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## On Bilevel Variational Inequalities

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### Abstract

A class of bilevel variational inequalities (shortly (BVI)) with hierarchical nesting structure is firstly introduced and investigated. The relationship between (BVI) and some existing bilevel problems are presented. Subsequently, the existence of solution and the behavior of solution sets to (BVI) and the lower level variational inequality are discussed without coercivity. By using the penalty method, we transform (BVI) into one-level variational inequality, and establish the equivalence between (BVI) and the one-level variational inequality. A new iterative algorithm to compute the approximate solutions of (BVI) is also suggested and analyzed. The convergence of the iterative sequence generated by the proposed algorithm is derived under some mild conditions. Finally, some relationships among (BVI), system of variational inequalities and vector variational inequalities are also given.

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### Within this Article

1. Introduction

2. Preliminaries
3. Existence Results for (BVI)
4. A Penalty Function Method for (BVI)
5. Algorithm and Convergence Analysis
6. Relationships Among (BVI), (SVI) and (VVI)
7. Concluding Remarks
8. References
9. References

## Related Content



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