

Operation Optimization Method on Saving Compressor Energy of Natural Gas Pipeline

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ABSTRACT

The long distance gas pipeline compressor is a large-scale energy consumption equipment, the operation energy consumption of compressor station account for more than 70% of the operation energy consumption of the gas pipeline system. In the practical operation, gas pipeline and compressor station is a unity. The operating parameters are interrelated, affected by factors such as changes in production capacity of gas source, seasonal fluctuation of user gas consumption and equipment service or failure. The operating conditions for pipelines are constantly changing over time, so the compressor unit may be at low-efficiency resulting in the rise in unit of gas energy consumption or gas pressure and flow rate less than the set values. On the basis of the security analysis in the joint operation of the pipelines, equipment components, Selecting the operating mode and status of the compressor unit, as well as the operating parameters of the gas stations (compressor stations, injection points, outlet points, the pipeline intersections, etc) as the optimization variables. Taking the operation energy consumption of pipe network as the objective function, establish the optimization model of energy-saving operation of long distance gas pipeline, and then put forward the model solution method based on bundle projection subgradient method combined with Lagrange multiplier. Through the computation and analysis of actual operation scheme for pipelines, the results show that it effectively reduce the operating energy consumption of the entire gas pipeline system.

KEYWORDS

Long distance gas pipeline; Compressor station; Energy conservation; Optimize operation; Application analysis

INTRODUCTION

The gas pipelines are wide complex systems in length (several hundred kilometers, and even of the thousands) intended for the transport of natural gas by pipe. There are three different kinds of topologies: linear, tree and cyclic. A gas pipeline is composed of compressing stations intended to provide the energy of pressure necessary to transport gas via a pipeline. A number of centrifugal compressors located in parallel are the principal equipments of the compressing stations. A part of the gas crossing through the station is used as fuel gas for the centrifugal compressors.

A major challenge is to operate the pipeline network at minimum cost, with minimum environmental emission, fulfilling variations in contractual nominations and maintaining a sufficient pipeline inventory to provide operational flexibility. This requires striking a delicate balance between high pressure and associated energy consumption for compressors on the one hand, and lower pressure and risk of losing gas sale on the other. It also calls for detailed knowledge of network integration and the relationship between customer nominations, pipeline flow and inventory, compressor station operation, and operational flexibility of the network.

In natural gas pipeline operations, the station operator is responsible for making two important decisions: increase or decrease compression in the pipelines, and start-up or shut down of centrifugal compressor units. Incorrect decisions made by the operator increases energy cost or may cause customer dissatisfaction. The main objective of this study is to provide a decision aid tool that assists operators to make the most appropriate decision within a short time.

Many researchers have tried to find a way to optimize operation of gas pipeline network. Mathematical modeling is a common tool for simulation and optimization of such system. Several optimization methods were developed, however, none of them have considered all aspects of the problem. The majority of them are based on the dynamic programming (Wong PJ et al., 1968; Peretti A et al., 1982; Carter R et al., 1998; Ríos-Mercado, R.Z et al., 2002) or gradient search techniques (Percell PB et al., 1987; Wu S et al., 2000).

The work of Batey et al. represents one of the early attempts to develop a rational control policy. Wong and Larson applied the DP technique for the first time to the linear systems in 1968, and then Wong and Larson used it for the ramified networks. Peretti and Toth used a DP formulation, which uses dominance properties and lower bounds. In 1987, Percell and Ryan applied a methodology based on a generalized reduced gradient (GRG) nonlinear optimization technique for non-cyclic structures. The most significant work on cyclic networks is due to Carter who used a non-sequential DP algorithm, but limited to a set of flows. Wu et al. presented two model relaxations, one for the feasible operating domain of compressor and another in the fuel cost function, and derive a lower bounding scheme. Cobos-Zaleta and

Ríos-Mercado presented a method based on an outer approximation with equality relaxation and augmented penalty algorithm (OA/ER/AP) for solving a MINLP problem. Chebouba and Smati proposed a model based on DP for operating compressor unit choice. Ríos-Mercado et al. used a two stage iterative procedure. At the first stage, gas flow variables are fixed and optimal pressure variables are found via dynamical programming. At the second stage, they search a new set of flow variables that improve objective function. Martin et al. applied a technique for a piece-wise linear approximation of the model non-linear.

The principal advantages of DP are that a global optimum is guaranteed and that the non-linearity can be easily treated. The disadvantages of DP are that its application is practically limited to the linear or tree network topologies and that computation increases in an exponential way with problem dimension. The advantages of GRG method consist of that dimension does not be a problem and, it could be applied to cyclic network schemes. However, as GRG method is based on a search gradient method, there is no guarantee to find a global optimum. In fact, with discrete decision variables, it can fix with the local optimum.

MATHEMATICAL MODEL OF OPERATION OPERATION OPTIMIZATION ON SAVING COMPRESSOR ENERGY OF NATURAL GAS PIPELINE

Description

The transportation system is defined by the set of all nodes V , the set of all arcs A , the set of compressor station arcs A_C and the set of pipe leg arcs A_P . The transportation system receives at its inlet a quantity of gas at some pressure conditions. For each station (i, j) , decisions variables are the discharge pressure (p_j) and number of operating centrifugal compressors (n_{ij}). It is necessary to take into account several constraints. Some of these are physical and represent the feasible operating domain for compressor station (the speed and flow rate). Others are management constraints such as the maximum pressure value inside pipes, the minimum and maximum pressure value at suction and discharge node of each station, the minimum pressure value at the outlet of the line, the maximum available number of centrifugal compressors (Ríos-Mercado, R.Z et al., 2002; Xu, et al., 2011). The subject of our work is the fuel consumption minimization problem of gas pipeline.

Formulation

$$\min \sum_{(i,j) \in A_C} Q_{ij} \frac{Z_i R T_i}{p_i} \ln \frac{p_j}{p_i} \quad (1)$$

$$q_{i,\min} \leq q_i \leq q_{i,\max} \quad i \in V \quad (2)$$

$$p_{i,\min} \leq p_i \leq p_{i,\max} \quad i \in V \quad (3)$$

$$p_j \geq 0 \quad j \in A_c \tag{4}$$

$$D_{ij} / n_{ij}, p_i, p_j \in D_{ij}, n_{ij} \in \{1, 2, \dots, N_{ij}\} \quad (i, j) \in A_c \tag{5}$$

$$\sum_{i \in A_c} a_{ij} Q_{ij} \leq q_i \quad (i, j) \in A_p \tag{6}$$

$$P_i^2 \leq P_j^2 \leq k(i, j) Q_{ij}^2 \quad (i, j) \in A_p \tag{7}$$

$$\frac{\max(p_i, p_j) d}{2et\sigma} \leq \frac{d}{F} \quad (i, j) \in A_p \tag{8}$$

At each arc $(i, j) \in A_c$, p_j and p_i are, respectively, the discharge pressure and the inlet pressure of station (i, j) . For each station (i, j) , Q_{ij} , n_{ij} and N_{ij} represent, respectively, the mass flow rate, the operating centrifugal compressors number and number of available centrifugal compressors in the compressor station (i, j) . The gas compressibility factor Z_i and gas temperature T_i are defined at suction conditions of station (i, j) . η_j is the centrifugal compressor adiabatic efficiency in station (i, j) . The gas constant R and the gas specific heat ratio γ are characteristics of the transported gas. $P_{i,\min}$, $P_{i,\max}$, $q_{i,\min}$ and $q_{i,\max}$ are the pressure and flow limits at node i , and represent respectively lower limit and upper limit. D_{ij} is the feasible operating domain for a single centrifugal compressor unit in compressor station (i, j) . Δ_j is the correlation coefficient between nodes and pipelines, k_{ij} represents the resistance of pipe (i, j) ($(i, j) \in A_p$). e , d , $[\sigma]$, t , σ and F represent, respectively, wall thickness, diameter, minimum pipe yield limit of pipe, temperature reduction coefficient, weld coefficient, design coefficient of pipe.

For measuring total power consumed by all the pipeline compressor stations, we use Eq. (1). The centrifugal compressor adiabatic efficiency η_j is obtained from Eq. (10). Eq. (2) and (3) defines flow rate and pressure requirement of gas source and customer. Eq. (4) defines the pressure as non-negative variable. Eq. (5) represents the feasible operating domain for a single compressor station unit. This equation defines that the operating point of the compressor station must belong to the feasible operating domain which is bounded by inequalities (8) and (9). Eq. (6) and (7) defines the gas flow dynamics in pipeline network and each pipe leg (i, j) . Eq. (8) defines the maximum operation pressure in the pipeline.

The compressor stations are constituted of several identical compressor stations, built in parallel, which could be stopped or started. The operation range of a compressor station (i, j) as a function of the variables Q_{ij} (flow through the compressor station unit), p_i (suction pressure) and p_j (discharge pressure) is given by the following equations:

$$\frac{h_{ij}}{s_{ij}^2} \leq a_0 \leq b_0 \left(\frac{Q_{ij}}{s_{ij}}\right) \leq c_0 \left(\frac{Q_{ij}}{s_{ij}}\right)^2 \tag{9}$$

$$\Delta_{ij} = c_1 \frac{Q_{ij}^2}{s_j} + b_2 \frac{Q_{ij}}{s_j} + a_1 \quad (10)$$

$$s_{min} \leq s_j \leq s_{max} \quad (11)$$

$$\text{surge} \leq Q_{ij} / s_j \leq \text{stonewall} \quad (12)$$

where a_0, b_0, c_0, a_1, b_1 and c_1 are constants which depend on the compressor unit and are typically estimated by applying the least squares method to a set of collected data of the quantities Q_{ij}, s_j, h_{ij} . Surge is lower bound of Q_{ij}/s_j and Stonewall is upper bound of Q_{ij}/s_j .

MODEL SOLUTION OF OPERATION OPTIMIZATION ON SAVING COMPRESSOR ENERGY OF NATURAL GAS PIPELINE

Mathematical analysis of optimization model

The energy consumption function is described using flow through the compressor station unit (Q_{ij}), suction pressure (p_i) and discharge pressure (p_j). The flow of compressor station, the suction pressure and discharge pressure is restricted to the feasible operating domain of the compressor, which lead to the non-linear, non-convex, noncontinuity and non-smoothness of the energy consumption function. There are two important kinds of constraint conditions: (1) nonlinear equality constraints of pressure drop and flow rate in each pipe section; (2) nonlinear, non-convex and noncontinuity constraints of the feasible operating domain of pressure and flow rate of compressor station. The optimization variables include node pressure, pipe flow rate, suction pressure, discharge pressure, and operating centrifugal compressors number. Except for operating centrifugal compressors number is integer variable, the rest are continuous variables. The dimension of variables is depended on the scale of pipeline network and the number of physical components.

From the aspects of mathematical programming, the question belongs to mixed integer nonlinear programming. The methods of model solution published in the literature have these disadvantages as below, the generality is not strong, the calculation efficiency is low and the result is unstable etc. This paper puts forward the solution methods based on bundle projection subgradient method combined with Lagrange multiplier (Xu, et al., 2011). The detailed algorithm and steps are as follow: Step1: the pipeline network structure is described, the boundary conditions, the hydraulic calculation model, the initial constraint and the corresponding control variables of the network optimization model are confirmed;

Step2: model initialization. The steady-state model of gas pipeline network in under given conditions is solved, taking the results of steady state simulation as the initial points u_0 ;

Step3: the iteration error $\epsilon > 0$ is allowed, choosing $N = 0$.

Step4: using Lagrange multiplier vector $\Delta = [\Delta(1), \Delta(2), \dots, \Delta(I)]$,

$L(\lambda, \mu, \nu)$, $\lambda(1), \lambda(2), \dots, \lambda(m)$, the equality constraint and inequality constraints of network operation model optimization are relaxed to get:

$$L(\lambda, \mu, \nu) = f(\lambda) + \sum_{i=1}^p \lambda_i h_i(\lambda) + \sum_{j=1}^q \mu_j [g_j(\lambda) - y_j^2] \quad (13)$$

$L(\lambda, \mu, \nu)$, $h_i(\lambda) = 0$, $g_k(\lambda) = 0$ represent Lagrange augmented function of the objective function, the equality constraints and the inequality constraints, respectively.

Step5: calculation of subgradient g_j^k :

$$g_{\lambda}(t)_j^k = p_d(t) + \sum_{i=1}^N p_i(t)_j^k \quad (14)$$

$$g_{\lambda}(t)_j^k = p_d(t) + p_r(t) + \sum_{i=1}^N \lambda_i(t)_j^k p_i^{\max}, \quad t = 1, \dots, T, j = 1, \dots, J^k ;$$

Step6: confirming the weight of subgradient. If previous value is more close to the current subgradient direction, it contains more information, and the weight is greater:

$$\lambda_j^k = \frac{\sum_{j=1}^{J^k} \bar{\lambda}_j^k}{\sum_{j=1}^{J^k} \bar{\lambda}_j^k} \quad (15)$$

$$\bar{\lambda}_j^k = \begin{cases} (L_{\min}^k - L_j^k) / \Delta & \text{if } L_j^k \leq L_{\min}^k + \Delta \\ 0 & \text{otherwise} \end{cases}$$

Step7: judge whether the angle posed by \bar{g}^k and the last search direction d^{k-1} is acute, and then constructing the search direction d^k , namely

$$d^k = \begin{cases} \bar{g}^k, & \text{if } d_{k-1}^T \bar{g}^k \leq 0 \\ \bar{g}^k + \Delta_k d_{k-1}, & \text{otherwise} \end{cases} \quad (16)$$

$$\Delta_k = \Delta_k \frac{|d_{k-1}^T \bar{g}^k|}{\|d_{k-1}\|^2}, \quad 0 \leq \Delta_k \leq 1 ;$$

Step8: update Lagrange multiplier, $\lambda^{k+1} = \lambda^k + \Delta_k d_{\lambda}^k$, $\lambda^{k+1} = \lambda^k + \Delta_k d_{\lambda}^k$,

$$\Delta_k = \Delta_k \frac{\bar{L} - L}{\|(d_{\lambda}^k)^T d_{\lambda}^k\|}, \quad \bar{L} \text{ is the estimated upper bound as the upper limit optimal}$$

value of the original problem. If \bar{L} doesn't increase after set number, then Δ_k halves;

Step9: judge whether $GAP \leq \epsilon$ or $N \leq N_{\max}$, or turn to Step5. when the Lagrange relaxation method is used to solve the network optimization problem, the dual problem optimal solution is always less than or equal to the corresponding original optimization problem of optimal solution, the difference in value called dual clearance, the size of the dual clearance directly reflects capability of the iteration process, and it can be used as the standard of the algorithms end. After the above iteration, relatively dual gap is gotten.

$$GAP \square \frac{f(\square) \square L(\square, \square, \square)}{L(\square, \square, \square)} \tag{17}$$

Step10: output the optimal operation scheme \square and node parameter of the pipeline network.

OPTIMIZATION OPERATION SCHEME AND RESULTS ANALYSIS

The length of natural gas pipeline is 2203 km, among the trunk line primary route is about 1700 km, and the pipe diameter is 1016 mm. The topological structure as is shown in Figure 1. The design throughput is $120 \times 10^8 \text{ m}^3/\text{a}$, and the design pressure is 10.0 MPa. The entire route contains 18 transporting stations, including 3 compressor stations. The configuration of the compressor units is two used and one standby centrifugal compressor, all in parallel operation, which total power is 11000 kW. The inlet pressure of the first station is 8.0 MPa, and the pressure at the end of the pipeline requires not less than 4.0 MPa. According to the transport and supply plan of $90 \times 10^8 \text{ m}^3/\text{a}$ and $120 \times 10^8 \text{ m}^3/\text{a}$ transport and supply plan, in definite situation of total throughput and customer demand, taking all the lowest energy consumption of compressor station energy consumption for target, the optimal calculating is solved to confirm the optimal operation schemes of two throughputs. The calculation results is are shown in Table 1 - Table 4 and Figure 2- Figure 5.

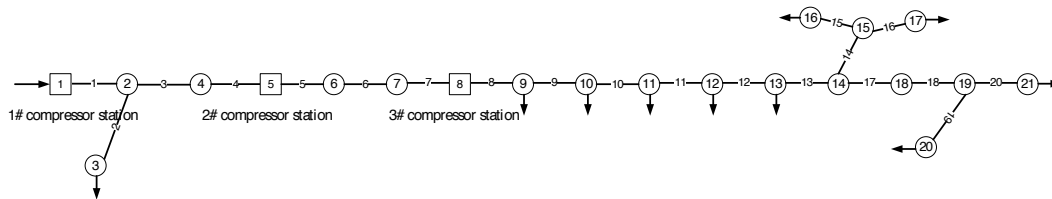


Figure 1. The topological structure of the gas pipeline network.

Table 1. The optimal pipeline operation scheme of $90 \times 10^8 \text{ m}^3/\text{a}$ throughput.

Node	Flow ($10^4 \text{ m}^3/\text{d}$)	Pressure (MPa)	Node	Flow ($10^4 \text{ m}^3/\text{d}$)	Pressure (MPa)
1# compressor station (suction)	2500.0	8.000	Node11	85.0	5.304
1# compressor station (discharge)	0.0	8.825	Node 12	10.0	5.052
Node 2	0.0	8.265	Node 13	40.0	4.641
Node 3	580.0	7.086	Node 14	0.0	4.490
2# compressor station (suction)	0.0	7.772	Node 15	0.0	4.432
2# compressor station (discharge)	0.0	8.485	Node 16	300.0	4.285
3# compressor station (suction)	0.0	6.904	Node 17	60.0	4.422
3# compressor	0.0	6.904	Node 19	400.0	4.089

station (discharge)					
Node 9	200.0	6.331	Node 20	60.0	4.076
Node 10	130.0	5.723	Node 21	635.0	4.074

Table 2. The optimal compressor station operation scheme of $90 \times 10^8 \text{m}^3/\text{a}$ throughput.

compressor station	compression ratio	Flow ($10^4 \text{m}^3/\text{d}$)	Power (kW)	efficiency	Max compression ratio	Min compression ratio	operating compressors number
1# compressor station	1.104	2500.0	3491.455	0.786	1.5	1.0	1
2# compressor station	1.092	1920.0	2635.181	0.712	1.5	1.0	1

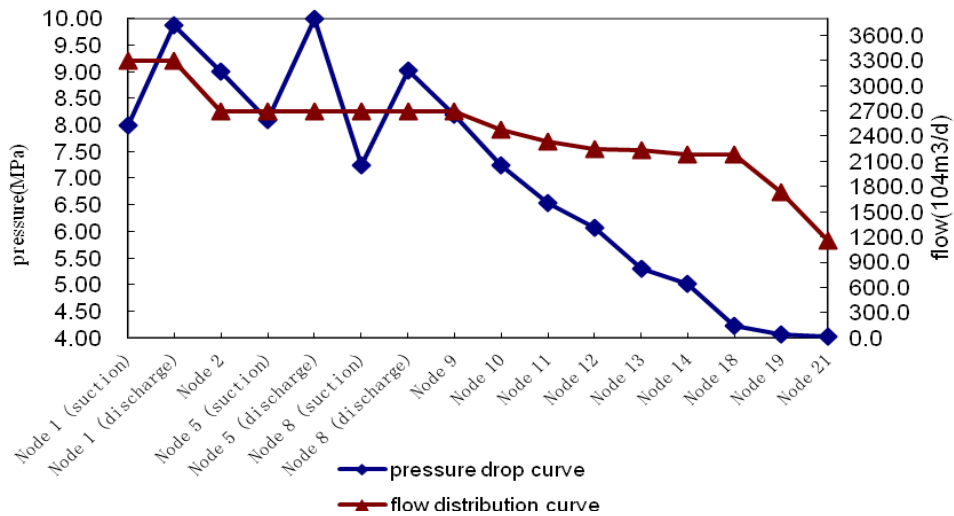


Figure 2. Pipeline pressure drop curve and flow distribution curve of $90 \times 10^8 \text{m}^3/\text{a}$ throughput.

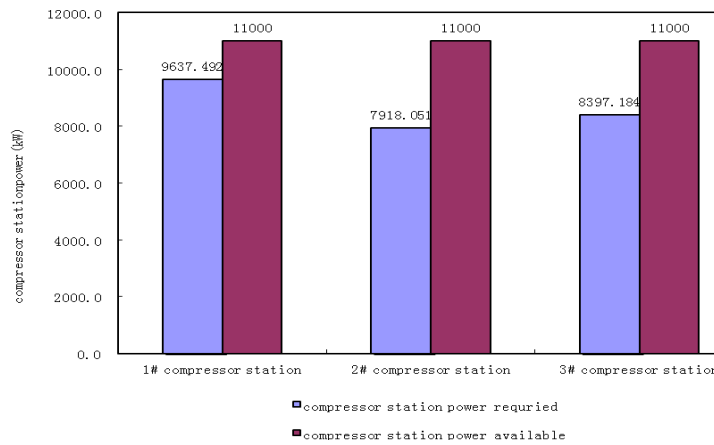


Figure 3. Compressor station load distribution of $90 \times 10^8 \text{m}^3/\text{a}$ throughput.

The analysis and discussions of calculation results:

(1) Under the $90 \times 10^8 \text{m}^3/\text{a}$ throughput conditions, running one compressor in 1# and 2# compressor station can complete transmission task. The total energy consumption is 6126.626 kW. Transmission ability still residue from pipe diameter specification and compressor configuration.

(2) The highest pressure point of gas is 8.825 MPa in 1# compressor station discharge node, the lowest pressure of gas is 4.074 MPa, each node pressure is within the limits of the maximum operating pressure (10.0 MPa) and lowest supply pressure (4.0 MPa), the optimal operation scheme can meet process constraints of the pipeline network transmission.

(3) In the optimal operation scheme, the compression ratio, pressure head and discharge pressure of 1# and 2# compressor station is close; the distribution of total compressor power between two stations is uniform. Combined with the analysis of energy consumption function of compressor, the relationship between power and the compressor ratio is exponential. The uniform distribution of compressor ratio between two compressor stations is a reasonable and economic way.

Table 3. The optimal pipeline operation scheme of $120 \times 10^8 \text{m}^3/\text{a}$ throughput.

Node	Flow ($10^4 \text{m}^3/\text{d}$)	Pressure (MPa)	Node	Flow ($10^4 \text{m}^3/\text{d}$)	Pressure (MPa)
1# compressor station (suction)	3300.0	8.000	Node11	85.0	6.526
1# compressor station (discharge)	0.0	9.877	Node 12	15.0	6.073
Node 2	0.0	9.004	Node 13	50.0	5.305
Node 3	600.0	7.869	Node 14	0.0	5.007
2# compressor station (suction)	0.0	8.096	Node 15	0.0	4.924
2# compressor station (discharge)	0.0	9.994	Node 16	370.0	4.724

3# compressor station (suction)	0.0	7.237	Node 17	85.0	4.906
3# compressor station (discharge)	0.0	9.033	Node 19	510.0	4.068
Node 9	228.0	8.188	Node 20	60.0	4.055
Node 10	140.0	7.233	Node 21	1157.0	4.018

Table 4. The optimal compressor station operation scheme of $120 \times 10^8 \text{m}^3/\text{a}$ throughput.

compressor station	compression ratio	Flow ($10^4 \text{m}^3/\text{d}$)	Power (kW)	efficiency	Max compression ratio	Min compression ratio	operating compressors number
1# compressor station	1.236	3300.0	9637.492	0.824	1.5	1.0	2
2# compressor station	1.234	2700.0	7918.051	0.814	1.5	1.0	2
3# compressor station	1.248	2700.0	8397.184	0.819	1.5	1.0	2

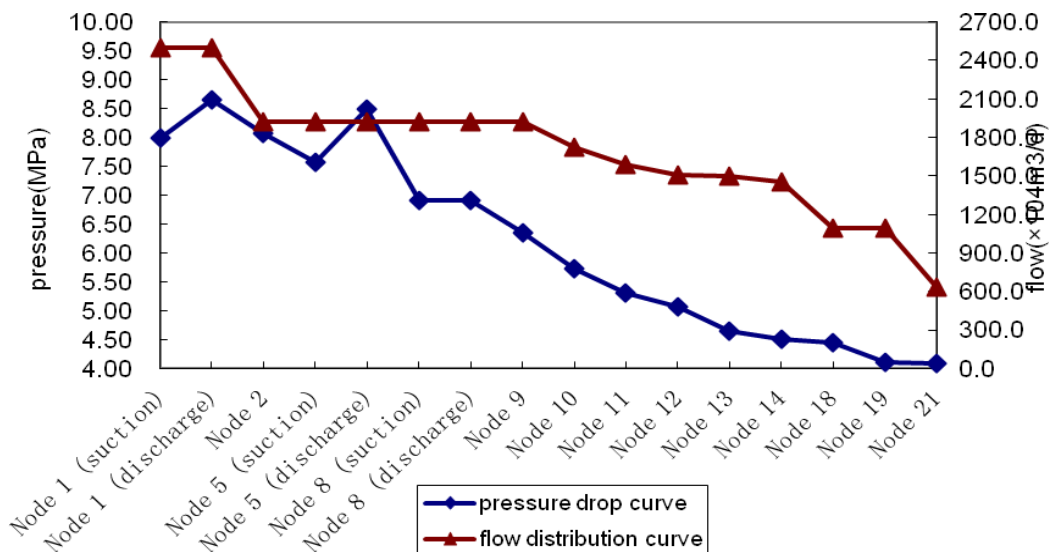


Figure 4. Pipeline pressure drop curve and flow distribution curve of $120 \times 10^8 \text{m}^3/\text{a}$ throughput.

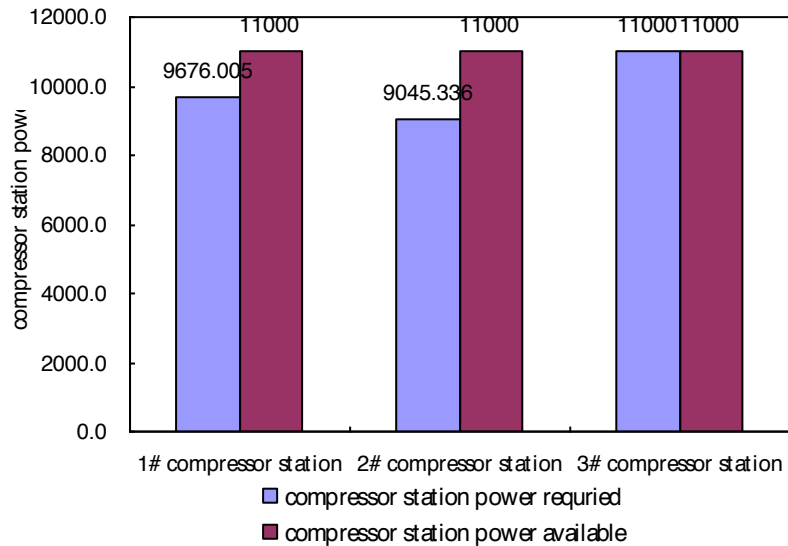


Figure 5. Compressor station load distribution of $120 \times 10^8 \text{m}^3/\text{a}$ throughput.

The analysis and discussions of calculation results:

(1) Under the $120 \times 10^8 \text{m}^3/\text{a}$ throughput conditions, running two compressor in 1#, 2# and 3# compressor station can complete transmission task, the total energy consumption is 25952.727 kW, pipeline transmission ability is almost close to the limitation from pipe diameter specifications and compressor configuration.

(2) The highest pressure point of gas is 9.994MPa in 2# compressor station discharge node, the lowest pressure of gas is 4.018MPa, each node pressure is within the limits of the maximum operation pressure (10.0 MPa) and lowest supply pressure (4.0 MPa), the optimal operation scheme can meet process constraints of the pipeline network transmission.

(3) The pressure ratio of the three compressors is close, the power of the compressors are close to the maximum power, the discharge pressure of the two station has almost reached the maximum operation pressure, the discharge pressure is relatively lower compared with the other two stations.

CONCLUSION

Based on analyzing the operation secure domain of pipeline and other equipment units, taken node parameters of pipeline network, compressor station flows, suction and discharge pressures as the optimization variables, optimizing operation mathematical model is established respectively based on the least energy consumption of the compressor station. On the basis of analysing the objective function, restrained conditions and decision variables of the optimizing operation mathematical model, the thesis adopts bundle projection subgradient combined with Lagrange multiplier method, the optimization strategy can alleviate the oscillation and speed up convergence.

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