MULTIOBJETIVE SUPPLY CHAIN DESING WITH DIRECT SUPPLY

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ABSTRACT

This paper reviews the problem of designing a multi-objective supply chain called Capacitated Fixed Cost Facility Location Problem with Transportation Choices (CFCLP-TC). The problem is formulated as a bi-objective optimization model with objective functions based on cost and time. The problem has two- echelons with plants that supply to distribution centers and these distribute the product to customers. In this problem there are various transportation channels between different nodes. This work contributes to extend the existing literature by incorporating the flow of products from plants to customers. We solve this bi-objective problem through the epsilon-constraint approach. We compare the sets of efficient solutions and computational time required. The models are implemented in GAMS and solved with CPLEX for a set of instances to evaluate the effectiveness of this option.

KEYWORDS: supply chain design, epsilon-constraint, bi-objective, transportation channel

1. - Introduction

The Capacitated Fixed Cost Facility Location Problem with Transportation Choices (CFCLP-TC) is a combinatorial optimization problem proposed by Olivares (2007). It has two objectives: to minimize cost and to minimize the time of transportation from plants to customers. The criterion of cost is an aggregate function that adds variable cost and fixed cost. The function of time represents the maximum time it may take to transport a product from any plant to any customer. The problem is based on a distribution system of two-echelons for a single product. This paper reviews the CFCLP-TC and proposing a variation where the model is modified such that in some cases the plants can supply to customers directly. We propose different sizes of variations for the instances during the implementation of this problem. The size of these instances is limited to those that can be solved in reasonable time in commercial software. This problem is an extension of the classical Capacitated Fixed Charge Facility Location Problem (CFLP), this problem have been classified as

NP-hard, (see Daskin (1995)), so with these new approaches are expected to be found also in this category.

We compare the results on the basis of two metrics, the first one is the R_{pos} (set of no dominated solutions) used by Altiparmak et al. (2006) and the second are called \overline{D} and D_{min} (metric of the mean and minimal difference) proposed by Olivares (2007), in order to make comparisons of sets from point to point. The problem is implemented in GAMS and solved in CPLEX. An additional contribution of this paper is to establish the difference in times for each instance between the original model and the change proposed to provide benchmarks for comparison. This paper is divided into seven sections as follows: the first section presents a general introduction to the work. In the second section, we review the literature of the topic and locating the knowledge vacuum that this work fills. The third section describes an overview of the model. In the fourth section an overview of the computational experiments is presented. The fifth section describes the metrics used to evaluate the Pareto fronts. In the sixth section the results of the computational implementation are reported. Finally, the seventh section presents the conclusions and proposals for future work.

2. - Literature review

Supply Chain Management (SCM) is the process for planning, implementing and controlling the operation of the supply chain efficiently. SMC spans all movements and storage of raw materials, work-in-process inventory, and finished goods from the point of origin to the point of consumption. Part of the planning processes in SCM aims at finding the best possible supply chain configuration so that all operation can be performed in an efficient way. The capacitated facility location problem (CFLP) is a well-known combinatorial optimization problem. It consists in deciding which facilities to open from a given set of potential facility locations and how to assign customers to those facilities. The objective is minimizing the total fixed and shipping cost. Applications of the CFLP include location and distribution planning, lot sizing in production planning, and telecommunication network design as mentioned by Klose and Gortz (2007). Numerous heuristics and exact algorithms for the CFLP have been proposed in the literature. Heuristic solution methods as well as approximation algorithms were proposed by Kuehn and Hamburger (1963), Khumawala (1974), and Korupolu et al. (1998). Tabu Search methods regarding to the p-median problem and the CFLP with single source were developed by Rolland et al. (1996) and Delmaire et al. (1999). Exact solution methods based on the Benders decomposition algorithm are considered in Magnanti and Wong (1981). Polyhedral results for the CFLP have been obtained by Leung and Magnanti (1989). Aardal (1998) uses these results in a branch and cut algorithm for the CFLP. A variety of heuristic and exact solution approaches for the CFLP, however, they use the method of Lagrangean Relaxation.

Moreover, several variants of the CFLP have been investigated. For example, Laporte et al. (1994) formulated stochastic integer linear programming model for the CFLP with stochastic demands. In other way a branch and cut approach was applied to find the optimal solution of the problem. Tragantalerngsak et al. (1997) formulated the mathematical model of the two-echelon SSCFLP and they considered six Lagrangian relaxation based approaches for the solution. In recent years, many meta-heuristic approaches have been applied to combinatorial optimization problems successfully, such as Simulated Annealing (SA), Genetic Algorithms (GAs), Tabu Search (TS) and Ant Colony Optimization (ACO). Some recent work in this field include those presented by Moncayo-Martinez et al. (2011) in which they use an ant colony approach for the design of a supply chain. Wei-Chang Yeh (2006) presented a presented a memetic algorithm for a problem of supply chain multi-stage. Rajesh et al. (2011) proposes a simulated annealing algorithm for allocation problem. The biobjetive location problems are an extension of classic locations problems. These problems are bio-

objetive median, knapsack, quadratic, covering, unconstrained, location-allocation, hub, hierarchical, competitive, network, undesirable and semi-desirable location problems. Considering capacities in location problems, there are capacitated and uncapacitated problems in the literature. For instance, Myung et al. (1997) have considered an uncapacitated facility location problem with two maxisum objectives (net profit and profitability of investment and modeled it as parametric integer program with fractional and linear objectives. Villegas et al. (2006) have modeled a supply network as a bi-objective uncapacitated facility location problem, with minisum and maxisum objectives (cost and coverage). In contrast, Galvao et al. (2006) developed an extension of the capacitated model to deal with locating maternity facilities with minisum objective (distance traveled and load imbalance). Costa (2008) has utilized a different bio criteria approach to the single allocation hub location problem. This approach have two objectives, the first is a minisum form (cost), while the second objective (process time) has two alternative forms.

The Capacitated Fixed Cost Facility Location Problem with Transportation Choices (CFCLP-TC) proposed by Olivares (2007) is an extension of the CFLP with bi-objective mixed-integer program approach (Cost and time), it is based in a two-echelon system for the distribution of one product in a single time period. This approach considers several alternatives to transport the product from one facility to the other in each echelon of the network. At difference from similar works in the literature the aim here is to provide to the decision maker with a set of non-dominated alternatives to allow them to decide. Some qualitative information only known by the decision maker may motivate the selection of one of these alternatives. This approach provides an alternative solution through a precise focus and further proposed the use of constructive method approach GRAPS.

In combinatorial optimization, the consideration of multiple objectives has received attention, in specific; the multi-objective combinatorial optimization (MOCO) has become a very active area of research Ehrgott and Gandibleux (2004). This approach has been extensively studied in the literature, Bornstain et al. (2012) develops an algorithm with re-optimization for one problem with a cost and several bottleneck objective functions, Bérubé et al. (2009) propose an exact ε-constraint method for a especial case of MOCO called bi-objective combinatorial optimization (BOCO) for the traveling salesman problem with profits.

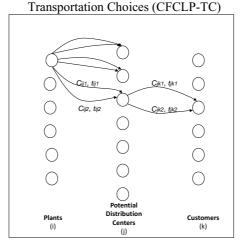
The ε-constraint method is based on a scalarization where one of the objective functions is optimized while all the other objective functions are bounded in the form of additional constraints Ehrgott (2005). The ε-constraint method guarantees to find weakly efficient solutions. However when we have an optimal solution, it is not easy to verify if this solution is either an efficient solution or it is not. Cardona-Valdés et al. (2011) propose an algorithm based on the fusion of the εconstraint and the L-shaped method for a bi-objective supply chain design problem with uncertainty. Likewise Salazar et al. (2011) uses the approach of ε-constraint for a bi-objective programming model for designing compact and balanced territories in commercial districting. In our implementation of the ε-constraint method we select the objective function given by Equation (1) as the main objective function and the objective function given by Equation (2) is transformed to another constraint. For variations in the direct flow to plants (i) to customer (k) compared with the original model, the same approach to generate the Pareto Front is used. Mula et al. (2010) establish as one of the main contributions that remain to be done in the field of mathematical models for designing supply chains, is the consideration of the different distribution channels in these models and considering the different types of configurations for product flow. The study of the variations of the problem (CFCLP-TC) will establish the importance of the choice of distribution channels down the assumptions described above so that this problem is closer to real situations that are demanded in the process chain supplies and location of modern facilities. Thus, the investigation in this paper contributes the development of state of art in an area that has not been sufficiently explored and will therefore close the gap between theoretical models and the practical application of them.

3. - Problem description

The CFCLP-TC is a problem based on two echelons to distribute a product in a single period of time. In the first echelon, manufacturing plants send the product to distribution centers. The second echelon corresponds to the flow of product from the distribution centers to the customers. In the problem, the number and location of plants and customers are known.

There is a set of potential locations to open the distribution centers. The number of distribution centers is not defined a priori. Each candidate site has a fixed cost for opening a facility. Each potential site has a limited capacity. The plants have limited manufacturing capacity. One constraint in the original model means that each customer is supplied by only one distribution center, which is called the single source constraint. However, the demand of each customer must be satisfied. Figure 1 shows the outline of the distribution network. From the above conditions the flow of products should be: A starting point in plants (i) to ship products to potential distribution centers (j), and then send them to customers (k), with diverse choices of transportation between plants, distribution centers and customers. At this point the one question arises: how costs and times behave for each instance by allowing a customer to be supplied by more than one DC?

Figure 1: Scheme of the Capacitated Fixed Cost Facility Problem with



The idea of this problem is to select suitable sites to open distribution centers and the flow between facilities. One objective is to minimize an aggregate function that combines transportation costs and fixed costs of distribution centers. In this problem we consider various transportation channels that has an impact on the transportation time from the plants to the customers. The cost-time balance must be considered by formulating the mathematical model to minimize both criteria simultaneously. Therefore, the problem is addressed with a bi-objective optimization model.

From the above conditions the flow of products should be: A starting point in plants (i) to ship products to potential distribution centers (j), and then send them to customers (k), with diverse choices of transportation between plants, distribution centers and customers. In the original model it is established as a restriction that each customer can be supplied only by a distribution center (DC). This paper proposes to compare the original model (CFCLP-TC) with our proposal allowing product flow directly from plants to customers. This proposal does not eliminate the role of distribution centers, but seeking to establish which approach provides better costs (minimum) under the proposed conditions. It is important to consider that the proposal is justified when clients are geographically closer to the plants of the distribution centers. Figure 2 shows the outline of the distribution network.

When planning the configuration of the supply chain down the places where customers will be located all the time, it is not possible to accurately predict future where they will reside geographically in the future new customers, and is clear that more costly to relocate distribution centers based on the geographical configuration of new customers. Therefore an alternative that would optimize the cost and time of the supply chain based on the ability to send directly from plants to customers is in itself an attractive alternative. The results obtained allow us to determine best ever obtained Pareto fronts with this approach, however a problem with this approach is the time required to process the request, since the increase of time is significant as we approach the larger instances that resembled real problems. The proposal explores a configuration of the new supply chain that has not been considered in the literature.

Cik1, tik1

Plants
(i)

Cik2, tik2

Cok2, tik2

Figure 2: Scheme direct flow between plants (i) and customers (k)

The formulation of the model is preceded by the notation shown below:

Sets:

I: Set of plants i

j : Set of potential distribution centers *j*

K: Set of customers k

 LP_{ij} : Set of arcs l between nodes i and j; $i \in I, j \in J$ LW_{jk} : Set of arcs l between nodes j and k: $j \in J, k \in K$ LV_{ik} : Set of arcs l between nodes i and k: $i \in I, k \in K$

Parameters:

 CP_{ijl} : cost of transporting one unit of product from plant i to distribution center j using the arc ijl; i and j; $i \in I, j \in J, l \in LP_{ij}$

 CW_{jkl} : cost of transporting one unit of product from distribution center j to customer k using the arc jkl; i and j, $j \in J$, $k \in K$, $l \in LW_{jk}$

 Cv_{ikl} : cost of transporting one unit of product from plant i to customer k using arc ikl: $i \in I, k \in K, I \in LV_{ik}$

 TP_{ijl} : time for transporting any quantity of product from plant i to the distribution center j using arc ijl; $i \in I, j \in J, l \in LP_{ii}$

 TW_{jkl} : time for transporting any quantity of product from distribution center j to customer k using arc jkl; $j \in J, k \in K, l \in LW_{jk}$

 TV_{ikl} : time for transporting any quantity of product from plant i to costumer k using arc ikl: $i \in I, k \in K, l \in LV_{ik}$

 MP_i : capacity of plant $i; i \in I$

MW; : capacity of distribution center $j: j \in J$

: demand of customer k; $k \in K$

: fixed cost for opening distribution center j; $j \in J$

Decision variables:

 X_{ijl} : quantity transported from plant i to distribution center j using arc ijl; $i \in I, j \in J, l \in LP_{ij}$

 Y_{ikl} : quantity transported form distribution center j to customer k using arc jkl; $j \in J$, $k \in K$, $I \in LW_{jk}$

 V_{ikl} : quantity transported form plant i to to customer k using arc ikl; $i \in I, k \in K, l \in LV_{ik}$: binary variable equal to 1 if distribution center j is open and equal to 0 otherwise; $j \in J$

: binary variable equal to 1 if arc ijl is used to transport product from plant I to distribution center j A_{iil} and equal to 0 otherwise; i y j; $i \in I, j \in J, l \in LP_{ij}$

: binary variable equal to 1 if arc jkl is used to transport product from distribution center j to customer k and equal to 0 otherwise; $j \in J$, $k \in K$, $I \in LW_{jk}$

: binary variable equal to 1 if arc ikl is used to transport product from the plant i to customer k and equal to 0 otherwise: i and k; $i \in I$, $k \in K$, $l \in LV_{ik}$

Auxiliary variables:

 \mathbf{T} : maximum time that takes sending product from any plant to any customer

 E_i^1 : maximum time in the first echelon of the supply chain for active distribution center j, $E_i^1 =$ $\max(TP_{ijl}A_{ijl})$; $i \in I, j \in J, l \in LP_{ij}$.

: maximum time in the second echelon of the supply chain for active distribution center j, $E_i^2 =$ $\max(TW_{jkl}B_{jkl})$; $j \in J, k \in K, l \in LW_{jk}$.

: maximum time that takes sending product i to k, , $E3 = \max(TV_{ikl}G_{ikl})$; $i \in I, k \in K, l \in LV_{ik}$

Model

 $min(f_1, f_2)$

$$f_{1} = \sum_{i \in I} \sum_{i \in I} \sum_{l \in I, l \in I} CP_{ijl} X_{ijl} + \sum_{i \in I} \sum_{k \in K} \sum_{l \in I, W_{ik}} CW_{jkl} Y_{jkl} + \sum_{i \in I} \sum_{k \in K} \sum_{l \in I, V_{ik}} CV_{ikl} V_{ikl} + \sum_{i \in I} F_{j} Z_{j}$$

$$\tag{1}$$

$$f_2 = T \tag{2}$$

Subject to

$$T - E_i^1 - E_i^2 \ge 0 \qquad \forall j \in J$$
 (3)

$$E_{j}^{1} - TP_{ijl}A_{ijl} \ge 0 \qquad \forall i \in I, j \in J, l \in LPij$$
 (4)

$$E_j^2 - TW_{jkl}B_{jkl} \ge 0 \qquad \forall j \in J, k \in K, l \in LWjk$$
 (5)

$$E3 - TV_{ikl}G_{ikl} \ge 0 \qquad \forall i \in I, k \in K, l \in LVik$$
 (6)

$$T - E3 \ge 0 \qquad \forall i \in I, k \in K, l \in LVik$$
 (7)

$$\sum_{i \in I} \sum_{l \in LW_{ik}} Y_{jkl} + \sum_{i \in I} \sum_{l \in LV_{ik}} V_{ikl} = D_k \qquad \forall k \in K$$
(8)

$$\sum_{i \in I} \sum_{l \in IP} X_{ijl} + \sum_{i \in I} \sum_{l \in IV} V_{ikl} \le MP_i \qquad \forall i \in I$$
 (9)

$$\sum_{j \in J} \sum_{l \in LW_{jk}} Y_{jkl} + \sum_{i \in I} \sum_{l \in LV_{ik}} V_{ikl} = D_k \qquad \forall k \in K$$

$$\sum_{j \in J} \sum_{l \in LP_{ij}} X_{ijl} + \sum_{i \in I} \sum_{l \in LV_{ik}} V_{ikl} \leq MP_i \qquad \forall i \in I$$

$$MW_j Z_j - \sum_{j \in J} \sum_{l \in LW_{jk}} Y_{jkl} \geq 0 \qquad \forall j \in J$$

$$(8)$$

$$\sum_{j \in J} \sum_{l \in LP_{ij}} X_{ijl} - \sum_{j \in J} \sum_{l \in LW_{ik}} Y_{jkl} = 0 \qquad \forall j \in J$$

$$(11)$$

$$\sum_{i}\sum_{k}G_{ikl}+\sum_{k}\sum_{l}B_{jkl}=1 \qquad \forall k \in K$$
(12)

$$\sum_{i \in I} \sum_{l \in LV_{ik}} G_{ikl} + \sum_{j \in J} \sum_{l \in LW_{jk}} B_{jkl} = 1 \qquad \forall k \in K$$

$$\sum_{l \in LP_{ij}} A_{ijl} \le 1 \qquad \forall i \in I, j \in J$$
(12)

$$\sum_{l \in I(W)} B_{jkl} \le 1 \qquad \forall j \in J, k \in K$$
 (14)

$$\sum_{l \in IV} G_{ikl} \le 1 \qquad \forall i \in I, k \in K$$
 (15)

$$X_{ijl} - A_{ijl} \ge 0 \qquad \forall i \in I, j \in J, l \in LP_{ij}$$
 (16)

$$Y_{jkl} - B_{jkl} \ge 0 \qquad \forall j \in J, k \in K, l \in LW_{jk}$$
 (17)

$$V_{ikl} - G_{ikl} \ge 0 \qquad \forall i \in I, k \in K, l \in LV_{ik}$$
 (18)

$$MP_{i}A_{ijl} - X_{ijl} \ge 0 \qquad \forall i \in I, j \in J, l \in LP_{ij}$$
(19)

$$\begin{aligned} \mathsf{M} W_{\mathbf{j}} \mathsf{B}_{\mathbf{j} \mathbf{k} \mathbf{l}} - \mathsf{Y}_{\mathbf{j} \mathbf{k} \mathbf{l}} &\geq 0 & \forall j \in J, k \in K, l \in LW_{jk} \\ \mathsf{M} \mathsf{P}_{\mathbf{i}} \mathsf{G}_{\mathbf{i} \mathbf{k} \mathbf{l}} - \mathsf{V}_{\mathbf{i} \mathbf{k} \mathbf{l}} &\geq 0 & \forall i \in I, k \in K, l \in LV_{ik} \end{aligned} \tag{20}$$

$$\sum_{i \in I} \sum_{I \in LP_{ij}} A_{ijl} - Z_{j} \ge 0 \qquad \forall j \in J$$

$$T, E_{j}^{1}, E_{j}^{2}, E3, X_{ijl}, Y_{jkl}, V_{ikl} \ge 0 \qquad \forall i \in I, j \in J, \ k \in K, \ l \in LP_{ij}, l \in LW_{jk}, l \in LV_{ik}$$
(23)

$$T, E_i^1, E_i^2, E_i^2, E_i^2, X_{ijl}, Y_{ikl}, V_{ikl} \ge 0$$
 $\forall i \in I, j \in J, k \in K, l \in LP_{ij}, l \in LW_{jk}, l \in LV_{ik}$ (23)

$$Z_{i}, A_{ijl}, B_{ikl}, G_{ikl} \in \{0, 1\}$$
 $\forall i \in I, j \in J, k \in K, l \in LP_{ij}, l \in LW_{ik}$ (24)

In this model the objective function (1) minimizes the transportation costs and the cost of opening up distribution centers. The objective function (2) minimizes the time of transportation, it considers alternative (i) - (j) - (k) and also directing the flow of (i) and (k). The restriction (3) enables the lowest total time taken between (i) - (j) - (k) and the direct flow of (i) - (k). The restriction (4) and (5) allow us to take the lesser travel time earlier of (i) and (j) and then from (j) and (k). Restriction (6) sets the time of the alternative of directing flow between (i) and (k). Restriction (7) calculates the longest time between (i) and (k). The restriction (8) allows the satisfaction of the needs of each client. The restriction (9) means that we cannot exceed the capacity of each plant (i). The restriction (10) cannot exceed the capacity of distribution centers (i). The restriction (11) allows the balance of flow between (i) - (j) and (j) - (k). The restriction (12) states that customers can only be provided by a source. The restriction (13) requires that at most it can be select an arc between each (j) and (k). The restriction (14) requires that at most you can select an arc between each (i) and (j). The restriction (15) requires that at most it can be selected an arc between each (i) and (k). The restriction (16) states that if the arc between (i) and (j) has no flow, then this will be inactive. The restriction (17) provides that if the arc from (j) and (k) has no flow, then this will be inactive. The restriction (18) states that if the arc between (i) and (k) has no flow, then this will be inactive. The restrictions (19), (20) and (21) states that the product flow can only be done through actives arcs. The restriction (22) states that the distribution centers (j) will be closed but have active arcs incident. The restriction (23) establishes the restriction or greater than zero. The restriction (24) establishes the required binary variables in the model.

4. Computational experiments

For the process of computational experiments five sets of instances are shown in Table 1, and each set of instances, generated randomly, for five different instances are tested, these were generated randomly:

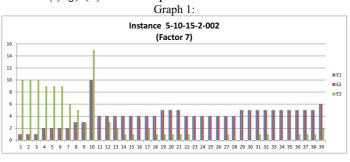
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Tab	nle -	١٠	Instances	SIZES

Instance sizes	Integer variables in the original model	Integer variables in the approach that allowing direct flow between (i) and (k)
5 - 5 - 5 - 2	105	205
5-5-5-5	255	505
5 - 10 - 10 - 2	320	510
5 - 10 - 15 - 2	410	710
5 - 10 - 20 - 2	510	910

The encoding of the instance is as follows: The first index indicates the number of plants (i), the second index indicates the number of distribution centers (j), the third index indicates the number of customers (k), and finally the fourth index indicates the number of arcs between nodes in each echelon. In each size five instances were tested. The implementation of the models was made in GAMS 23.6.2 and solved with CPLEX 12. Instances used in the original model were proposed and described by Olivares (2010); these instances were used to test the original model. The same instances were used as a basis to suit the proposal. For the approach presented, we proceeded with the modification of the instances as follows: The costs were generated and additional times for the direct flows between (i) and (k). Times are generated randomly in agreement to a normal distribution with values between 5-50; this is in order that the values are competitive compared with the values of the original instance. The time and cost of additional workflows are correlated in a negative way. The idea of a negative correlation in the additional arcs in this variation is the same as in the original approach. It is assumed that the costs of complimentary alternatives are related so that the longer in the transport, the lower associated cost, and the shorter transportation, the higher associated cost. The unit cost of transportation is a floating point variable calculated as follows:

$$Cost = \frac{(7)(50)}{Time} \tag{25}$$

The value of 7 is a constant that allows us a uniform distribution of the arches used either by the flows (i)-(j)-(k) or variation proposed (i)-(k), so that the proposed instance is consistent and not easy to be resolved by the model. We present the following graphs which showing how the option used by the model (arches) are distributed between the flow (i)-(j)-(k) in conjunction with the variation proposed (i)-(k). We are considering a factor of 7 for all levels of compensation allowing a more homogeneous distribution. The Graph (1) shows the distribution of the arches of the original alternative (i)-(j)-(k) and the proposed variation (i)-(k). As shown in Graph (1) the model has a homogeneous distribution of the selection of arcs between the two alternatives, which means that instance proposal is competitive with the original instance. In this graph E1 and E2 represent the selection of alternative arcs (i)-(j)-(k) and E3 represents the selection of the alternative arcs (i)-(k).



5. - Metrics for evaluations

To make the comparison between the Pareto fronts with the single source and the Pareto fronts without single source constraint, the metric R_{pos} (P_i) proposed by Altiparmark et al. (2006) was used. Additionally, we registered the average number of Pareto-Optimal solutions in each front. To calculate the R_{pos} (P_i) let P_1 and P_2 be the sets of Pareto-Optimal solutions obtained from each model, and let P be the union of the sets of Pareto-Optimal solutions (i.e., $P = P_1 \cup P_2$) such that it includes only non-dominated solutions Y. The ratio of Pareto-Optimal solutions in P_i that are not dominated by any other solutions in P is calculated as follows:

$$R_{pos}(P_i) = \frac{|P_i - \{X \in P_i | \exists T \in P: Y < X\}|}{P_i}$$
Where $Y < X$ means that the solution X is dominated by solution Y . The higher the ratio $R_{pos} = (P_i)$

Where Y < X means that the solution X is dominated by solution Y. The higher the ratio $R_{pos} = (P_i)$ is, the better the solution set P_i is. Similarly, we used the metrics proposed by Olivares (2007) called \overline{D} and D_{min} . These were developed to give practical meaning to the comparison of sets point by point. The discretization of objective f_2 and the number of objectives allows proceeding as follows for a pair of sets S_1 and S_2 .

Let f_1 and f_2 are the objective functions of the problem, s1 and s2 sets of nondominated solutions associated with the true Pareto fronts and approximate respectively. By discretization of function f2 we can construct a set T, such that its elements are those values of f_2 that exist in s_1 and s_2 .

$$T = \{ f_2(s) \lor f_2(s'), s \in S_1, s' \in S_2 | \exists f_1(s) \land \exists f_1(s') \land f_2(s) = f_2(s') \}$$
 (27)

Then computes an average rate deviation of the objective function f_1 for each value of f_2 , which is in the set T.

$$\overline{D} = \frac{\sum_{t \in T} \frac{f_1(s): f_2(s) = t}{f_1(s'): f_2(s') = t}}{|T|} \quad \forall s \in S_1, s' \in S_2$$
 (28)

$$D_{min} = min_{t \in T} \frac{f_1(s): f_2(s) = t}{f_1(s'): f_2(s') = t}$$
 $\forall s \in S_1, s' \in S_2$ (29)

The metric \overline{D} indicates the quality of a set compared to another. The following relationship can be established:

If
$$\overline{D}$$
 $\left\{ \begin{array}{l} <1~S_1 is~better~than~S_2 \\ >1~S_1 is~worse~than~S_2 \\ =1~S_1 is~similar~to~S_2 \end{array} \right.$

It is important to establish the roll of the computational time required to solve each model for an instance. The experiments were performed on a workstation machine with a Core TM 2 Duo T8300 CPU at 2.40 GHz with 12 GB of RAM, all under an operating system of 64 bits windows (seven).

6. Results

In the table (2) we show the results of the comparison between the original and the variation that allows the direct flow between the plants (i) and customers (K). Here we present the evaluation of 5 instances of each size with the values of R_{pos} , \overline{D} and D_{min} , and the processing time in seconds for each case. In the table (2), column $|S_i|$ shows the number of solutions that structure Pareto fronts for each approach respectively. We observe that in all cases \overline{D} is smaller than 1, this indicates that the Pareto fronts of the variation are best compared to the original model. D_{min} in all cases indicates

the smallest difference compared both fronts, and it provides a measure of the difference of the fronts compared. Respecting for to R_{pos} is observed that the variation values of a model presented in all cases compared to the original model. The values of R_{pos} in the original model are on average 53% for instance 5-5-5-2, 73% for instance 5-5-5-5, 64% for the instances 5-10-10-2, 26% for instance 5-10-15-2 and 18% for instance 5-10-20-2. This indicates that in all cases the direct flow variation between (i) and (k) always presents best Pareto fronts compared with those obtained in the original model.

Table 2: Results for the instances

Table 2: Results for the instances										
		Original Model		Direct flow between (i) and (k)						
Instance	Size	Si	Rpos (Si)	Si	Rpos (Si)	Davg	Dmin	Time with Original Model (Seconds)	Time Direct flow between (i) and (k) (Seconds)	
	1	27	0.87096774	31	1	0.9975774	0.95965946	8.838	9.263	
	2	15	0.42857143	35	1	0.98096094	0.91788812	9.197	10.276	
5_5_5_2	3	18	0.54545455	33	1	0.96701778	0.85386306	9.606	10.456	
	4	1	0.03225806	31	1	0.95896192	0.91764374	14.514	12.989	
	5	23	0.79310345	29	1	0.99362365	0.96388563	9.006	9.018	
	1	38	0.92682927	41	1	1	1	40.998	46.599	
	2	33	0.76744186	43	1	0.99936964	0.99153492	55.89	55.641	
5_5_5_5	3	16	0.38095238	42	1	0.98025659	0.93238372	68.772	67.826	
	4	35	0.81395349	43	1	0.99788515	0.97854988	55.651	65.761	
l [5	34	0.79069767	43	1	0.99839478	0.96884197	41.012	44.064	
	1	18	0.46153846	39	1	0.98846976	0.94669285	356.355	320.192	
	2	32	0.7804878	41	1	0.99766759	0.97732813	193.005	190.587	
5_10_10_2	3	27	0.675	40	1	0.99658614	0.96356955	417.547	407.858	
	4	31	0.775	40	1	0.99855269	0.98577745	233.841	245.735	
	5	19	0.52777778	36	1	0.98881179	0.89438732	259.442	247.134	
5_10_15_2	1	2	0.0555556	36	1	0.91196956	0.7597195	8042.069	316.244	
	2	11	0.28205128	39	1	0.98962601	0.94587785	11207.506	10616.087	
	3	11	0.275	40	1	0.97893561	0.86283473	16262.752	14671.503	
	4	14	0.35	40	1	0.97651925	0.87874916	23225.921	14748.102	
	5	18	0.45	40	1	0.98074666	0.91927882	8734.642	6973.83	
5_10_20_2	1	3	0.07692308	39	1	0.94659341	0.80255363	149028.323	137238.826	
	2	9	0.24324324	37	1	0.95456087	0.83359301	112315.372	112106.767	
	3	10	0.26315789	38	1	0.96070312	0.83766502	246814.212	129141.434	
	4	8	0.20512821	39	1	0.96055131	0.83801063	240520.165	246519.6	
	5	9	0.23076923	39	1	0.96889729	0.86548166	78406.285	467645.918	

Regarding to the processing time by comparing the original model with the variation that allows the direct flow of (i) and (k) we have the following: For instance 5-5-5-2 is increased on average by 1.6%. For instance 5-5-5-5 is increased on average 6.2% over the original model. For size 5-10-10-2 shows a decrease of 3.4% on average. But in instance 5-10-15-2 decreases 22% and in the instance 5-10-20-2 increases 16%.

7. Conclusions and Directions for Future Research

Melo et al. (2009) define the implementation of the supply chain as the process of planning, implementation and control of the operations of the supply chain of an efficient way. This aspect is defined in a context of tactical decisions that allow save the complete manufacturing cycle. The work carried out in this research explores an area which has not been sufficiently worked and incorporated into mathematical models of the design of supply chains distribution channel selection Mula et al. (2010) and Bozart et al. (2009). The CFCLP-TC problem proposed by Olivares (2007) and Olivares et al. (2010) incorporate novel way the selection of alternatives of transport in the context of a problem (two-echelon) whereas: plants, distribution centers and customers, however variations proposed in this paper allow to approach the theoretical models to real applications, considering situations that could occur in real-life situations. In this proposal the model allows that in some cases the plants product flows go directly to customers without necessarily going through distribution centers, generally it gets best cost (minimal) compared with the original proposal in the same time points. It must be considered that this approach does not imply the elimination of distribution centers, and the proposal would only be justified when customers are geographically closer to the plants to distribution centers. When it is planned on the configuration of the supply

chain and it determines the points where unlocated customers that project at that point in time, is not possible to predict exactly where is located geographically in the future new customers to future, and is clear that it would be more expensive to relocate distribution centers based on the new geographical configuration of clients. Therefore an alternative that allowing to optimize the costs and time of supply chain based on the possibility of sending the plants directly to customers is an attractive alternative. The results obtained allow us to get always the best Pareto fronts with this approach, however a problem with this approach is the time required to process instances, as the increase in time is considerable when approaching larger bodies that resemble real problems. This proposal makes it possible to explore a configuration of innovative supply chain that has not been considered in the revised literature. It is clear that new approaches presented an increase in the required processing time. The findings and conclusions presented are based only on instances that were tested and they are in a context of real implementation, are small. It is clear that the results of the Pareto front for instance higher could be different, therefore it is important to determine these fronts, and however the impossibility of doing so with exact methods requires us to try to get them through heuristics and metaheuristics: In the revised literature Lagrangian decomposition method is widely used to solve similar problems Lidestam et al. (2011), the results show the feasibility of a future implementation to evaluate the behavior of instances of large size to the problem raised in this work. Another approach used is that of genetic algorithms, Gen et al. (2006) for a similar problem found in the literature, in this reported satisfactory results for proven instances of large size, this could extend to the problem treated for large instances. Evolutionary algorithms for optimization approach is widely used in similar trouble Lin et al. (2009), such as multi-objective Non-dominated Sorting Genetic Algorithm (NSGA-II) and Stregth Pareto Evolutionary Algorithm (SPEA - II) for individual cases are therefore a natural consequence in the development of this research.

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