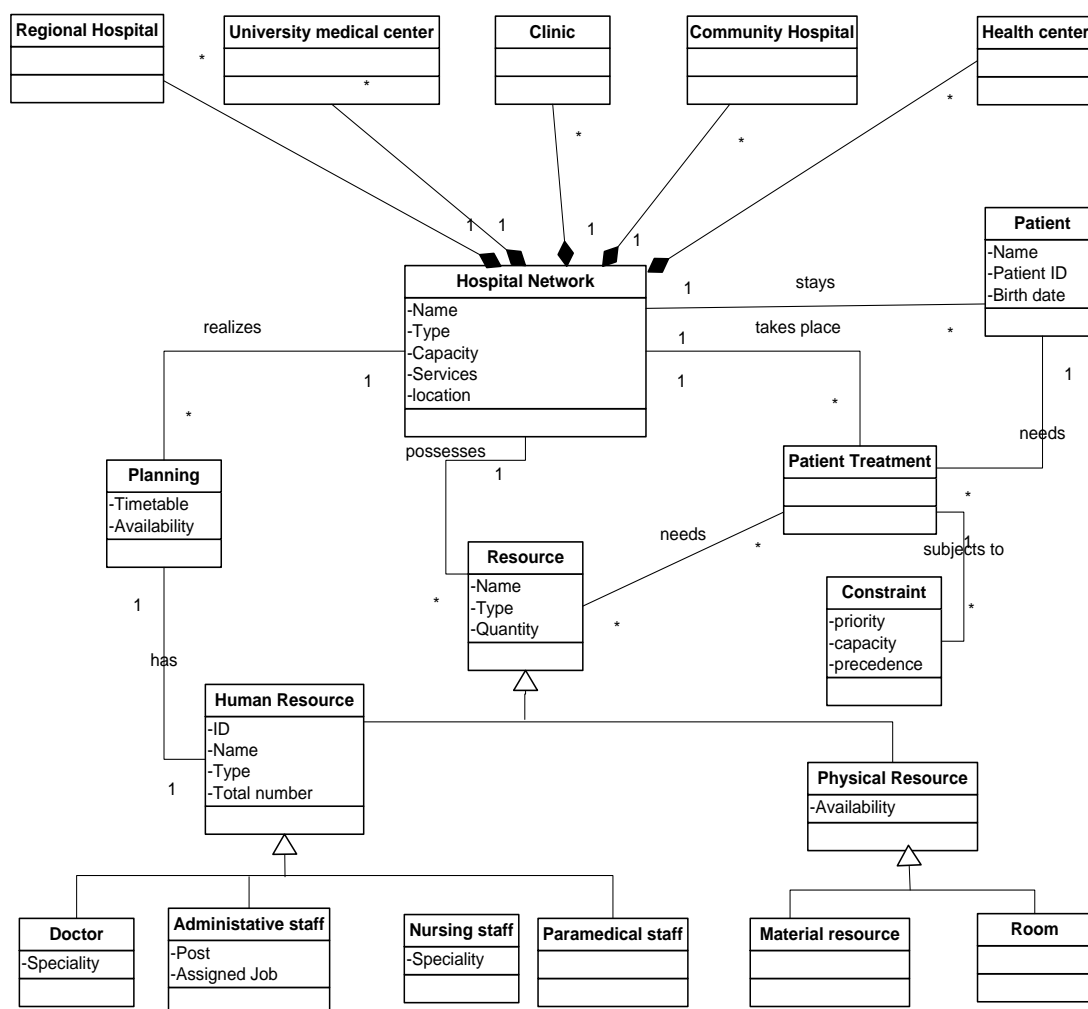
Figure 3.2: Class diagram of Health Care Network

3.2 Proposition of an Unified Approach

Congestion at hospitals, the increase in demand for care and the aging of the population are all factors that lead to reconsider the organization of the health care system and the emergence of new structures for the health care system. Due to the large number of actors involved in the process of care, the variety of clinical and organizational decisions and the difficulty of synchronizing human and physical resources, management of health care system is a difficult problem. Our problem is to find the more efficient system not only in terms of costs but it also includes other criteria like bio-medical effect, quality of life and patient's satisfaction. For this purpose, we can introduce an intermediate structure called Evaluation Center between hospitalization and home health care.

If we combine the health care network (Figure 3.2) and home health care (Figure 3.1) into a unified structure as shown in Figure 3.3 and Figure 3.4, this can result in a much better use of health care system. The Evaluation Centers consisting of a team of case managers, operation research (OR) specialists and doctors for each population locality can be added. We can align health care case managers with family physicians through a formalized and structured partnership to create health teams uniquely equipped to provide optimal patient care. Their objective is to assign patients either to a traditional hospital network or home health care service. They can study each patient case in coordination with his family physician and patient himself. After completing the care in a chosen service, the patient file can be re-assessed to analyze for further proceeding which can be continuity of care in the same service or change of service or End of Treatment (if the patient is recovered).

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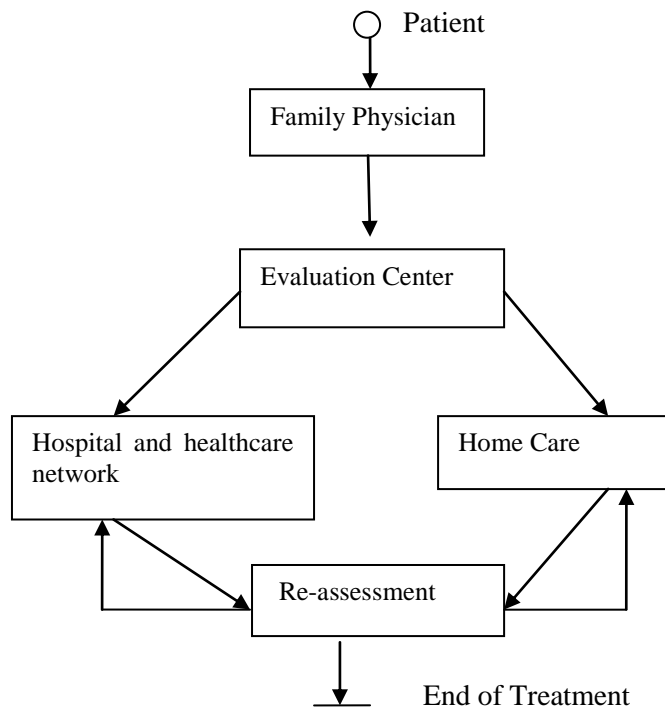
Figure 3.3: Unified approach of Health Care System

3.2.1 Criteria for Choosing the appropriate Service

For the evaluation purpose of the patient or client demand, there should be clearly defined criteria. These can include availability of uniform equitable health services, recent information of resources and demand, implementing of new information technology system, patient's choice, clinical criteria, social criteria, residential criteria and so on.

Good level of communication among different members of health care team and interdisciplinary collaboration are needed for this integrated approach. Sharing information among different members is the key element to build this unified approach. The effective information technology systems are required to connect patients, health care providers and

payer for the process of care. The role and duties of each participating member should be reconsidered. This process requires time and patience but the results will improve the efficiency by using everyone's professional skills in a better way.
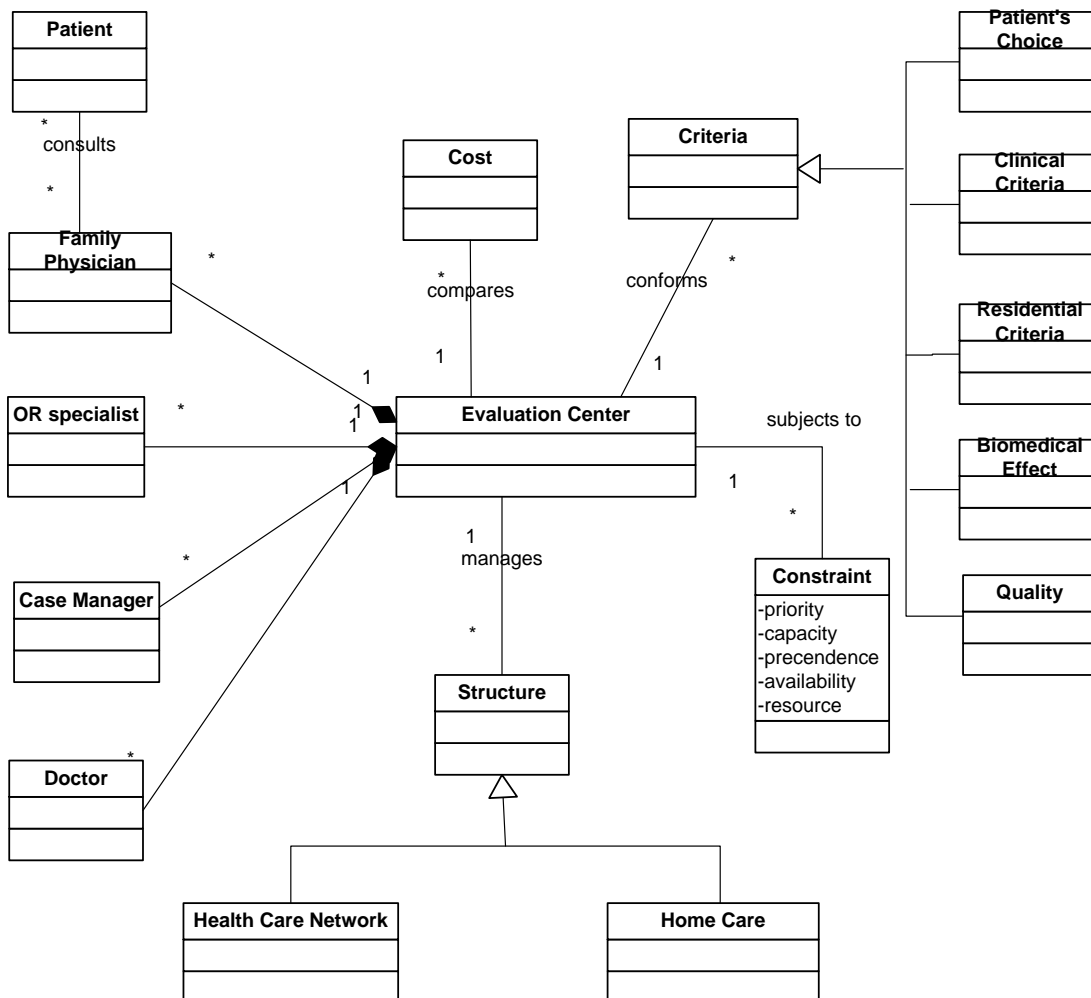
Figure 3.4: Class diagram of Unified health system

3.2.2   Benefits of the unified approach

This approach is relevant for all type of health care applications. Research suggests a number of outcomes as a result of the successful unification. There are benefits for the patients as well

as for healthcare provider and healthcare system. For instance, research indicates that integrated care can increase the satisfaction of patients by improving the quality of their interaction with the health care workers. The patients' confidence in the health system may increase. It will help to improve the quality of life of patients, greater self-management and support for the patient, their families and healthcare workers. This can improve the access to health care services and people can get the right service at the right time and place by improving the navigation of the health care system. These effects can result to better health levels of populations and communities in general.

Increased health care worker satisfaction and professional fulfilment can result in increased recruitment and retention of health care staff. The cross training and greater understanding, maximizing skills and fostering mutual respect for complementary skills will increase health care workers' knowledge. The communication, collaboration and shared accountability for patient outcomes can be increased. There will be a team-based and integrated approach to decision-making and care delivery process. This integrated approach establishes trusting partnerships where interaction is based on understanding and familiarity. Physicians can send increased number of patients to various more appropriate health care services when they work in association with a case manager. They need to spend less time in locating the required information. They are able to actively participate in important decisions related to the health of their patients. This can help to achieve balanced workload for health care workers in different healthcare sectors.

Health care system may use the resources in an optimal way. The shared decision making process and good communication lead to better care, better service choices for patients (both within home health care and within the hospital and health care network), and better utilization of scarce resources. The system can improve support of patients at health transition points. This integration may change health care process from a reactive approach to a proactive approach. The disease management with an emphasis on health promotion and earlier interventions can help to prevent the necessity of using more costly acute care and institutional services. The dependence on acute care for preventable health issues can be reduced. The overall system evaluation may lead to evidence based resource allocation and more effective health care spending. On the other hand, the weakness of this approach is the

need of high set up costs in the start for opening these evaluation centers and the implementation of policy change.

3.3 The role of OR in Home health care

The work of operation research (OR) specialists included in Evaluation Centers is to help in this decision making process by providing optimal solutions for the relevant combinatorial problems. Operations research techniques can help reduce health care costs by improving resource utilization, hospital patient flow, medical supply chain, staff scheduling, and medical decision making. Some of them are given as Location-Allocation problem, Problem of resource allocation, Problem of assignment of resources, Assigning health professional to a home health care patient, Care worker scheduling problem and Nurse routing problem etc.

OR specialists can use their knowledge to build a decision-making approaches for HHC structures in order to find the most efficient organisation in terms of resources. It consists in reducing costs related to staffs, transportation, inventory, etc. and maximising satisfaction level of clients and nurses. The HHC is characterized by many sources of uncertainties. In this setting, OR specialists can deal with a multi-criteria and stochastic problem to optimise resources. We have studied the Location- Allocation Problem and derived a particular case of LA problem which is given in the following.

3.4 Proposed application of LA problem

The problem with relying on LA problems for improving accessibility lies in the political, financial and logistical difficulty of reconfiguring a system of health facilities. Analysts are rarely faced with a green field situation. Often, health facilities are established in certain locations due to other factors besides maximal coverage of population, such as availability of an adequate labour pool, existing infrastructure or strategic government mandates. There are many reasons a region cannot reconfigure locations of a system of health facilities, rendering Location–Allocation models of limited use in these scenarios. So we assume that the health facilities are fixed in location.

The problem of allocating patients to the health facility is in our consideration. Among the all possible choices compatible in terms of required service and territory, we allocate the patients to health facilities in order to increase the accessibility and reduce the costs. We call this modified LA problem as patient assignment problem. This represents the traditional health care network system. We have given a class diagram of Health Care Network in Figure 3.5. This modified LA problem is more applicable in advanced countries than its original problem. Here we have presented that how we have deduced a simplified form of LA problem for healthcare. We shall give its application for home health care in the Chapter 4.

3.5   Conclusion and perspectives

The objective of this unified approach is to enhance the access to health care services, improve quality of care and lower overall health care expenditures. Through this integration, the goals of improved efficiency and better organizational management of the health care system can be achieved.

In this chapter, we have presented a unified approach for LA problem and home health care problem. This unification approach needs systematic planning to design strategies for policy change and to deal the difficulties in implementation. Also it needs more efforts for defining criteria for evaluation purpose. We think this is most appropriate approach to respond the increasing demand and challenges faced by the health care system.

The disadvantage of this approach is that its implementation is difficult to effect and slow to happen and sustainability of unifying strategies requires a willingness to change throughout the system. There is a change in policy as well as practice.

The perspectives of our work are resource dimensioning, mathematical or any other modeling approach for this system and optimization algorithm or a decision tool which can help us in assigning patients.

Chapter 4

4 HOME HEALTHCARE WORKERS ASSIGNMENT PROBLEM

4.1 Introduction

Home health care (HHC), i.e. visiting and nursing patients in their homes, is a growing sector in the medical service business. In this type of method, the care is provided to patients while living at their homes. The care providers and the patients can be at different geographical positions. One of the important tasks of HHC is to assign health professionals to patients at minimum cost, while satisfying various constraints. Since the health care workers travel to patients' homes, so costs can be minimized by reducing distances among the location of home healthcare workers' residence and patients' homes.

In literature we find different names for scheduling process of home healthcare workers. Some of these are delivery problem, assignment problem, planning and scheduling problem, routing problem, etc. The definitions of these problems are variable. Usually, every author describes the problem in terms specific to his real life organization. We have given a review of related works in Chapter 2 (from section 2.1.4 and section 2.1.5). The analysis of literature reveals that the scheduling of home healthcare workers in HHC involves two main decisions which can be performed hierarchically or simultaneously. First decision, home healthcare workers are assigned to patients and the second decision, the detailed schedules are prepared for all home healthcare workers who travel among different patients. In this chapter, we deal the first decision i.e. patient to home healthcare worker assignment and the second is dealt later in the Chapter 5.

Most of the works in literature perform these two decisions simultaneously. In doing so the assignment of health care workers is done with respect to skill level requirements and patient preferences for a particular home healthcare worker. But to our knowledge, none of them

considers the assignment process on the basis of geographical positions of patients and home healthcare workers. The feature that we add to this problem is to consider their geographical positions along with other constraints of skill level, workload balance and preferences. For this purpose, we divide all patients into groups on the basis of their locations. The transportation costs are one of the major cost factors for HHHC organizations. The consideration of geographical location during first decision (assignment of patients to home healthcare workers) can help to reduce the travel time and thus transportation costs as well. We also take into account the constraints of availability of home healthcare workers.

The healthcare workers assignment problem (AHHCW) is formulated as binary integer programming. The objective of our model is to minimize three types of costs (travelling costs, hiring costs and assigning costs). This model is solved by LINGO 6.0. Computational tests are performed for different scenarios in order to analyze the tradeoffs among different performance measures.

Facility location studies are generally devoted to the location of a set of resource or service facilities to optimally serve a given set of existing customer or demand facilities. We have used facility location model to find optimal number of home healthcare workers and to assign them to patients for one or more periods on the basis of their geographical positions in HHHC. HHC is the way of treatment in which a patient receives all his care at his home by health professionals. It replaces the traditional hospitalization or helps in the early discharge of a patient.

This chapter is organized as follows. Section 4.2 provides the description of the problem. Section 4.3 gives the assumptions of our problem and defines the relevant decision variables and parameters. Section 4.3 gives the constraints of the problem and mathematical formulations is given in section 4.4. Section 4.5 gives representation of solution. Section 4.6 provides the results of a rigorous computational testing of the model. Finally the concluding remarks and future research directions are given in Section 4.7.

4.2 Description of Problem

Now we describe home healthcare workers assignment problem (AHHCW) which consists of assigning patient groups to healthcare workers in different periods of the planning horizon. The objective is to find an optimal assignment with respect to travelling costs, hiring costs and assignment costs while taking into account of many constraints. In each period of planning horizon, we have a set of available health workers. AHHCW is the problem of hiring an optimal number of available home healthcare workers and then assigning some of them to different patient groups depending on the demand of each period. From the total number of available home healthcare workers, only subset of home healthcare workers with a specified number is assigned during a given period.

4.3 General assumptions for the problem

In this section, we propose the mathematical formulations for the AHHCW problem for which we present assumptions adopted and notations used in our case.

4.3.1 Assumptions

In our proposed model, we would consider the following assumptions without loss of generality:

A.1. A patient group is a set of patients living in the same location typically in the zip code area, postal area, geo-code address, etc. A patient group can contain one or more patients.

Indeed, patients whose therapeutics projects have similarities in terms of the expected duration of care, type, number and average duration of visits are assembled into the same group. The patients are divided into groups on the basis of the similar health care requirements so that a patient group can be assigned to the same home healthcare worker.

A.2. The distance that separates two patients of the same patient group is negligible.

A.3. A patient health care needs are assumed to be known for one period and does not change during that period.

A.4. The number of patients admitted to HHC is known in advance and does not change while considering the assignment problem.

A.5. All patients admitted to HHC in a period have to be assigned to required number of home healthcare workers in that period.

A.6. We assume that a home healthcare worker cannot be assigned to patient (i.e. incompatible for that patient) due to several reasons:

a) A home healthcare worker does not have the required skills to treat a patient.

b) Existence of geographical obstacles between them.

c) Difficulty or impossibility to travel from location of home healthcare worker residence to that of patient residence by the means of transportation used by home healthcare workers (public transportation, private cars, etc.).

4.3.2 Decision Variables/Output Variables

x_{ijk} : assignment decision variable and it is equal to 1 if a home healthcare worker j is assigned to patient group i in period k and 0 otherwise.

y_j is equal to 1 if a home healthcare worker j hired 0 otherwise.

v_{ik} is 1 if a patient group i is treated in period k and 0 otherwise.

u_{jk} is equal to 1 if a home healthcare worker j is assigned in period k and 0 otherwise.

4.3.3 Notations and Input Parameters

The necessary parameters to formulate the model are listed below.

I Set of patient groups

J   Set of available home healthcare workers, J= {1, 2... p}

p   Maximum number of available home healthcare workers

K   Set of periods, K= {1, 2... $\bar{k}$ }

$\bar{k}$ Total number of periods

$p_{k-\max}$ Maximum number of home healthcare workers hired in period k

$p_{k-\min}$ Minimum number of home healthcare workers hired in period k

w_{ik} Number of patients of a patient group i in period k

n_{ik} Number of home healthcare workers required for a patient group i in period k

o_{jk} Cost of assigning a home healthcare worker j in period k

t_{ijk} Transportation cost of travelling of home healthcare worker j to patient group *i* in period *k*

e_{ij} Compatibility index in terms of required qualification e_{ij} is equal to 1 if home healthcare worker j has required qualification to treat a patient group i and 0 otherwise.

L_j Minimum workload necessary to hire a home healthcare worker j

f_j Cost of hiring home healthcare worker j

4.4   Mathematical Formulation

$$\text{Min} \quad \mathbf{Z} = \sum_{i \in I}\sum_{j \in J}\sum_{k \in K} w_{ik} t_{ijk} x_{ijk} + \sum_{j \in J} f_j y_j + \sum_{k \in K}\sum_{j \in J} o_{jk} u_{jk} \qquad (1)$$

Subject to:

$$\sum_{j \in J} x_{ijk} = n_{ik} \qquad i \in I \qquad k \in K \tag{2}$$

$$x_{ijk} \leq u_{jk} \qquad i \in I \qquad j \in J \qquad k \in K \tag{3}$$

$$x_{ijk} \leq v_{ik} \qquad i \in I \qquad j \in J \qquad k \in K \tag{4}$$

$$u_{jk} \leq y_j \qquad j \in J \qquad k \in K \tag{5}$$

$$\sum_{j \in J} y_j \leq p \tag{6}$$

$$\sum_{j \in J} u_{jk} \leq p_{k-\max} \qquad k \in K \tag{7}$$

$$\sum_{j \in J} u_{jk} \geq p_{k-\min} \qquad k \in K \tag{8}$$

$$x_{ijk} \leq e_{ij} \qquad i \in I \qquad j \in J \qquad k \in K \tag{9}$$

$$\sum_{i \in I} w_{ik} x_{ijk} \geq L_j u_{jk} \qquad j \in J \qquad k \in K \tag{10}$$

$$x_{ijk} \qquad y_j \qquad u_{jk} \qquad v_{ik} \qquad \text{0-1 binary variables} \tag{11}$$

The objective (1) is to minimize the total cost, which includes three components: (i) the travelling cost of all the home healthcare workers, (ii) the fixed cost of hiring the home healthcare workers, and (iii) the costs of assignment of a given number of home healthcare workers during each period. Of course, the hiring costs may sometimes not be taken into consideration.

Constraints (2) show that each patient group must be assigned to the required number of home healthcare workers in a period. In constraints (3), patient groups are only assigned to the hired home healthcare workers, and constraints (4) shows that we should assign a home healthcare worker to a patient group who are admitted in HHC structure for that period. The constraints (5) show that we can hire only those home healthcare workers in all periods that are available. Constraints (6) show the total number of home healthcare workers available and constraints (7) specify the maximum number of home healthcare workers to hire during each period and constraints (8) specify the minimum number of home healthcare workers to hire during each period. Constraints (9) describe that a home healthcare worker j can be assigned to a patient group i only if that home healthcare worker is compatible with that group.

Constraints (10) describe the workload threshold that is required to hire any home healthcare worker j during period k. Constraints (11) show that the decision variables are 0-1 binary variables.

The model as introduced allows decision makers to pre-specify a number of hired home healthcare workers during the different periods while finding their best locations. If $k = 1$ and $p_k = p$ for all periods k then the description of the problem can be reduced to the uncapacitated facility location (UFL) problem. Since G. Cournejols *et al.* [117] had shown that UFL is an NP-hard problem; we deduce that our problem home healthcare workers assignment problem (AHHCW) as described is also NP-hard. The present formulation may also be viewed as a multi-period version of the UFL. It is a static formulation tackling a dynamic problem because of the presence of the different demands of each period. Since the location of the demand over time is known in advance, we do not deal with the dynamic facility location problem considered by T. J. Van Roy and D. Erlenkotter [118], in which the location in the next period depends on the previous location.

4.5   Solution Representation

A solution to AHHCW problem consists of a list of assigned patient groups for each home healthcare worker or is empty in case the home healthcare worker is not assigned for that particular period. We employ a simple solution generation heuristic that produces a random home healthcare worker-job assignment. The generated solution only guarantees that all jobs are executed by a home healthcare worker of adequate qualification. We receive an initial solution from this solution generation heuristics. However, such a solution is typically of rather poor quality as some aspects of the soft constraints in the objective function (such as short travel times) have not yet been considered appropriately. Additionally the instances of problem are not large therefore we have opted to solve AHHCW problem by means of LINGO which can give the optimal solution very quickly.

4.6   Computational Results

We have initially used the data given by M. Ndiaye and H. Alfares [119] for locating health care facilities for moving population groups. This is given in Table 4.1-Table 4.2. They used this data to solve a facility location problem. But here our problem (AHHCW) considers assignment of healthcare workers which considers additional issues of required number of healthcare workers and their qualification level. Since each patient group requires different number and qualification levels of healthcare workers. Therefore we edited their data to take into account these additional issues. For this purpose we generated the remaining data randomly that is explained in the following.

The compatibility matrix e (I,J) E(I,J) is generated as follows:

- We fix a weight r_max that represents, for each patient group i, the maximum ratio of incompatibilities with the home healthcare worker j (j=i+1): r_max {0, 0.05, 0.1,…, 0.3, 0.35 } (used a set or one fixed value).We randomly generate eij (j=i+1) such that the ratio of the number of zeros in the right part of the line i is less or equal to r_max.

Patient Group i	Location coordinates of Period 1		Location coordinates of Period 2		Number of Patients in group i	
	x	y	x	y	Period 1	Period 2
1	236	360	169	289	525	401
2	306	356	296	298	337	641
3	352	364	400	225	594	526
4	496	197	455	156	542	345
5	485	238	423	202	530	364
6	491	247	470	293	680	431
7	548	344	478	375	479	444
8	595	239	576	261	359	429
9	550	323	595	271	674	383
10	604	386	557	491	696	400
11	638	265	633	292	392	394
12	633	366	673	281	382	583
13	667	384	672	486	339	394
14	736	372	709	508	491	447
15	761	395	761	433	428	606
16	768	391	829	378	673	624
17	797	396	812	465	305	439

Table 4.1: Location coordinates and size of patient groups

- For each patient group i, we randomly generate number of home healthcare worker required in every period from a uniform distribution DU (0, 3).

Home healthcare worker j	Location coordinates		Workload threshold	Assignment costs o_{jk}		Hiring costs
	x	y	L_j	Period 1	Period 2	f_j
1	169	289	3023	110000	66000	115700
2	306	356	3518	56000	120000	125000
3	400	225	1633	130000	62000	270000
4	496	197	3297	65000	130000	110000
5	557	491	3204	100000	60500	175000
6	550	323	2883	46000	110000	230000
7	633	292	1761	120000	56000	110000
8	667	384	4103	46000	110000	170000
9	829	378	1887	120000	62000	180000
10	768	391	2472	62000	140000	240000

Table 4.2: Location coordinates and costs for home healthcare workers

To test the model, we do not focus on solving large instances of the problem. We rather try to describe the properties of the model, looking at the behaviour of the optimal solution under different conditions. A C++ program is used to generate the model in the format required by LINGO 6.0, which is used to solve different instances of the model. Using an Intel computer with 2.5 GHz CPU, the run times ranged between 1 and 2 s. The program is run for different

values of the vector (p, $p1$, $p2$) in order to analyze the optimal solutions and their general behaviour depending on the considered parameters. The problem is solved both with and without considering the hiring costs given in Table 4.2. Including hiring costs, the number of variables becomes 404with 1098 constraints.

We also consider the possibility for the decision makers to state either the minimum number denoted by ($\_j{\in}J$ $u_{jk} \geq p_{k-\min}$ $\forall k \in K$) or the maximum number ($\_j{\in}J$ $u_{jk} \leq p_{k-\max}$ $\forall k \in K$) of home healthcare workers to be assigned.

 The different computational results are summarized as in Table 4.3-4.8. We have solved this model in three following scenarios.

- − First scenario is with zero hiring costs, we call it Scenario 1
- − Second scenario is with zero hiring costs and relaxing compatibility constraints, we call it Scenario 2
- − Third scenario is with non-zero hiring costs, we call it Scenario 3

In each scenario, we considered two cases. First case is with limit on maximum number of home healthcare workers assigned and second case is with limit on minimum number of home healthcare workers assigned.

| (p,p1,p2) | Home healthcare workers hired | Home healthcare workers   assigned | | Total cost |
		Period 1	Period 2	
(5,4,3)	No feasible sol.			
(5,3,4)	(1,7,8,9,10)	(1,9,10)	(1,7,8,9)	4589093
(6,3,4)	(1,4,7,8,9,10)	(4,9,10)	(1,7,8,9)	4517406
(7,3,4)	(1,2,4,7,8,9,10)	(4,9,10)	(1,7,8,9)	4517406
(7,3,5)	(1,3,4,7,8,9,10)	(4,9,10)	(1,3,7,8,9)	4466731
(10,3,5)	(1,3,4,7,8,9,10)	(4,9,10)	(1,3,7,8,9)	4466731

Table 4.3: Zero hiring costs and limit on maximum number of home healthcare workers assigned (first case of Scenario 1)

| (p,p1,p2) | Home healthcare workers hired | Home healthcare workers   assigned | | Total cost |
		Period 1	Period 2	
(5,2,3)	(1,3,4,7,9)	(3,4,7,9)	(1,3,4,7,9)	4565043
(6,3,4)	(1,3,4,7,8,9)	(3,4,7,9)	(1,3,7,8,9)	4488125
(7,3,4)	Feasible+ no Opt			
(7,3,5)	(1,3,4,7,8,9,10)	(4,9,10)	(1,3,7,8,9)	4466731
(10,3,5)	(1,3,4,7,8,9,10)	(4,9,10)	(1,3,7,8,9)	4466731

Table 4.4: Zero hiring costs and limit on minimum number of home healthcare workers assigned (second case of Scenario 1)

First scenario is with zero hiring costs. We consider that there are no hiring costs for healthcare workers and we have only assigning costs to be considered.

In this scenario we have performed two types of cases. First case is with the limits on maximum number of home healthcare workers assigned (we take into account constraints (7) but not constraints (8) the results of first case are presented in Table 4.3.). The second case is with limits on minimum number of home healthcare workers assigned (we take into account constraints (8) but not constraints (7)). The results of second case with zero hiring costs and limits on minimum number of home healthcare workers assigned are given in Table - 4.4.

We suppose that the home healthcare workers are multi tasked. They can perform the different tasks related to the treatment of all the patient groups included. So we can relax compatibility constraint (i.e. constraints (9)). Our second scenario is with zero hiring costs and relaxing compatibility constraints (i.e. constraints (9).The results of this scenario for zero hiring costs and limits on maximum (first case) are presented in Table 4.5 and for zero hiring costs and limits on minimum number of home healthcare workers(second case) are presented

in Table 4.6. This scenario assumes that if a home healthcare worker is available, it can be assigned without extra costs.

Table 4.3 shows the computational results for the case without hiring cost (f_j =0 for all j ∈ *J)* and a maximum limit on the number of assigned home healthcare workers.

From Table 4.3 it can be seen that increasing the number of hired home healthcare workers *p* will improve the solution if we keep the same number of assigned home healthcare workers during the two periods. This result is not unexpected as the pair of constraints (3) and (4) forces the model to assign at most *min {p, p*1 *+ p*2} home healthcare workers.

(p,p1,p2)	Home healthcare workers hired	Home healthcare workers assigned		Total cost
		Period 1	Period 2	
(4,3,4)	(4,7,8,9)	(4,7,9)	(4,7,8,9)	3758365
(5,3,3)	(3, 4, 7, 9)	(7,9,4)	(3,7,9)	3894704
(5,3,4)	(3, 4, 7, 8, 9)	(7,9,4)	(3,7,8)	3703330
(6,4,4)	(2,3, 7, 8, 9,10)	(2,7,9,10)	(3,7,8,9)	3683413
(7,5,5)	(2,3, 4, 7, 8, 9,10)	(4,7,9)	(3,7,8)	3683413

Table 4.5: Zero hiring costs and limit on maximum number of home healthcare workers assigned and relaxing compatibility constraints (first case of Scenario 2)

The computational results in Table 4.5 – Table 4.6 s are for the case without hiring cost (f_j =0 for all j ∈ *J*) and a maximum limit on the number of assigned home healthcare workers and relaxing compatibility constraints.

| (p,p1,p2) | Home healthcare workers hired | Home healthcare workers   assigned | | Total cost |
		Period 1	Period 2	
(4,3,3)	(4,7,8,9)	(4,7,9)	(4,7,8,9)	3758365
(5,2,3)	(3,4,7,8,9)	(4,7,9)	(3,7,8)	3703330
(5,4,3)	(2,4,7,8,9)	(2,4,7,9)	(4,7,8,9)	3744507
(6,3,4)	(2,3,7,8,9,10)	(2,7,9,10)	(3,7,8,9)	3683413
(7,3,5)	(1,2,3,4,7,8,9,10)	(2,7,9,10)	(1,7,9,3,4)	3714376
(10,3,5)	Infeasible sol.			

Table 4.6: Zero hiring costs and limit on minimum number of home healthcare workers assigned and relaxing compatibility constraints (second case of Scenario 2)

In third scenario, we have used non-zero hiring costs. Here also we have considered two cases: first case is with limit on maximum number of home healthcare workers assigned and second case is with limit on minimum number of home healthcare workers assigned. This scenario represents the real life application in which there are usually some hiring costs and HHC organizations want to limit the number of hired home healthcare workers to contain costs. The results of first case are in Table 4.7. For limit on minimum number of home healthcare workers assigned we also relax compatibility constraints to analyse the effect of this constraint which is our third scenario. The results of second case are shown in Table 4.8.

(p,p1,p2)	Home healthcare workers hired	Home healthcare workers assigned		Total cost
		Period 1	Period 2	
(5,4,3)	No feasible sol.			
(5,3,4)	(1,7,8,9,10)	(1,9,10)	(1,7,8,9)	5404793
(6,3,4)	(1,7,8,9,10)	(1,9,10)	(1,7,8,9)	5404793
(7,5,5)	(1,3,4,7,9)	(3,4,7,9)	(1,3,4,7,9)	5350743
(7,3,5)	(1,3,4,7,9)	(3,4,9)	(1,3,4,7,9)	5404284

Table 4.7: Non-zero hiring costs and limit on maximum number of home healthcare workers assigned (first case of Scenario 3)

The results of table 4.5 and table 4.6, table 4.7 are with assumption of zero hiring costs. Since our numerical analysis is not based on a real case study but aims at better understanding the behaviour of the models we have developed, we chose to generate randomly several parameters necessary for the models. In particular, the random variables which we consider are generated from a uniform distribution. The use of this distribution is arbitrary, meaning that other types of distributions can also be chosen in the numerical tests. An interesting perspective to this chapter would be to consider other distributions such as normal distribution to generate these parameters and to see if the results found in this chapter are still valid when we change the distributions from which we generate the data.

(p,p1,p2)	Home healthcare workers hired	Home healthcare workers assigned		Total cost
		Period 1	Period 2	
(4,3,4)	(4,7,8,9)	(4,7,9)	(4,7,8,9)	4328365
(5,2,3)	No Feasible solution			
(5,4,3)	(2,4,7,8,9)	(2,4,7,9)	(4,7,8,9)	4632301
(6,3,4)	(1,2,4,7,8,9)	(4,7,9)	(4,7,8,9)	4569065
(7,3,4)	(1,2,4,5,7,8,9)	(4,7,9)	(8,7,9,4)	4744065
(10,3,5)	(1,2,3,4,5,6,7,8,9,10)	(4,7,9)	(8,7,9,3,4)	5429030

Table 4.8: Non-zero hiring costs and limit on minimum number of home healthcare workers assigned and relaxing compatibility constraints (second case of Scenario 3)

4.7 Conclusions

A location allocation model has been adopted to solve home healthcare worker assignment problem for different patient groups while taking into account the locations of both home healthcare workers and patients. The feature that we add to this problem is to consider their geographical positions along with other constraints of skill level, workload balance and preferences. For this purpose, we divide all patients into groups on the basis of their locations. The transportation costs are one of the major cost factors for HHHC organizations. The consideration of geographical location during first decision (assignment of patients to home healthcare workers) can help to reduce the travel time and thus transportation costs as well. We also take into account the constraints of availability of home healthcare workers.

The healthcare workers assignment problem (AHHCW) is formulated as binary integer programming. The objective of our model is to minimize three types of costs (travelling costs, hiring costs and assigning costs). This model is solved by LINGO 6.0. Computational tests are performed for different scenarios in order to analyze the tradeoffs among different performance measures. The model allows decision makers to choose the compatible home healthcare workers and assign them to patient groups and also to consider the trade off between hiring costs and assigning cost of HHC facilities when patient groups reside in different locations during a period. All hired home healthcare workers must satisfy a minimum workload requirement during the period in which they are assigned.

The required qualification of home healthcare workers is guaranteed via compatibility constraints. Computational tests have been performed on different versions of the problem in order to analyze the main characteristics of the model.

The model can be extended to allow budget planning constraints such as having a budget related limitation to determine the number of assigned home healthcare workers. A maximum distance threshold for the travelling distances can be considered to limit the costs of the home healthcare workers travelling to treat the patients at their homes.

The possible research direction suggested by the chapter is the development of a heuristic solution approach which can provide good feasible solutions for real size instances within a reasonable computational time. Another extension route may be to integrate the model in a planning support system that can assist on key strategic decision planning processes. This model constitutes a first step towards the design of a complete healthcare solution for HHC patients.

Chapter 5

5 ROUTING AND SCHEDULING HOME HEALTHCARE WORKERS PROBLEM

5.1 Introduction

The Routing and Scheduling Home healthcare workers Problem (RSHHCW) has a highly constrained search space. This chapter deals with the personnel scheduling problem in the home health care (HHC) services. There are a lot of articles focused on the standalone Nurse Rostering Problems or on the Vehicle Routing Problems, while the number of articles that handle the combination of these problems is very small. Here we have considered the problem that consists of assigning healthcare works to patients and daily scheduling of these healthcare workers for the given planning horizon. The problems most frequently addressed in the home healthcare worker routing and scheduling literature are often modelled as multiple travelling salesperson problems with time windows.

E. Cheng and Rich [149] addressed the daily scheduling problem as a multi-depot vehicle routing problem with time windows and compatibility information. They distinguished between the healthcare workers on the basis of working time, one who work full-time and the second who work half-time. The objective of this daily scheduling was to minimize the total costs associated with the amount of overtime hours of both types of healthcare workers considering the constraints such that each patient is visited exactly once; each healthcare worker visits at least one patient, starts and ends her route at her home and takes a lunch break within the lunches' time windows. The constraints related to human resources such that the maximum healthcare workers' shift length, healthcare workers' qualification requirements, visits' and healthcare workers' time windows must be respected. The problem was formulated as a mixed integer linear programming model.

The approach developed in S.V. Begur et al. [150] used a simple route construction approach based on the savings and nearest neighbour heuristics to determine routes for each home healthcare worker for each day of the planning horizon. Schedulers could use a geographic information system (GIS) map-based visual interface to modify the routes to address concerns such as balanced workload across days and matching home healthcare workers' skill levels to patient needs. Eveborn et al. [12] also developed an interactive tool that created initial solutions to maximize the number of patients served and minimize distance travelled subject to hard constraints for critical time windows and soft constraints for patient provider preferences. Schedules were determined by applying a heuristic solution approach to a set partitioning model, and schedulers could make new solutions by adding staff members or relaxing constraints.

Akjiratikarl et al. [9] developed home healthcare workers routes by using particle-swarm optimization in combination with local improvement techniques to solve a tightly constrained multiple travelling salesperson problem with time windows for each day of the planning horizon. Time windows were formed around specified appointment times and vary between plus or minus 5 and 15 minutes, but constraints on patient healthcare worker preferences and home healthcare workers' skill level requirements were not included.

Bertels and Fahle [7] used a hybrid heuristic that combined techniques such as simulated annealing, tabu search, and constraint programming to assign staff to patient visits. They strictly enforced skill level requirements, work time limitations, and vital time windows, and modelled less critical time windows and various patient and provider preferences through the use of a single penalty function. Once visit-to-staff assignment decisions had been made, a hybrid linear and constraint programming model was used to optimize each staff member's daily work plan.

The research described above does not address the complicating dynamic and periodic aspects of the home healthcare workers scheduling problem. One dissertation from Germany partially incorporated the periodic component by addressing the visit day assignment decision. J. M. Steeg [151] used constraint programming and adaptive large neighbourhood search to construct a set of schedules for each day of the week that satisfied visit day combination constraints for patients which were known in advance. The paper also partially addressed

problem dynamics by developing a tabu search algorithm that could be used to incorporate periodic visits for a new patient into partial schedules. However, the repeatability of visit times throughout the patient's duration of care was not enforced, and same day request arrivals were not handled. The problem studied does not fully address the periodic or dynamic components.

In the classic Period Vehicle Routing Problem (PVRP), the classic single-period Vehicle Routing Problem (VRP) was extended to multiple periods, where each patient must be visited by a vehicle a number of times over a given study period using a selection from a set of allowable visit day combinations for each patient. The problem was to simultaneously assign visit days to patients and to create daily vehicle routes for each day of the planning period that minimizes total travel costs. Variants of periodic routing problems may also include time constraints, such as tour duration constraints or patient time windows.

Two early papers on periodic routing problem variants that achieved especially good computational results on test problems were Chao et al. [152] and Cordeau et al. [153]. The first paper, studying the periodic travelling salesman problem (PTSP), used a binary integer program to assign patients to visit days and solved the resultant TSP on each day of the planning horizon using a modified Clarke-Wright algorithm. Local search methods were used to improve the initial visit day assignments and routes. Alternatively, Cordeau et al. [153] presented a tabu search algorithm for the PVRP that relied on very few parameters and used a generalized-insertion heuristic to perform route construction and improvement.

More recent examples of work on the periodic travelling salesman problem include Bertazzi *et al*. [154] and Polacek et al. [155]. In the first paper, a cheapest-insertion method was used to assign visit day combinations to patients and insert them into corresponding tours. Routes were improved through a combination of local search techniques, such as removing a set of scheduled patients from their tours, assigning them to new visit day combinations, and inserting them into corresponding tours. In the second paper, a variable neighbourhood search heuristic was used to assign visit days to patients and determine tours. Penalty costs were used to enforce consistency of inter-visit time for individual patients over the planning horizon. Hemmelmayr et al. [156] presented a variable neighbourhood search algorithm that obtains

competitive results for both the PVRP and PTSP; the proposed heuristic often outperforms other methods.

Other recent work extends the periodic routing literature to allow patient service frequency to be a decision variable. Francis *et al.* [157] studied the PVRP with service choice, where all patients had a minimum frequency with which they required visits but would accept more frequent visits. Baptista *et al*. [158] studied a waste collection problem where feasible visit frequencies and patterns were defined by patient demand rates. Both papers developed heuristic methods to assign patients to visit frequencies and visit day combinations before developing routes for each day of the planning horizon.

In research surveyed above, methods were developed to create routes that visit each patient on regular visit days. However, no research to date investigated problems that constrain the visit times on those days to be consistent or that patients were visited by the same vehicle from week to week; these features were important in home health care scheduling problems.

Patient demands were revealed over time in dynamic extensions to routing problems. In stochastic variants of dynamic routing problems, it was assumed that probabilistic information describing future patient requests was known when planning. Existing research focused on solution heuristics that performed route planning throughout the planning horizon to develop routes that served known patients and were flexible to serve future arriving patients.

In some cases, the planning process was event driven, where decisions were made each time an event (*e.g.*, new patient arrival) occurred. The approach in Ichoua et al. [160] used a tabu search heuristic to plan vehicle routes that served a set of patients which had time windows. Each route specified a planned sequence of patients, and a vehicle was committed to a specific patient only after it had departed for that patient. The vehicle would not leave its current location if it arrived at the next patient location early. This waiting strategy implicitly accounted for future request arrivals by preserving the ability to incorporate new request arrivals. A separate heuristic was used to determine when a vehicle should depart for the next patient, based on the probability of a new patient arriving.

Similarly, Van Hemert and LaPoutre [159] developed an evolutionary approach that was used to determine whether a vehicle should wait in a region that is likely to produce a new patient.

Their approach allowed anticipatory moves to new regions. Bent and van Hentenryck [162] proposed a multiple plan approach for dynamic routing problems that maintained a set of alternate routing plans at each execution step for the currently arrived patient requests. When a new patient request arrived, the approach selected a plan that could feasibly incorporate the new patient. For a dynamic and stochastic routing problem variant, they extended the multiple plan approach to a multiple scenario approach, where each scenario contained simulated future requests in addition to the currently arrived patient requests.

An alternative to event-driven planning approaches was to divide the planning horizon into discrete time intervals at which to perform planning, such as in Hvattum et al. [161]. This paper formulated a multistage stochastic model, developed a hedging solution method to generate sample scenarios, and solves each heuristically as a static VRP. Common features from the solutions to each scenario were combined to form a solution to the original problem.

A.Coppi et al. [163] solved a problem concerning both the planning of health care services and the routing of vehicles for patients' transportation. An integrated approach, based on the column generation technique, was proposed to solve the planning and routing problem.

In each of the above papers, final decisions regarding patient service requests were delayed as much as possible. Both the patients assigned to a vehicle and the sequence of those patients could change at any time, as long as the service windows were not violated for any associated patients.

Our goal is to build a generic model which can be used for several home health care organizations. In this chapter, we have presented a new mixed linear integer programming formulation for routing and scheduling home healthcare workers problem. RSHCW problems deal two decisions (i.e. assignment of patients to home healthcare workers and preparing routing plan for each home healthcare worker to visit assigned patients). The mathematical model for the problem provides many operational rules and requirements which are needed in many organizations. We have formulated mathematical model in a generalized way by gathering many constraints from different organizational environments in a single formulation. For this model the objective function is also generalized and contains many criteria that can be adopted for specific problems by choosing the values of weighted

constants. For the purposes of this chapter, we define the problem for a finite study period H of d days. Although we recognize that the real-world problem continues beyond this period, we will attempt to minimize horizon end effects in our computational study.

The rest of the chapter is organized as follows. Section 5.2 provides formal definition of the Routing and Scheduling Home healthcare workers Problem (RSHHCW), considered assumptions, notions, parameters and mathematical model. Section 5.3 illustrates the mathematical model by explicitly describing the objective function and constraints of the RSHHCW problem. Section 5.4 gives the solution representation and the benefits of using this representation. This is followed by the initialization method in Section 5.5. We develop Variable Neighbourhood Search algorithm in Section 5.6 by giving all its steps in detail. Section 5.7 provides a variant of previous algorithm called Fixed Neighbourhood Search algorithm. The next two sections (5.8 and 5.9) explain how we implement Simulated Annealing algorithm and Genetic Algorithm for the above described RSHHCW problem. Section 5.10 presents the data generation. Section 5.11 addresses the performance of all proposed algorithms by comparing their computational results. Section 5.12 finally presents the discussion and the concluding remarks.

5.2   Problem description

RSHHCW problem deals with the two decision problems simultaneously. The first is assigning patients to healthcare workers and second is preparing rout plan to fulfil these assignments. The objective is to perform these two decisions in an optimal way that can reduce costs and improve quality of healthcare service provided to patients and healthcare workers as well.

 In this section, we propose mathematical formulation for the assignment and routing problem of healthcare workers called RSHHCW problem. First, we present assumptions adopted, decision variables, parameters and notations used.

5.2.1 Assumptions

In the model we have developed, we would assume, without loss of generality, that:

(i) Every job involves exactly one patient, but one patient may create several jobs, e.g., washing in the morning, application of medication or cleaning of the house. Since all jobs are patient related, we can always identify the patient corresponding to a job. In the following, we will refer them as patient jobs.

(ii) A patient health care needs are assumed to be known for the planning horizon and do not change during that period.

(iii) The number of patient jobs admitted to HHC structure is known in advance and does not change while considering the problem.

(iv) The number of patient jobs leaving HHC structure is known in advance and does not change while considering the problem.

(v) All patient jobs admitted to HHC structure in a period have to be assigned to required number of healthcare workers in that period.

(vi) For every assignment of a healthcare worker to a patient job we define validity of this assignment such that a healthcare worker cannot perform a patient job (i.e. this assignment is not valid) due to several reasons:

   a. Existence of geographical obstacles between them.

   b. Difficulty or impossibility to travel from location of healthcare worker residence to that of patient residence by the means of transportation used by home healthcare workers (public transportation, private cars, etc.).

(vii) Patients considered in this study suffer from different diseases. At this level, it is important to mention that most of HHC structures classify patients' therapeutic projects into categories named "profiles". Indeed, patients whose therapeutic projects have similarities in terms of the expected duration of care, type, number and average duration of visits are grouped into the same profile.

(viii) Patients who live in a given basic unit can have different profiles.

(ix) A patient profile is assumed to be known when he/she is admitted to HHC structure and does not change during his/her stay within HHC system.

(x) The number and average duration of patient jobs that are required by each patient are known and are the same among the patients who have the same profile.

(xi) Human resources delivering care to patients are of three different types. Doctors, nurses and medical assistants. We have three types of health care workers and in each type there are several qualification levels.

(xii)   There are an enough number of healthcare workers available. Each healthcare worker has a predetermined capacity (i.e. he/she can handle a certain volume of workload). This capacity is variable between the different healthcare workers.

(xiii)   The speed pertaining to the travel between two patient jobs is considered as deterministic.

(xiv)   The distance metric in our experiments is the network distance since it reflects the real time spent by a healthcare worker between the patient jobs.

(xv)   The duration of every patient job consists of travelling time to reach that patient and time to provide service for that patient. Therefore the duration of a patient job can be one or more time slots. The travelling time for each job is chosen by considering the distance among patients and healthcare workers home in such a way that this is sufficient enough time to travel between any two destinations. Here our objective is to reduce the overall travelled distance and we relax the travelling time.

(xvi)   This scheduling is done once for a given period of time which corresponds to the planning horizon H.

5.2.2 Decision Variables

x_{hjt} is equal to 1 if a healthcare worker h performs a patient job j at time slot t and 0 otherwise.

Since the duration of a patient job can be one or more than one i.e. it can take more than one time slots. Also a patient job can require one or more healthcare workers to perform this patient job. Therefore more than one x_{hjt} can be equal to 1.

y_{hj} is equal to 1 if a healthcare worker h is assigned to a patient job j

$z_{hj_1j_2}$ is equal to 1 if a healthcare worker h performs patient job j_2 immediately after patient job j_1 and 0 otherwise.

5.2.3 Notations and Parameters

The necessary parameters to formulate the model are listed below.

H planning horizon

$D = \{1, 2... d\}$ Set of days in planning horizon H

$J = \{1, 2... m\}$ Set of patient jobs

$HC = \{1, 2... n\}$ Set of healthcare workers

$T = \{1, 2... p\}$ Set of time slots

T_d Set of time slots in a day d *i.e.;* $T_d = \{1, 2... p_d\}$

The time is divided into time slots small enough such that any job needs at least ONE time slot.

5.2.3.1 Input Parameters

n_j Number of healthcare workers required to perform a patient job j

dur_j Duration of patient job j

For our problem, we have considered travelling cost in terms of distance. We want to minimize the distance of each tour. But we have assumed travelling time for each patient job in advance and this travelling time is included in the duration of each patient job.

f_j Frequency of patient job j *i.e.* number of occurrence of patient job in the planning horizon H

req_j Required qualification level of a home healthcare worker to perform patient job j

hs_j Hard start time of patient job j or beginning of hard time window for job j

he_j Hard end time of patient job j or end of hard time window for job j

ss_j Soft or desired start time of patient job j or beginning of soft time window for job j

se_j Soft or desired end time of patient job j or end of soft time window for job j

We choose the soft time windows and hard time windows in such a way that these values should satisfy following equation.

$$hs_j \leq ss_j \leq se_j \leq he_j \qquad \forall \qquad j \in J \tag{A}$$

The equation (A) shows the relation of hard time window and soft time window for every patient job j.

RD_j Release date of a patient job j i.e. the date from which a patient job is available to perform.

DD_j Deadline or due date of patient job j i.e. the date after which a patient job is expired or this job cannot be performed.

This release date and deadline of a patient job are suggested by the healthcare experts according to his treatment plan.

T_j Set of time slots which satisfy release date and due date for a patient job j

$t_{j_1 \cdot j_2}$ Transportation cost of travelling of patient job j_1 to patient job j_2 . This cost is measured in terms of distance from home of one patient or healthcare worker to home of other patient or healthcare worker

v_{hj} Validity of performing a patient job j by a healthcare worker h

v_{hj} is equal to 1 if assignment of healthcare worker h to treat a patient job j is valid and 0 otherwise; This validity of occurrence is defined in above section of assumptions.

nd_j Number of different healthcare workers performing the patient job j

Q_h Qualification level of a healthcare worker h

$Avl_h = (a_{ht})$ is availability matrix of a healthcare worker h

a_{ht} is equal to 1 if healthcare worker h is available at time slot t and 0 otherwise

$Desirability = (des_{hj})$ is desirability matrix of a healthcare worker h for a patient job j and vice versa

This desirability index shows the preference of a healthcare worker h for performing a patient job j or the preference of a patient job j (as jobs are related to patients so this is patient's preference) for a certain healthcare worker. This index can be positive or negative. For instance a patient may have preference for a healthcare worker gender and a healthcare worker with dog's allergy may refuse to treat a patient who keeps pets at his home.

$\min W_{hd}$ Minimal working hours per day of a healthcare worker h

$\max W_{hd}$ Maximal working hours per day of a healthcare worker h

l_{h-max} Maximum number of hours of a healthcare worker h in the planning horizon

C_h Cost of each overtime unit of a healthcare worker h

p_d Maximum number of time slots per day

J_{pre} Set of pre-allocated patient jobs

5.2.3.2 Output variables

$seq_{j_1 j_2}$ is sequence of occurrence of job j_1 and job j_2 and is equal to 0 if of job j_1 and job j_2 occur in desired sequence and 1 otherwise. This is defined to ensure the required precedence among patient jobs

s_j Start time of patient job j

W_{hd} Workload of healthcare worker h on day d

W_h Total workload of healthcare worker h

W_{av} Average workload

gap_h Gap from average workload for healthcare worker h

ltw_j Deviation of start time of patient job j from soft start time of patient job j

utw_j Deviation of end time of patient job j from soft end time of patient job j

0_h Total overtime cost of a healthcare worker h

5.2.4 Mathematical Model

Objective function

$$\text{Min}\quad \mathbf{Z} = \beta_1 \cdot \sum_{h \in HC} \sum_{j_1, j_2 \in J} t_{j_1 j_2} \cdot z_{h j_1 j_2} + \beta_2 \cdot \sum_{h \in HC} O_h + \beta_3 \cdot \sum_{j \in J} (ltw_j + utw_j) - \beta_4 \cdot \sum_{h \in HC} \sum_{j \in J} des_{hj} y_{hj}$$

(1)

$$+ \beta_5 \cdot \sum_{h \in HC} gap_h + \beta_6 \cdot \sum_{j \in J} \sum_{\substack{h \in HS \\ Q_h > req_j}} y_{hj} + \beta_7 \cdot \sum_{j_1, j_2 \in J} seq_{j_1 j_2}$$

Where $O_h = C_h \cdot (W_h - l_{h-\max})$, $\qquad h \in HC$ (2)

$$W_h = \sum_{j \in J} \sum_{t \in T} x_{hjt} \; \forall \qquad h \in HC \tag{3}$$

$$ltw_j = ss_j - s_j \qquad \forall j \in J \tag{4}$$

$$utw_j = (s_j + dur_j) - se_j \qquad \forall \; j \in J \tag{5}$$

$$gap_h = \left| \frac{W_{av} - W_h}{W_{av}} \right| \qquad \forall \; h \in HC \tag{6}$$

$$W_{av} = \frac{\sum_{h \in HC} W_h}{n} \tag{7}$$

Given that if patient job j_1 is predecessor of patient job j_2 i.e. $j_1 \prec j_2$,

$$s_{j_1} + dur_{j_1} - H.seq_{j_1 j_2} \leq s_{j_2} \; \forall \quad j_1, j_2 \in J \tag{8}$$

$$utw_j \geq 0, ltw_j \geq 0 \qquad \forall \qquad j \in J \tag{9}$$

$$gap_h \geq 0 \;, O_h \geq 0 \qquad \forall \qquad h \in HC \tag{10}$$

Constraints:

The objective function (1) is minimized subject to following constraints:

$$\sum_{t \in T_j} x_{hjt} = f_j . dur_j \qquad \forall\, j \in J \qquad \forall \qquad h \in HC \qquad (11)$$

$$\sum_{h \in HC} x_{hjt} = n_j \qquad \forall\, j \in J \qquad \forall \qquad t \in T \qquad (12)$$

$$x_{hjt} \le a_{ht} \qquad \forall\, t \in T \qquad \forall \qquad h \in HC \qquad (13)$$

$$hs_j . y_{hj} \le s_j \le s_j + dur_j \le he_j \qquad \forall\, j \in J \,, \quad h \in HC \qquad (14)$$

If patient job j_1 and patient job j_2 are the two jobs performed by the same healthcare workers h on the same day such that job j_1 is performed earlier than job j_2,

$$s_{j_1} + dur_{j_1} \le s_{j_2} + p_d (1 - z_{hj_1 j_2}) \quad \forall j_1, j_2 \in J \qquad (15)$$

$$s_j > t + H.(x_{hjt+1} - x_{hjt} - 1) \qquad (16)$$

$$s_j < t - H.(x_{hjt+1} - x_{hjt} - 1) \qquad (17)$$

$$req_j \le Q_h . y_{hj} \qquad \forall \quad j \in J \qquad \forall \quad h \in HC \qquad (18)$$

$$\min W_{hd} \le \sum_{j \in J} \sum_{t=1}^{p_d} x_{hjt} \le \max W_{hd} \qquad \forall \quad h \in HC \; \forall \; d \in D \qquad (19)$$

If a patient job j is pre-assigned to a healthcare worker h_j at a time slot t_j,

$$x_{h_j j t_j} = 1 \qquad \forall \; j \in J_{pre} \quad , \quad h_j \in HC \quad t_j \in T \qquad (20)$$

$$dur_j \cdot \sum_{h \in HC} (x_{hjt} - x_{hjt+1}) - \sum_{h \in HC} \sum_{q=hs_j+1}^{t} x_{hjt} \le 0 \quad j \in J \; , \; t \in \{hs_j + 1, ..., he_j\} \qquad (21)$$

$$\sum_{j \in J} x_{hjt} \le 1 \qquad\qquad h \in HC \quad , \quad t \in T \qquad (22)$$

$$x_{hjt} \le v_{hj} \cdot y_{hj} \qquad \forall \quad j \in J \quad \forall \quad h \in HC, \; t \in T \qquad (23)$$

$$z_{hj_1 j_2} \le \sum_{t \in T} (x_{hj_1 t} + x_{hj_2 t}) \qquad \forall \quad j \in J, \qquad h \in HC \qquad (24)$$

$$x_{hjt}, y_{hj}, z_{hj_1 j_2} \in \{0,1\} \qquad j \in J \qquad h \in HC \qquad t \in T \qquad (25)$$

5.3 Explaination of mathematical model

Now we give explicit description of the above mathematical model in two parts: objective function and constraints.

5.3.1 Objective Function

The objective function (1) is a multi objective function. Now we explain its all seven terms on right hand side of equation (1) in the following:

(i)   Total travelling cost of all healthcare workers

(ii)   Overtime cost of all healthcare workers

(iii)   Deviation from soft time window of all patient jobs

(iv)   Desirability of healthcare workers for patients and vice versa

(v)   Workload balance among healthcare workers

(vi)   Number of over qualified assignments

(vii)   Number of violations of desired sequence of patient jobs

Travelling cost (i) and overtime cost (ii) should be kept as minimum as possible. This will reduce the cost for the employer organization. We want to minimize deviation from soft time windows (iii) and number of violations of desired sequence of patient jobs (vii) to maximize satisfaction of healthcare workers. We also minimize the number of over qualified assignments (vi). It shows the number of patient jobs which are assigned to a healthcare worker with a higher qualification level than necessary. This helps to use the limited human resources in an efficient way. Since it reduces the under utilization of human resources so it will help in minimizing the healthcare service cost or overall cost for the employer organization. We want to improve the workload balance among different healthcare workers so we minimize the sum of deviations of all patient jobs from the average workload (v). On the other hand we want to maximize desirability of healthcare workers for a patient or vice versa (iv). This desirability covers the preferences stated by healthcare workers and patients. These preferences are handled as soft constraints, since all jobs have to be accomplished (even if preferences are violated). As the whole objective function is minimization. We shall subtract this fourth term in order to maximize it. The constants from β_1 to β_7 are non negative constants. Their values can be chosen by decision makers according to their weighting or importance for different objective functions.

5.3.2 Constraints

Constraints of the RSHHCW problem are divided into two parts. The first part is stated as hard constraints that have to always be fulfilled. On the other hand, soft constraints can be violated, but their non-fulfillment is penalized in the cost function.

5.3.2.1 Hard Constraints

The RSHHCW problem contains following hard constrains:

Constraints (11) show that very patient job should be performed the required number of times i.e. equal to its given frequency during the time slots of planning horizon H which satisfy release date and due date for that patient job. This shows that a patient job cannot be performed before its release date or after its deadline. Constraints (12) assure that each patient job should be performed by the required number of healthcare workers. Constraints (13) are availability constraints i.e. a healthcare worker can perform a job at a time slot t if he is available at that time. Constraints (14) give hard time window for every patient job. This hard time window for every occurrence of patient job should be satisfied. In constraints (15), we require enough time (i.e. equal to the duration of a patient job) to travel and provide the service for a patient job before the next patient job starts. Constraints (16) and constraints (17) show the relationship of variables s_j and x_{hjt}. These show that when a patient job start at *t,* i.e. s_j =t, the value of x_{hjt} becomes equal to 1(x_{hjt} =1).

Constraints (18) guarantee that every patient job should be performed by a well qualified healthcare worker. Constraints (19) show that working times cannot exceed the daily maximal working time. Here maximal working time shows the shift length for healthcare worker. Working times cannot be less than the daily minimal working times since it is needed to assign the healthcare worker for that day or not. Healthcare workers have some pre-assigned jobs. These can be administrative jobs so we cannot move these jobs to any other time slot or healthcare worker given by Constraints (20). Constraints (21) are non pre-emption constraints. We cannot pre-empt a patient job. If a patient job is started, it should not be interrupted for a time period equal to its duration. Constraints (22) show that every healthcare worker can perform only one patient job at any time slot. So there should not be double allocation for any health care worker. Constraints (23) define the validity of assignment of a healthcare worker to a patient job. These show that a patient job can occur at any period *t* if this job is already assigned to a healthcare worker. Constraints (24) show the relationship of decision variables and constraints (25) are binary constraints.

5.3.2.2 Soft Constraints

On the other hand, the following requirements are only soft, i.e. violations of them are not desired but tolerated (with corresponding penalization in the objective function). The

constraints given following are soft constraints. Equations (A) give soft time window violations for ever job. Each patient states a time window in which the job shall be started. Violations of these time windows are penalized in the objective function. However, deviations of three hours and above are considered equally bad. Each customer states a desired start time for each job i.e. a concrete point in time at which the job should be started. Deviations from the start time are penalized, where, similar to time window violations, deviations of one hour and above are assumed to be equally bad. Constraints (8) show precedence constraints between some patient jobs. This constraint imposes that patient job j_1 is performed before patient job j_2. i.e. patient job j_1 is predecessor of patient job j_2. If patient job j_1 is performed before j_2 then the treatment of patient job j_2 does not begin before the end of treatment of patient job j_1. Constraints (2) define overtime cost for each healthcare worker. If a healthcare worker has spent number of hours more than $l_{h-\max}$ in the planning horizon H, they are considered as overtime. Since for every day we have defined maximal working hours which show shift length for any healthcare worker (Maximum number of hours a healthcare worker can work each day).This constraint is fulfilled via constraints (19). But to calculate overtime, we have defined an input parameter $l_{h-\max}$ and we compare working hours of a healthcare worker over the whole planning horizon. Constraints (3) give the total workload of a healthcare worker. Constraints (7) and constraints (6) give average workload and deviation from this average workload respectively. These constraints (3, 6, 7) help in maintaining a workload balance among various healthcare workers. This workload balance results in more satisfied healthcare workers. In constraints (4) and constraints (5), ltw_j and utw_j give the deviation of start time of a patient job from its soft start time and the deviation of end time of a patient job from its soft end time respectively. Constraints (9) and (10) are non negativity constraints.

5.4 Solution Representation

HHC service has to prepare a schedule for the given planning horizon. This schedule contains information about healthcare workers' assignment to patients, at any hour of the day, on any

day of the given planning horizon. This is a collection of many separate schedules, one for each healthcare worker.

So the potential solution of RSHHCW problem is a collection of personal schedules. We represent one personal schedule by a two dimensional matrix as shown in Figure 5.1. Every schedule stores information that which healthcare worker is assigned to which patient at what time on which day of the given planning horizon, each assignment is one gene. If there is no assignment at any time slot, we leave that time slot empty. For genetic algorithm, a population consists of n (n = population size) number of schedules. We also store information about schedules like less costly schedule, most costly schedule, average cost, average number of violations of constraints and the total number of schedules in population.

We are using two-dimensional matrix (i.e. grid) representation for our solution and each cell in this representing is an empty slot or having at most one event. Benefit of using this representation is that it reduces the search space significantly. From permutation of mathematics, if there are m patient jobs to schedule in l places then the total number of possible ways to schedule m patient jobs in l places is l^m where l places equal to total number of healthcare workers multiplied by total number of time slots. Now the claim of reduction of search space by using the method of this encoding can be proved in this way. The number of

ways to assign m patient jobs to l places as we have used in our chromosome design are $\dfrac{l!}{(l-m)!}$. This mathematical expression is elaborated in the following.

$$\frac{l!}{(l-m)!} = \frac{l \times (l-1) \times (l-2) \times ..... \times (l-(m-1))(l-m)(l-(m+1)) \times ...... \times 2 \times 1}{(l-m) \times (l-(m+1)) \times ......2 \times 1} = l \times (l-1) \times (l-2) ...... (l-(m-1))$$

(A)

H1					
	T1	T2	....	....	Tp
D1	J1		J5	J20	J9
D2	J5		J7	J20	J9
. .	. .	. .	. .	. .	. .
Dd	J2	J3	J1	J5	

.
.
.
.
.

Hm					
	T1	T2	....	....	Tp
D1	J7		J1	J10	
D2	J11		J4	J10	
. .	. .	. .	. .	. .	. .
Dd	J8		J3	J6	

Figure 5.1: Solution representation for schedule of healthcare workers

Now comparing l^m with (A), one can prove above claim.

$$l^m = l \times l \times ....m(time)...........\times l$$ Which is always greater than (A).

$$l^m = l \times l \times ....m(time)...........\times l > l \times (l-1) \times (l-2)...........(l-(m-1)) \cdot$$

Where l is always equal or greater to m. (i.e. $l \geq m$) because this is a necessary condition to get feasible solution otherwise feasible solution could not be achieved.

One benefit of this presentation is that each patient job can be placed uniquely to any slot easily or clash of double healthcare worker booking can be simply finished by using this representation. So by using this method on encoding one can avoid one important hard constraint.

5.4.1 Benefits of using this representation

As well as making use of a two-dimensional matrix representation for schedules in solution of RSHHCW problem, now we shall explain how many matrixes can be made by using this representation.

5.4.1.1 Patient job-Healthcare worker Matrix

This matrix is used to indicate that which patient job is suitable for which healthcare worker. This is a Boolean matrix and can easily be calculated that which healthcare worker h satisfies the conditions to perform patient job j. Thus if healthcare worker h satisfies the conditions to perform patient job j then the element (j,h) marks as true otherwise it marks as false.

5.4.1.2 Conflicts Matrix

This matrix is very similar to adjacency matrix used for representing graphs and this indicates that which pair of patient jobs have conflicts (so these cannot be scheduled in the same time slot). For example if any two patient jobs j_1 , j_2 have any conflict then element (j_1, j_2) in the matrix marks as true otherwise marks as false.

By using this encoding counting of violations is easy and inexpensive. To check violations are easy now, for example if one wants to check that each patient should have at most one patient job assigned in any time slot, it can be evaluated by checking each column whether this column is true more than one entry or not (rows of the matrix are patient jobs). If one wants to check that proper healthcare worker type h has been assigned to a patient job j in schedule, simply it can be verified by checking entry (j,h) is true in patient job-healthcare worker matrix.

We have suggested some more matrices for RSHHCW problem after following the suggestions of M. Carter [100].

5.4.1.3 Healthcare worker – Time slot Matrix

This matrix indicates that healthcare worker h is available at time slot t or not, if healthcare worker h is available at time slot t then element (t,h) marks as true otherwise it marks as false because in our problem healthcare worker are not available for some time slots. This can also help to find penalties of soft time windows of the healthcare workers.

5.4.1.4 Patient job-Time slot Matrix

This matrix shows the relationship between time slots and patient jobs because some patient jobs cannot be scheduled in some time slots. If patient job j can be scheduled in time slot t then element (j, t) will be marked true otherwise it will be marked false. This can also help to find violation of hard time windows of patient jobs.

5.4.1.5 Patient-Time slot Matrix

This matrix indicates patient- time slot relationship and benefit of this matrix is that because some patients have soft time windows. Thus this matrix helps us to identify the position of patient with respect to time windows. If patient k can be assigned in time slot t then element (k,t) marks as true otherwise it marks as false.

5.5   Initial Solution

Initially, we used two methods for initial solution generation: construction heuristics and random solution generation and then applying a repair strategy. But the preliminary results of our algorithms showed that use of heuristics has little or no significant effect on the final solution obtained. So we implemented the technique of random initial solution generation for most of our experiments. No side constraints, such as time window restrictions are considered. We do not impose the condition of feasibility at this stage. The reason is to have larger search space and to evaluate the influence of the initial solution during the later

improvement phase. And we do not want to invest too much into the initial solution generation, since this step is just the first of several steps in the whole optimization approach.

We have chosen random initialization method. For this purpose we start by a random initial solution. After getting the random initial solution repairing is mainly done on violation of hard constraints. First of all we know about the location of the offending slots and replace them iteratively with valid slots. The benefit of this procedure is that there would be a large diversity in solutions of search space but on the other hand algorithm can face problem of quick convergence due to this randomness.

For RSHHCW problem, we have generated our initial solution randomly and then repaired it to satisfy a chosen hard constraint of frequency of every patient job. It is performed in two steps to fulfill two hard constraints.

- Every patient job should be assigned.
- The number of assignments of every patient job should be equal to its given frequency.

In first step we see if a patient job is assigned more than its given frequency, we remove the extra assignments randomly. In second step we see that if any patient job is missing or it has assigned less than its given frequency, we find a new slot randomly and assign it. At the end of this two step verification procedure every patient job is assigned and its number of assignments is equal to its given frequency. This is explained in following Figure 5.2.

Repair procedure

1. **Step1**
2. for every patient job j
3. if Num_ assigned $> f_j$
   a. while (job_occurence $> f_j$) do
      i. Randomly remove an assignment
   b. endwhile
4. end if
5. end for
6. **Step2**
7. for every patient job j
8. if Num_ assigned $< f_j$
   a. while (job_occurence $< f_j$) do
      i. Choose a healthcare worker-time slot pair randomly
      ii. Assigne this patient job
   b. endwhile
9. end if
10. end for

Figure 5.2: Pseudo code of repair procedure

5.6 Variable Neighbourhood search algorithm (VNS)

VNS guides the search process through search space among local optima by using a set of neighbourhood structures. VNS algorithm starts with an initial solution. The process of building this initial solution is explained in section 5.5. Then we define the set of neighbourhood structures to be used later. Each iteration of VNS consists of three main steps: move, local search and shaking.

5.6.1 Search space and solution space

We propose a variable neighbourhood search (VNS) algorithm to solve RSHHCW problem. For this problem we define the *solution* space as the set of all the feasible schedules of healthcare workers, but the *search* space as the set of all the ways that all patient jobs can be assigned to healthcare workers. Now we define the neighbourhood structures to be used in

VNS algorithm. A neighbourhood is a relation among different points of the search space. We can move or explore the search space by using this relation.

5.6.2 Steps of VNS algorithm

We can divide VNS algorithm into the following main steps:

− Find an initial solution
− Define neighbourhood moves
− Define order for applying different neighbourhood structures
− Develop a local search and decision to accept a solution found or not
− Apply shaking to maintain diversity in solution space
− Choose a termination criterion

In the following, we explain the components of our VNS algorithm: neighbourhood moves, the proposed local search, acceptance criterion, shaking and termination criterion.

5.6.3 Neighbourhood Moves

The success of VNS largely depends on the fact that how the neighbourhoods are defined. A neighbourhood is defined by first defining a set of *moves*. *Move* is a way of converting one solution to another. We have used the following five types of neighbourhood moves. These are explained one by one.

5.6.3.1 Move 1:

Exchange the positions of two patient jobs assigned to the same healthcare worker on the same day. This is a local modification in schedule of one day of one healthcare worker. We illustrate move 1 in the Figure 5.3.

Figure 5.3: Move 1 example

5.6.3.2 Move 2:

Exchange two patient jobs assigned to the same healthcare worker on different days. This is a local modification in schedule of more than one days of one healthcare worker. Figure 5.4 shows an example for this move 2.

Figure 5.4: Move 2 example

5.6.3.3 Move 3:

Choose two healthcare workers of same qualification level and swap any of two patient jobs randomly. This is a local modification in schedule of one day of more than one healthcare worker. Move 3 is displayed in the Figure 5.5.
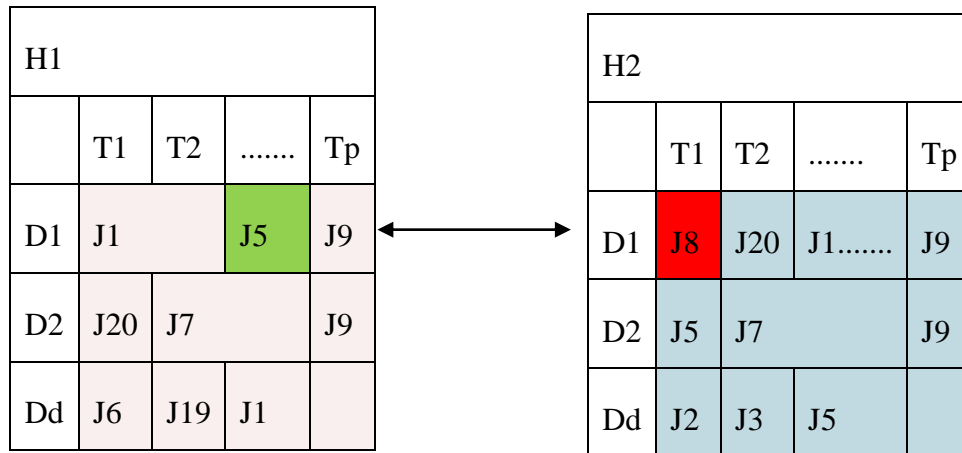
Figure 5.5: Move 3 example

5.6.3.4 Move 4:

Choose two healthcare workers of same qualification level and swap all patient jobs assigned to first healthcare worker on any random day with all patient jobs assigned to second healthcare worker on the same day. This is a local modification in schedule of one day of more than one healthcare worker. An example is shown in Figure 5.6 to explain this move 4.
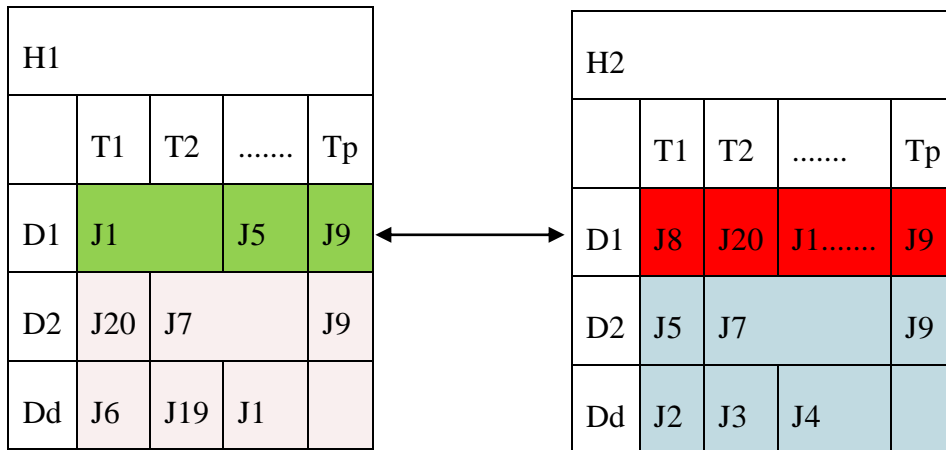
Figure 5.6: Move 4 example

5.6.3.5 Move 5:

Choose two healthcare workers of same qualification level and swap all patient jobs assigned to first healthcare worker on any random day with all patient jobs assigned to second healthcare worker on any random different day. This is a local modification in schedule of more than one day of more than one healthcare worker. Figure 5.7 illustrates move 5.
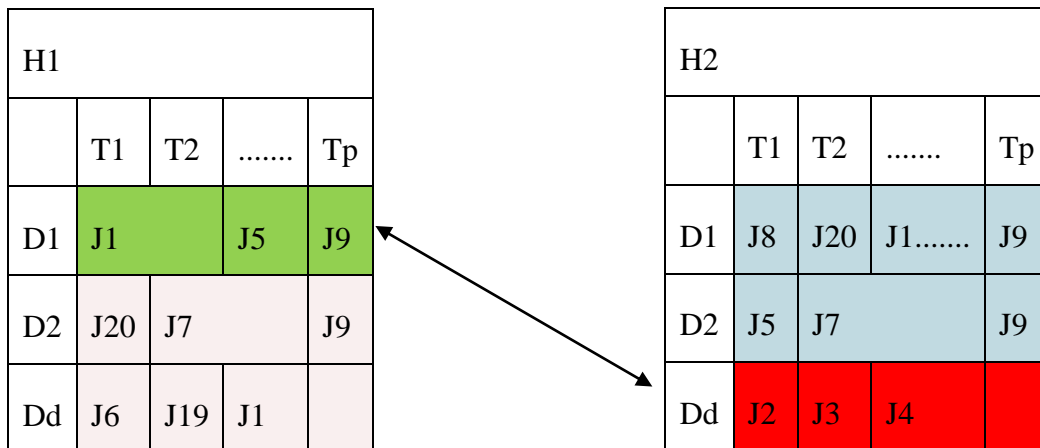
Figure 5.7: Move5example

5.6.4 Proposed Local search

The local search techniques in VNS algorithms are used for improving the quality of solutions. In our case, we take one or two patient jobs randomly and swap them by using our neighbourhoods moves defined in section 5.6.3. If it reduces the cost function, we accept it otherwise we reject it. If the cost is adjusted, the solution which is the result of local search is replaced with the primary solution; otherwise that primary solution is identified as the best solution in its neighbourhood and remains unchanged. So the accepting criterion i.e. decision to accept solution by local search is based on improvement of the solution found. The pseudo code for the Local Search is described in the Figure 5.8.

5.6.5 Termination Criterion

We have used elapsed time as termination criterion. VNS algorithm continues until the elapsed time reaches the time limit defined or a solution of defined minimum cost is obtained.

The procedure of local search

1. input: start with a solution s_o found by selected neighbourhood move

2. while termination condition not achieved do

3. if cost of solution s_o is greater than minimum cost defined then

    a. choose n patient jobs randomly from the set of the all patient jobs

        i. for $i = 1$ to n do

        ii. choose a solution s in the neighbourhoods of s_o by Move 1 (explained above in Figure 5.3)

        iii. end for

    b. examine the solution s

    c. if the cost of solution $s <$ the cost of solution s_o then

    d. Set $s_o = s$ and register s as local minimum found.

        i. for $i = 1$ to n do

        ii. choose a solution s_1 in the neighbourhood of s_o by Move 2(explained above in Figure 5.4)

        iii. end for

    e. examine the solution s_1

    f. if the cost of solution $s_1 <$ the cost of solution s_o then

    g. Set $s_o = s_1$ and register s_1 as local minimum found.

    h. end for

4. end if

5. end while

6. output: A possibly improved solution

Figure 5.8: Pseudo code for local search used in VNS

5.6.6 Shaking

Shaking is used when VNS algorithm is held stuck at some local minima. The purpose is to maintain diversity in search space. In shaking, we apply random moves in larger neighbourhoods. Usually the randomness in shaking is increased each time the local search

process does not improve the fitness value of current best solution. In our case, shaking is applied after every twenty iterations of VNS algorithm. The procedure of shaking is described in Figure 5.9.

Pseudo code for Shaking

1. for each healthcare worker h1
2. select another healthcare worker h2 randomly
3. if (qualification level of h1= qualification level of h2)
   a. for $i =1$ to d
      i. for $j =1$ to p
      ii. swap current patient job with a randomly chosen patient job
      iii. end for
   b. end for
4. end if
5. else
6. keep the schedule of healthcare worker h1 unshaken.
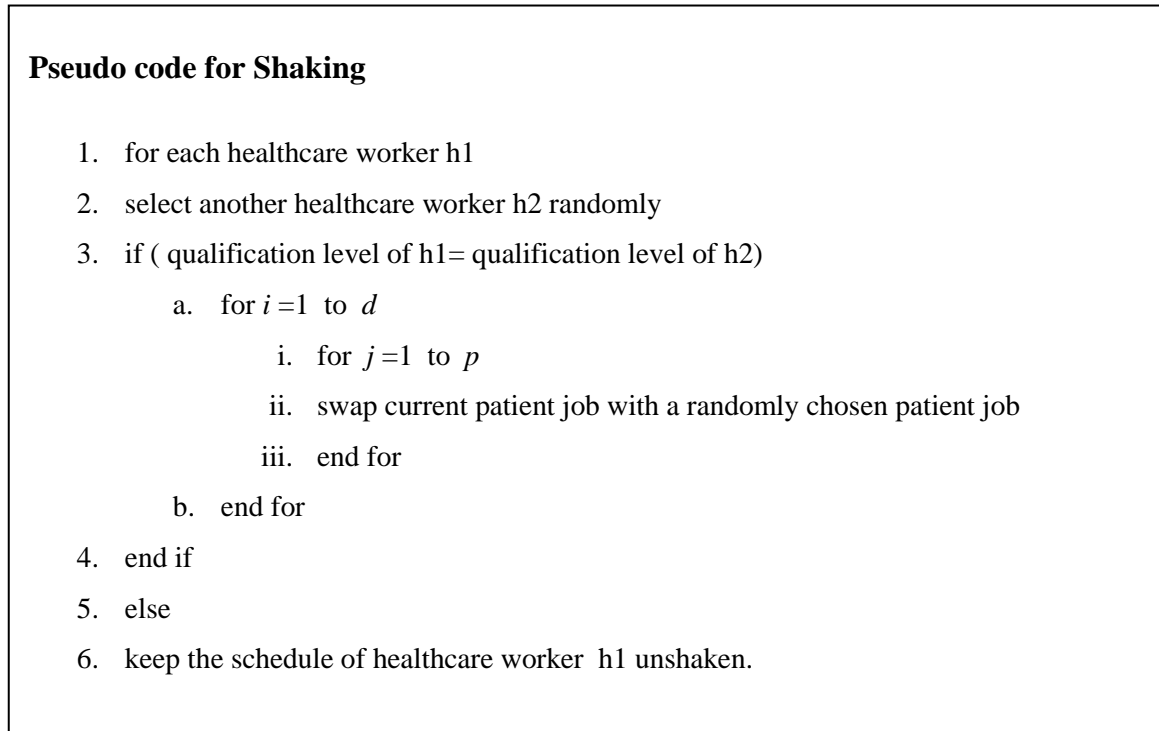
Figure 5.9: Pseudo code for Shaking

5.6.7 Pseudo code of proposed VNS algorithm

Here we describe the basic algorithm for our proposed VNS algorithm by combining all above steps. First we find an initial solution s by adopting the same procedure explained above in section 5.5. Then we use five moves given above for the set of neighbourhood structures. In the start we take this initial solution s as best found solution. After it, we select a solution *s'* in first neighbourhood of initial solution *s*. Next we apply local search on *s'*. The local search gives new solution s_0, if cost of s_0 is less than cost of previously best found solution we save it as new best found solution and we continue the search focused around s_0 again with the first neighbourhood until the maximum number of applying every neighbourhood structure in one iteration. The maximum number of applying every neighbourhood structure in one iteration is defined and the value of this parameter is equal to

10. If cost of s_0 is less than cost of *s'* but greater than the cost of previously best found solution, we save it as local minimum and in this case the search procedure is iterated using the next neighbourhood. If cost of s_0 is greater than cost of *s'*, we abandon first neighbourhood and select the next neighbourhood for search procedure. After *p* iterations of this procedure, shaking is applied on the best found solution, where *p* is number of iterations after which we repeat this shaking procedure. Shaking helps search procedure to escape from local minima. The detailed pseudo code of VNS algorithm is given in figure 5.10. The values of parameters used in VNS algorithm are given in Table 5.1.

Parameter	Setting
n (number of patient jobs for local search)	2
k_{max} (number of neighbourhood structures)	5
p (gap between two shaking operations)	20

Table 5.1: Parameters for VNS algorithm

Pseudo code for VNS algorithm

1. Generate an initial solution *s* randomly i.e. a schedule for each healthcare worker

2. Repair this solution *s* by repair procedure given in Figure 5.2

3. Select the set of neighbourhood structures $N_k, \forall k = 1,...,k_{\max}$

4. Select the set of neighbourhood structures $N_k, \forall k = 1,...,k_{\max}$

5. Set number of iterations, t=0.

6. While termination criteria or best solution not achieved

    a. For every $N_k, \forall k = 1,...,k_{\max}$

        i. Find a neighbouring solution s' at random by applying N_k, the kth neighbourhood of s (i.e. $s' \in N_k(x)$)

        ii. Apply proposed local search on s' given in Figure 5.8. The new solution obtained is denoted as s_0.

        iii. If the local optimum s_0 is better than the incumbent *s*, continue search with N_k

    b. Set k=k+1

7. If t mod $p = 0$, where p denotes number of iterations after which shaking is applied;

    a. Perform Shaking on s as given in Figure 5.9.

8. number of iterations , *t=t+1*.

9. End while

10. output: best found solution

Figure 5.10: Pseudo code for VNS algorithm

5.7   Fixed Neighbourhood Search (FNS) algorithm

FNS algorithm is a variant of variable neighbourhood search. The term fixed represents that we fix the size of all neighbourhoods. We describe the differences between VNS algorithm and FNS algorithm in Table 5.2. We have implemented these variations in VNS as a result of our preliminary experiments. These experiments show that these variations lead towards a more efficient algorithm which we name as fixed neighbourhood search (FNS) algorithm.

VNS algorithm	FNS algorithm
Continue examining the neighbour points until finding a local minimum in each neighbourhood	Examine a fixed number of neighbour points in each neighbourhood
Apply all defined neighbourhoods	Apply only improving neighbourhoods
Accept bad solutions too	Accept only good solutions
After p iterations, apply shaking	No shaking at all
Cycling may occur	No cycling

Table 5.2: Differences between FNS algorithm and VNS algorithm

Now we explain in detail our proposed FNS algorithm. First we find an initial solution adopting the same procedure as explained above in section.

Then we use five moves given above for the set of neighbourhood structures. We compare the performance of these five moves and selected the three better performing for RSHHCW problem. We choose the size of every neighbourhood equal to l_{max}. The parameters used are given in Table 5.3.

Parameter	Setting
n (number of patient jobs for local search)	1
k_{max} (number of neighbourhood structures)	3
l_{max} (size of neighbourhood)	10

Table 5.3: Parameters for VNS algorithm

We start by first neighbourhood and take l_{max} random points in this neighbourhood of initial solution s and compare their costs. If we find a solution better than initial solution s, we save it as we select as best solution found denoted by s'. The same procedure is iterated with next neighbourhood and best found solution is updated if found. We continue until we have used all better performing neighbourhoods defined. Note that contrary to VNS algorithm, there is no shaking and local search applied here. We induce diversification in search process by applying random moves and skipping the use of greedy local search. The detailed pseudo code is given in Figure 5.11.

Pseudo code for FNS algorithm

1. Generate an initial solution s randomly i.e. a schedule for each healthcare worker

2. Repair this solution s by repair procedure given in Figure 5.2.

3. Select the set of neighbourhood structures $N_k, \forall k = 1,...,k_{max}$

4. Set number of iterations, t=0 and best found solution = s

5. While termination criteria or best solution not achieved

    a. For every $N_k, \forall k = 1,...,k_{max}$

        i. For $l = 1, . . . ,l_{max}$

        ii. Find a neighbouring solution s' at random by applying N_k, the

        kth neighbourhood of s (i.e. $s' \in N_k(x)$)

    b. Set k=k+1

    c. If s' is better than best found solution

    d. best found solution = s'

6. number of iterations , $t=t+1$.

7. end while

8. output: best found solution

Figure 5.11: Pseudo code for FNS algorithm

5.8 Simulated Annealing (SA) algorithm

We have developed a simulated annealing algorithm for RSHHCW problem. This problem generally has a very large search space with difficult constraints. SA algorithm needs an initial solution and annealing schedule as input. The initial solution is generated in the same way as for VNS algorithm given in section 5.5. The main features of SA are the annealing schedule, the acceptance probability function and the criteria for move acceptance.

The annealing schedule is composed of defining initial maximum temperature, final minimum temperature and cooling rate (i.e. how much temperature is decreased at each iteration). In SA algorithm, few parameters are required to be set and for our proposed SA algorithm the values used for these parameters are given in Table 5.4. We have used cooling schedule which is linear. This is given by following equation A.

$$T = T - T * \gamma \qquad \text{(A)}$$

Here T is temperature at given iteration and γ is cooling rate. At a particular level of temperature, only one iteration is performed.

Parameters	Setting
Maximum Temperature T_{max}	1000
Minimum Temperature T_{min}	0.0005
Cooling rate γ	0.0025
Stopping criteria	Final minimum temperature or time elapsed

Table 5.4: Parameters for SA algorithm

At each iteration of SA, we apply move 3 (described in detail in section 5.6.3.3) to generate a neighbour of current solution. This choice is random and other moves can equally likely be used. If this move decreases the cost, neighbour solution is always accepted. On the other hand, if the neighbour generated by this move has higher cost we accept it with a given probability. This probability is called acceptance probability function which depends on the

temperature at given iteration and difference in the cost of the current solution and the generated neighbouring solution. This is given in equation (B).

P (delta, T) = exp ((-1.0*delta*log (t+1))/ (1000*T))   (B)

Pseudo code SA algorithm

1. Input: Initial solution and annealing schedule
2. Let S be the initial solution, S_{best} denotes the best solution and S_c denotes the current solution, T_{max} be maximum temperature.
3. Set S_{best} = S= S_c, T= T_{max} and γ in (0,1) is cooling rate and number of iterations t , t=0
4. Repeat
   a. At given temperature *T*,
   b. Generate a neighbour solution by applying Move 3 given in Section 5.6.3.3
   c. if the cost of neighbour solution is less than cost of current solution
      i. Current solution= neighbour solution
      ii. if the cost of neighbour solution is less than cost of best solution
         1. Best solution= neighbour solution
   d. Else if the cost of neighbour solution greater than cost of current solution
      iii. Calculate probability P(delta,T)
         P(delta,T) = exp((-1.0*delta*log(t+1))/(1000*T));
         where delta= cost of neighbour solution - cost of current solution.
      iv. Select a random number *r in (0, 1)*
      v. If r < P(delta,T)
         a. Current solution= neighbour solution
   e. Set T= T - $T * \gamma$, number of iterations , *t=t+1*.
5. If $T= T_{min}$ or stopping criterion reached , then Stop.
6. Otherwise, repeat Step 4.
7. output: Best solution S_{best}

Figure 5.12: Pseudo code for SA algorithm

In equation (B), delta denotes the difference in the cost of the current solution and the cost of generated neighbour solution. This function is defined in such a way that it accepts worse solution more in the start of algorithm and the probability of accepting worse solution decreases as number of iterations increases or temperature decreases. The search process succeeds in avoiding local optima due to the probabilistic acceptance of a non improving neighbour solution. We lower temperature gradually as given in equation (A). After each iteration, we decrease temperature and save the best solution found so far. This process continues until the final minimum temperature is achieved or stopping criteria is attained. The stopping criterion is specified in terms of maximum limit on elapsed time. For the sake of completeness, the pseudo code for SA algorithm is summarized in Figure 5.12.

5.9 Genetic Algorithm

Genetic Algorithm is a search algorithm that imitates the process of natural selection and natural genetics. It uses selection, crossover, and mutation operators for the search of the best solution the given problem. Genetic algorithm continues until a specified termination criterion is reached.

The solution of a problem is called a chromosome. The chromosome representation for the problem considered is given in section 5.4. A chromosome is made up of a collection of genes which represents the characteristics of the solution. A classical genetic algorithm creates an initial population (a collection of chromosomes), evaluates this population, and then evolves the population through multiple generations (using the genetic operators discussed above) to find the good solution of the problem considered.

5.9.1 Initial population

Population based algorithms are generally used with an initial population. This initial population sometimes is generated randomly and sometimes by using special techniques to make a higher quality initial solutions for population. The size of population depends on problem. It can contain several hundreds or thousands of possible solutions depending upon the problem nature but for our problem it varies from 10 to 50.

5.9.2 Genetic operators

Evolutionary algorithms are good tools for a big space optimization problems but their performance depends a lot on the type of genetic operator used and the values of parameters such as mutation rate, crossover rate and population size. Crossover operator is used for exploration and mutation is used to avoid from local minima in these algorithms. In the following we explain the genetic operators used for our proposed genetic algorithm.

5.9.2.1 Selection

We have used both elitism and roulette wheel selection (RWS) for choosing parents for reproduction purpose. Elitism selects the best chromosome or a few best chromosomes for the next population. This procedure can increase algorithm performance very quickly because it preserves the best found solutions.

The roulette wheel selection is the proportionate reproduction operator where a chromosome is selected for mating with a probability proportional to its fitness. The chromosomes with better fitness values have more chances to be selected for mating than the chromosomes with worse fitness values. We explain it by showing a roulette wheel for a population of six chromosomes. Their fitness values are given in Table 5.5.

Chromosomes	Fitness
1	15
2	15
3	4
4	5
5	27
6	34

Table 5.5: example population used to explain RWS

The Figure 5.13 shows a roulette-wheel for each chromosome having different fitness values. Since the *fifth and sixth chromosomes have* higher fitness values than any other, it is expected that the roulette-wheel selection will choose the *fifth and sixth chromosomes* more than other chromosomes. But there are also chances that it can select third or any other chromosome with lower fitness value. So this selection method maintains the diversity of search space by choosing some bad solutions.

To implement RWS, we generate a random number between 0 and n (n = population size). If the number generated is less than 20, we select the chromosome with best fitness value, if the number is between 20 and 30, we choose the chromosome with second best fitness value and if generated number is greater than 30 we choose any other chromosome with a probability corresponding to its fitness value. So our probability for the best chromosome is 0.4, for the second best 0.2 and 0.4 is divided among all other chromosomes.
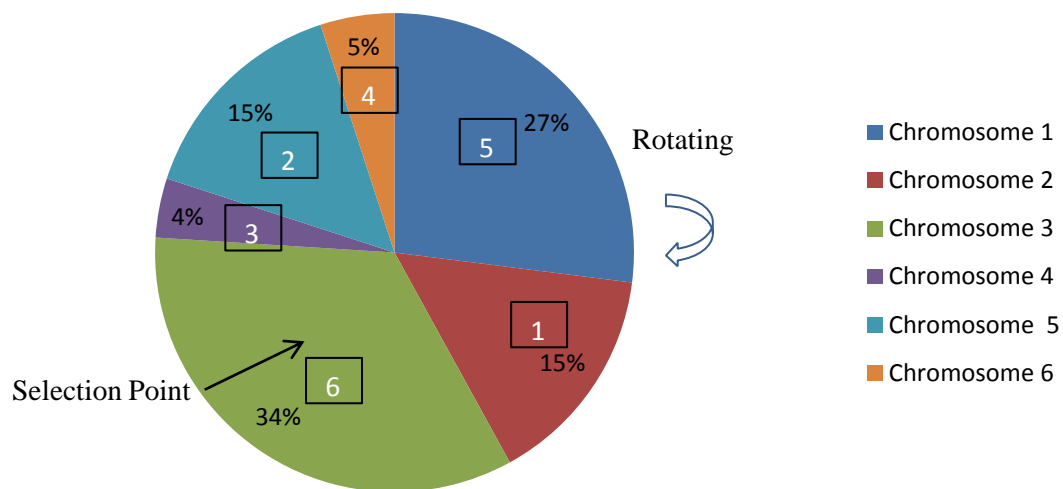
Figure 5.13*: A roulette-wheel marked for six chromosomes according to their fitness values.*

5.9.2.2 Crossover

In crossover, the two parent solutions are mated to produce the child solution. The role of crossover operators is to inherit some characteristics of the two parents to the children of next

generation. We have implemented uniform crossover in the mating step of the proposed genetic algorithm. The procedure is shown in Figure 5.14 for the schedule of one healthcare worker. One chromosome is collection of as many such schedules as the total number of healthcare workers.

The uniform crossover evaluates each gene in the parent chromosome for exchange with a probability of 0.5. There exists no satisfactory theory that explains the reasons of performance differences between uniform crossover and traditional one point crossover. The empirical evidence suggests that uniform crossover is more exploratory approach to crossover than the traditional one point exploitative approach. This results in a more complete search of the design space with maintaining the exchange of good information.

The crossover rate represents the proportion of parents on which a crossover operator acts. The best parameter value for crossover rate is related to other parameters among them such as the population size, the mutation probability and the selection procedure. There is choice to apply crossover to some percentage of population and not to all. But we have chosen to apply cross over operator to all population with crossover rate equal to 0.5.

H1					
	T1	T2	....	....	Tp
D1	J1		J5	J20	J9
D2	J20	J7		J5	J9
.	.	.	.	.	.
.	.	.	.	.	.
Dd	J6	J19	J1	J5	

Schedule of H1 from parent1

H1					
	T1	T2	....	....	Tp
D1	J5	J20	J1		J9
D2	J5	J7		J20	J9
.	.	.	.	.	.
.	.	.	.	.	.
Dd	J2	J3	J5		

Schedule of H1 from parent1

H1					
	T1	T2	....	....	Tp
D1	J5	J20	J1		J9
D2	J20	J7		J5	J9
.	.	.	.	.	.
.	.	.	.	.	.
Dd	J6	J3	J5		

Schedule of H1 for new child
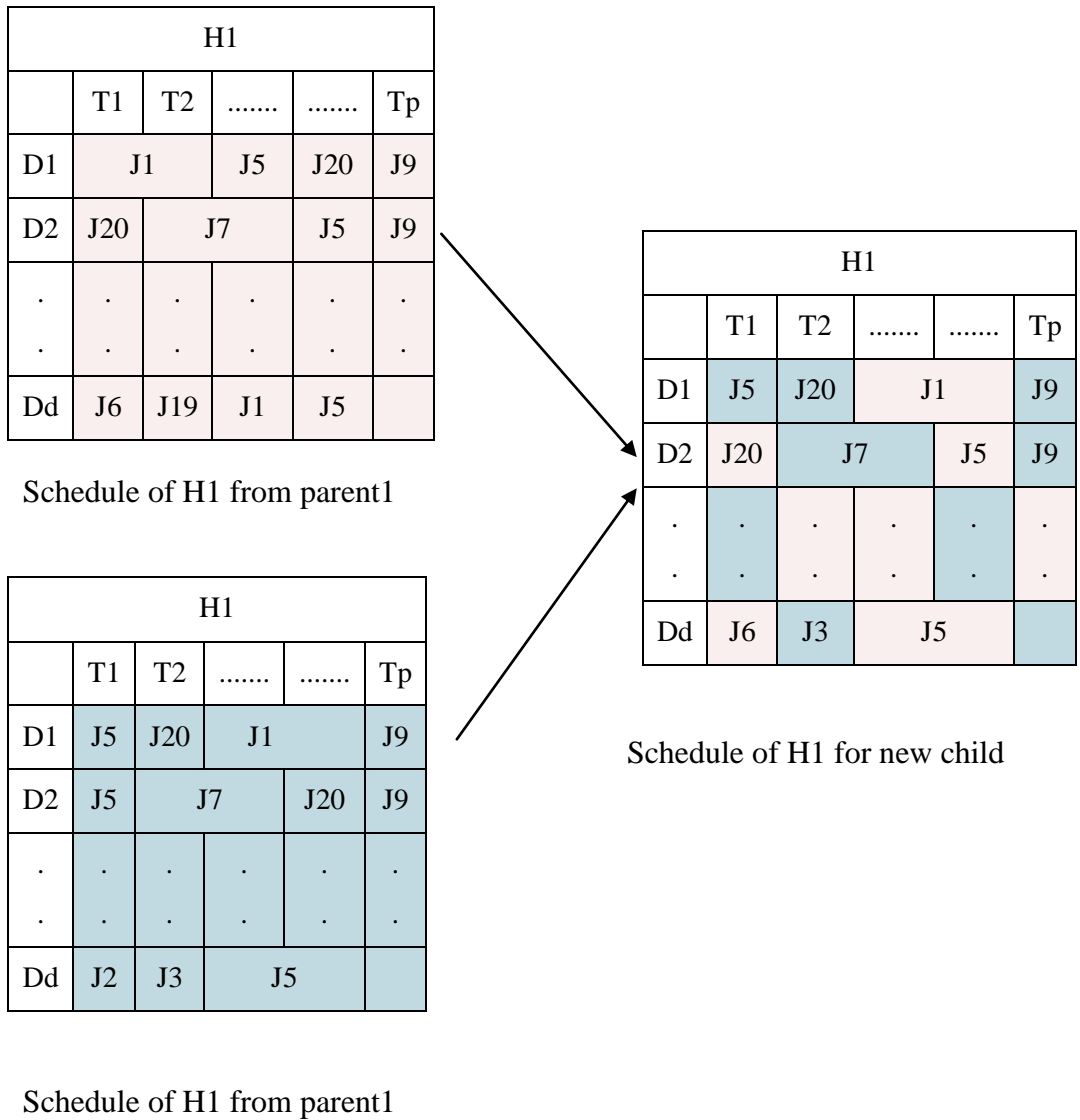
Figure 5.14: Crossover operator applied on schedule of one healthcare worker

5.9.2.3 Mutation

Mutation operator is used normally to do the random alteration of genes. This is done during the process of copying a chromosome from one generation to the next. This results in the children having different genetic material from those of their parents. Mutation is used to add new information to genetic search process and also helps to avoid from being trapped at local minima. The purpose of mutation is to add diversity in the population. Thus the population does not become homogeneous due to repeated crossover applied over the same population

pool. We used a random mutation operator and mutation rate equal to 0.05. This is explained in Figure 5.15 for the schedule of one healthcare worker.

	H1				
	T1	T2	....	....	Tp
D1	J5	J20	**J1**		**J9**
D2	J20	**J7**		J5	J9
.	.	.	.	.	.
.	.	.	.	.	.
Dd	**J6**	J3	J5		

(a)Before mutation

	H1				
	T1	T2	....	....	Tp
D1	J5	J20	**J7**		**J6**
D2	J20	**J1**		J5	J9
.	.	.	.	.	.
.	.	.	.	.	.
Dd	**J9**	J3	J5		

(b) After Mutation
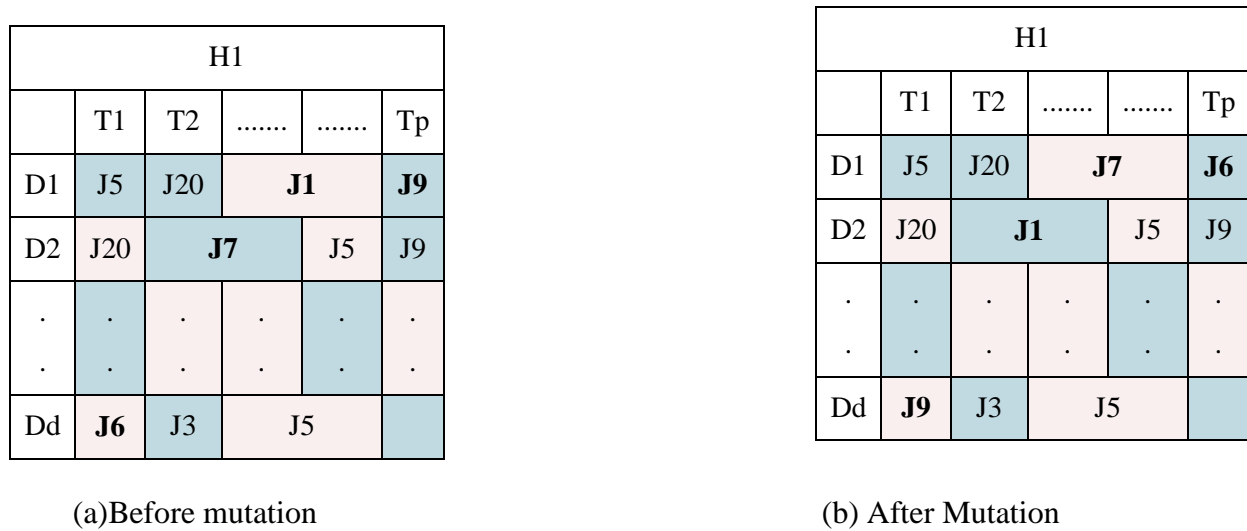
Figure 5.15: Mutation operator applied on schedule of one healthcare worker

5.9.2.4 Replacement

This is the replacement procedure which decides about child survival or extinction. Since the aim of algorithm is to improve the objective function value in each generation. For this purpose we have used replacement with elitism. In replacement with elitism, we keep best n members of the population and kill all other ones. By this way, good solutions are prevented from becoming extinct. The choice of the number n and the decision of extinction of chromosomes from the current population are important aspects of genetic algorithm.

In our proposed algorithm, we killed 20% of total population and these are the solutions with worse fitness values. These are replaced with 20% new solutions generated in each generation.

We use steady state algorithm and general genetic algorithm in our preliminary studies and then compare them. We find general replacement algorithm more efficient as compared to steady state algorithm in our case. The parameters used for our proposed algorithm are given in Table 5.6.

Parameters	Settings
Population size	50
Selection	Roulette wheel selection
Crossover	Uniform crossover
Crossover rate	0.50
Mutation	Exchange two patient jobs of same duration randomly from the schedule of two healthcare workers of same qualification level
Mutation rate	0.05
Stopping Criterion	Time limit defined for each type of dataset

Table 5.6: Parameters used for the proposed genetic algorithm

5.9.2.5 Termination criterion

Termination is the criterion is used to decide whether the genetic algorithm should continue or stop creating new generations. Different criteria can be enabled for example maximum number of generations, maximum elapsed time or the maximum fitness achieved. Each of the enabled termination criterion is checked after each generation to see if it is time to stop. In our case, we have used time limit to stop the search process of genetic algorithm.

5.9.3 Pseudo code of genetic algorithm

The pseudo code for proposed genetic algorithm is shown in Figure 5.16.

Pseudo code for Genetic Algorithm (GA)

1. input : A problem instance I

2. set the generation counter g := 0

3. while (solution_colony. population_size < n)

    a. Initialize randomly a schedule for each healthcare worker

    b. Repair this schedule by repair procedure given in Figure 5.2.

    c. calculate the cost

    d. enter schedule to the population colony

4. end while

5. compare the cost of all schedules and choose the one with minimum cost as s_{best}

6. while time limit is not reached

    a. kill 20 % members of the colony

        i. while (solution_colony. population_size < n)

        ii. choose two parents via RWS

        iii. s_i child solution generated by applying the uniform crossover operator with a crossover rate p_c

        iv. s_i child solution after mutation with a mutation rate p_m

        v. Repair this schedule by repair procedure given in Figure 5.2.
        vi. Calculate the cost of s_i
        vii. enter this schedule to the population colony
    b. end while
7. compare the cost of all schedules and choose the one with minimum cost as s_{best}
8. g := g + 1
9. end while
10. output : The best achieved solution s_{best} for the problem instance I

Figure 5.16: The pseudo code for genetic algorithm

5.10 Data generation

In this section, we describe the procedure for generating datasets for the numerical analysis of our proposed algorithms applied to solve RSHHCW problem. For this purpose, we have used three types of datasets called small dataset, medium dataset and large dataset. For every dataset we have generated 10 instances randomly. The parameters are divided into two kinds. The first kind is the parameters whose values are different for each dataset. These include the number of home healthcare workers n, the number of patients, the number of patient jobs m, the number of working days d, the number of hours in a working day p, the number of pre assigned jobs, the number of patient jobs with precedence, the number of un-availabilities for each patient and for each healthcare worker, the number of incompatibilities for each healthcare worker. These are given in Table 5.7.

The second kind is those parameters which are randomly generated for every instance of all three datasets. But their procedure and ranges are same for small datasets, medium datasets and large datasets. These are explained in the following:

- There are 4 categories of healthcare workers and in each category we have healthcare workers of different qualification levels. This qualification level is randomly assigned from 1 to 4. Typically the qualification levels are hierarchical, e.g., a home healthcare worker possesses all qualifications of a medical assistant but the opposite is usually not true. However, these hierarchies can occur in several dimensions: Assume a HHC service employs a nursing staff and a cleaning staff, the member of the cleaning staff cannot perform any medical treatments, while a home healthcare worker is not supposed to clean. So it is not possible to assign a healthcare worker having a qualification level lower than the qualification level required for the patient job. The converse is possible but it mayl result in increase of cost for HHC service because the human resource skills are not optimally used.

Datasets		Small	Medium	Large
Parameter	No. of patient jobs	20	50	100
	No. of Patients	20	20	30
	No. of healthcare workers	5	10	30
	No. of working days	3	4	5
	Hours in a working day	5	5	7
	No. of pre assigned jobs	2	5	10
	No of patient jobs with precedence	8	20	40
	Number of un availabilities of every healthcare worker	2	4	5
	Number of invalid assignments	5	10	20
	Computational time	10 seconds	20 seconds	60 seconds

Table 5.7: Table showing values for three kinds of datasets (Small, Medium and Large)

– Distance of a patient from healthcare worker home or any other patient's home is from 0 to 10.

- Average duration of each patient job is 60 minutes.

- Frequency of each patient job is 1 to 3.

- The number of healthcare workers required to perform a patient job j is n_j. For simplicity, we have taken $n_j=1$ i.e. every patient job is performed by a single healthcare worker.

- We randomly generate release date and deadline for every patient job over the planning horizon. The release date is the date from which a patient job is available to schedule and deadline is the date after which the patient job cannot be performed. This can be due to the some short time valid medicine or material or availability of equipment. The minimum difference between these two dates (i.e. the number of days during the job should occur) is taken equal to 3.

- Time window for each patient job is randomly generated in taking into account of release date and deadline of that patient job. The width of each time window is at least double of its duration time.

- Soft time window for each patient is randomly generated over the whole day. If a patient is related to only one patient job then time window for patient and for patient job are taken same (as in case of small dataset). But if a patient has more than one relative patient jobs, time window for patient and related patient jobs are taken different. But the time windows of all patient jobs should be included in the soft time window of the relative patient. For example if a patient has a soft time window $[a,b]$, then all patient jobs related to this patient can occur in $[a+s, b-s]$, where $s \geq 0$.

- The total number of healthcare workers is n. The working time window for n/2 healthcare workers are [0, p], where p is number of hours in a working day. For the remaining n/2 healthcare workers, the working time window is [0, $p/2$] or [$p/2$, p]. This can represent the real case situation, where some of the healthcare workers are permanent or full time employees and some other are hired for part time (either for morning shift or afternoon shift).

- The minimum and maximum working hours in a day for each healthcare worker are randomly generated from uniform distribution U(1,3) and U(3,p) respectively, where p is number of hours in a working day .

- We also consider maximum number of working hours of a healthcare worker over the whole planning horizon. The time exceeding this limit is considered as over time. The cost of over time for each healthcare worker is randomly chosen from 5 to 20 for category 1, category 2 and from 20 to 50 for category3, category 4.

- Transportation cost for travelling from one patient's home to another patient's home is taken equal to distance covered during this travelling. The distance between any two patients' homes is generated randomly from uniform distribution U (0, 20). It is assumed that the distance matrix is symmetric and satisfies the triangle inequality, i.e., $t_{jk} < t_{ji} + t_{ik}$ for i, j, k any three patient jobs.

- We have used a desirability index of healthcare workers for patients and vice versa. In this way, each patient can describe a ranking order for all healthcare workers and similarly healthcare workers can show their desires to treat a specific type patient or not. To generate this index we assign randomly from 0 to 5. Here number 0 represents the least desired and number 5 for most desired. Our aim is to maximize the sum of this desirability index for all patient jobs performed. The higher value of this sum means increased satisfaction level of both patients and healthcare workers. This helps to improve the quality of the healthcare service.

- To achieve a workload balance among healthcare workers, we have defined an admissible difference from average workload of all healthcare workers. This admissible difference is taken as 10% of average workload.

- We now explain the procedure of generating matrices validity matrix V= (v_{hj}) i.e. validity of performing a patient job j by a healthcare worker h. For each dataset we define a value of number of invalid assignments. We randomly generate v_{hj} such that the number of zeros in the whole matrix is equal to maximum number of invalid assignments.

- We have $Avl_h = (a_{ht})$ is availability matrix of a healthcare worker h. This is generated randomly in the following way. For every dataset, we define a value of number of unavailabilities of every healthcare worker. Then we randomly assign a_{ht} is equal to zero or 1 such that number of zeros for in availability matrix of every healthcare worker is equal to number of un availabilities.

5.11 Computational Results

In this section, we have given results obtained by fixed neighbourhood search algorithm, variable neighbourhood search algorithm, genetic algorithm and simulated annealing algorithm across 10 runs on each of the 10 data instances of small, medium and large size problems. For the sake of brevity we use the abbreviated name for each algorithm. These abbreviations are GA algorithm for genetic algorithm, VNS algorithm for variable neighbourhood search algorithm, FNS algorithm for fixed neighbourhood search algorithm and SA algorithm for simulated annealing algorithm. In these tables bold entry shows the best result attained.

5.11.1 Minimum cost Comparison

This section presents minimum cost comparison for three types of datasets (small datasets, medium datasets and large datasets) solved by the algorithms (Table5.8 - Table 5.10). We use the term of cost for objective function given in equation (1). This is our multi criteria objective function. So cost means sum of following criteria:

  (i)   Total travelling cost of all healthcare workers
 (ii)   Overtime cost of all healthcare workers
(iii)   Deviation from soft time window of all patient jobs
(iv)   Desirability of healthcare workers for patients and vice versa
 (v)   Workload balance among healthcare workers
(vi)   Number of over qualified assignments
(vii)   Number of violations of desired sequence of patient jobs

The performance of GA algorithm is best for small datasets as compared to other algorithms. The second better performing algorithm for small datasets is VNS algorithm. The possible reason can be the large number of population and the special genetic operators which provide diversity to genetic algorithm and also help to avoid from local minima.

For some data instances, the results of GA algorithm and VNS algorithm are close to each other (see instance3, instance4 and instance8). SA algorithm produces good results as

compared to FNS algorithm but its performance is not better than GA algorithm and VNS algorithm. Overall FNS algorithm produces worst results and GA algorithm gives best results.

	FNS	VNS	GA	SA
Instance1	54	35	**30**	44
Instance2	66	37	**29**	42
Instance3	57	37	**36**	43
Instance4	63	31	**29**	44
Instance5	53	36	**34**	43
Instance6	79	44	**35**	52
Instance7	64	38	**32**	42
Instance8	51	32	**31**	40
Instance9	66	44	**41**	53
Instance10	67	37	**25**	41

Table 5.8: Minimum cost comparison of all algorithms for small data sets

For medium datasets, performance of FNS algorithm is clearly improved as it has given best results for six data instances out of ten. Similarly VNS algorithm has also improved its performance by giving best results for two instances. However, the performance of genetic algorithm has relatively decreased because it can produce best results for only two data instances. But its performance is best amongst all algorithms for small datasets. SA algorithm performance is also decreased for medium datasets. While both neighbourhood search algorithms show better performance as the size of problem increases.

	FNS	VNS	GA	SA
Instance1	139	150	**131**	158
Instance2	147	**134**	148	172
Instance3	**148**	161	160	155
Instance4	**143**	169	**143**	171
Instance5	144	166	**140**	171
Instance6	**129**	165	160	174
Instance7	**147**	165	165	176
Instance8	171	**143**	154	161
Instance9	**119**	153	158	173
Instance10	**145**	160	165	172

Table 5.9: Minimum cost comparison of all algorithms for medium data sets

	FNS	VNS	GA	SA
Instance1	**650**	928	927	947
Instance2	**602**	909	902	905
Instance3	**490**	827	808	828
Instance4	**692**	970	959	977

Instance5	**538**	903	856	857
Instance6	**596**	891	884	861
Instance7	**698**	960	945	964
Instance8	**632**	993	973	991
Instance9	**577**	891	852	906
Instance10	**535**	875	883	886

Table 5.10: Minimum cost comparison of all algorithms for large data sets

For large datasets FNS algorithm has shown best performance for all data instances. While other three algorithms show poor performance in comparison of FNS algorithm. If we analyze their performance GA algorithm is relatively better than VNS algorithm and SA algorithm. GA algorithm has given better results for nine instances than VNS algorithm and GA algorithm.

5.11.2 Maximum cost comparison

This section gives maximum cost comparison for three types of datasets solved by the algorithms (Table 5.11 - Table 5.13).

The performance of GA algorithm is best for small datasets as compared to other algorithms. The second better performing algorithm for small datasets is VNS algorithm. SA algorithm produced good results as compared to FNS algorithm but its performance is not better than GA algorithm and VNS algorithm. Overall FNS algorithm produces worst results and GA algorithm produces best results. One can notice that this trend is similar to the trend for minimum cost comparison.

	FNS	VNS	GA	SA
Instance1	101	65	**39**	75
Instance2	109	48	**38**	76
Instance3	111	57	**40**	75
Instance4	112	47	**39**	54
Instance5	99	49	**39**	62
Instance6	134	65	**48**	96
Instance7	100	60	**48**	67
Instance8	110	53	**46**	66
Instance9	123	62	**52**	76
Instance10	131	49	**41**	58

Table 5.11: Maximum cost comparison of all algorithms for small data sets

For medium datasets, performance of GA algorithm is again best for all data instances. We observe that all algorithms follow the same pattern as in small datasets.

	FNS	VNS	GA	SA
Instance1	208	186	**166**	197
Instance2	213	178	**172**	200
Instance3	186	189	**176**	199

Instance4	206	200	**189**	215
Instance5	233	196	**174**	210
Instance6	247	202	**169**	199
Instance7	203	198	**186**	211
Instance8	239	183	**166**	200
Instance9	182	192	**171**	209
Instance10	243	190	**185**	206

Table 5.12: Maximum cost comparison of all algorithms for medium data sets

For large datasets FNS algorithm has shown best performance for all data instances. While other three algorithms show poor performance in comparison of FNS algorithm. If we analyze their performance GA algorithm is relatively better than VNS algorithm and SA algorithm. GA algorithm has given better results for all ten instances than VNS algorithm and SA algorithm.

	FNS	VNS	GA	SA
Instance1	**690**	980	963	979
Instance2	**725**	964	932	976
Instance3	**578**	865	842	864
Instance4	**757**	994	985	994
Instance5	**638**	955	910	926

Instance6	**669**	913	913	924
Instance7	**797**	1010	994	1023
Instance8	**769**	1026	1015	1028
Instance9	**659**	938	914	945
Instance10	**620**	926	928	946

Table 5.13: Maximum cost comparison of all algorithms for large data sets

5.11.3Average Cost Comparison

This section illustrates average cost comparison for three types of datasets (small datasets, medium datasets and large datasets) solved by the algorithms (Table 5.14 - Table 5.16).

GA algorithm is performing best for small datasets as compared to other algorithms. The second better performing algorithm for small datasets is VNS algorithm. SA produces good results as compared to FNS but its performance is not better than GA algorithm and VNS algorithm. On the whole FNS algorithm produces worst results and GA produces best results.

	FNS	VNS	GA	SA
Instance1	79.4	45.6	**33**	54
Instance2	90.4	41.6	**33.6**	59.4
Instance3	85.2	45.6	**39.2**	57.4
Instance4	77.4	40.6	**34.4**	48.4
Instance5	69.6	45	**35.8**	59.8

Instance8	194.2	165.6	**158.4**	178.8
Instance9	**151.2**	173.2	163.2	191.6
Instance10	196.2	177.8	**175.2**	186.8

Table 5.15: Average cost comparison of all algorithms for medium data sets

FNS algorithm has performed best for large datasets. As we have seen that minimum cost of FNS algorithm is best for large datasets, so this algorithm produces best average cost as well. When we compare the other three algorithms (VNS, GA and SA), we find that GA algorithm has shown relatively better performance. It gives better results for all ten instances in comparison with VNS algorithm and SA algorithm.

	FNS	VNS	GA	SA
Instance1	**665.4**	958.8	950	955.4
Instance2	**660**	937	913.4	943.6
Instance3	**551.4**	843.8	827.8	847.4
Instance4	**719.2**	980.6	971	984
Instance5	**589**	922.4	898.6	884.6
Instance6	**626**	907.4	897.8	890.8
Instance7	**745.8**	985.8	968.4	989.4
Instance8	**728**	1010.2	994.6	1013
Instance9	**605.6**	906.6	889.4	919

Instance10	**574.6**	910.6	900.2	919

Table 5.16: Average cost comparison of all algorithms for large data set

5.12 Discussion and conclusion

This section illustrates the behavior of the algorithms when applied to solve RSHHCW problem. We have deduced these behaviors from the results presented in the previous section 5.11.

We have used certain time limits on our algorithms during testing of our datasets and we consider this is fair for these sizes of problems. These are 10, 30 and 60 seconds of CPU time for the small, medium and large datasets respectively. We have used an Intel computer with 2.5 GHz CPU and 4GB RAM under windows operating system for these experiments.

Our analysis is on the basis of cost value. We use the term of cost for objective function given in equation (1). This is our multi criteria objective function. So cost means sum of following criteria: total travelling cost of all healthcare workers, overtime cost of all healthcare workers, deviation from soft time window of all patient jobs, desirability of healthcare workers for patients and vice versa, workload balance among healthcare workers, number of over qualified assignments and number of violations of desired sequence of patient jobs.

Firstly we analyze the behaviour of the components of proposed algorithms and then give an overall comparison among all algorithms. These graphs are drawn for results obtained across 1 run on one data instance chosen from three types of datasets. We shall specify the size of dataset with each graph.

We start this discussion by giving the analysis of neighbourhood moves given in section 5.6.3. Since these moves are very important and their behavior can affect the performance of two neighbourhoods based algorithms (VNS algorithm and FNS algorithm).

We have used five neighbourhood moves for our preliminary experiments and found that move2 and move4 are not much efficient for the considered problems, so we have used

remaining three moves (move1, move3, move 5) for our final experiments. In the following we compare these three moves for various size problems.

Figure 5.17 shows the behaviour of three neighbourhood moves on small dataset. This graph is drawn for results obtained across 1 run on one data instance. This data instance is chosen from small dataset.

We have found that move 1 perform best for this size of problem. Move 3 is not performing well in the start but its performance improves in the end of the search process. Move 5 has shown lower performance than other two moves. It begins with good starting value but after certain time its performance becomes stagnant.
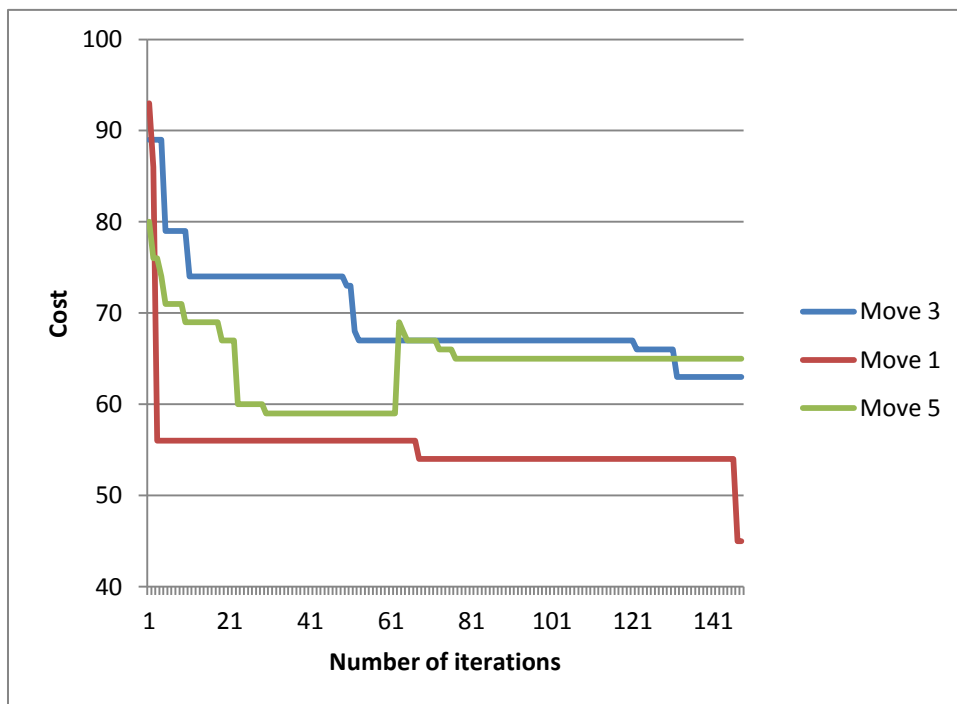
Figure 5.17: Graph representing behaviour of Neighbourhood moves on small dataset. Here X-axis represents number of iterations and Y-axis represents the cost.

Figure 5.18 shows the behaviour of neighbourhood moves on medium dataset. Here we can see that move 3 has shown overall good performance than other two moves. Move 1 has shown quick decrease in the beginning but it does not prove to be the best as number of iterations increases. Move 5 shows bad performance in this case.

Figure 5.19 represents behaviour of neighbourhood moves on large dataset. It shows the move 5 to be the best here. It has a significant difference of performance with respect to other moves. Move 1 is performing well than move 3. So these two moves cannot produce best result for large dataset.

As we have seen that none of these moves is dominant on others because for small dataset move 1 was best, for medium dataset move 3 is best and for large dataset move 5 is best. That is why we decide to use them together for exploration of search space in VNS algorithm and FNS algorithm.
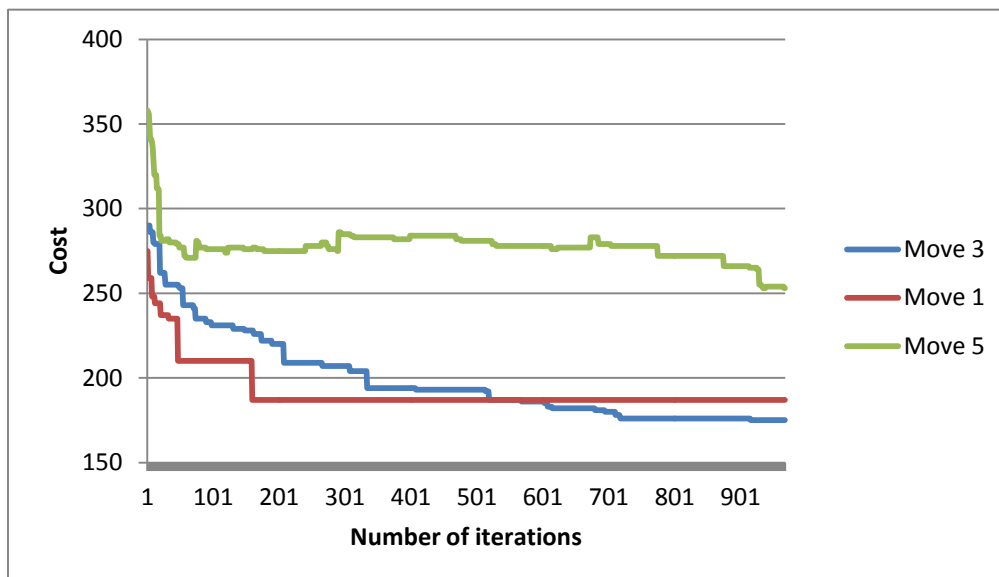
Figure 5.18: Graph representing behaviour of Neighbourhood moves on medium dataset. Here X-axis represents number of iterations and Y-axis represents the cost.
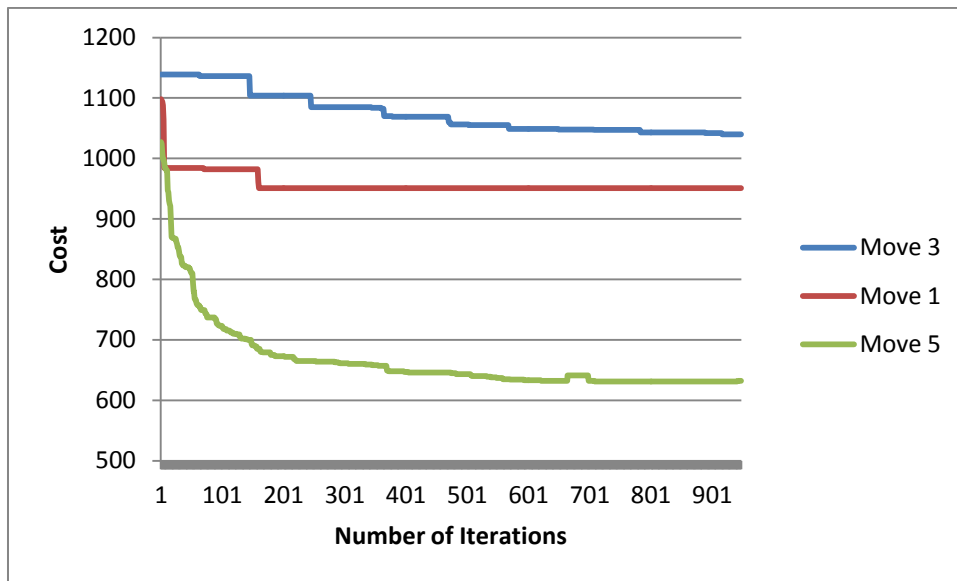
Figure 5.19: Graph representing behaviour of neighbourhood moves on large dataset. Here X-axis represents number of iterations and Y-axis represents the cost.

The second important decision in neighbourhood based algorithm is the criteria of accepting the solution produced by neighbourhood moves. Here we have analysed two types of criteria: greedy and random. The greedy criterion means to accept only the better solutions produced by the move and worse solutions are rejected. The random criterion means to accept all solutions produced by the move.

Now we show the comparison of these two acceptance criteria for our neighbourhood based algorithms. Figure 5.20 represents the behaviour of greedy FNS algorithm and random FNS algorithm for small dataset. We have used their names on the basis of applied decision criteria. The performance of random FNS algorithm is clearly better than greedy FNS algorithm. We think that this is due to lack of randomness and greedy FNS gets stuck in local minima. There are two reasons of this deficiency in randomness, absence of shaking process and greedy decision criterion in greedy FNS algorithm. So we have used random FNS algorithm for our detailed experiments.
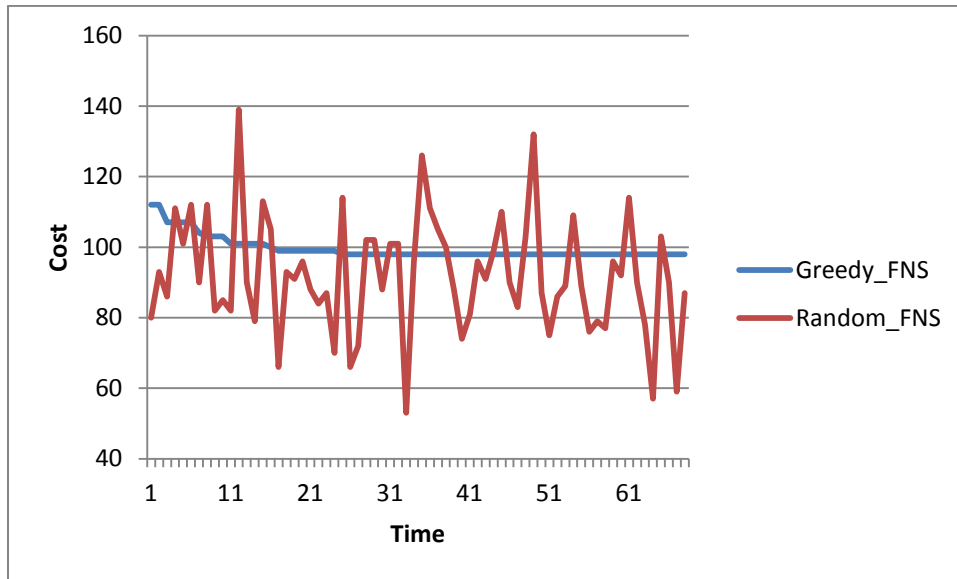
Figure 5.20: Graph representing behaviour of greedy and random FNS algorithm. Here X-axis represents the time elapsed and Y-axis represents the cost.

Figure 5.21 represents comparison of greedy VNS algorithm and random VNS algorithm for medium dataset. Contrary to FNS algorithm, here greedy VNS algorithm proves much better than random VNS algorithm. The reason may be applying this random decision criterion induces much randomness in VNS algorithm because shaking process in this algorithm already gives randomness in search space. Due to this over randomness the search process fails to converge towards good solutions.
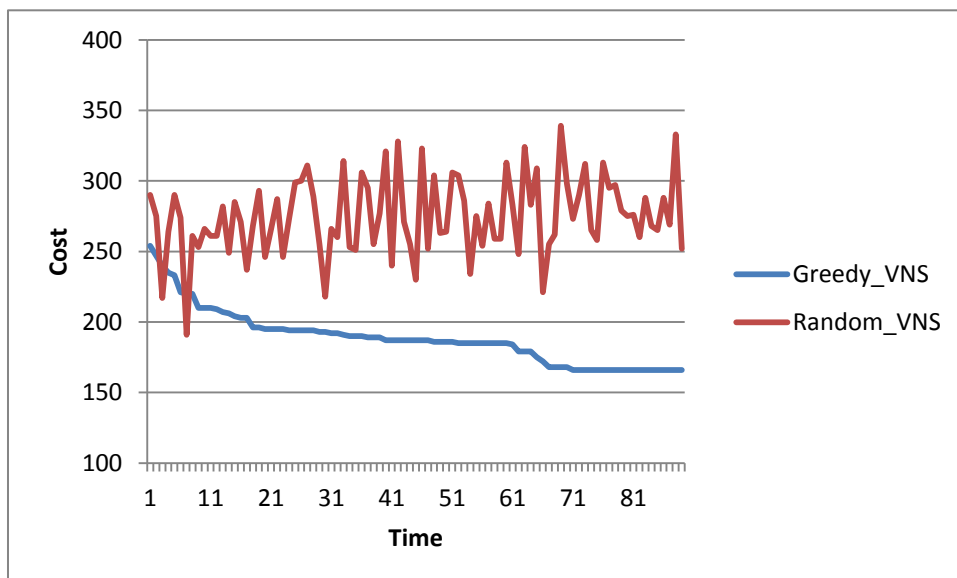
Figure 5.21: Graph representing behaviour of greedy and random VNS algorithm. Here X-axis represents the time elapsed and Y-axis represents the cost.

Now we are presenting the analysis performed for the tuning of parameters for the proposed genetic algorithm (GA). The purpose is to choose appropriate values for the parameters and good performing operators.

Figure 5.22 shows comparison of GA algorithm for different values of population size. It ranges from 10 to 100. Population size 50 produces best values for cost but for 10 its value is lower than 50 and good than 100. We think that population size 10 decreases the diversity and so search process can get struck to any local minima. On the other hand population size 100 makes the search space very wide and due to this search process gets slow down.
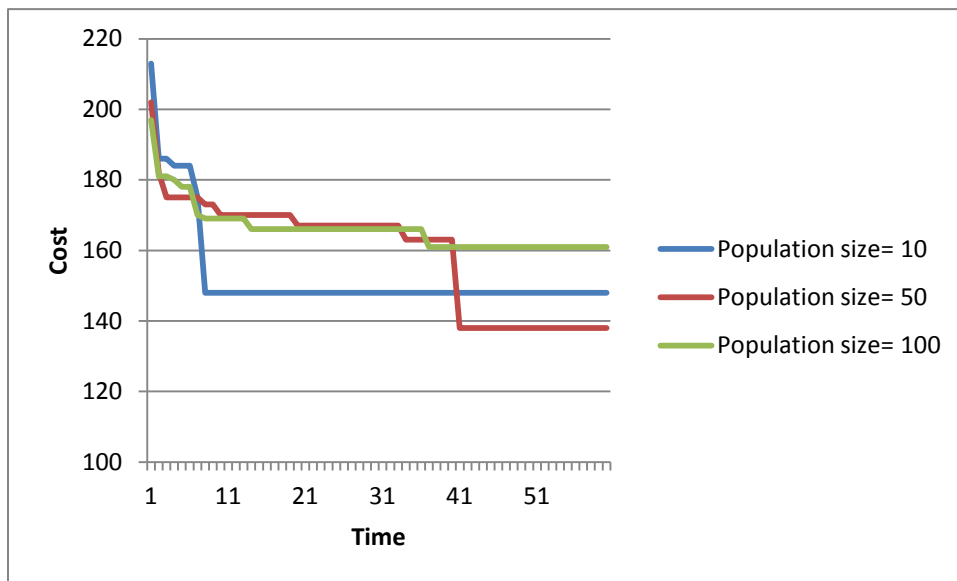
Figure 5.22: Graph representing comparison for different population size on GA algorithm. Here X-axis represents time and Y-axis represents the cost.

In Figure 5.23, we present the comparison for different values of mutation rate in GA. Mutation rate represents mutation operator can be applied on how many members of population colony. The value of mutation rate ranges from zero to hundred. Here mutation rate equal to hundred represents mutation applied to all members of population colony and mutation rate equal to zero represents no mutation at all. For RSHHCW problem, mutation rate equal to one gives the best results. If we increase mutation rate, the performance of GA

gradually decreases and shows worst result for mutation rate equal to hundred. Likewise, if mutation rate is decreased than one, the performance also deteriorates. Less than one mutation rate case corresponds to no mutation applied in the Figure 5.23.

In Figure 5.24, we give the comparison of three selection operators (elitism, roulette wheel selection RWS, random) in GA. One can note that elitism performs best and random performs worst. RWS shows intermediate results for this problem. This behaviour leads us to choose elitism and RWS for selection of parents in genetic algorithm. The purpose of combining RWS with elitism is to provide randomness in population colony while retaining good solutions by means of elitism. Another advantage of the roulette wheel selection is that it provides more chance to survive to fittest individuals than weaker ones. Thus there is a good probability that fitter individuals can survive for the next generation through the mating pool. At the same time weaker individuals also have chances of being selected which may also be useful for future generations.
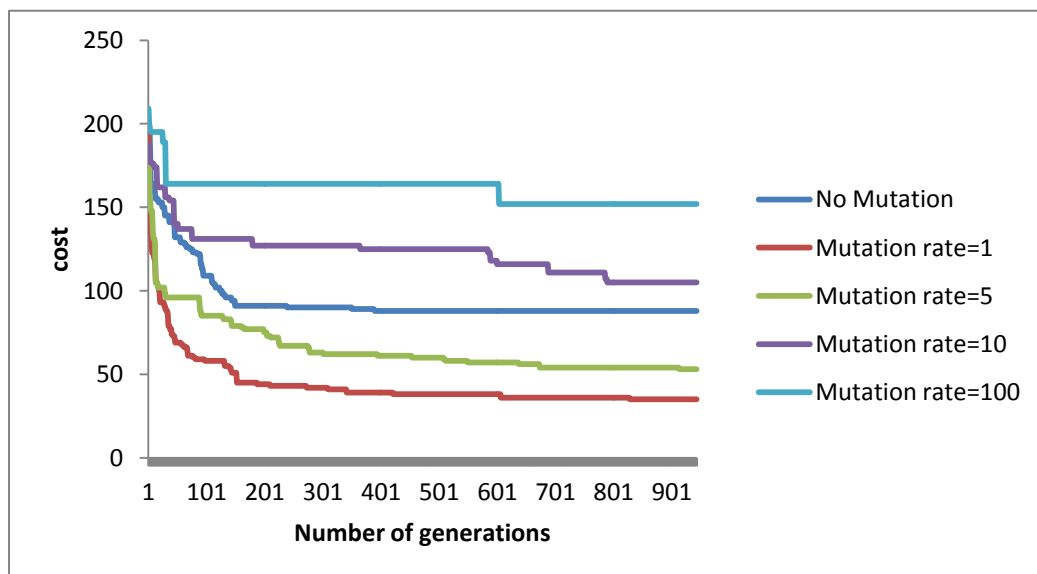
Figure 5.23: Graph representing effect of different mutation rates in GA. Here X-axis represents number of generations and Y-axis represents the cost.
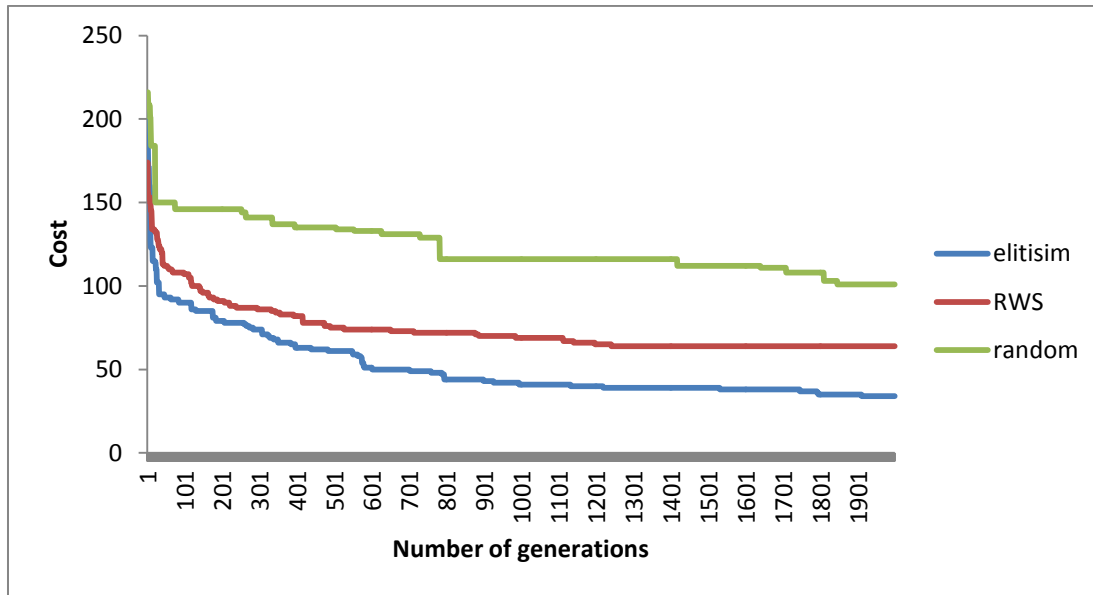
Figure 5.24: Graph representing effect of different selection operators in GA. Here X-axis represents number of generations and Y-axis represents the cost.

Now at the end we discuss the overall behaviour of all algorithms to solve RSHHCW problem. In Figure 5.25, we present a graph to compare the behaviour of four proposed algorithms (FNS, VNS, GA and SA) for large dataset. We have chosen one instance of large dataset for this comparison. From this graph, it is evident that FNS algorithm is performing best. The second best performing algorithm is GA algorithm. While VNS algorithm and SA algorithm are behaving in a same manner. We have shown a glimpse of algorithm's behaviour by this graph, but one can see their detailed comparison from our experimental work given in section 5.11. We think the performance of FNS algorithm is better due to its ability to explore and exploit the search space simultaneously.
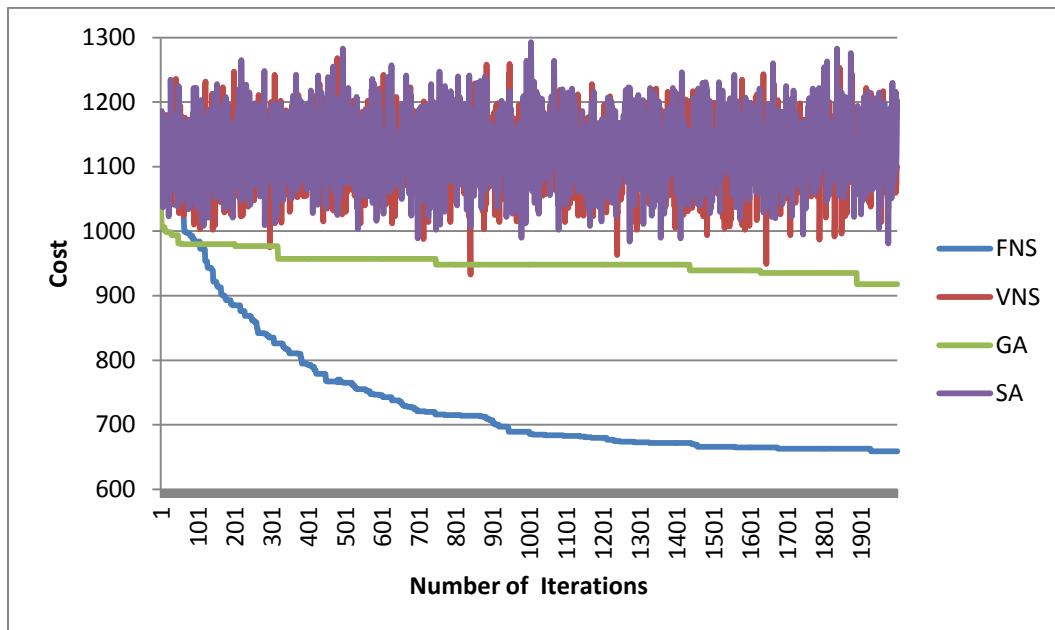
Figure 5.25: Graph displaying the behaviour of four proposed algorithms (FNS, VNS, GA, and SA) for large datasets. Here X-axis represents number of generations and Y-axis represents the cost at each generation.

We conclude that the behavior of these algorithms with the small, medium and large datasets is different in each case for our experiments. Their performance is well for small dataset and there is less difference of objective function value. But for medium dataset we have seen mixed pattern of their performance. The performance of VNS algorithm and FNS algorithm improves while performance of SA algorithm and GA algorithm declines. Finally, in large dataset FNS algorithm produces the best results.

Chapter 6

6 HOME HEALTH CARE DISTRICTING PROBLEM

Districting is the process of partitioning a large region into smaller areas for organisational or administrative purpose. This process is carried out in many applied fields such as political districting, school districting, salt spreading, design of territories for salesmen, police districting, health care districting, districting for emergency services and districting for the organisation of the distribution or collection operations. Most districting approaches in literature are of agglomeration type. This means a large geographical region to be divided into districts is pre-partitioned into smaller basic areas (or units) which are aggregated into districts afterward. So districting becomes finding groups of smaller areas or units to optimise certain criteria or objectives while satisfying some side constraints. Some objective functions used for districting problem can be minimization of cost, minimization of response time, minimization of travelled distance and balanced workload. The usual constraints used are indivisibility of basic units, contiguity or connectivity of districts, compactness, respect of administrative boundaries etc.

6.1 Introduction

Districting means the dividing of a large geographical region into sub-areas (districts) according to some criteria, to facilitate the organization of the operations within that region. A common approach to deal this problem is to partition the region into a large number of elemental areas (or basic units and to aggregate these units into districts, while an objective function is optimized and some constraints are satisfied. In general, the constraints ensure the contiguity, compactness and balance of the districts, while it is usually not allowed to split a unit over several districts.

In this chapter we focus on the problem of districting for home health care services. The home health care districting (HHCD) problem is a tactical planning problem which can influence RSHHCW problem solution quality. A set of geographic zones must be partitioned into districts to be served by home healthcare workers, such that workload is balanced across districts and home healthcare workers travel is minimized. A set partitioning model is formulated for HHCD and two types of metaheuristics are developed which use the neighbourhood moves, local search and evolutionary approach to explore the search space. Test instances are generated randomly to see the performance of the proposed metaheuristics.

We propose a mathematical formulation which integrates the various soft constraints into a multi criteria objective function. This work concerns an application of the metaheuristics techniques to a graph–theoretic formulation of the districting problem in home healthcare context. As HHCD problem is computationally very hard to solve; thus it makes sense to look at metaheuristics, in order to find good feasible solutions with a modest computational effort. Our objective is the comparison of different metaheuristics in terms of quality of the solution found and computation time.

6.2 Literature review

This problem has been widely studied in the operations research literature in variety of applications. Typical applications of this problem include the design of political districts (e.g. [128, 129, 130]), sales territories [126, 135], emergency and health care districts [136], etc. We present the related works in different domains and we particularly focus on the problem of districting for home health care services.

6.2.1 Political districting

In the political districting problem, a territory is divided into a given number of districts from which political candidates are elected. The basic requirement for political districting is to ensure the "one-man-one-vote" principle. As one member is elected from each district, the districts designed must have almost equal number of voters while satisfying some other criteria. This problem is formulated mathematically for the first time by Hess *et al.* [123] as a

location-allocation problem. The authors considered the criteria of contiguity and compactness for designed districts which respect the indivisibility of basic units' constraints and whose populations must lie within a predetermined interval.

After that, [128] addressed the problem of political districting as a set partitioning problem. They have presented a two-stage enumerative procedure which minimizes the maximum deviation of each district size from the average size. In the first stage, they have generated feasible districts based on criteria related to population equality (total voters within an interval), compactness and contiguity. In the second stage, they have determined the set of optimal districts that could minimize the maximum deviation of each district's population from the average population while satisfying the constraint of indivisibility for basic units.

More recently, [127] applied a three-stage location-allocation approach to divide a territory into a given number of districts while respecting criteria related to contiguity, compactness and population equality. By using this methodology, district centers were determined. Then, basic units were allocated to those districts. Finally the basic units that were divided between two districts were reassigned to only one district.

Furthermore, [129] built their work based on the previous work of [128] and formulated the problem as a set-partitioning of the districts. These latter were characterized by population equality, contiguity, non-splitting of the basic units and respect of administrative boundaries as much as possible. [130] also proposed a weighted multi-criteria approach based on five criteria: contiguity, population equality, compactness, socio-economic homogeneity and similarity with the existing plan where the first criterion is considered as a hard constraint while the others are combined in a weighted additive multi-criteria objective function.

After that, [131] have formulated the political districting problem as a multi-criteria set partitioning problem. The criteria considered are: indivisibility of basic units, contiguity, population equality, compactness and conformity to administrative boundaries. The main objective of this problem is to minimize one or a convex combination of the three last criteria.

6.2.2 Sales territory alignment

In sales, the region is divided into small groups (consisting of smaller sales units) to balance the workload or salesmen's responsibilities in terms of number of customers. These groups must have almost equal size. [124] first applied a location-allocation model in order to maximize the total compactness of all districts while minimizing the changes of the existing boundaries. The criterion was to balance the activity or workload of the salesmen. The authors suggested different "activity" measures such as the number of sale calls or sale potential and highlighted the importance of selecting well the "activity" measure since it could influence the quality of the solution. [137] also proposed a heuristic approach for first constructing sales regions and then subdividing each region into sales districts. These districts were built by starting with their centers and were extended so that the workload of each salesman was approximately equal to the average workload of the region without splitting the basic units between districts and by respecting administrative boundaries while integrating managers' preferences. The shortcoming of this approach is that it does not provide a methodology for partitioning the territory since it represents a manual adaptation of managers' preferences.

Another heuristic approach based on managers' preferences which maximizes the profit while balancing the workload had been suggested by [138]. Here profit is dependent on the time spent in each district and the number of trips made in each district. Their heuristic procedure determined the optimal sales call frequencies and also the optimal partitioning of the region into districts.

[139] suggested a mixed integer programming model for the sales territory alignment problem. The objective considered in this case was the minimization of the total travel driving distance of salesmen while respecting the balance of the travel driving distance between districts, indivisibility of basic units, compactness and contiguity of districts. This model had been solved via an interactive heuristic which provided the possibility of changing the assignments of basic units to districts to the managers so that they could take into account their preferences.

After this, [126] developed four properties of the sales territory design which are the indivisibility of the basic units, activity balance according to predefined attributes, contiguity

of the districts and compatibility with geographical obstacles. The authors suggested that the respect of these properties would lead to good partitioning of the region. In order to satisfy these four properties, they proposed a location-allocation model whose objective would be the minimization of the travel time or the maximization of the profitability. [135] had also approached the sales territory alignment problem by a location-allocation model which satisfied the workload balance, compactness of the districts and indivisibility of basic units.

More recently, [140] have suggested a location-allocation model where the objective is the maximization of the total compactness while balancing activity measures such as the number of customers, product demand and workload among the contiguous districts.

We found application of districting approach for different types of services in the literature. Among these studies, we can identify the school districting problem, salt spreading application, electrical power and police districting problems.

6.2.3 School districting

In school districting problem, the student for each school are specified. It means we find the students who would attend a specific school. It is different from the previous application because here it is not the salesmen/service providers that travel to the customers but it is the students that have to travel to schools. Here the criterion of the minimizing the distances between school and students homes is more important in this area than in the other domains.

For school districting application, [141] formulated the problem of assigning students to schools as a location-allocation model whose objective consisted in minimizing the total weighted distance associated with the assignment of students to schools while respecting capacity limitations and racial balance constraints. [142] also developed an interactive decision support system that included multiple heuristics so as to design contiguous districts which guarantee that students could attend the same school from year to year while respecting the capacity constraint of each school. In fact, they had not presented an optimization model but the users had the possibility to interact with the system to modify solutions in order to improve the contiguity, homogeneity or respect of school capacities for some grades criteria.

This system also allowed a very rapid, precise and easy analysis of scenarios in order to determine the most suitable solutions.

Using an approach similar to those proposed by [126], authors in [148] identified seven criteria for the good school districting: indivisibility of the basic units, respect of grades' capacities, contiguity, compatibility with geographical obstacles, compactness related to the total distance traveled by all students, assignment of students to the same school for all the grades and similarity with the existing districting pattern. Furthermore, the authors considered an additional criterion related to the maximum distance walked by students in order to guarantee the individual satisfaction. The objective function considered in the optimization model proposed was the minimization of the total walking distance. This model had been coupled with a commercial Geographic Information System (GIS) whose integration allowed an interaction between the user and the model in different manners which permitted the involvement of other issues not considered in the model such as teachers' availability and opinion or the parents' point of view. The advantage of this interaction was that it allowed the users to solve highly subjective problems where it was important to handle complexity and human's intuition and experience.

Note that it may also be interesting to develop such systems for solving the home health care districting problem that would integrate the home health workers' point of view and availability in such a way that an adequate equilibrium between this qualitative criterion and the best solution obtained through the mathematical formulation can be reached.

6.2.4 Salt spreading services

The design of districts for salt spreading and road maintenance operations involves the partition of a large geographical region into districts in order to facilitate the organization of the operations to be performed within this region. This case had been studied in the literature by [146, 147] who assumed that the partitioning of the territory's road network into districts must favor the contiguity, compactness, non-splitting of basic units criteria but also the centrality of the depots (whose locations were given) such that each route started and ended at a depot. The objective function to optimize can be: the minimization of the number of trucks,

minimization of the total distance, minimization of the number of vehicles required or the balance of the workload.

6.2.5 Electrical power districting problem

Another type of services for which districting models have been developed is the electrical power problem. Within this context, the districting problem involves grouping electricity users' units into districts of approximately equal revenue. A mathematical programming approach developed by [143, 144] was based on a multi criteria model that could minimize both the total compactness and the total deviation of revenue potential in each district from a target value.

6.2.6 Police districting

Hess et al. [123] gave the original location-allocation (LA) formulation for the political districting problem, which Hess and Samuels [124] later applied for the sales territory alignment problem.

Hess and Samuels [124] solved LA by relaxing the integrality constraints, solving the resultant linear programming, and rounding the solution so that subunits are not split between districts. The disadvantage of this approach was that the rounded solution may not be optimal, and wais also likely to violate attribute balancing constraints. Additionally, district contiguity was encouraged by the objective function but was not guaranteed. Linear district contiguity constraints which could be incorporated in LA are given in Shirabe [125], but the author demonstrated through a computational study that problem instances containing more than 100 subunits were not manageable.

Zoltners and Sinha [126] developed an alternate approach for incorporating contiguity constraints into a location-allocation formulation. An adjacency graph was created which consisted of nodes associated with each subunit and district center. The graph included edges between pairs of adjacent subunits and adjacent district centers.

Hojati [127] developed a facility location formulation that did not require a predefined set of district centers, but instead allowed district centers to be selected from population subunits.

 Garfinkel and Nemhauser [128] proposed a set partitioning formulation for a political districting problem and developed an enumerative approach for its solution. The set of all districts satisfying contiguity, shape compactness, distance compactness, and population compactness criteria were enumerated. Each of the compactness measures are specified as nonlinear functions of the subunit groupings. For example, shape compactness was defined as the ratio of the maximum distance between any two included subunits and the area of the district.

Mehrotra et al. [129] presented an optimization based solution approach for solving set partitioning which did not require enumerating all feasible districts in advance. They instead developed a branch-and-price procedure, using column generation to generate new districts on an as- needed basis. The procedure first solved the linear relaxation of set partitioning on an initial subset of feasible districts. Then, a pricing sub problem which used a linear cost function was used to find negative reduced costs districts which might improve the solution. Solving the pricing problem to optimality required solving a two-sided knapsack problem with additional contiguity side constraints for each subunit, where each problem found the minimum cost district centered at the associated subunit. While this approach was optimization-based, it was not exact, because the contiguity constraints the authors employed in the pricing sub problem excluded some portions of the feasible region.

Bozkaya et al. [130] proposed a set partitioning model for the political districting problem that included a contiguity constraint and an objective function which incorporates various criteria with nonlinear representations: population equality, compactness, socio-economic homogeneity, similarity to the existing plan, and integrity (non-splitting) of communities. They solved the model using a tabu search procedure that included two neighbourhoods. The first was a shift neighbourhood, which included all feasible solutions that could be reached from the current solution by moving a subunit from its currently assigned district to another district. The second was a swap neighbourhood, which included all feasible solutions that could be reached by swapping a pair of subunits between a pair of districts.

Ricca and Simeone [131] also developed a heuristic procedure to solve a set partitioning for-mulation of the political districting problem. Four heuristic methods (descent, tabu search, simulated annealing, old bachelor acceptance) which used shift neighbourhoods were

evaluated under a variety of objective functions (population equality, compactness and administrative conformity). Excluding descent, each method demonstrated good performance on their test problem.

In literature, the districting problem within the police patrol context had been studied by [145]. They modelled the problem as a set-partitioning problem subject to constraints of compactness, contiguity and also quality of service considerations related to the response time to calls for service which had to be minimized and/or lies within an interval. Note that the "goodness" of a district was related to the disparity between the maximum workload and the minimum workload of the patrol officers and also to the average response time to a call. After the design of the districts, the optimal number of patrol cars was determined for each district.

Based on this literature review, we assume that despite the various applications of the districting problem, there are many similarities between them. More precisely, the HHC districting problem share common features with the applications characterized by the importance of the human factor namely: the sales territory alignment and the service districting problem. Indeed, the partitioning proposed would have an important impact on the employees (salesmen and service providers) and the customers. The criteria used within these areas whose respect allows the improvement of the service quality towards the employees and/or the customers as well as the improvement of the service delivery or sales processes' efficiency (e.g. compactness, indivisibility of basic units, activity balance, etc.) must thus been considered for the HHC districting problem in order to enhance the objectives of adopting this approach within the HHC context. Nevertheless, we assume that the criteria used in the political field are identical (such as compactness, contiguity, indivisibility of basic units, etc.) or analogous to the ones used in the others applications in the sense that they aim at balancing an attribute which can be population number for the political area or workload, number of customers, etc. for the sales and service areas.

The approaches proposed to solve districting problem in literature can be categorized in two types: the managerial approach and the exact methods based on mathematical programming techniques. In the following, we present some of the works dealing with the districting problem.

6.3 Home health care districting

In this work, we have considered the districting problem for home health care provider agencies. Home health care (HHC) agencies often provide services in large geographical areas. The services are provided to patients in their residences by healthcare workers. The problem faced by these agencies is first to assign one or more patients to every healthcare worker and then to prepare a visit schedule and the days and times on which these visits will occur. Since every healthcare worker can visit a limited number of patients every day. The service provided by a healthcare worker is affected by the size of the region in which their assigned patient requests are distributed. The quality of these schedules can be improved by dividing the whole service region into sub regions or districts to be served by every healthcare worker.

We define home health care districting problem for a region that consists of a set of smaller basic units. The aim is to group these smaller basic units into districts and deploy a healthcare worker team dedicated to each district. This can reduce the travel times of healthcare workers and thus can play role to reduce the costs of service. The benefits of better districting in HHC service can be more efficient service for the patients in terms of costs and quality and more balanced workload for the healthcare workers. Also by assigning a healthcare worker team dedicated to every district can help in the continuity of care due to the fact that patients receive the care from the same team and thus do not have to change continuously their relationships with a new team.

Finally, this approach may improve healthcare workers' working conditions through the workload balance and reinforced collaboration inside the team which would enhance the satisfaction of healthcare workers. Within this chapter, we consider the workload balance as criteria for improving working conditions.

The main objectives of this chapter are to formulate the home health care districting problem (HHCD) by considering the continuity of care, the workload balance and minimizing the distances travelled by home healthcare workers. We find a district design for a connected service region that includes a set of subunits, *e.g.*, zip codes, and a staff of healthcare workers

with limited capacities that must be deployed to serve patients' demand within the region. HHC service districts must be designed such that each zip code is assigned to exactly one district and the workload of each district must be within the allowable workload bounds of the healthcare worker or team of healthcare workers serving the district. To limit distance travel between patient visits, districts should be geographically compact and contiguous. Additionally, the number of healthcare workers serving each district should be small. So the number of different healthcare workers received by any patient during their process of care is limited. Consistency of healthcare service provider, referred as continuity of care in the health literature, has positive effects on health care outcomes. Studies have shown a correlation among continuity of care, increased patient satisfaction, decreased hospitalizations and less emergency room visits [101]. Although we are not aware of a study directly linking improved HHC outcomes with continuity of care, the correlation is expected because correlation between continuity of care and improved health outcomes is most consistently indicated for patients with chronic conditions, which comprise the majority of patients receiving HHC services.

In this chapter, we present a set partitioning model and optimization-based metaheuristic for the home health care districting problem. The goal of the approach is to create home health care districts which are contiguous, compact, balance the expected workload across districts and minimize expected operational routing and scheduling costs.

The plan of the remaining chapter is as follows. In Section 6.2, we provide the description of HHCD problem and types of mathematical models being used to formulate the districting problem. Section 6.3 gives the definitions of basic terminology used here and Section 6.4 presents the criteria considered for this problem. The next section provides the notations and parameters used to develop mathematical model. Section 6.6 contains the description of HHCD problem in the form of a graph. Next we present decision variables, binary integer mathematical model for HHCD problem and its explanation in Section 6.7. We address the complexity issues of the home health care districting problem in Section 6.8. In Section 6.9, four basic metaheuristics (Variable Neighbourhood Search, Fixed Neighbourhood Search, Simulated Annealing, and Genetic Algorithm) are presented and details about their implementation for the above problem are provided. Section 6.10 provides the details about

data used to solve HHCD problem. Section 6.11 describes the results of our computational experiments. Final discussion and conclusions are presented in Section 6.12.

6.4   Description of the problem

The political districting studies in literature developed approaches to group subunits into contiguous and compact districts such that each subunit is assigned to exactly one district and voter population count is balanced across districts. The primary differences between these applications and the home health care districting (HHCD) problem are as follows.

−   The attribute balanced across districts cannot be represented as a simple count of the patients in each district. The objective is to balance workload of healthcare workers, which includes time spent visiting patients and time spent travelling between patient locations. Home health agencies often serve geographic regions comprised of both metropolitan and rural areas, where travel between patient locations is expected to be higher in the less densely populated portions of the region.

−   The compactness measures used in the political districting literature favour districts of regular shape. We adopt district cost measure that favours districts which maximize the productivity of a mobile workforce by minimizing the travel required to serve patients' demand.

Accurately modeling these problem characteristics requires nonlinear representations of both district cost and district workload balance constraints.

We now state the districting problem in home health care. In this study, we assume that demand information is available at the basic unit level. The region in which home healthcare workers have to provide services contains a set of n basic units. Where every basic unit has demand measured in the number of visits needed for patients living in that subunit.

6.4.1 Mathematical models for districting problem

Two types of mathematical models have been used to model districting problem. These are location allocation models and set partitioning models. When location-allocation models are

used, it is assumed that a set of fixed district centers or a finite set of possible district centers is given. The required decisions include selecting districts' centers, if necessary, and assigning subunits to the selected district centers. These decisions are made taking into account of given constraints.

Modeling the districting problem by set partitioning model does not require that the centre of a district is specified. Instead, the set of all feasible districts is assumed to be available, and required decisions include selecting a subset of districts such that each subunit is included in exactly one district. Location-allocation and set partitioning models from the districting literature are reviewed in section 6.2.1.

6.5   Terminology

Before giving the mathematical model, we present the definition of some of important terms which are used to describe HHCD problem.

6.5.1 Basic unit

A basic unit is an aggregation of patients living in the same location. Typically, a basic unit can be a zip code area, postal area, geo-code address, etc. The distance that separates two patients living in the same basic unit is negligible.

6.5.2 Service region

There is a connected service region which contains a set of n basic units, where each basic unit has demand that is measured in terms of the number of visits required to patient living in that basic unit. The specific locations to which visits are required each day are not known in advance, but are assumed to be independent and uniformly distributed throughout the service region. The service region is covered means that healthcare workers should be assigned to all patients living in that region.

6.5.3 Health care worker

Human resources delivering care to patients are of the same type, namely the health care workers, who are multi-skills, i.e. able to treat the different types of patients among all the basic units. There are an enough number of health care workers available. Each health care worker has a predetermined capacity (i.e. he/she can handle a certain volume of workload). This capacity is identical between the different health care workers. This capacity includes time spent on visiting patients and travelling between patient locations.

6.5.4 District

Each district is under the responsibility of a unique health care workers' team. We want to construct p contiguous districts, one for each team of health care workers, such that the daily workload is balanced across districts. The daily workload of a district is defined as the time required for *health* care workers to serve the demand of each basic unit comprising the district, including time spent visiting patients and travelling between patient visits. Workload is balanced if the workload of each district is within an allowable percent deviation from the target workload specified by parameter τ. Smaller values of τ allow less disparity in district workload. The demand of a district is equal to the sum of the demands of each included basic unit.

6.5.5 District demand

The demand of a district is equal to the sum of demands of all basic units assigned to this district. The demand of a basic unit i is defined as sum of number of visits required for different types of patient living in that basic unit.

6.5.6 District feasibility

For a district to be feasible, it should be either contiguous or the distance between any of the two basic units assigned to it should be less than a given max limit (d_{max}) and all the basic units assigned to the same district should be compatible. The basic units of a district can be

connected directly (share at least one common boundary) or via another basic unit of the same district (there exists another basic unit of the same district between these two basic units). The expected daily workload for healthcare workers assigned to serve the district should be within the allowable bounds of the average workload of all districts.

6.5.7 Different types of patients

The patients considered in this study are of three types. The number and average duration of visits that characterize the patient type are known. A patient is of type I if and only if its care workload in each basic unit cannot be equal to zero. Type II represents the profiles whose workload can be equal to zero for any basic unit. However, if the care workload of a basic unit i is not null in the start of period of optimization, it cannot be null during the optimization period. Indeed, both profile types I and II correspond to profiles characterized by a long duration of stay within HHC system. In other words, patients that are admitted in HHC system would be treated for the whole planning period. These profiles generally correspond to continuous care such as breathing assistance that is delivered for an undetermined duration in order to maintain current functioning levels to patients. The difference between them is that type I represents the case where, at the beginning of period, there are already patients having these profiles in all the basic units covered by the HHC system. On the contrary, profile type II corresponds to the profiles that do not exist in all the basic units.

Finally, a profile is of type III if there is no restriction on number of visits in each basic unit. This type of profile corresponds to the punctual care such as chemotherapy, anti biotherapy which are intended for patients having not stabilized pathologies and treated for periods determined beforehand. Care can be reiterated frequently. Indeed, we need to distinguish between the different types of patients in order to be able to model the criterion related to the change of the basic units' assignments to districts.

6.6 Criteria considered

We consider the following criteria for the home health care districting problem:

• The compactness is formulated as a hard constraint by limiting the maximum distance between two basic units that would be assigned to the same district.

- The workload balance is essential for the design of "good" districts. It consists in having almost the same workload in the different districts.

- The continuity of care is crucial for guaranteeing a good districting process. As we have explained before, the continuity of care constraint is modelled by the indivisibility of the basic units and the fact that changes in the assignments of these basic units to districts are not allowed.

- The compatibility criterion (related to the accessibility of the basic units and conformity of the districts to the administrative boundaries) in the model is considered by introducing a compatibility index for every basic unit with respect to all other basic units.

As explained earlier, the human aspect is a major characteristic of HHC services. In our model, the human aspect related to the satisfaction of home healthcare workers is captured via the workload balancing criteria. Other factors such as the motivation and learning aspects that can be associated with the structure of health care worker teams are not explicitly integrated to our model.

The human aspect related to the satisfaction of patients is captured via the continuity of care constraint which ensures that the patient receives care from the same home healthcare workers team responsible of the district which the patient belongs to.

6.7   Notations

We give all the notations used to formulate HHCD problem mathematically.

6.7.1 Input Parameters

The necessary parameters to formulate the model are listed below.

n Number of basic units $i, j \in V = \{1, 2, ..., n\}$

p Number of districts

i, j Basic unit indices

k District index; $k \in K = \{1, 2, ..., p\}$

E Edge set of basic units

$M^i = (\{ j \in V : (i, j) \in E \vee (j, i) \in E \})$ Set of nodes which are adjacent to node i; $i \in V$

L Number of types of patients considered. These types are defined on the basis of patients' disease and their required treatment plan.

dur_l Average duration of one visit of patient of type $l=\{1, 2, ..., L\}$

b_l Number of visits required by a patient of type $l=\{1, 2, ..., L\}$ during the period he/she is admitted in HHC system

o_{il} Number of patients of type l living in the basic unit $i \in V = \{1, 2, ..., n\}$

d_{ij} Euclidean distance between the basic unit i and basic unit j : $i, j \in V = \{1, 2, ..., n\}$

d_{max} Maximum distance allowed between two basic units that can be assigned to the same district

D Set of basic unit pairs (i, j) where $(i, j) \in D$ if and only if $d_{ij} > d_{max}$

B Set of basic unit pairs (i, j) where $(i, j) \in B$ if and only if basic unit i and basic unit j are neither connected to each other directly and nor via another basic unit,

τ Admissible tolerance of difference of the workload associated to a given district in comparison with the average workload among all districts $\tau \in [0, 1]$

c_{ij} Compatibility index, $c_{ij} = 1$ if the basic units i and j are compatible and 0 otherwise.

C Set of basic units pairs (i, j) where $(i, j) \in C$ if and only if $c_{ij} = 0$

6.7.2 Output variables

w_k Size of a district k with respect to workload

w_{av} Average workload among all districts

V_k Set of basic units included in district k

6.8   Problem description in graph form

The HHCD problem can be modelled by a graph $G = (V, E)$, where a basic unit i is associated with a node, and an arc connecting nodes i and j exists if basic units i and j are adjacent to each other. Now each node $i \in V$ has several associated parameters such as geographical coordinates (cxi, cyi) and size with respect to workload. A district is a subset of nodes $Vk \subset V$. The number of districts is given by the parameter p. It is required that each node is assigned to only one district. Thus, the districts define a partition of V. One of the properties sought in a solution is that the districts are balanced with respect to workload.

Due to the discrete structure of the problem and to the unique assignment constraint, it is practically impossible to have perfectly balanced territories with respect to workload. To account for this, we measure the balance degree by computing the relative deviation of each district from its average size.

Another important feature is that all of the nodes assigned to each district are connected by a path contained totally within the district. In other words, each of the districts Vk must induce a connected sub graph of G. In addition, HHC agency demands that in each of the districts, basic units must be relatively close to each other. One way to achieve this is to define a distance measure such as d_{max} which is maximum distance allowed between two basic units that can be assigned to the same district and D is set of basic unit pairs (i, j) where $(i, j) \in D$ if and only if $d_{ij} > d_{max}$, where d_{ij} represents the Euclidean distance from node i to nod j. So maximizing compactness is equivalent to minimizing this distance. All parameters are assumed to be known with certainty. The HHCD problem can be thus described as finding a

p-partition of *V* satisfying the specified planning criteria of balancing and contiguity that minimizes the above distance-based dispersion measure.

We want to partition the edges of G into p disjoint subsets Ek. The subgraph induced by edges of each subset Ek is serviced by only one home healthcare worker or team. The aim is to find a distinct configuration which incurs minimum costs.

6.9 BIP formulation / Mathematical Model

Here we present decision variables and binary integer mathematical formulation for home health care districting problem.

6.9.1 Decision Variables

x_{ik} is equal to 1 if a basic unit *i* is assigned to a district *k* and 0 otherwise.

We have opted for mathematical model of type set partitioning for home health care districting (HHCD) problem. Set partitioning formulation provides the flexibility to find feasible districts and calculate their cost outside of the core optimization problem. Therefore, such approaches are often used to handle complicating constraints on district structure and complex objective functions.

Given a set of contiguous districts which are feasible with respect to workload bounds, the model selects *m* districts which cover each basic unit exactly once and minimize the maximum distance between two basic units assigned to the same district.

$$\text{Min} \quad \mathbf{Z} \quad = \quad \max_{i,j \in V} \{ \ d_{ij}.x_{ik}.x_{jk} \ \}$$

We add a new variable *Maxi* in order to linearize our objective function. This variable should respect the additional constraints (2). So our objective function becomes linear given in the following:

$$\text{Min} \quad \mathbf{Z} \quad = \quad \textit{Maxi} \tag{1}$$

Subject to:

$$Maxi \geq d_{ij}.(x_{ik} + x_{jk} - 1) \qquad \forall i, j \in V \quad \forall k \in K \qquad (2)$$

$$\sum_{k=1}^{p} x_{ik} = 1 \qquad\qquad \forall \quad i \in V \qquad (3)$$

$$x_{ik} + x_{jk} \leq 1 \qquad \forall (i, j) \in C, k \in K \qquad (4)$$

$$w_k = \sum_{i=1}^{n} \sum_{l=1}^{L} dur_l.b_l.o_{il} x_{ik} \qquad \forall \ k \in K \qquad (5)$$

$$w_{av} = \frac{\sum_{k=1}^{p} w_k}{p} \qquad (6)$$

$$w_k \leq (1 + \tau).w_{av} \qquad \forall \ k \in K \qquad (7)$$

$$w_k \geq (1 - \tau).w_{av} \qquad \forall \quad k \in K \qquad (8)$$

$$x_{ik} + x_{jk} \leq 1 \qquad \forall (i, j) \in D, k \in K \qquad (9)$$

$$\sum_{\substack{i \in \cup (M^v - S) \\ v \in S}} x_{ik} - \sum_{i \in S} x_{ik} \geq 1 - |S| \qquad i \in V_k \quad , k \in K, S \subset V - (M^i \cup \{i\}) \qquad (10)$$

$$x_{ik} + x_{jk} \leq 1 \qquad \forall (i, j) \in B, k \in K \qquad (11)$$

$$x_{ik} \in \quad \{0, 1\} \qquad i \in V \quad , \quad k \in K \qquad (12)$$

6.9.2 Explanation of Mathematical model

The objective is to minimize the total cost of all districts, where the cost of a district is defined as the maximum travel distance between two basic units assigned to the same district. The objective function (1) guarantee the minimization of the maximum distance travelled among all districts. This objective function would help to improve the reactivity of home healthcare workers and to reduce the waiting time of patients as much as possible.

Constraints (3), together with constraint (10) show that every basic unit should be assigned to one and only one district. Constraint (4) guarantees the compatibility of any two basic units assigned to the same district. Constraints (5) and (6) define respectively the care workload of each district and the average care workload among all districts. The workload balance is being considered in constraints (7) and (8) which defines the minimum and maximum tolerance of care workload within each district. Finally, constraint (9) is related to the compactness criterion where the distance between two basic units assigned to the same district is bounded by d_{max}. This upper bound guarantees the containment of the travel time and thus enables a better reactivity within each district.

Constraints (10) and constraints (11) show that every district should be connected. For a basic unit i and a basic unit j belonging to the same district k are either connected to each other directly (share at least one common boundary) or via another basic unit of the same district k (there exist another basic unit of the same district between basic unit i and basic unit j), Constraint (12) shows that decision variables are the 0-1 binary variables.

Constraints (4), constraints (9) and constraints (11) are same kind of constraints as all of these constraints restrict the inclusion of a pair of basic unit in the same districts. The difference among them is that in each case the criterion of restriction is different. For example in constraints (4), the criterion is compatibility and in constraints (11), it is connectedness. We can combine these three constraints into one constraints (13) given in the following.

$$x_{ik} + x_{jk} \leq 1 \qquad\qquad \forall (i,j) \in C \cup D \cup B, k \in K \qquad\qquad (13)$$

Visiting people's mobility and workload balance are potential criteria for HHCD problem. These two criteria are combined in objective function and problem is solved by VNS and GA. Then the results are compared to see the relative performance of these algorithms.

6.10 Computational complexity

Puppe and Tasnád [102] established that determining an optimal districting is a computationally intractable NP complete problem. In order to prove this, they reduced a well known variant of set packing problem to districting problem. Where set packing problem is a proven NP-complete problem [134]. So if it exists a polynomial time algorithm for optimal districting one would obtain a polynomial time algorithm for set packing problem as well.

They showed these complexity results for political districting problem. Since the decision problem in HHCD problem is also districting with respect to different criteria and different set of constraints. So we can conclude that HHCD problem is also NP complete problem. This means that the computational effort required for finding the optimal solution grows exponentially as the problem size increases. For solution of such problems, approximation algorithms and metaheuristics are more appropriate choices.

6.11 Solution Methods for Districting Problem

This section provides details on solution methodologies used to solve HHCD problem. We have already applied these algorithms to solve routing and scheduling home healthcare workers problem described in chapter 5. Here we want to apply these metaheuristics to solve home health care districting problem. Since the home health care districting problem is quite different than routing and scheduling home healthcare workers problem, we have to modify these algorithms. In the next section, we illustrate these modifications specific for districting problem. For the sake of completeness the pseudo codes are also presented for each algorithm.

6.11.1 Problem specific input

We start by giving problem specific input for districting problem which are following:

- Initial solution
- Search space and solution space
- Neighbourhood Moves

6.11.1.1   Initial Solution

For HHCD problem, we have applied the same procedure for initialization as given in chapter 5. Here we describe in detail how this procedure is modified to make it suitable for HHCD problem. We start by generating our initial solution randomly and then repair it to satisfy the following criteria.

- All basic units should be assigned.
- Every basic unit should be assigned to exactly one district.

This repair procedure is performed in two steps. In first step, we count for each basic unit, number of districts to which it is assigned in initial solution. If a basic unit is assigned to more than one district, we remove the extra assignments randomly and made its assignment to only one district. In second step we see that if any basic unit is missing, we assign it randomly to a district. At the end of this two step verification procedure every basic unit is assigned and its number of assignments is equal to exactly one. This is explained in following Figure 6.1.

6.11.1.2   Search space and solution space

For districting problem in home health care we define the *solution* space as all the feasible partitions of set of basic units, but the *search* space as the power set P (V) of set of basic units V. The power set P (V) is the set of all subsets of V including empty set and V itself.

```
Repair procedure

    1.  Step1
    2.  for every basic unit i
    3.  if  Num_ districts > 1
            a.  while (Num_ districts >1) do
                    i.  Randomly remove an assignment
            b.  endwhile
    4.  end if
    5.  end for
    6.  Step2
    7.  for every basic unit i
    8.  if  Num_ districts <1
            a.  while (Num_ districts <1) do
                    i.  Assigne this basic unit i randomly to a district k
            b.  endwhile
    9.  end if
    10. end for
```
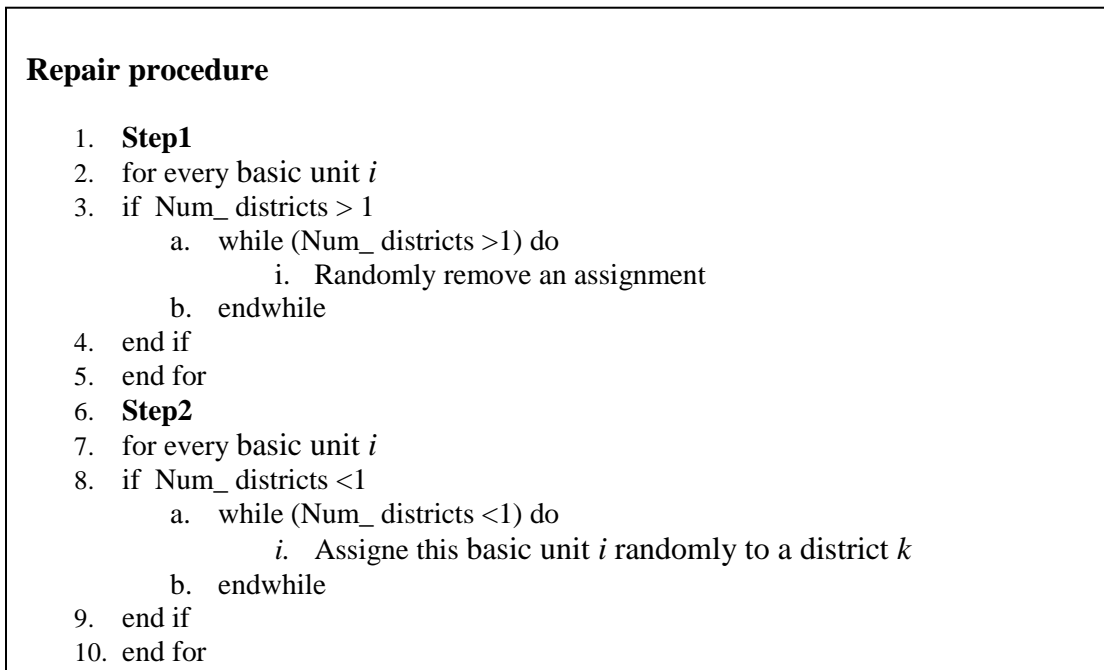
Figure 6.1: Pseudo code of repair procedure for home health care districting problem

6.11.1.3   Neighbourhood Moves

We have used the following three types of neighbourhood moves. These are explained one by one.

6.11.1.3.1   *Move 1:*

Transfer one basic unit from a district to another district satisfying the constraints of workload. Only those moves are performed which satisfy the workload constraint for both districts. Move 1 is displayed in the Figure 6.2. Here basic unit b is transferred from district A to district B to form new districts A' and B'. Note that districts are simply sets of basic units.

(a) Before applying move   (b) After applying move

Figure 6.2: Move 1 example

Note that we have not any restriction on the shape of districts. In the figures behind the rectangular shapes are only symbolic. The designed districts can be of any shape.

6.11.1.3.2   *Move 2:*

Exchange a pair of basic units between a pair of adjacent districts. The two basic units are chosen randomly. Here basic unit a and basic unit b are swapped between district A and district B to form new districts A' and B'. Figure 6.3 illustrates the Move 2.

(a) Before applying move   (b) After applying move
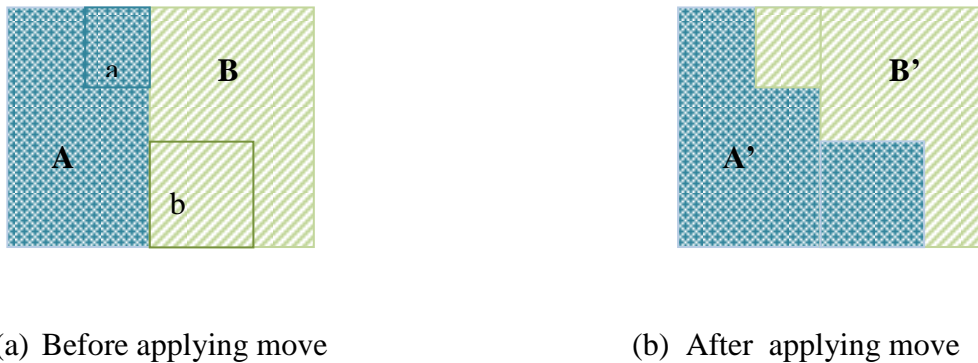
Figure 6.3: Move 2 example

6.11.1.3.3   *Move 3:*

Choose two districts and swap two basic units between first district and second district without violating the compatibility constraint. This is a local modification in assignment of two basic units. Let *A* be a district which contains basic unit a, and let *B* be a district containing basic unit b. Consider performing Move 3 and new districts A' and B' are created by swapping basic unit a and basic unit b. Now we see if this move is feasible with respect to compatibility constraints or not. This requires only the verifying the feasibility of the new district A' and B'. This is done by checking that all basic units of A' are compatible to each other. Similarly all basic units of B' should also be compatible to each other.

6.11.2   Variable Neighbourhood Search algorithm

To apply VNS algorithm for solving home health care districting problem, we need an initial solution and a set of neighbourhood structures. The initialization phase is given in section 6.11.1.1. Now we define set of neighbourhood structures specific for HHCD problem. Three main steps of VNS are neighbourhood moves, local search and shaking. The moves are earlier explained in section while local search, shaking and termination criterion are described in the following.

6.11.2.1   Proposed local search

To apply local search in VNS algorithm for solving HHCD problem, we modify the local search applied to RSHHCW problem given in Figure 5.8. We have used the neighbourhood moves specific for districting problem. This is explained in the following. We take any two adjacent districts and swap two basic units between them by using move 2 defined in section 6.11.1.3.2. If it reduces the cost function, we accept it otherwise we reject it. If the cost is adjusted, the solution which is the result of local search will be replaced with the primary solution; otherwise that primary solution is identified as the best solution in its neighbourhood and will remain unchanged. So the accepting criterion i.e. decision to accept solution by local search is based on improvement of the found solution. For the purpose of illustration, the pseudo code for the Local Search is described in the Figure 6.4.

<div style="border:1px solid black; padding:1em;">

The procedure of local search used in VNS algorithm

1. input: start with an initial solution s_o

2. while termination condition not achieved do

3. if cost of solution s_o is greater than minimum cost defined then

    a. choose two adjacent districts

        i. choose a solution s in the neighbourhoods of s_o by Move 2 (explained above in Figure 6.3)

    b. examine the solution s

    c. if the cost of solution s < the cost of solution s_o then

    d. Set $s_o = s$ and register s as local minimum found.

4. end if

5. end while

6. output: A possibly improved solution

</div>

Figure 6.4: Pseudo code for local search used in VNS for home health care districting problem

6.11.2.2   Shaking

Shaking is used when VNS algorithm is held stuck at some local minima. The purpose is to maintain diversity in search space. In shaking, we apply random moves in larger neighbourhoods. Usually the randomness in shaking is increased each time the local search process does not improve the fitness value of current best solution. In our case, shaking is applied after every ten iterations of VNS algorithm. The procedure of shaking is described in Figure 6.5.

```
Pseudo code for Shaking

    1.  for each district k1
    2.  for each basic unit assigned
            a.  select another district k2 randomly
            b.  Move the chosen basic units between the  two districts k1 and
                k2
    3.  end for
    4.  end for
```

Figure 6.5: Pseudo code for Shaking for home health care districting problem

6.11.2.3   Termination criterion

We have used elapsed time as termination criterion. VNS algorithm continues until the elapsed time reaches the time limit defined or a solution of defined minimum cost is obtained.

6.11.2.4   Pseudo code of proposed VNS algorithm

Now we describe VNS algorithm for home health care districting problem. We start by an initial solution s found as given above in section 6.11.1. Then we use three moves given above for the set of neighbourhood structures. In the start we take this initial solution s as best found solution. After it, we select a solution *s'* in first neighbourhood of initial solution *s*. Next we apply local search on *s'*. The local search gives new solution s_0, if cost of s_0 is less than cost of previously best found solution we save it as new best found solution and we continue the search focused around s_0 again with the first neighbourhood which is maximum number of applying every neighbourhood structure in one iteration. If cost of s_0 is less than cost of *s'* but greater than the cost of previously best found solution, we save it as local minimum and in this case the search procedure is iterated using the next neighbourhood. If cost of s_0 is greater than cost of *s'*, we abandon first neighbourhood and select the next neighbourhood for search procedure. After *p* iterations of this procedure, shaking is applied

on the best found solution, where p is number of iterations after which we repeat this shaking procedure. Shaking helps search procedure to escape from local minima. The detailed pseudo code is given in Figure 6.6. The values of parameters used in VNS algorithm are given in Table 6.1.

Parameter	Setting
n (number of basic units for local search)	2
k_{max} (number of neighbourhood structures)	5
p (gap between two shaking operations)	20

Table 6.1: Parameters for VNS algorithm for home health care districting problem

Pseudo code for VNS algorithm

1. Generate an initial solution *s* randomly i.e. a partition of set V

2. Repair this solution *s* by repair procedure given in Figure 6.1

3. Select the set of neighbourhood structures $N_k, \forall k = 1,...,k_{max}$

4. Set number of iterations, t=0.

5. While termination criteria or best solution not achieved

    a. For every $N_k, \forall k = 1,...,k_{max}$

        i. Find a neighbouring solution s' at random by applying N_k, the kth neighbourhood of s (i.e. $s' \in N_k(x)$)

        ii. Apply proposed local search on s' given in Figure 6.4. The new solution obtained is denoted as s_0.

        iii. If the local optimum s_0 is better than the incumbent *s*, continue search with N_k

    b. Set k=k+1

6. If t mod $p = 0$, where p denotes number of iterations after which shaking is applied;

    a. Perform Shaking on s as given in Figure 6.5.

7. number of iterations , $t=t+1$

8. End while

9. output: best found solution

Figure 6.6: Pseudo code for VNS algorithm for home health care districting problem

6.11.3 Fixed Neighbourhood Search algorithm

Now we give FNS algorithm applied to solve home health care districting problem. First we find an initial solution adopting the same procedure as explained above in section 6.11.1.

Then we use three moves given above for the set of neighbourhood structures. We compare the performance of these three moves and rank them according to their performance for

HHCD problem. We choose the size of every neighbourhood equal to l_{max}. The parameters used are given in Table 6.2.

We apply the best neighbourhood and take l_{max} random points in this neighbourhood of initial solution s and compare their costs. If we find a solution better than initial solution s, we save it as best solution found denoted by s'. The same procedure is iterated with next neighbourhood in the list and best found solution is updated if found. We continue until we have used all neighbourhoods ranked in the list. Note that contrary to VNS algorithm, shaking and local search are not applied here. We induce diversification in search process by applying random moves and skipping the use of greedy local search. The detailed pseudo code is given in Figure 6.7.

Pseudo code for FNS algorithm

1. Generate an initial solution s randomly i.e. a partition of set V
2. Repair this solution s by repair procedure given in Figure 6.1.
3. Select the set of neighbourhood structures $N_k, \forall k = 1,...,k_{max}$
4. Set number of iterations, t=0 and best found solution = s
5. While termination criteria or best solution not achieved
   a. For every $N_k, \forall k = 1,...,k_{max}$
      i. For $l = 1,...,l_{max}$
      ii. Find a neighbouring solution s' at random by applying N_k, the kth neighbourhood of s (i.e. $s' \in N_k(x)$)
   b. Set k=k+1
   c. If s' is better than best found solution
   d. best found solution = s'
6. number of iterations , $t=t+1$
7. end while
8. output: best found solution s'

Figure 6.7: Pseudo code for FNS algorithm for home health care districting problem

Parameter	Setting
n (number of basic units for local search)	2
k_{max} (number of neighbourhood structures)	3
l_{max} (size of neighbourhood)	10

Table 6.2: Parameters for FNS algorithm

6.11.4 Simulated Annealing algorithm

SA algorithm needs an initial solution and annealing schedule as input. The initial solution is generated in the same way as described in section. The main features of SA are the annealing schedule, the acceptance probability function and the criteria for move acceptance. We have applied the simulated annealing algorithm given in chapter in section to solve the home health care districting problem. We modify some parameters of annealing schedule to make it convenient for solving HHCD problem. These problem specific parameters for districting problem are given in Table 6.3. The remaining algorithm follows the same procedure as used for RSHHCW problem in the Chapter 5. To avoid repetition we refer to the pseudo code given in Figure 5.12 in Chapter 5.

Parameters	Setting
Minimum temperature	0.00005
Cooling rate γ	0.00125
Stopping criteria	Final minimum temperature

Table 6.3: Parameters for SA algorithm for home health care districting problem

6.11.5 Genetic Algorithm

For home health care districting problem, one chromosome is a partition of set V. We start genetic algorithm by finding random partitions of set V. These random partitions are then repaired using the repair procedure given in Figure 6.1. The number of these partitions is equal to population size n. We compare the cost of all individuals or solutions of this population and one with minimum cost is saved as best solution.

Parameters	Settings
Population size	50
Selection	Elitism + Roulette wheel selection
Crossover	Uniform crossover
Crossover probability	0.50
Mutation	Apply Move 2 given in Figure 6.3.
Mutation probability	0.1
Replacement	Worse individuals
Killing ratio	0.2
Stopping criterion	Time elapsed or defined minimum cost attained

Table 6.4: Parameters used for the proposed genetic algorithm

Then we select parents for breeding. Here the mother is chosen via elitism operator and the father is chosen via roulette wheel selection (RWS) operator. The purpose of combining these

two operators is to get benefits of both. Since elitism helps in retaining good solutions and RWS provides randomness in population. Next we apply uniform crossover to produce child solution and mutate it via mov2 given in Figure 6.3 before entering it into population colony. So one generation of genetic algorithm is created. Now we rank all individuals (solutions) in population colony in descending order with respect to cost. If we find an individual whose cost is less than best solution achieved so far, we update the best solution. Before creating next generation, we kill a part of population colony. The number of individuals to be killed is equal to killing ratio multiplied by population size. Here we replace or kill worse solutions or individuals of population colony. For the sake of completeness the pseudo code for proposed genetic algorithm is displayed in Figure6.8.

The tuning of parameters is very important in genetic algorithm. The purpose is to find reasonable combination of these parameters such as population size, crossover probability, the mutation probability and killing ratio for the problem being solved. For example, if we use a very small mutation rate, it can cause genetic drift. While a too high mutation rate may cause the loss of good solutions. We can combine a higher mutation rate with the elitism (selection operator) to avoid this loss of good solutions. Too high crossover rate is linked with premature convergence of the genetic algorithm. The values of the parameters used in genetic algorithm for solving HHCD problem are given in Table 6.3.

Pseudo code for Genetic Algorithm (GA)

1. input : A problem instance I

2. set the generation counter g := 0

3. while (solution_colony. population_size < n)

    a. find an initial solution (i.e. a partition of set V) randomly

    b. repair this solution by repair procedure given in Figure 6.1.

    c. calculate the cost

    d. enter this solution to the population colony

4. end while

5. compare the cost of all solutions and choose the one with minimum cost as s_{best}

6. while time limit is not reached or defined minimum cost is attained

    a. kill 20 % members of the colony

        i. while (solution_colony. population_size < n)

        ii. choose mother via Elitism and father via RWS

        iii. s_i child solution generated by applying the uniform crossover operator with a crossover rate p_c

        iv. s_i child solution after mutation with a mutation rate p_m

        v. repair this solution by repair procedure given in Figure 6.1.

        vi. Calculate the cost of s_i

        vii. enter this child solution to the population colony

    b. end while

7. compare the cost of all solutions and choose the one with minimum cost as s_{best}

8. g := g + 1

9. end while

10. output : The best achieved solution s_{best} for the problem instance I

Figure 6.8: The pseudo code for genetic algorithm

6.12 Data generation

A computational experiment is carried out in order to check the performance of our solution methods. Since we did not find any data set from the literature specifically designed for HHCD problem, we cannot compare our results with others. We have generated our own instances for the experiment using following method.

In this section, we want to analyze the behaviour of our proposed algorithms for the home health care districting problem. For the numerical analysis, we start by setting the values of the number of basic units n, the number of districts to design p, the number of types of patients L, number of patient of each type in each basic unit o_{il}, the number of visits b_l and the average duration of the visits for patients of each type dur_l, the maximum distance between two basic units assigned to the same district $d_{\max}$, the admissible difference in workload τ, the distance matrix D and compatibility matrix C for three sets of data. We called these sets as small dataset, medium dataset and large dataset. For every dataset we generate 10 instances randomly. We have used uniform distributions. The parameter values for each data set are given in Table 6.4.

We now explain the procedure of generating the used matrices. The three matrices (compatibility matrix C, distance matrix D, connectedness matrix B) have been created in the same way for all datasets.

We have generated the distance matrix D(n, n) as follows:

- For each basic unit i, we randomly generate an abscissa X_i and an ordinate Y_i from a uniform distribution DU (0, 200).

- For each pair of basic units i and j, the distance d_{ij} is then calculated according to the formula: $\sqrt{(X_i - X_j)^2 + (Y_i - Y_j)^2}$

Datasets		Small	Medium	Large
Parameter	No. of basic units	20	50	500
	No. of districts	3	4	10
	Types of Patients	3	3	3
	No. of patients of each type	DU (1, 50)	DU (1, 50)	DU (1, 50)
	Duration of visit	DU (1, 4)	DU (1, 4)	DU (1, 4)
	No. of visits	DU (1, 10)	DU (1, 10)	DU (1, 10)
	Maximum distance limit	100	150	250
	Workload permissible deviation	10 % of average workload	10 % of average workload	10 % of average workload
	Maximum number of basic units assigned to a district	15	30	60
	Computational time	10 seconds	30 seconds	60 seconds

Table 6.5: Table showing values for three kinds of datasets (Small, Medium and Large)

The compatibility matrix C(n, n) is generated as follows:

- For each basic unit *i*, we fix the maximum number of incompatibilities with the other basic units j (j=i+1…n) . In HHCD problem we have taken the number of incompatibilities equal to 20.

- For each line *i* of C:

a) We fix c_{ii}=1.

b) We randomly generate c_{ij} (j=i+1…n) such that the number of zeros in the right part of the line i is less or equal to maximum number of incompatibilities.

- For each column *i* of C:

a) c_{ij} (j=i+1…n) = c_{ji}

Since we have not used a real life data for our numerical analysis but a randomly generated data is used to understand and compare the performance of our algorithms. In real case data, we can use geographical connectedness on the basis of either two basic units share some common boundaries or not. But here in our data sets we have randomly generated this connectedness. For every basic unit i, we fixed a parameter called *Nadj* = number of nonadjacencies. i.e. a basic unit can be connected to maximum of *(n - Nadj)* other basic units. Then for every basic unit i, we randomly fix these (*n*- Nadj) other connected units. In HHCD problem we have taken the number of nonadjacencies equal to 30 (*Nadj*= 30).

6.13 Computational Results

In this section, we have given results obtained by fixed neighbourhood search algorithm, variable neighbourhood search algorithm, genetic algorithm and simulated annealing algorithm across 10 runs on each of the 10 data instances of small, medium and large size problems. Our analysis is on the basis of cost value. We use the term of cost for objective function given in equation (1) in section 6.9. The objective function is the minimization of

the maximum distance travelled among all districts. So here cost shows minimum of the maximum distance travelled among all districts.

For simplicity we replace the names of algorithms with their abbreviations. These abbreviations are GA algorithm for genetic algorithm, VNS algorithm for variable neighbourhood search algorithm, FNS algorithm for fixed neighbourhood search algorithm and SA algorithm for simulated annealing algorithm. In these tables bold entry shows the best result attained.

6.13.1.1 Minimum cost comparison

This section illustrates minimum cost comparison for three types of datasets (small datasets, medium datasets and large datasets) solved by the algorithms (Table 6.5 - Table 6.7).

For small dataset, we notice that the performance of SA algorithm is better for home health care districting HHCD problem in comparison with routing and scheduling home healthcare workers problem. Here it produces best results for six out of ten data instances. In the remaining four data instances, GA algorithm gives best results. Here FNS and VNS algorithms cannot perform well in comparison of SA algorithm and GA algorithm. There is not any significant dominancy among FNS algorithm and VNS algorithm.

	FNS	VNS	GA	SA
Instance1	83	81	81	**71**
Instance2	106	100	**90**	97
Instance3	102	111	**98**	106
Instance4	119	122	114	**107**
Instance5	105	99	104	**97**
Instance6	97	101	91	**90**

Instance7	102	102	**88**	96
Instance8	99	104	93	**92**
Instance9	83	92	**81**	87
Instance10	116	100	104	**97**

Table 6.6: Minimum cost comparison of all algorithms for small data sets

The performance of SA algorithm decreases for medium datasets as it do not produce best results for any data instance. While the performance of VNS algorithm and FNS algorithm has improved in case of medium datasets. VNS algorithm has shown best results for five out of ten data instances and FNS algorithm is best for two data instances. GA algorithm has decreased its performance little bit and has produced best results for three data instances.

	FNS	VNS	GA	SA
Instance1	**257**	273	280	291
Instance2	340	**327**	**323**	372
Instance3	275	**259**	273	301
Instance4	270	263	288	287
Instance5	277	285	**260**	272
Instance6	271	287	**270**	283
Instance7	258	**250**	282	285
Instance8	279	**257**	264	290

Instance9	273	**254**	259	271
Instance10	**222**	260	258	284

Table 6.7: Minimum cost comparison of all algorithms for medium data sets

For large datasets the performance of GA algorithm and SA algorithm has more decreased, because both of these algorithms can give best results only for one data instance for each algorithm. On the other hand VNS algorithm has shown good performance here. It produced best results for seven out of ten data instances.

	FNS	VNS	GA	SA
Instance1	1031	**979**	1062	1045
Instance2	1007	1004	1057	**957**
Instance3	990	**936**	1064	1080
Instance4	1042	**911**	1064	1025
Instance5	**969**	988	1042	1021
Instance6	1037	**1024**	1087	1071
Instance7	1058	1017	**969**	1062
Instance8	1055	**1042**	1094	1066
Instance9	1024	**968**	1081	1058
Instance10	1016	**1001**	1012	1064

Table 6.8: Minimum cost comparison of all algorithms for large data sets

6.13.1.2 Maximum cost comparison of all algorithms for large data sets

This section presents maximum cost comparison for three types of datasets solved by the algorithms (Table 6.8 - Table 6.10).

In case of small datasets GA algorithm has given the lowest maximum costs for eight out of ten data instances. SA algorithm gives the highest maximum cost for nine data instance. FNS algorithm has produced lowest cost for only one data instance. But VNS algorithm always gives higher maximum costs.

	FNS	VNS	GA	SA
Instance1	113	110	**108**	124
Instance2	**131**	133	132	140
Instance3	157	145	**140**	157
Instance4	153	153	**139**	142
Instance5	137	135	**130**	135
Instance6	145	133	**114**	141
Instance7	137	147	**116**	132
Instance8	127	135	**116**	133
Instance9	119	127	113	**109**
Instance10	170	158	**146**	168

Table 6.9: Maximum cost comparison of all algorithms for small data sets

GA algorithm shows lower maximum cost for only one data instance while SA algorithm, FNS algorithm and VNS algorithm have produced lower minimum costs for three data instance for each algorithm.

	FNS	VNS	GA	SA
Instance1	**324**	351	354	328
Instance2	**387**	391	398	392
Instance3	358	337	**330**	359
Instance4	308	**307**	353	320
Instance5	325	**324**	328	336
Instance6	344	338	343	**330**
Instance7	318	**314**	333	315
Instance8	333	342	324	**314**
Instance9	336	323	346	**314**
Instance10	**304**	316	305	336

Table 6.10: Maximum cost comparison of all algorithms for medium data sets

For large dataset, VNS algorithm has given lower maximum costs for seven data instances which shows an improvement in the behavior of VNS algorithm. FNS algorithm has given lower maximum cost for two data instances. GA algorithm and SA algorithm produces relatively higher maximum costs for all data instances.

	FNS	VNS	GA	SA
Instance1	1196	**1158**	1411	1226
Instance2	1190	**1139**	1244	1150
Instance3	1219	**1192**	1378	1311
Instance4	1246	**1083**	1278	1338
Instance5	**1191**	1221	1281	1290
Instance6	1318	**1282**	1457	1321
Instance7	1195	**1160**	1350	1172
Instance8	**1275**	1349	1345	1345
Instance9	1209	**1209**	1363	1254
Instance10	1302	**1246**	1373	1354

Table 6.11: Maximum cost comparison of all algorithms for large data sets

6.13.1.3 Average Cost Comparison

This section illustrates average cost comparison for three datasets solved by the algorithms (Table 6.11 - Table 6.13).

For small dataset, GA algorithm performs best in comparison with other algorithms. It gives best average costs for eight data instances. SA algorithm is the second performing algorithm by giving best average cost for two data instances. The average cost produced by VNS algorithm and FNS algorithm are relatively higher for all data instances.

	FNS	VNS	GA	SA
Instance1	98.8	103	**92.4**	101
Instance2	118.4	115	**109.2**	115.4
Instance3	128.4	133.6	**119.2**	125
Instance4	134.6	130	**122.6**	125.4
Instance5	121.8	122.2	**117.4**	117.6
Instance6	121.8	119.8	121.6	**116.4**
Instance7	119.4	124.8	**100.2**	118.6
Instance8	117.8	120.8	**103.2**	114.6
Instance9	98	107	98.2	**96.4**
Instance10	132.8	134.8	**120.6**	132.6

Table 6.12: Average cost comparison of all algorithms for small data sets

For medium dataset, FNS algorithm has given best average cost for five data instances, while VNS algorithm cannot give best average cost for any data instance. Here GA algorithm and SA algorithm have shown a relative lower performance with respect to small dataset. Since GA algorithm has given best average cost for four data instances and SA for only one data instance. These numbers are eight and two in case of small dataset.

	FNS	VNS	GA	SA
Instance1	**297.6**	306.4	309.4	309.6
Instance2	358.6	364.6	**349.6**	377.2
Instance3	**299.2**	301.2	310.4	324.6
Instance4	**290.4**	296.4	319.4	298.8
Instance5	302	305.4	**286.8**	303.6
Instance6	311.8	316.2	403.2	**303.4**
Instance7	**287.6**	288.6	294.8	295
Instance8	308.2	297.2	**295.4**	303
Instance9	305.6	292.8	**290.6**	303.2
Instance10	**270.6**	291.8	285.4	308.6

Table 6.13: Average cost comparison of all algorithms for medium data sets

For large dataset, the performance of VNS algorithm has significantly improved. Since it produces best average cost for eight data instances. We have already noticed that this algorithm has produces best minimum cost for large dataset. The performance of FNS algorithm has decreased with respect to average cost. Since it can produce best average cost only for one data instance. SA algorithm and GA algorithm have become worse for large dataset because both of them showed the higher average cost for all data instances.

	FNS	VNS	GA	SA
Instance1	1096.2	**1076.6**	1172.6	1134
Instance2	1079	1068.6	1131.4	**1056**
Instance3	1081.6	**1052.8**	1152.8	1251
Instance4	1128.6	**996.8**	1147	1190.8
Instance5	1106.6	**1072.2**	1124	1184.6
Instance6	**1116.4**	1105.8	1244	1249
Instance7	1104.4	**1084.6**	1169.2	1126.2
Instance8	1159.8	**1139.6**	1194.2	1208.6
Instance9	1100	**1057**	1178.6	1145.8
Instance10	1149.2	**1107.6**	1161	1190.2

Table 6.14: Average cost comparison of all algorithms for large data set

6.14 Discussion and conclusion

This section illustrates the behavior of the algorithms when applied to solve the home health care districting problem. This discussion is on the basis of the results presented in the previous section 6.13. These graphs are drawn for results obtained across 1 run on one data instance chosen from three types of datasets. We shall specify the size of dataset with each graph.

Termination criterion of our algorithms is the limit on time elapsed during testing of our datasets. These are 10, 30 and 60 seconds of CPU time for the small, medium and large datasets respectively. We have used an Intel computer with 2.5 GHz CPU and 4GB RAM under windows operating system for these experiments.

The analysis of neighbourhood moves (defined in section 6.11.1.3) is given first which follows the work done for the tuning of parameters for GA algorithm. At the end, we present a comparison of all algorithms.

Neighbourhood moves affect the performance of neighbourhood based algorithms (VNS algorithm and FNS algorithm). So we should choose them with lots of care. In the following we compare the three moves for various size datasets of home health care districting problem.

Figure 6.9 shows the behaviour of three neighbourhood moves on small dataset.We have found that move 1 performed best for this size of problem. Move 2 and move 3 show good performances in the beginning but as number of iteration increases their performances become stagnant.
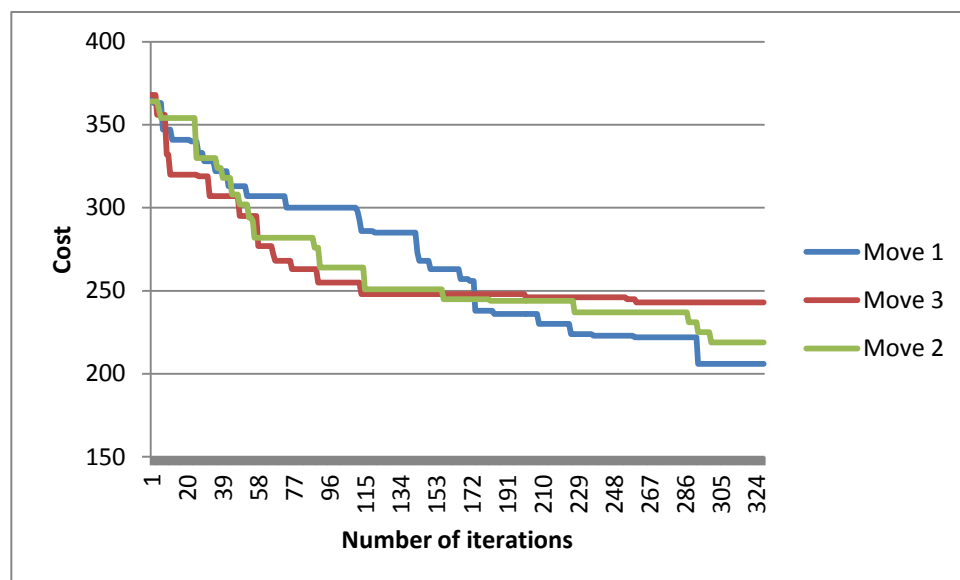

Figure 6.9: Graph representing behaviour of Neighbourhood moves on small dataset of home health care districting problem. Here X-axis represents number of iterations and Y-axis represents the cost.

Figure 6.10 shows the behaviour of neighbourhood moves on medium dataset. Here we can see that move 1 has again shown overall good performance than other two moves. Move 2 and move 3 follow the same pattern as given for small size data instance.
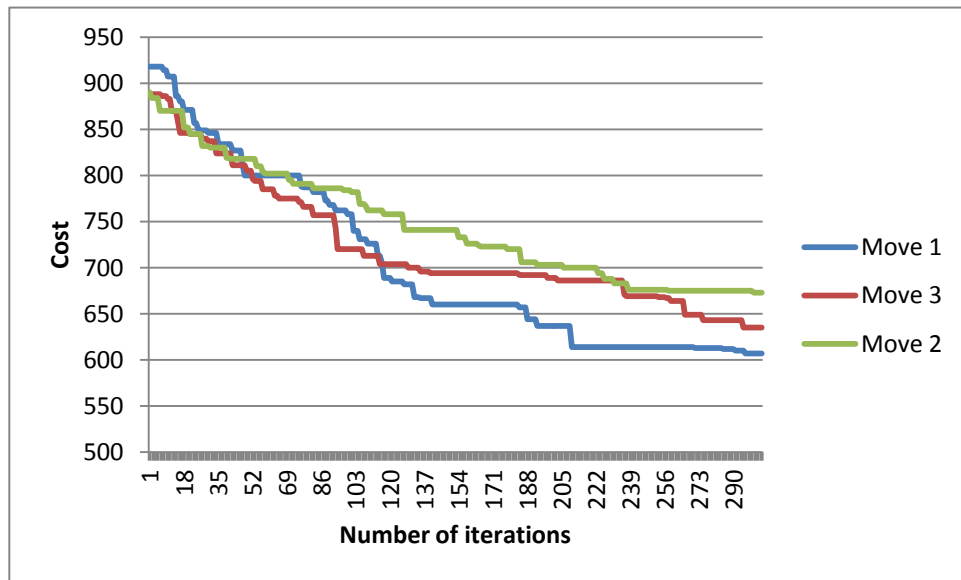

Figure 6.10: Graph representing behaviour of Neighbourhood moves on medium dataset of home health care districting problem. Here X-axis represents number of iterations and Y-axis represents the cost.

In Figure 6.11, we give the comparison of neighbourhood moves on large dataset. It shows that move 1 to be the best here. It has a significant difference of performance with respect to other moves towards the end of search process. Move 3 is performing well than move 2.
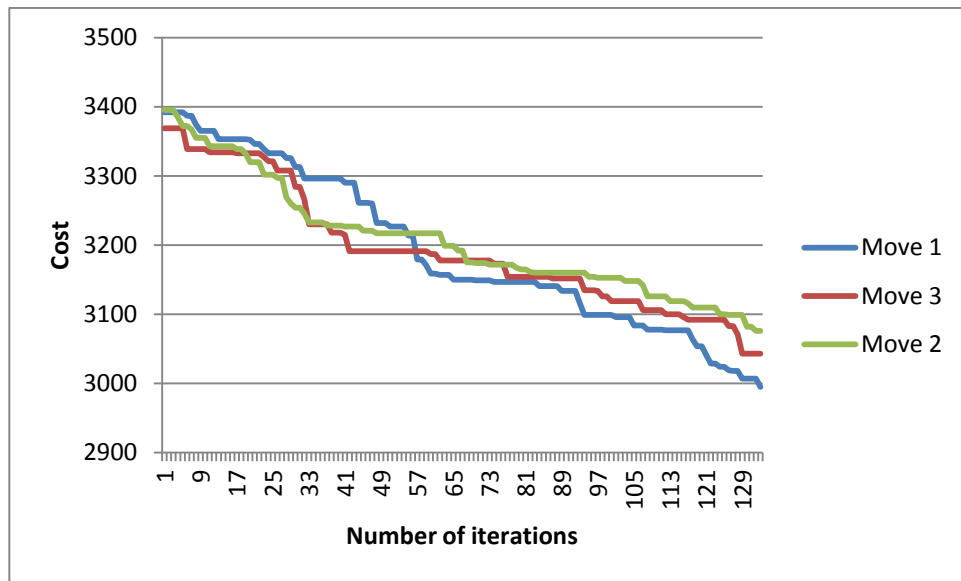
Figure 6.11: Graph representing behaviour of Neighbourhood moves on large dataset of home health care districting problem.. Here X-axis represents number of iterations and Y-axis represents the cost.

One can conclude that move 1 has shown best performance for all types of datasets (small dataset, medium dataset and large dataset). Move 2 and move 3 alone cannot produce best results for the problem considered but when these moves are applied together in set of neighbourhoods in VNS algorithm and FNS algorithm; they help in the exploration of search space.

In Figure 6.12, we give the comparison of three selection operators (elitism, roulette wheel selection RWS, random) in GA algorithm to solve home health care districting problem. One can notice that RWS performs best and random performs worst. Elitism gives intermediate results for HHCD problem. That is why we combined elitism and RWS for selection of parents in GA algorithm.
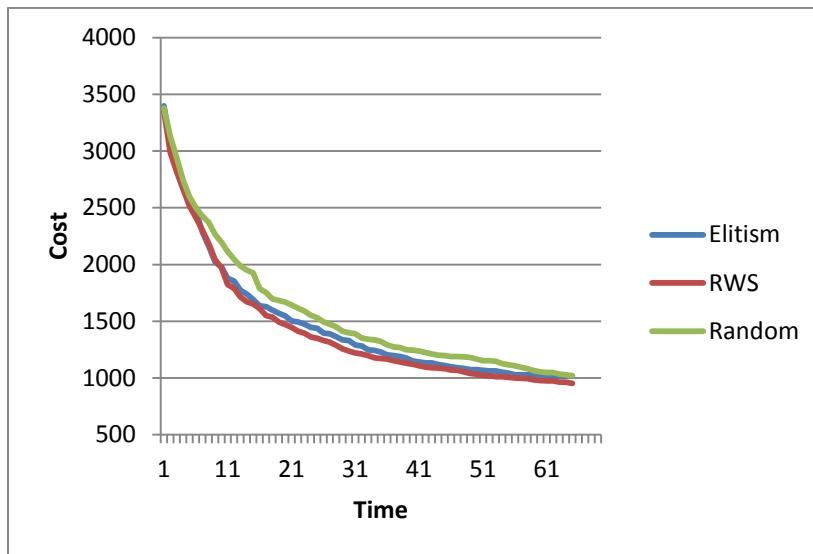
Figure 6.12: Graph representing comparison of three selection operators

Figure 6.13 represents comparison of greedy VNS algorithm and random VNS algorithm for large dataset. Greedy VNS algorithm shows much better performance than random VNS algorithm.

Figure 6.13: Graph representing comparison of random VNS and greedy VNS

Figure 6.14 illustrates the comparison for different values of mutation rate in GA algorithm. We perform this experiment for mutation rate equal to zero and increase its value till hundred. For the home health care districting problem, mutation rate equal to one produces the best

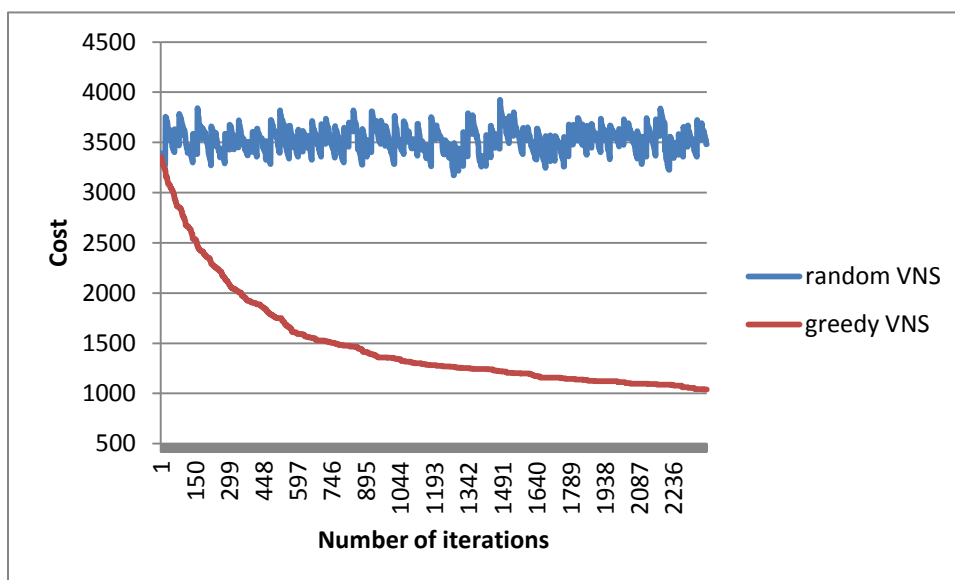results and mutation rate equal to hundred produces worst result. For mutation rate equal to five, the performance of GA algorithm decreases. But it is improved little bit exceptionally for mutation rate equal to ten. On the other hand, if mutation rate is less than one, the performance also deteriorates. Less than one mutation rate case corresponds to no mutation applied in the Figure 6.14.
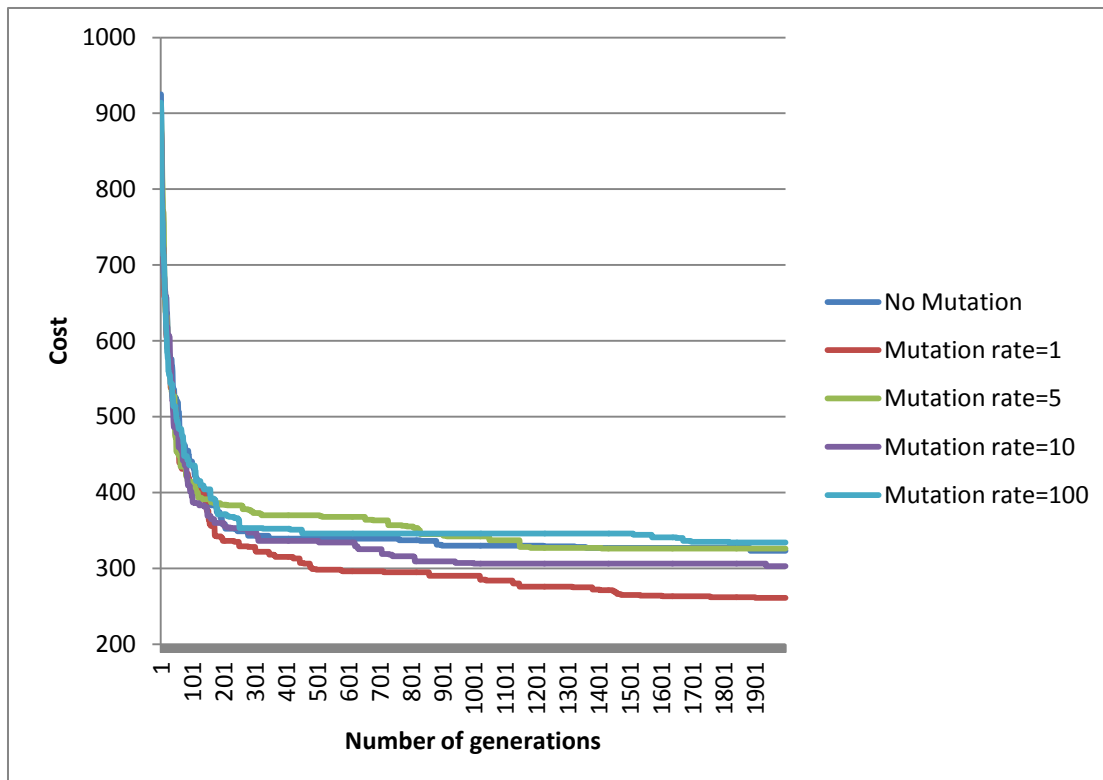
Figure 6.14: Graph showing effect of different mutation rates in GA algorithm. Here X-axis represents number of generations and Y-axis represents the cost.

Now at the end we discuss the behaviour of all algorithms to solve home health care districting problem. In Figure 6.15, we present a graph to compare the behaviour of four proposed algorithms (FNS, VNS, GA and SA) for large dataset. This comparison is done for each algorithm performed once on one instance of large dataset.

Our initial motivations for designing these algorithms were as follows:

(1) Many researchers have solved home health care districting problem by means of solvers like CPLEX. We have seen use of metaheuristics in other types of districting problem like

political districting and school districting etc. This motivates us to apply different metaheuristics to solve this problem.

(2) The purpose of applying neighbourhood based algorithms (VNS algorithm and FNS algorithm) is their robustness and generality.

We can see that the performance of VNS algorithm is best. The second best performing algorithm is GA algorithm. We think it is due to large number of population and the special genetic operators; these genetic operators provide diversity to genetic algorithm and also help to avoid from local minima. FNS algorithm and SA algorithm are showing lower performance here. We have shown a glimpse of algorithm's behaviour by this graph, but one can see their detailed comparison from our experimental work given in previous section. The advantage to use VNS is to use its ability to explore and exploit the search space simultaneously which we think is the reason of its better performance with respect to other proposed metaheuristics.

The reason for overall lower performance of SA algorithm may be its dependence on annealing schedule and acceptance probability. In our case, we could not choose proper values for SA algorithm parameters (cooling rate, maximum temperature and probability function). We think there is a need to do detailed experiments to finely tune these parameters. This can be an important perspective of our work.
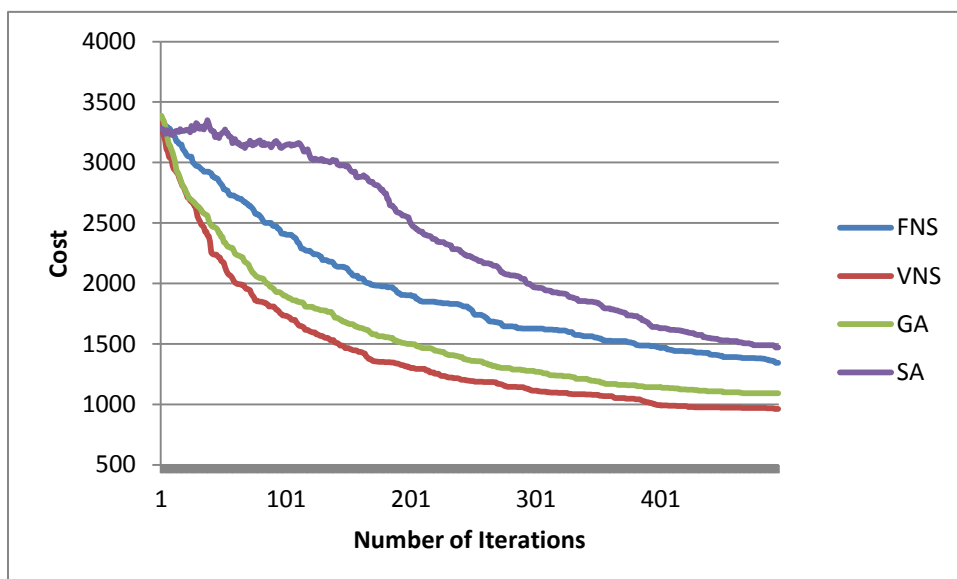
Figure 6.15: Graph illustrates the behaviour of four proposed algorithms (FNS algorithm, VNS algorithm, GA algorithm and SA algorithm) applied to solve home health care districting problem. Here X-axis represents number of iterations and Y-axis represents the cost.

General Conclusions and Perspectives

Home health care (HHC) has increasingly been regarded as one of the essential components of comprehensive health care. We see a rapid growth in the demand for HHC in the past few decades. The major reasons are the demographic change, development in technology, therapeutic evolutions and increasing healthcare costs. HHC organisations are facing many organisational and logistical challenges. Operation research specialists can play an important role to address some of these issues such as the management of complex HHC organizations and the efficient healthcare delivery. This can help to fulfil the increasing demands or expectations of an ageing population, utilizing the limited budgets and advancing medical technology.

We have carried out a qualitative study which identifies the potential optimization problems faced by HHC organisations. We have categorized them on the basis of attention got by researchers and then highlighted some problems for further research investigation. In Chapter 2, we have reviewed these problems and their solution methodologies. We have focused on the home health care operations problems and studied two of them in detail. We have proposed some efficient solution algorithms for home health care planning and districting problems. Our contributions can be comprised of following points:

- We identify the key OR challenges faced to HHC organisations. Then we highlight the combinatorial optimization problems that can be solved by applying exact or approximation methods.
- We present class diagrams for healthcare network and home health care. These class diagrams elaborate the healthcare system and help to better comprehend the relationship of different entities of the system.
- We describe the home healthcare workers assignment problem for assigning home healthcare workers to patients or vice versa considering the widely dispersed geographical locations of patients and the constraints related to the required skills of home healthcare workers for different patients. We develop mathematical model for this problem and test it on data which is partially derived from literature and partially randomly generated.

- We incorporate the routing constraints and reformulate this problem mathematically as routing and scheduling home healthcare workers problem in Chapter 4. We propose neighbourhood based and population based metaheuristics for this extended problem. We also consider the precedence constraints for various patient jobs. We generate random data and compare the performance of these metaheuristics. We conclude that neighbourhood based metaheuristics produced good feasible daily schedules for home healthcare workers.

- We develop the home health care districting problem. This problem has been studied in the literature in other domains such as political districting and school districting but in home health care perspective it did not get much attention. This motivates us to study districting problem in home health care context. We thus propose a binary integer programming as a set portioning problem. The variants of Variable Neighbourhood Search algorithm, Fixed Neighbourhood Search algorithm, Simulated Annealing algorithm and Genetic algorithm are implemented to solve home health care districting problem. According to our knowledge these metaheuristics are not used earlier in literature to solve home health care districting problem. We have done detailed experiments to find good districting approach via different metaheuristics.

Now we present some avenues for future research and perspectives of our work in the context of routing and scheduling home healthcare workers problem and home health care districting problem.

The home health care planning problems are idiosyncratic in nature i.e. their definition is highly dependent on the environment. Most of the studies found in literature are also specific to the situations. A possible extension can be the proposition of a more generic model which can be adapted to large number of real case situations. Every real case situation can be a sub problem of this generic model by choosing the required constraints and objective function.

Another limitation of home health care related research is the lack of standard bench mark data. Since often these problems are solved for home health care organizations which are extremely different in terms of constraints and data. There is no standard data which can be used by researchers to see performance of their solution techniques. So the proposition and

publication of benchmark data for home health care planning and districting problems will be an important area of further research.

The mathematical models developed in this work have considered the deterministic values for most of their parameters such as travelling time between two patients' homes, duration of the patient jobs and demand of home health care in a subunit. These are estimated in advance and are kept same through the planning process. But in real life application, there are many uncertainties both in demand and forecasting process which may deteriorate the quality of the obtained solutions. So developing a stochastic version of these problems by taking into account the uncertainties is the perspective of our work.

The proposed metaheuristics always start from a random initial solution in our case. One of the possible improvements in their performance can be implementation of a guided search construction of initial solution.

Here we have focused two decisions in the realm of home health care independently in two different situations (i.e. planning and districting). In reality these two decisions are closely related to each other. Districting decision is strategic or tactical in planning horizon and planning decision is operational. We first divide a territory into districts and then patients in each district can be assigned to one or more home healthcare workers. Thus integrating these two decisions in one joint model may be a good further step in this research.

We remark that in addition to basic operational planning, we can also use optimization techniques for other important issues in home health care such as personnel skill management and service planning. These can be worth investigating for future study.

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