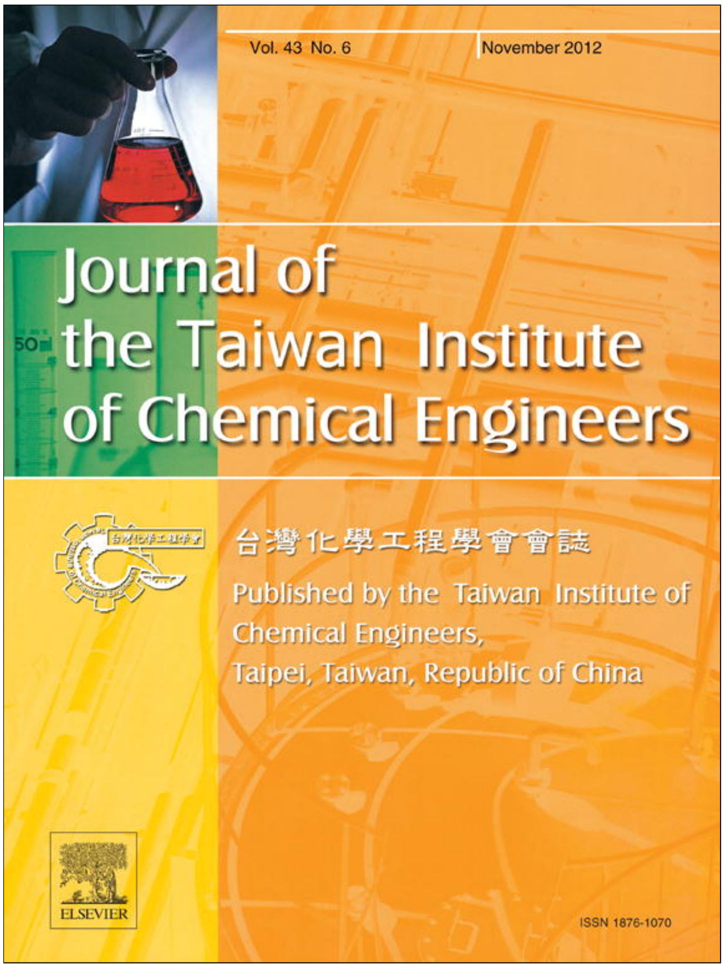
Vol. 43 No. 6
November 2012
Journal of
the Taiwan Institute
of Chemical Engineers
台灣化學工程學會會誌
Published by the Taiwan Institute of
Chemical Engineers,
Taipei, Taiwan, Republic of China
ELSEVIER
ISSN 1876-1070

Journal of the Taiwan Institute of Chemical Engineers 43 (2012) 904–917

Contents lists available at SciVerse ScienceDirect

Journal of the Taiwan Institute of Chemical Engineers

journal homepage: www.elsevier.com/locate/jtice

Minimization of fuel consumption in cyclic and non-cyclic natural gas transmission networks: Assessment of genetic algorithm optimization method as an alternative to non-sequential dynamic programing

Sepehr Sanaye *, Javad Mahmoudimehr

Energy Systems Improvement Laboratory (ESIL), Department of Mechanical Engineering, Iran University of Science and Technology (IUST), Iran

ARTICLE INFO

Article history:
Received 28 December 2011
Received in revised form 31 March 2012
Accepted 27 April 2012
Available online 15 June 2012

Keywords:
Cyclic and non-cyclic natural gas transmission networks
Optimization of fuel consumption
Non-sequential dynamic programing
Genetic algorithm

ABSTRACT

For minimization of fuel consumption of natural gas transmission networks, non-sequential dynamic programing (NDP) method guarantees to find the global optimal solution, however NDP method cannot be used for analysis of cyclic networks in which the flow rate values are not known in priori. Therefore modified NDP method is proposed in this paper which is capable of being applied to the cyclic network problems. Still a drawback remains with the proposed modified NDP which is impractical computing time except for simple cyclic networks. To solve this basic problem, the genetic algorithm (GA) method was selected as an alternative method. Then the modified NDP and GA methods were applied to three types of natural gas transmission network problems including linear, branched and cyclic structures and their results were analyzed and compared. The results showed that for three mentioned network structures, the difference values in objective function (rate of fuel consumption) which were obtained from NDP and GA methods were within acceptable range of 0–0.55%. Furthermore, it was observed that while the computing time required by the NDP method exponentially depended on pressure and flow rate step sizes, the GA computing time did not show such a dependency on these parameters.

1. Introduction

Natural gas is increasingly used as a source of energy all over the world and the estimations show that its worldwide consumption in 2030 will be as twice as its present rate [1].

As natural gas travels through the transmission pipelines, the gas pressure drops due to both the friction with the pipe walls and heat transfer to the surroundings. Therefore there is need to compensate the pressure by a number of compressor stations located along the pipeline.

As an accepted rule of thumb, about 3–5% of the transmitted natural gas is consumed for gas turbine drivers to generate power for the compressor stations [2–4] which amounts to a huge cost because of the large quantities of natural gas transmitted through extensive networks. For example, based on the information presented by reference [4], the natural gas consumption in the United States was more than 60 billion cubic feet per day, and its worldwide consumption was three times of this amount. Based on the prices in 1998, the cost of the fuel burned to run the natural gas compressor stations in the United States came to about two billion dollars per year. Therefore, even a slight improvement in the performance of the gas transportation system can result in great savings.

The dynamic programing (DP) method was invented by an American mathematician named Richard Ernest Bellman [5]. DP is a powerful method for the minimization of fuel consumption of natural gas networks, and its advantage is that, in contrast to the gradient-based methods, it is insensitive to the non-linearity, non-convexity and existing discontinuities of the problem under study. DP is also guaranteed to find the optimal solution [3].

DP was first applied to the linear (also called gun-barrel) gas network in 1968 by Wong and Larson [6]. Then, researches in [7–9] applied DP to branched (also called tree) networks.

Ref. [10] applied the ant colony meta-heuristic optimization (ACO) method to a gun-barrel natural gas network. To verify the results of the ACO method, they were compared to those obtained by the DP method. This comparison showed a minimum/maximum relative difference of about 0.1%/0.66% between the two approaches.

The most significant work on cyclic networks, known so far, has been undertaken by Carter (1998) [4] who developed the non-sequential DP (NDP) algorithm. This method can be applied to gas networks with cyclic or non-cyclic structure, but is limited to a fixed set of flows.

* Corresponding author at: Energy Systems Improvement Laboratory, Mechanical Engineering Department, Iran University of Science and Technology, Narmak, Tehran 16844, Iran. Tel.: +98 21 77240192; fax: +98 21 77240192.
E-mail addresses: sepehr@iust.ac.ir (S. Sanaye), mahmoudimehr@iust.ac.ir (J. Mahmoudimehr).

1876-1070/$ – see front matter © 2012 Taiwan Institute of Chemical Engineers. Published by Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.jtice.2012.04.010

Nomenclature

ACO	ant colony optimization
CS	compressor station
D	pipe diameter (mm)
DP	dynamic programing
G	specific gravity
H	isentropic head (J/kg)
LHV	lower natural gas heating value (J/kg)
$\dot{m}$	mass flow rate (kg/s)
MAOP	maximum allowable operating pressure (Pa)
MMSCMD	million standard cubic meters per day
NDP	non-sequential dynamic programing
P	pressure (Pa)
Q_{ac}	actual volumetric flow rate passing through a compressor (m^3/s)
R	gas constant (J/kg K)
S	compressor rotational speed (rpm)
t	resistance of pipeline $(Pa\ s/kg)^2$
T	temperature (K)
TC	turbo-compressor
X	mass flow rate (kg/s)
Z	compressibility factor
Greek symbols	
η	efficiency
σ	isentropic exponent
Subscripts	
ac	actual
b	base value
c	natural gas compressor
d	discharge
f	fuel
is	isentropic
$mech$	mechanical
s	suction
th	thermal

In non-cyclic networks (gun-barrel or branched networks), the rate of flow passing through each existing pipeline or compressor station (CS) is uniquely specified, and the NDP method can be applied [11]. By contrast, in cyclic networks, the rate of flow passing through each part of the existing cycles is not pre-specified, and therefore, a set of flow variables is added to the decision variables of the optimization problem. This makes the NDP method multi-dimensional; and consequently it will require unreasonable and impractical computation time, except for the cases with very few cycles.

The research work performed by Luongo *et al.* [12] is among the early works in optimization of fuel consumption in cyclic networks. By assuming some fixed values for flow variables first they obtained optimal corresponding unknown pressure values *via* DP method. Then, they updated the flow variables which improved the objective function through direct search method.

There are also a few other two-stage iterative optimization methods which have been proposed in the literature for optimizing the operational conditions of cyclic networks. These algorithms assume some fixed values for flow variables at first, and find the corresponding optimal pressure values *via* NDP. Then they search for a new set of flow variables to improve the objective function *via* methods such as the heuristic [13] or Tabu search methods [2].

GA, as one of the evolutionary algorithms, is an efficient optimization tool that begins the search for the optimal solution from different points in the solution domain, thereby reducing the probability of being trapped in the local optima. GA only uses the objective function value for the optimization, it does not need information like derivatives and other auxiliary knowledge, and the aforementioned complexities do not complicate this algorithm [14]. The evolutionary algorithms are simple to write, and powerful, from the standpoint of setting the control parameters.

Refs. [14,15] have given a review of the research works that have used GA method. These references have also employed GA to optimize the CS problem.

Also Ref. [16] used GA method to determine the optimal pipe size for a gas network with predefined structure.

In this paper, the results and capability of finding lower values for fuel consumption rates using NDP and genetic algorithm (GA) methods for linear, branched and cyclic structures are analyzed and compared. Through this comparison, the GA method was verified as an appropriate alternative method for optimization of all three above mentioned gas network structures.

Our literature survey showed that the results of the proposed methods for optimization of operational conditions in cyclic networks were mainly verified by comparison with the results of gradient-based methods or personal experiences (which none of them necessarily lead to the optimal solution). However, in this paper, the optimization results obtained by GA for both the cyclic and non-cyclic (linear and branched) network structures are compared to the results of modified NDP method. This method of verification is very reliable, due to the fact that the modified NDP method certainly reaches the optimal solution.

Other contributions of this paper which are not usually referred to in literature are taking into account the effects of the ambient temperature and the driver (gas turbine) part load operation and shaft speed on the driver efficiency.

This paper is organized as follows: the modeling of various natural gas network structures with their governing equations are described in Section 2, the description of the optimization problem as well as a brief introduction to the NDP and GA methods are presented in Section 3. The case studies investigated in this paper are introduced in Section 4, and the fuel optimization results are presented and discussed in Section 5.

2. Modeling and the governing equations

Gas networks are composed of pipelines and CSs as the main components. A schematic diagram of a CS located between two pipelines, with a number of turbo-compressor (TC) units in parallel, is shown in Fig. 1. This figure shows that each TC consists of a natural gas compressor (responsible for compensating the pressure drop) and a gas turbine (also called turbine engine, as the driver of the natural gas compressor). The studied double-shaft gas turbine is composed of air compressor, combustion chamber, and low and high pressure (LP, HP) turbines, which are discussed in detail in Section 2.3. The "natural gas compressor" is henceforth shortened to "compressor". The modeling and the governing equations of typical natural gas network components are described as follows.

2.1. Pipeline

By assuming isothermal and steady-state conditions, the pressure drop in natural gas pipeline was obtained from a conservative flow equation named Weymouth equation [17–22] as follows:

$$Q = 3.7435 \times 10^{-3}\left(\frac{T_b}{P_b}\right)\left(\frac{P_1{}^2 - P_2{}^2}{GT_{av}LZ}\right)^{0.5} D^{2.667} \quad (1)$$

In the above equation, Q is the standard volumetric flow rate (in standard m^3/day), T_b, P_b are the base temperature and pressure, respectively (288.15 K, 100 kPa), T_{av} is the average gas flow temperature (K), P_1, P_2 are the upstream and downstream pressures (kPa), L is the length of pipeline segment (km), Z is the compressibility factor, and G is the natural gas specific gravity (the ratio of the density of natural gas to the density of air at standard conditions (101.325 kPa and 15 °C)).

Eq. (2) was used to determine the compressibility factor (Z) in Eq. (1), as a function of the reduced pressure (the ratio of the actual pressure to the gas critical pressure, P/P_c) and reduced temperature (the ratio of the actual temperature to the gas critical temperature, T/T_c) [23–25]:

$$Z = 1 + 0.257\left(\frac{P_{av}}{P_c}\right) - 0.533\left(\frac{P_{av}}{P_c}\right)\left(\frac{T_c}{T_{av}}\right) \quad (2)$$

The average gas flow pressure (P_{av}) was computed as $P_{av} = 2/3(P_1 + P_2 - (P_1 \cdot P_2/(P_1 + P_2)))$ [20].

2.2. Natural gas compressor

Eqs. (3)–(9) show the governing equations of gas flow passing through a typical compressor:

The ratio of compressor isentropic head to the square of rotational speed:

$$\frac{H}{S^2} = b_1 + b_2\left(\frac{Q_{ac}}{S}\right) + b_3\left(\frac{Q_{ac}}{S}\right)^2 \quad (3)$$

Compressor isentropic efficiency:

$$\eta_{c,is} = b_4 + b_5\left(\frac{Q_{ac}}{S}\right) + b_6\left(\frac{Q_{ac}}{S}\right)^2 \quad (4)$$

Eqs. (3) and (4) are empirical equations proposed by [26,27] for compressor operating points, in which b_1 to b_6 are constants that could be obtained from a specific compressor map in a pipeline.

Compressor isentropic head in terms of compressor pressure ratio:

$$H = \frac{Z_sRT_s}{\sigma}\left[\left(\frac{p_d}{p_s}\right)^{\sigma} - 1\right] \quad (5)$$

The compressor power consumption in terms of compressor mechanical efficiency ($\eta_{c,mech}$, which was considered equal to 0.98 in this paper):

$$Power_{shaft} = \frac{H \cdot \dot{m}_d}{\eta_{c,is} \cdot \eta_{c,mech}} \quad (6)$$

The required fuel mass flow rate in gas turbines for running the compressors:

$$\dot{m}_f = \frac{Power_{shaft}}{LHV \cdot \eta_{th,gas\,turbine}} \quad (7)$$

Eq. (8) indicates the mass balance among the mass flow rate of natural gas entering the turbo-compressor unit ($\dot{m}_s$), the gas turbine fuel consumption rate ($\dot{m}_f$), and the mass flow rate of natural gas passing through the compressor ($\dot{m}_d$) as indicated in Fig. 1.

$$\dot{m}_f + \dot{m}_d = \dot{m}_s \quad (8)$$

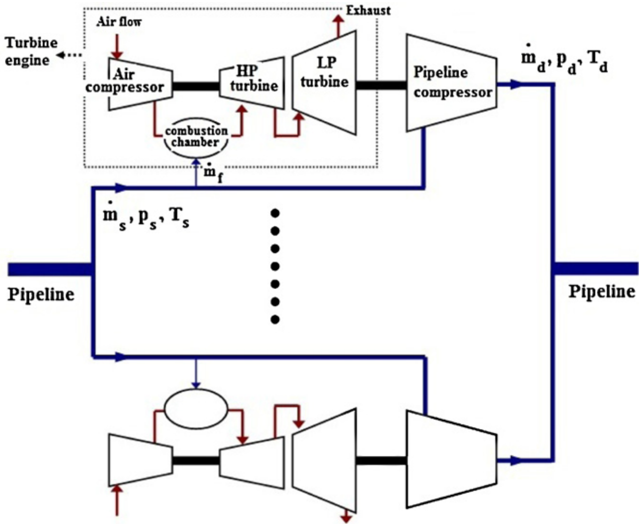

Fig. 1. Schematic diagram of a natural gas compressor station.

In the above equations, subscriptions s and d indicate compressor suction and discharge points.

The actual volumetric flow rate passing through a compressor as a function of mass flow rate, pressure and temperature, was also obtained from:

$$Q_{ac} = \frac{\dot{m}_d Z_s RT_s}{p_s} \quad (9)$$

2.3. Gas turbine (driver)

Generally, two-shaft gas turbines are used in gas pipeline applications because of their operational flexibility [28]. Part of Fig. 1 confined in a dashed box shows a schematic diagram of a two-shaft gas turbine. A gas turbine (including air compressor, combustion chamber, and low and high pressure turbines) provides power to run the air compressor by the high pressure turbine and to run the pipeline compressor by the low pressure turbine (also called power turbine).

For any shaft speed and power required by the pipeline compressor, the driver operational condition should be adjusted. Generally the technical information regarding the gas turbine includes the base load maximum power output ($Power_A$) and efficiency ($\eta_{th,A}$) at ISO conditions for point with A in Fig. 2, as well

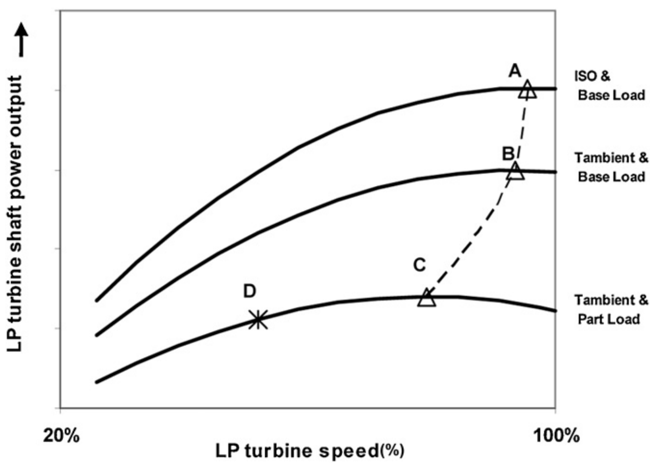

Fig. 2. Schematic diagram of a typical gas turbine performance map.

as its corresponding shaft speed (S_A). Ambient temperature and pressure values of 15 °C and 101.325 kPa were considered for ISO conditions.

In most cases though, the gas turbine does not operate at ISO conditions due to the elevation or temperature of the site in which the engine is installed as well as running at the partial load and/or out of design speed. Therefore, some corrections are required for computing the gas turbine overall performance as follows.

2.3.1. Correction for the ambient temperature

The ambient temperature variation changes the values of output shaft speed, power and efficiency [29,30] at the design point. For example, movement of point A to B in Fig. 2 shows the effect of a typical change in atmospheric temperature from T_{ISO} to $T_{ambient \neq ISO}$.

Therefore, some equations are required to estimate these effects. These equations can be extracted using the map or data available for a specific turbine which have the following forms:

$$\frac{Power_A}{Power_B} = f_1\left(\frac{T_{ambient}(K)}{T_{ISO}(K)}\right) \tag{10}$$

$$\frac{\eta_{th,A}}{\eta_{th,B}} = f_2\left(\frac{T_{ambient}(K)}{T_{ISO}(K)}\right) \tag{11}$$

$$\frac{S_B}{S_A} = f_3\left(\frac{T_{ambient}\ (K)}{T_{ISO}\ (K)}\right) \tag{12}$$

2.3.2. Correction for the part load operation

The turbine part load operation decreases the optimal values of output shaft speed, power and efficiency [29,31]. For example, movement of point B to C in Fig. 2 shows the effect of a typical part load operation. Equations which consider these effects are:

$$\frac{\eta_{th,C}}{\eta_{th,B}} = f_4\left(\frac{Power_C}{Power_B}\right) \tag{13}$$

$$\frac{S_C}{S_B} = f_5\left(\frac{Power_C}{Power_B}\right) \tag{14}$$

2.3.3. Correction for the operation out of design rotational speed

For any operating condition of a gas generator (part of the gas turbine which includes air compressor, combustion chamber and HP turbine as was shown in Fig. 1), there is a rotational speed at which the output shaft power and efficiency values are the highest. If the power turbine deviates from this speed, the power and efficiency decrease with the same proportions (Eq. (16)) [29]. For example, movement of point C to D in Fig. 2 shows the effect of a typical change in power turbine (LP turbine) shaft rotational speed from its optimal value at which it operates at its highest power and efficiency.

Eqs. (15) and (16) are proposed by [29,32] to take this effect into consideration:

$$\frac{Power_{shaft}}{Power_C} = 2\left(\frac{S}{S_C}\right) - \left(\frac{S}{S_C}\right)^2 \tag{15}$$

$$\frac{\eta_{th,D}}{\eta_{th,C}} = \frac{Power_{shaft}}{Power_C} \tag{16}$$

$Power_{shaft}$ and S in the above equations are the required power and speed (point D) by the pipeline compressor.

The set of nonlinear equations, including Eqs. (3)–(16) for each turbo-compressor unit, was solved using Newton–Raphson method in this paper.

3. Optimization

In this section, a description of the optimization problem as well as a brief introduction to the applied NDP and GA methods (whose codes were developed by the authors at the Energy Systems Improvement Laboratory, ESIL) are presented.

3.1. Optimization problem

The optimization problem, constituting the minimization of fuel consumption of a typical gas network composed of a set of pipeline arcs (A_P), a set of CS arcs (A_C), and a set of nodes (V) (starting and end points of pipeline and CS arcs), is mathematically expressed by Eqs. (17)–(21) [2,3,10,11].

$$\text{Minimize} \sum_{(i,j)\in A_C} J_{ij}\cdot g_{ij}(X_{ij}, n_{ij}, P_i, P_j), \quad J_{ij}\in\{0,1\} \tag{17}$$

$$\sum_{j:(i,j)\in A} X_{ij} - \sum_{j:(j,i)\in A} X_{ij} = S_i, \quad i\in V \tag{18}$$

$$P_i^2 - P_j^2 = t_{ij}X_{ij}^2, \quad (i,j)\in A_P \tag{19}$$

$$P_i\in[P_i^L, P_i^U], \quad i\in V \tag{20}$$

$$\text{if } J_{ij}=1 \Rightarrow (X_{ij}/n_{ij}, P_i, P_j)\in D_{ij}, \quad n_{ij}\in\{1,2,\ldots,N_{ij}\}, \quad (i,j)\in A_C \tag{21}$$

The decision variables are the rate of flow passing through each arc (X_{ij}, $(i,j)\in A$, $A = A_P \cup A_C$), pressure value at each node (P_i, $i \in V$), on/off status of each CS (J_{ij}, $(i,j)\in A_C$, where the value of 0/1 for J_{ij} shows the on/off status of the corresponding CS), and the number of active TCs in each CS (n_{ij}, $(i,j)\in A_C$).

Eq. (17) shows the objective function (sum of the rates of natural gas consumed as fuel by the drivers at the CSs), where the fuel consumption of each CS (g_{ij}, $(i,j)\in A_C$) is a function of the rate of flow passing through it (X_{ij}), number of active TCs (n_{ij}), and its suction and discharge pressures (P_i, P_j).

Eqs. (18)–(21) indicate the constraints of the problem.

Constraint (18) represents the balance of mass for each node. At each node ($i \in V$), there is a known parameter (S_i), which is called the net flow through that node. $S_i > 0$ ($S_i < 0$) implies that node i is a source (delivery) node, whereas $S_i = 0$ means that node i is just a transshipment node.

Constraint (19) represents the pipeline flow equation. In this equation, t_{ij} denotes the resistance of the pipeline segment $(i,j)\in A_P$ obtained from Eq. (1).

Constraint (20) expresses the pressure limits in each node.

Finally, constraint (21) implies that the operating point of each TC in an active CS should be in a feasible domain. In this equation, n_{ij}/N_{ij} is the number of active/available TCs at the CS, and the ratio of X_{ij}/n_{ij} indicates the mass flow rate passing through each active TC at that CS.

The feasible operating domain of a typical compressor bounded by maximum and minimum speed, surge and stonewall lines is shown in Fig. 3.

3.2. Non-sequential dynamic programing (NDP)

The NDP method was elaborately described in [4,33]. A brief description of the essence of this method is presented in this section. For the fixed values of flow variables in the gas network, the NDP method searches for the optimal set of nodal pressure values.

The NDP procedure takes two connected CSs and replaces them with a virtual composite element that represents the optimal operation of both CSs. These two connected elements can be

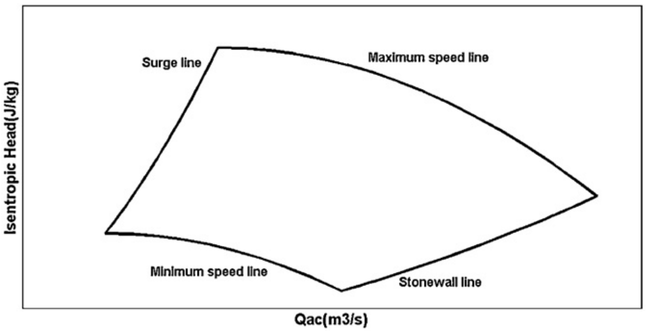

Fig. 3. Feasible operating domain for a typical compressor.

chosen from anywhere in the system. Following this step, the system has been replaced with an equivalent system (which has one CS lower). The procedure continues until only one virtual element, which fully characterizes the optimal behavior of the entire pipeline system, is left. Afterwards, the optimal set of pressure variables can be obtained by a straight-forward back-tracking process.

For the fixed values of flow variables, the NDP procedure can be applied to a network with any structure (cyclic or non-cyclic) by using three kinds of combinations of two connected CSs, as shown in Fig. 4. In this figure, $g(P_I, P_J)$ is the minimum value of fuel consumption of the CS with the suction and discharge pressures of P_I and P_J. This minimum value is obtained by finding the best number of active TCs through a search of all possible numbers. M_I is the number of elements in a discretized pressure range at point I.

The NDP technique reduces the computational complexity of the problem from $O(M_P{}^{\text{Number of CSs}})$ to less than $O(\text{Number of CSs} \cdot M_P{}^3)$, where M_P is the maximum number of elements in a discretized pressure range.

3.3. Genetic algorithm (GA) [16,34–36]

Evolutionary algorithms are random search methods that mimic the natural evolution. These algorithms start with a population of possible solutions and repeatedly generate a new population from the last one based on the survival of fitter solutions, with the hope of finding solutions with better objective functions. The steps of GA (as one of the evolutionary algorithms) are briefly described below:

3.3.1. Chromosome formation and the first population

As described in Section 3.1 of the manuscript, on/off status of each CS, the number of active TCs in each CS, and pressure value at each node form the decision variables of non cyclic problems. For the problems with cyclic structure, flow variables are also added to the decision variables.

It should be noted that the optimal number of active TCs in each CS, corresponding to the specified values of suction and discharge pressures as well as the rate of flow passing through that CS, was obtained exhaustively (*i.e.* by finding the best number of active TCs through a search of all possible numbers) in both NDP and GA methods, and therefore was excluded from the chromosome formation of GA.

Fig. 5 represents the chromosome formation. As shown in Fig. 5, the first part of the chromosome relates to the on/off states of the existing CSs in a typical gas network in which one binary bit is assigned to each CS so that the value of 1/0 in each bit represents the on/off status of its corresponding CS.

Also the second and third parts of the chromosome relate to the pressure and flow variables respectively.

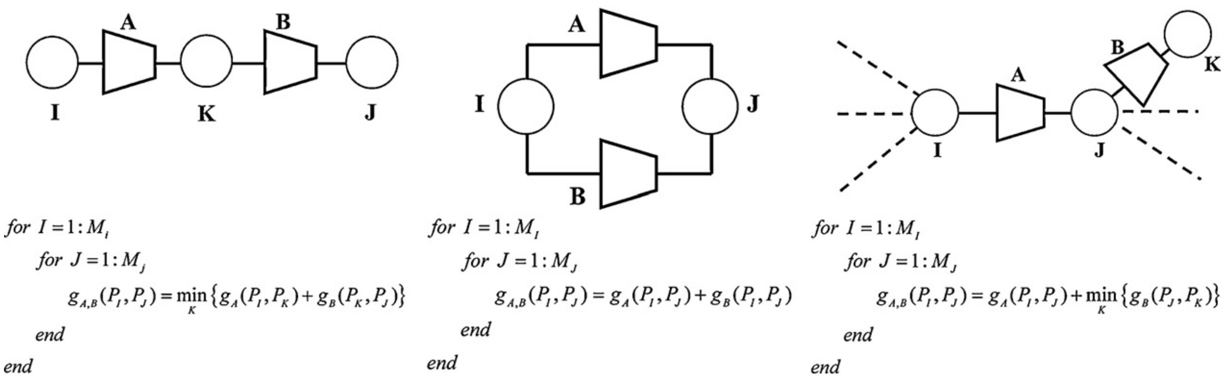

Fig. 4. Three different combinations of two connected CSs in the NDP method.

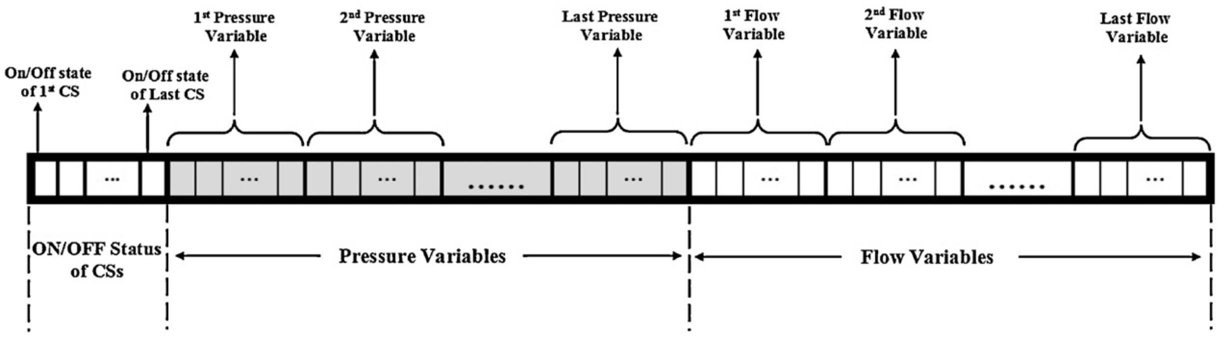

Fig. 5. Schematic diagram of the chromosome formation in GA.

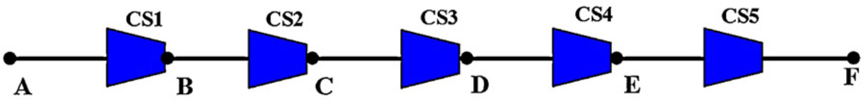

Fig. 6. Schematic diagram of the first case study (linear network).

As shown in Fig. 5, a number of bits are considered for each pressure variable depending on its lower and upper limits (P_i^L, P_i^U) as well as the pressure step size (ΔP).

Eqs. (22)–(26) are related to the coding and decoding processes of each pressure variable.

$$M_i = \lfloor \frac{P_i^L - P_i^U}{\Delta P} \rfloor + 1, \quad i \in V \tag{22}$$

$$\varepsilon = \frac{M_i - 1}{2^{N_{bit,i}} - 1} \tag{23}$$

$$u_i = \sum_{m=1}^{N_{bit,i}} J_m \cdot 2^{N_{bit,i} - m}, \quad J_m \in \{0, 1\} \tag{24}$$

$$Y_i = 1 + \varepsilon \cdot u_i \tag{25}$$

$$P_i = P_i^L + (||Y_i|| - 1)\Delta P \tag{26}$$

In Eq. (22), M_i denotes the number of elements in the discretized pressure range.

As the numbers 1, 2, ..., and M_i represent the 1st, 2nd, ..., and last elements in the discretized pressure range respectively, the range of 1 to M_i was coded with a number of binary bits ($N_{bit,i}$). Eqs. (23)–(25) show the decoding process of each set of the binary bits to the decimal value within the range of 1 to M_i. As shown in Eq. (25), ε is the minimum possible distance between the two decoded decimal values which was constrained to be less than 0.01 to give an almost uniform chance to each integer value between 1 and M_i to be selected when the decoded decimal value (Y_i) is returned to the nearest integer value ($||Y_i||$).

Finally, the pressure value corresponding to the obtained integer value is calculated using Eq. (26).

The flow variables are also coded with the same approach explained above which is not repeated here.

The formation of the first population (generation) is the first step in GA procedure. A population consists of a number of chromosomes (individuals), each a string of coded bits (genes), which represents a single solution of the problem under study. The first population is created by randomly choosing the binary value of 0 or 1 for each bit.

3.3.2. Selection process and the mating pool

The next step is to select some individuals to produce the offspring (children) for creating the new generation. This selection process is based on the fitness function that represents the objective function, and the individuals with greater fitness functions have a better chance of being selected. A collection of the selected individuals makes up the mating pool.

3.3.3. Crossover and mutation

After parents are randomly chosen from the selected individuals in the mating pool, they undergo the crossover procedure in order to produce offspring. Crossover produces offspring that inherit their genes from both parents. Then, mutation is applied on the children, which alters the initial values of some of their genes. Mutation makes it possible for the children to have some different gene values than their parents. It also prevents the optimization process from being trapped in the local optima.

3.3.4. Elitism

Elitism is a method which copies the best chromosome (or a few best chromosomes) to a new population without any change.

GA operators selected and used in this paper were the Roulette-Wheel selection, single point crossover, uniform mutation rate, and the Elitism. Also no improvement in the objective function for a sequence of 50 consecutive generations was considered as the stop criteria.

4. Case studies

To consider all types of gas network structures, the following three natural gas networks with linear, branched and cyclic structures were investigated in this paper.

4.1. First case study: linear network

The first case study, which is schematically shown in Fig. 6, is a linear network with five CSs, each including six parallel TCs. Table 1 shows the information regarding the first case study. Also the compressor constant values (required by Eqs. (3) and (4)), and the data pertaining to its feasible operating domain are listed in Table 2.

Table 1
Information regarding the first case study (linear network).

Diameter of each pipeline segment	Length of each pipeline segment	Rate of flow passing through the network	Gas pressure at point A
1.4224 mm	120 km	70 MMSCMD (601 kg/s)	55 bar

Average gas flow temperature	Minimum required gas pressure at point F	Pipeline maximum allowable operating pressure (MAOP)	Specific gravity
20 °C	50 bar	72 bar	0.6137

Table 2
The constant values for compressors.

b_1	b_2	b_3	b_4	b_5	b_6	Surge $(Q/S)_{min}$	Stonewall $(Q/S)_{max}$	S_{max}	S_{min}
8.294×10^{-4}	1.898	-2.532×10^3	13.929	2.54×10^5	-2.289×10^8	3.76×10^{-4}	8.53×10^{-4}	7700	4500

Table 3
Constant coefficients of functions f_1 to f_3 and f_5 in Eqs. (10)–(12) and (14).

Function	a_1	a_2	a_3
f_1	−4.3115	6.6618	−1.3618
f_2	−2.4918	4.4951	−1.0074
f_3	−0.4275	0.6710	0.7566
f_5	−0.397	1.0165	0.3777

The values of 7350 rpm, 25.4 MW, and 35.1% were considered for the parameters S_A, $Power_A$ and $\eta_{th,A}$, respectively, and applied in Eqs. (10)–(12).

The constant coefficients of four second-order polynomial equations ($a_1x^2 + a_2x + a_3$), which were obtained (by curve fitting) for functions f_1 to f_3 and f_5 in Eqs. (10)–(12) and (14), are shown in Table 3. Also, a logarithmic function was obtained for f_4 ($f_4 = 0.2457\ \ln(x) + 1$).

It is worth mentioning that the above equations have been verified by the authors in Ref. [37].

4.2. Second case study: branched network

The second case study, which is schematically shown in Fig. 7, is a branched network with eight CSs, each including six parallel TCs. The length, diameter and maximum allowable pressure (MAOP) are 150 km, 1.4224 mm and 72 bar for each pipeline segment of the incoming branch (branch with the flow rate of 70 MMSCMD), and also 100 km, 0.994 mm and 68 bar for each pipeline segment of the two outgoing branches (branches with the flow rates of 30 and 40 MMSCMD). Other information regarding the second case study is shown in Table 4. The same type of TCs as in the first case study was considered in the second case study.

4.3. Third case study: cyclic network

The third case study, which is schematically shown in Fig. 8, is a cyclic network with six CSs, each including six parallel TCs. The information regarding the pipeline segments of the third case study is given in Table 5 and the other relevant information is shown in Table 6. The type of TCs considered in the third case study was identical to that of the first and second case studies.

5. Results and discussion

5.1. Results of the first case study (linear network)

The flow variables are fixed throughout the network due to the non-cyclic structure of the first case study. Therefore, the decision variables include the values of pressure at nodes B to F (in Fig. 6), on/off status of each CS, and the number of active TCs at each CS.

The results obtained by NDP including the optimal decision variables and the objective function (fuel consumption) are shown in Table 7 for various pressure step sizes (0.25–2 bar). As Table 7 shows, for finer pressure step sizes (difference in two sequentially discrete pressure values), better solutions (solutions yielding lower fuel consumption) were obtained.

Table 7 also shows that in the optimal solution, the 3rd and 5th CSs were predicted to be bypassed (at off status), while the remaining CSs were active.

Fig. 9 shows the variation of natural gas pressure along the network of the first case study, based on the optimal values of the decision variables presented in Table 7 for the pressure step size of 0.25 bar (the smallest pressure step size which led to the lowest value of the objective function). This figure also compares the optimal pressure values obtained at nodes B to F with their upper and lower permissible bounds. As shown in Fig. 9, except for the end node (node F), the optimal values of nodal pressure (each encircled with an ellipse) were close to their upper bounds. This was due to the fact that for a specified mass flow rate of natural gas through the pipeline segment, the higher pressure values resulted in lower volumetric flow rates or gas velocities and therefore, lower values of pressure loss were obtained. The optimal pressure value for the end node (node F) was close to its lower bound, so that the gas flow leaves the pipeline with lower pressure which has lower amount of availability (physical exergy).

To compare the results obtained by NDP and GA methods, the GA program was developed.

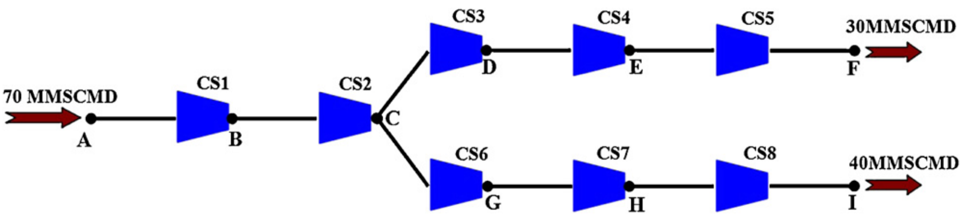

Fig. 7. Schematic diagram of the second case study (branched network).

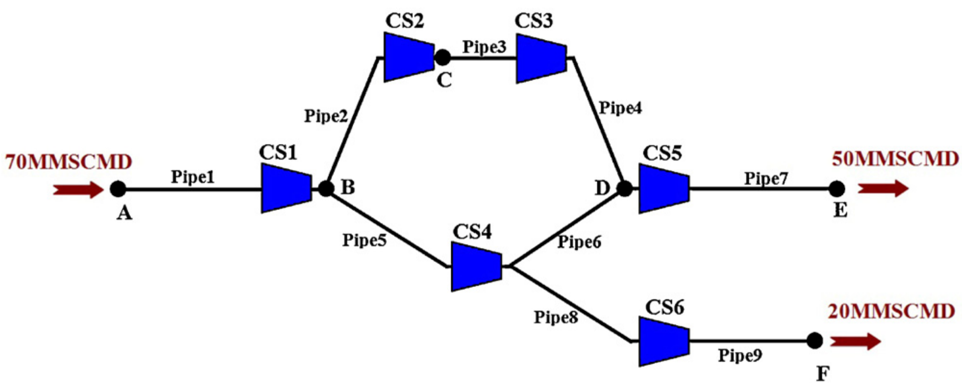

Fig. 8. Schematic diagram of the third case study (cyclic network).

Table 4
Information regarding the second case study (branched network).

Net flow through the supply node A	Net flow through the delivery node F	Net flow through the delivery node I	Gas pressure at point A	Minimum required pressure at point F	Minimum required pressure at point I

Table 5
Information regarding the pipeline segments of the third case study (cyclic network).

	Pipe number								
	1	2	3	4	5	6	7	8	9
Length (km)	150	100	100	100	100	100	150	100	100
Diameter (mm)	1422	994	994	994	994	794	1167	794	794
MAOP (bar)	72	68	68	68	68	68	80	68	68

Table 6
Information regarding the third case study (cyclic network).

Net flow through the supply node A	Net flow through the delivery node E	Net flow through the delivery node F	Gas pressure at point A	Minimum required pressure at point E	Minimum required pressure at point F
70 MMSCMD	50 MMSCMD	20 MMSCMD	55 bar	50 bar	42 bar

Table 7
Optimal values of decision variables and the objective function obtained by the NDP method in the first case study.

ΔP (bar)	2	1	0.5	0.25
CS1 state/number of operating units	On/4	On/4	On/4	On/4
CS2 state/number of operating units	On/3	On/3	On/3	On/3
CS3 state/number of operating units	Off/0	Off/0	Off/0	Off/0
CS4 state/number of operating units	On/3	On/3	On/3	On/3
CS5 state/number of operating units	Off/0	Off/0	Off/0	Off/0
P_B (bar)	66	67	67.5	67.75
P_C (bar)	72	72	72	72
P_D (bar)	63.12	63.12	63.12	63.12
P_E (bar)	72	71	70.5	70.5
P_F (bar)	52.52	51.08	50.35	50.35
Rate of fuel consumption (kg/s)	7.8087	7.7156	7.6699	7.6627

The value of 0.8 was considered for the crossover rate.

To obtain the appropriate values of the mutation probability and population size (as the two key factors in the optimization process) the sensitivity analysis was carried out for each pressure step size in each case study.

In the first step of the sensitivity analysis, the population size was assumed to be 100, and GA program was executed ten times for each mutation probability of 1%, 3%, 5%, 7% and 9%. The mutation probability with the minimum average rate of fuel consumption (or minimum average difference with NDP solution) was chosen as the best selection.

In the next step, the mutation probability was assumed equal to the best value obtained from the first step, and GA program was executed ten times for each population size of 50, 100, and 150. The population size with the minimum average rate of fuel consumption (or minimum average difference with NDP solution) was chosen as the best selection.

As an example, the first and second steps of the sensitivity analysis for the first case study (at the pressure step size of 2 bar) are shown in Fig. 10(a) and (b) respectively.

As Fig. 10(a) shows, the value of 9% was the best mutation probability by which each of ten GA executions resulted in the

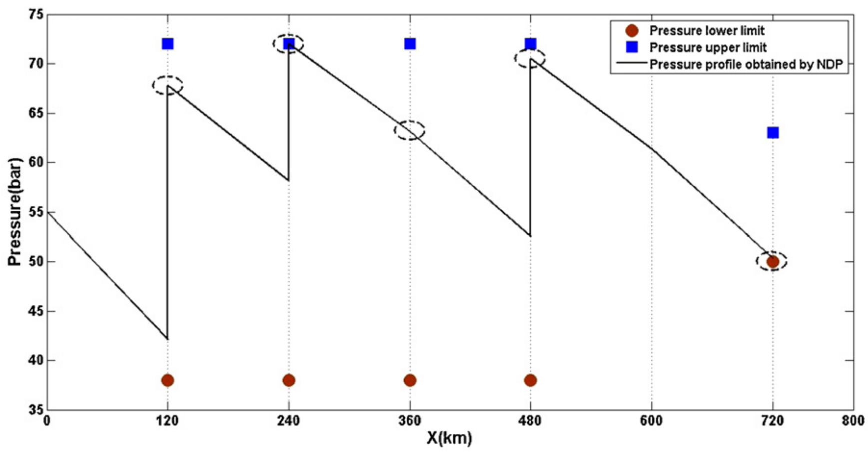

Fig. 9. Variation of natural gas pressure along the network of the first case study obtained by NDP method.

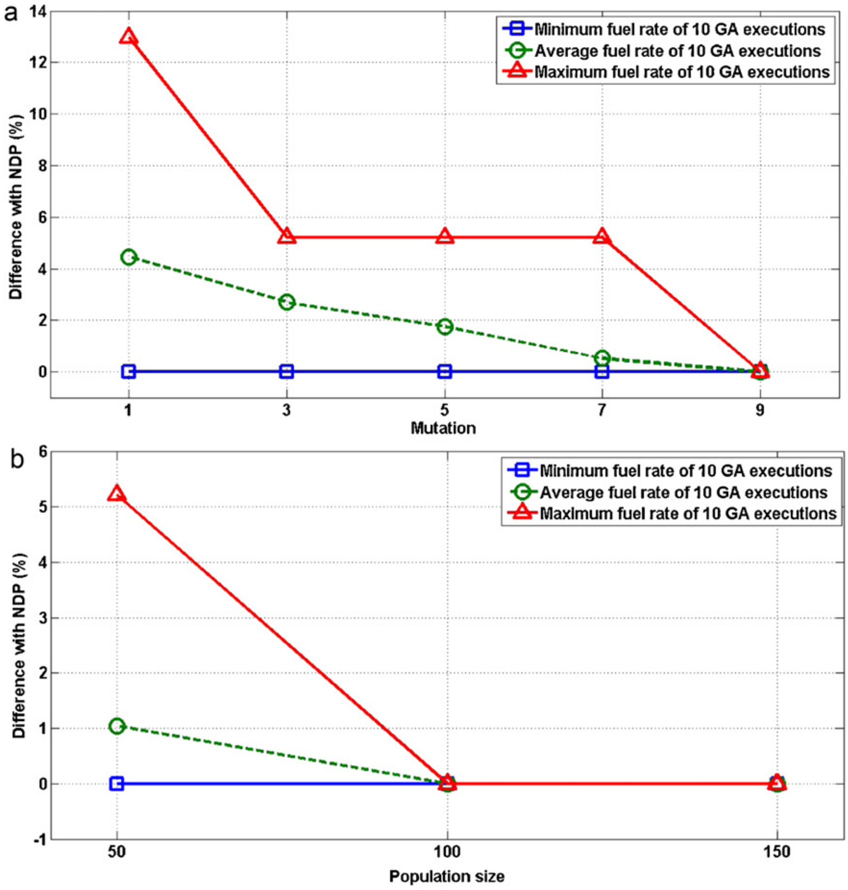

Fig. 10. Sensitivity analysis of the results of GA to the variation of (a) mutation probability and (b) population size.

optimal solution (NDP solution). Furthermore, 4, 4, 7 and 9 of ten GA executions resulted in the optimal solution (NDP solution) for mutation probabilities of 1%, 3%, 5% and 7%, respectively.

As Fig. 10(b) shows, at population sizes of 100 and 150, all ten GA runs resulted in the optimal solution (furthermore, eight of ten GA runs resulted in the optimal solution at the population size of 50). However, the selected population size of 100 required the lower computing time in comparison with the population size of 150.

Besides finding the appropriate GA parameters, the above discussed sensitivity analysis shows that the optimal solution could be correctly found in the wide ranges of mutation probability and population size with different frequency of obtaining the optimal solution.

The selected values of mutation rate and population size, the length of the chromosome, the relative standard deviation of ten GA executions, as well as the differences of the best, worst and the average fuel consumption rates obtained through ten GA executions with the optimal fuel consumption rate obtained by NDP method are shown in Table 8 for various pressure step sizes.

Table 8 shows that for each pressure step size, all ten GA executions resulted in the rate of fuel consumption exactly equal to that obtained by the NDP method. It should be noted that the same values of objective function implies the same values predicted for the decision variables.

The average computation time of ten GA executions as well as the computation time required for the NDP method on an Intel (R) Core (TM) i5 2.53 GHz processor are shown and compared in Fig. 11 for each pressure step size. As Fig. 11 shows, while the computation time required for the NDP method increased exponentially with decreasing the pressure step size, the computation time required for the GA method did not show such a dependency on this parameter.

Fig. 12 shows the progress of convergence for the objective function values in ten executions of GA *versus* the number of generations for the pressures step size of 2 bar, and compares them to the optimal objective function value obtained by the NDP method. It should be noted that there are similar diagrams for other pressure step sizes and also for the other case studies discussed in the following sections, which are not presented here only for conciseness.

Table 8
Comparison between objective function values obtained by the NDP and GA methods in the first case study.

ΔP (bar)	Number of bits	Mutation rate (%)	Population size	Difference of best/average/worst rate of fuel consumption obtained by ten GA executions with NDP solution (%)	Relative standard deviation (%)
2	59	9	100	0–0–0	0
1	64	7	100	0–0–0	0
0.5	69	7	100	0–0–0	0
0.25	74	7	150	0–0–0	0

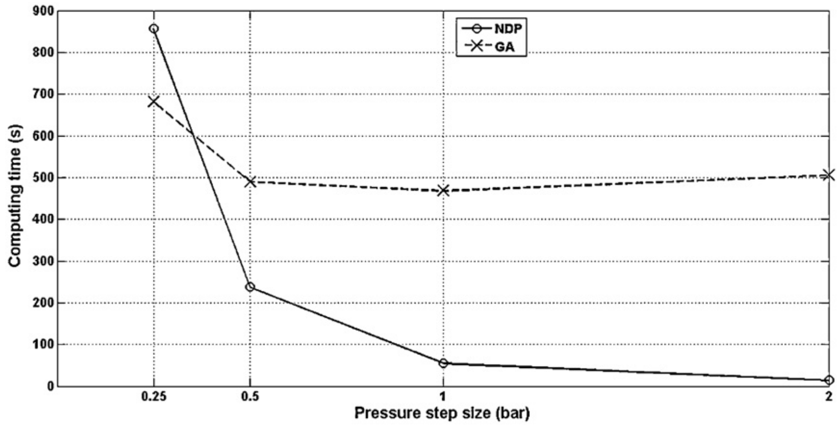

Fig. 11. Comparison of computing times required for the NDP and GA methods in the first case study.

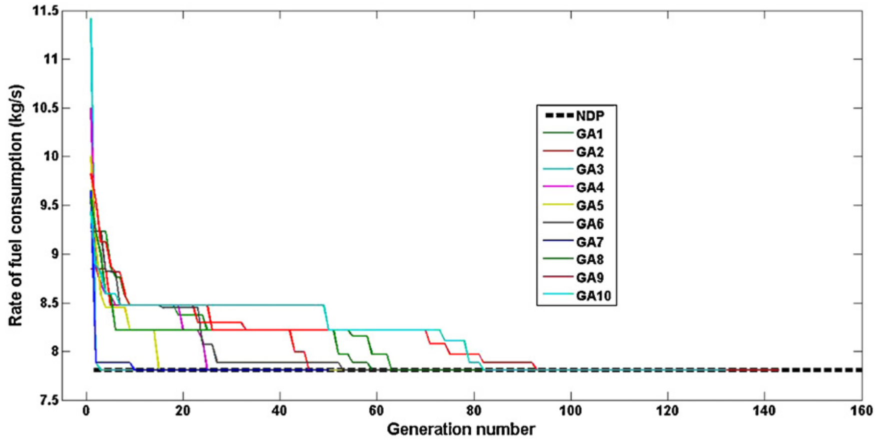

Fig. 12. Progress of the convergence for the objective function values in ten executions of GA *versus* the number of generations, and their comparison with the optimal objective function obtained by the NDP method for the pressures step size of 2 bar, in the first case study.

To verify the efficiency of applying GA optimization method for larger size networks, both NDP and GA methods were applied to a linear network with 15 compressor stations for the pressure step size value of 0.25 bar. NDP solution showed the optimal value of 19.55 kg/s for the rate of fuel consumption. With the selected values of 5%, and 100 for mutation probability and population size based on the sensitivity analysis, the best-average-worst values of fuel consumption rate obtained by ten GA executions showed the difference percentage point of about 0–1.8–2.4% with NDP solution. Also the computation time required by NDP method and the average computation time required by GA were about 3603 and 2846 s respectively.

Fig. 13 shows the optimal variation of gas pressure along the network. Furthermore, the optimal values of decision variables are shown in Table 9.

5.2. Results of the second case study (branched network)

Similar to the first case study, the second case study has a non-cyclic structure (Fig. 7), and therefore the flow variables are

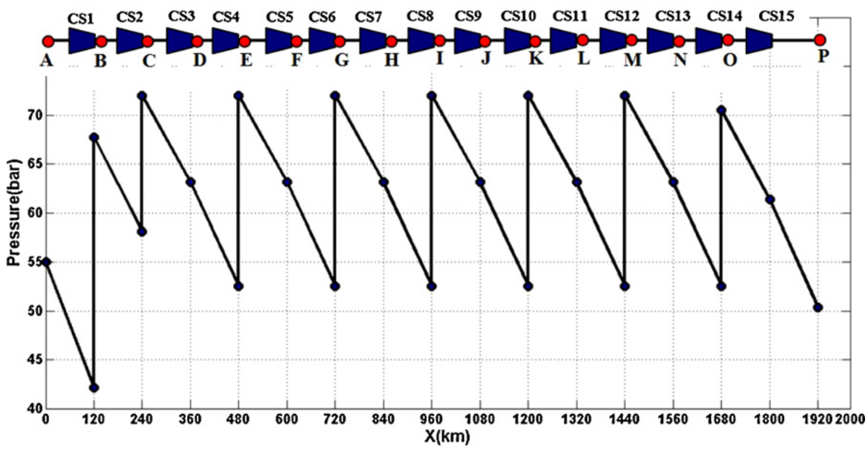

Fig. 13. Variation of natural gas pressure along the linear gas network with 15 compressor stations obtained by NDP method.

Table 9
Optimal values of decision variables obtained by NDP method for the linear network with 15 CSs for the pressure step size of 0.25 bar.

CS number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Status/number of active units	On/4	On/3	Off/0	On/3	Off/0	On/3	Off/0	On/3	Off/0	On/3	Off/0	On/3	Off/0	On/3	Off/0
Point labeled with	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
Pressure (bar)	67.75	72	63.1	72	63.1	72	63.1	72	63.1	72	63.1	72	63.1	70.5	50.35

Table 10
Optimal values of decision variables and the objective function obtained by the NDP method in the second case study.

ΔP (bar)	2	1	0.5	0.25
CS1 state/number of operating units	On/5	On/5	On/5	On/5
CS2 state/number of operating units	On/3	On/3	On/3	On/3
CS3 state/number of operating units	Off/0	Off/0	Off/0	Off/0
CS4 state/number of operating units	On/2	On/2	On/2	On/2
CS5 state/number of operating units	Off/0	Off/0	Off/0	Off/0
CS6 state/number of operating units	On/2	On/2	On/2	On/2
CS7 state/number of operating units	On/2	On/2	On/2	On/2
CS8 state/number of operating units	On/2	On/2	On/2	On/2
P_B (bar)	62	62	62.5	62.75
P_C (bar)	68	68	68	68
P_D (bar)	58.047	58.047	58.047	58.047
P_E (bar)	66	66	66	65.75
P_F (bar)	42.425	42.425	42.425	42.041
P_G (bar)	68	68	68	68
P_H (bar)	68	68	68	68
P_I (bar)	42	42	42	42
Rate of fuel consumption (kg/s)	12.2054	12.2054	12.1805	12.1587

pre-specified throughout the network. The decision variables include the values of pressure at nodes B to I in Fig. 7, the on/off status of each CS, and the number of active TCs at each CS.

The results obtained by NDP including the optimal values of decision variables and the objective function (fuel consumption) are shown in Table 10 for various pressure step sizes (0.25–2 bar).

Table 10 shows that in the optimal solution, the 3rd and 5th CSs were predicted to be bypassed (at off status), while the other CSs were active.

Fig. 14 illustrates the variation of natural gas pressure along the network of the second case study, based on the optimal values of the decision variables presented in Table 10 for the pressure step size of 0.25 bar. This figure also compares the optimal pressure values obtained at nodes B to I with their upper and lower permissible bounds. As shown in Fig. 14, based on the same explanations for the first case study, the optimal pressure values are close to their upper bounds at the transshipment nodes (nodes except for A, I and F in Fig. 14 which neither supply nor demand flow) and close to their lower bounds at the end nodes (nodes F and I).

It is worth mentioning that with the same hardware introduced in Section 4.2, different values of pressure at the upper and lower

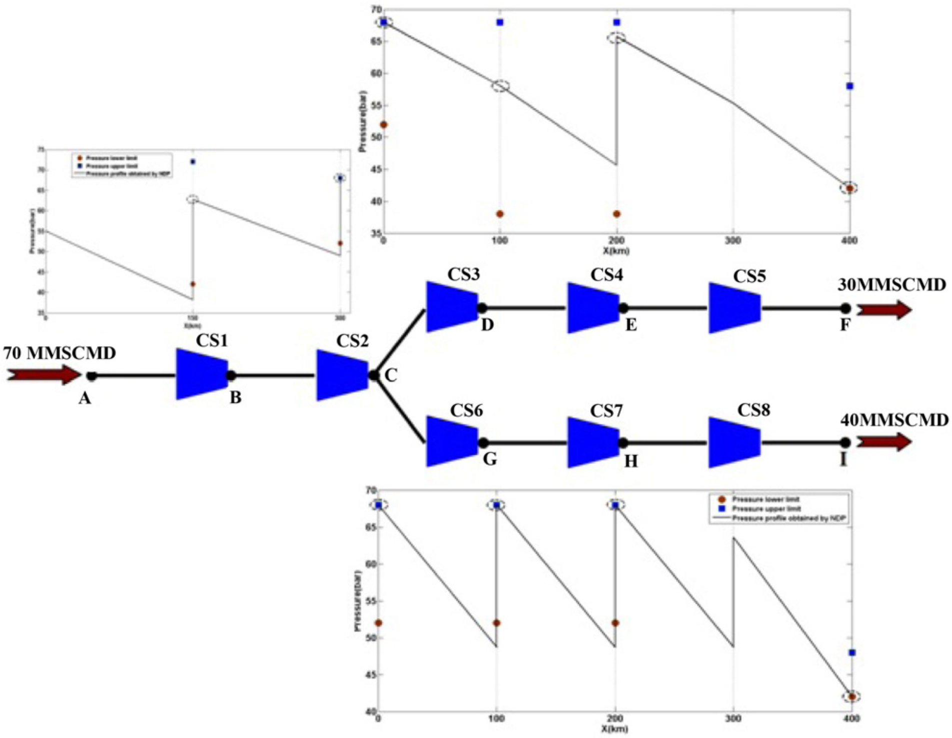

Fig. 14. Variation of natural gas pressure along the network of the second case study obtained by NDP method.

Table 11
Comparison between objective function values obtained by the NDP and GA methods in the second case study.

ΔP (bar)	Number of bits in each chromosome	Mutation rate (%)	Population size	Difference of best/average/worst rate of fuel consumption obtained by ten GA executions with NDP solution (%)	Relative standard deviation (%)
2	90	7	50	0–0–0	0
1	98	7	50	0–0.05–0.46	0.14
0.5	106	7	50	0–0.03–0.16	0.07
0.25	114	7	50	0–0.05–0.16	0.08

outgoing branches (shown in Fig. 14) are caused due to different rates of gas flow passing through those branches.

The selected values of mutation rate and population size, the length of the chromosome, the relative standard deviation of ten GA executions, as well as the differences of the best, worst and the average fuel consumption values obtained through ten executions of GA with the optimal fuel consumption value obtained by NDP method are shown in Table 11 for various pressure step sizes.

Table 11 shows that for each pressure step size, the best fuel consumption value obtained by ten GA executions is exactly equal to those obtained by the NDP method. Also, the difference between the average/worst value of the objective function for ten executions of GA and the optimal objective function value (obtained by the NDP method) was within the range of 0–0.05%/0–0.46% for various pressure step sizes.

The average computation time of ten GA executions as well as the computation time required for the NDP method are shown and compared in Fig. 15 for each pressure step size. As Fig. 15 shows, while the computation time required for the NDP method increased exponentially with the degree of precision (*i.e.* with the decrease in pressure step size), the computation time required by the GA method did not show such a dependency on the pressure step size.

5.3. Results of the third case study (cyclic network)

For the cycle existing in the third case study (pipeline segments 2–6 in Fig. 8 form a cycle), it is not known in advance how the rate of flow through pipeline segment 1 should be divided between pipeline segments 2 and 5. Therefore, one flow variable (the volume flow rate through pipeline segment 2) was added to the decision variables.

As was noted, the application of NDP is limited to networks with fixed values of flow variables. Therefore the procedure of optimizing the present cyclic network using the NDP was modified in this paper such that, the possible range of flow through pipeline 2 (within 0–70 MMSCMD) was discretized and then the NDP method was separately applied for each of the discrete values of flow in pipeline 2. It should be noted that while the computational complexity of the NDP method for non-cyclic structures is about $O(\text{the number of CSs} \cdot {M_P}^3)$, the above mentioned modified procedure imposes a computational complexity of about $O((\text{the number of CSs} \cdot {M_P}^3) \cdot ({K_m}^{\text{Number of cycles}}))$ for the cyclic structures; where K_m is the maximum number of elements in the discretized flow range. Therefore, this procedure imposes impractical computation time, except for cases with very few cycles.

To assess the GA capability in optimization of such a pipeline with cyclic network configuration, this method was applied to the third case study as well.

The results obtained by NDP are shown in Table 12 for various step size values of pressure (0.25–2 bar) and volume flow rate (0.25–2 MMSCMD). As Table 12 shows, the better solutions (solutions yielding lower fuel consumption values) were obtained with finer pressure and volume flow rate step sizes. Also Table 12 shows that all CSs should be active to reach the optimal solution.

The selected values of mutation rate and population size, the length of the chromosome, the relative standard deviation of ten GA executions, as well as the differences of the best, worst and the average fuel consumption values obtained through ten executions of GA with the optimal fuel consumption value obtained by NDP method are shown in Table 13 for various volume flow rate and pressure step sizes.

According to Table 13, the comparison between the best, average, and worst values of the objective function for ten GA executions and the optimal objective function value (obtained by the NDP method) shows the corresponding differences to be within the ranges of 0–0.55%, 0–1.43% and 0–4.2%, respectively, for various pressure and flow step sizes.

The computation time required for the NDP method and the average computation time of ten GA executions are shown in Fig. 16(a) and (b) respectively. These figures show that while the required computation time for the NDP method changed

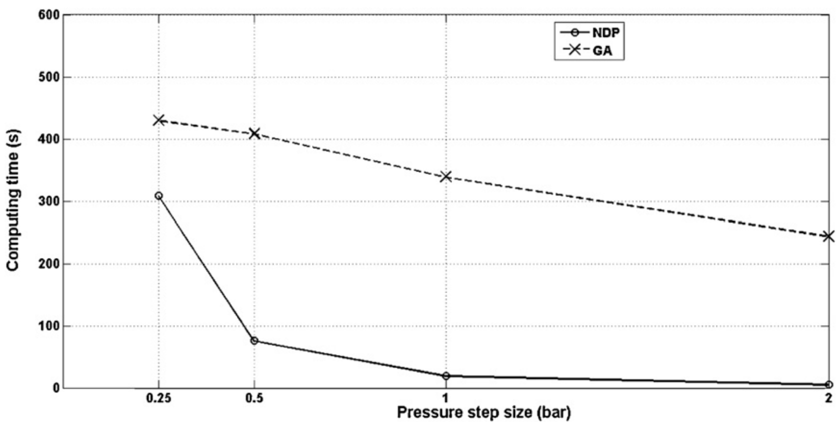

Fig. 15. Comparison of computing times required for the NDP and GA methods in the second case study.

Table 12
Optimal values of decision variables and the objective function obtained by the NDP method in the third case study.

$\Delta\dot{m}$ (MMSCMD)	2				1				0.5				0.25			
ΔP (bar)	2	1	0.5	0.25	2	1	0.5	0.25	2	1	0.5	0.25	2	1	0.5	0.25
CS1 state/number of operating units	On/5	On/5	On/5	On/5	On/5	On/5	On/5	On/5	On/5	On/5	On/5	On/5	On/5	On/5	On/5	On/5
CS2 state/number of operating units	On/2	On/2	On/2	On/2	On/2	On/2	On/2	On/2	On/2	On/2	On/2	On/2	On/2	On/2	On/2	On/2
CS3 state/number of operating units	On/2	On/2	On/2	On/2	On/2	On/2	On/2	On/2	On/2	On/2	On/2	On/2	On/2	On/2	On/2	On/2
CS4 state/number of operating units	On/2	On/2	On/2	On/2	On/2	On/2	On/2	On/2	On/2	On/2	On/2	On/2	On/2	On/2	On/2	On/2
CS5 state/number of operating units	On/2	On/2	On/2	On/2	On/2	On/2	On/2	On/2	On/2	On/2	On/2	On/2	On/2	On/2	On/2	On/2
CS6 state/number of operating units	On/1	On/1	On/1	On/1	On/1	On/1	On/1	On/1	On/1	On/1	On/1	On/1	On/1	On/1	On/1	On/1
P_B (bar)	61	62	62.5	62.75	61	62	62.5	62.75	61	62	62.5	62.75	61	62	62.5	62.75
P_C (bar)	67	67	68	68	67	68	68	68	67	68	68	68	67	68	68	68
P_D (bar)	54	54	55.5	55.75	54	55	55.5	55.75	54	56	56	55.75	54	56	56	56.25
P_E (bar)	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50
P_F (bar)	42	42	42	42	42	42	42	42	42	42	42	42	42	42	42	42
$\dot{m}_2$ (MMSCMD)	34	32	32	32	33	33	32	32	33	32.5	32.5	32	32.75	32.5	32.25	32.25
Rate of fuel consumption (kg/s)	10.04	9.838	9.711	9.68	9.971	9.813	9.711	9.68	9.971	9.75	9.707	9.68	9.959	9.75	9.698	9.668

Table 13
Comparison between objective function values obtained by the NDP and GA methods in the third case study.

$\Delta\dot{m}$ (MMSCMD)	ΔP (bar)	Number of bits in each chromosome	Mutation rate (%)	Population size	Difference of best/average/worst fuel consumption obtained by ten GA executions with NDP solution (%)	Relative standard deviation (%)
2	2	74	7	100	0–0–0	0
	1	79	7	100	0–0.94–4.2	1.74
	0.5	84	7	100	0–0.51–1.2	0.43
	0.25	89	7	150	0–0.37–0.52	0.24
1	2	75	7	100	0–0–0	0
	1	80	7	100	0–0.46–1.94	0.79
	0.5	85	7	100	0.42–1.43–3.74	1.29
	0.25	90	7	150	0–0.39–0.52	0.23
0.5	2	76	7	100	0–0.61–2.75	0.97
	1	81	7	100	0.41–0.82–3.32	0.83
	0.5	86	7	100	0.46–1.1–2.83	0.94
	0.25	91	7	150	0–0.69–2.89	0.86
0.25	2	77	7	100	0–0.72–2.88	1.24
	1	82	7	100	0–0.27–0.52	0.16
	0.5	87	7	100	0.55–0.9–2.92	0.82
	0.25	92	7	150	0–0.94–2.63	0.93

exponentially with the pressure step size, and linearly with the volume flow step size (due to the existence of only one cycle), such a dependency was not observed between the required computation time for the GA method and the pressure or the flow step sizes. As Fig. 16 shows, due to the multi-dimensionality of the third case study (existence of a flow decision variable in addition to the pressure decision variables), except for the cases with coarse step sizes (in which the required computation times for the GA and NDP methods are of the same order), the NDP computation time is much longer than that of the GA method (*e.g.* about 20 times longer for the pressure and flow step size values of 0.25 bar and 0.25 MMSCMD).

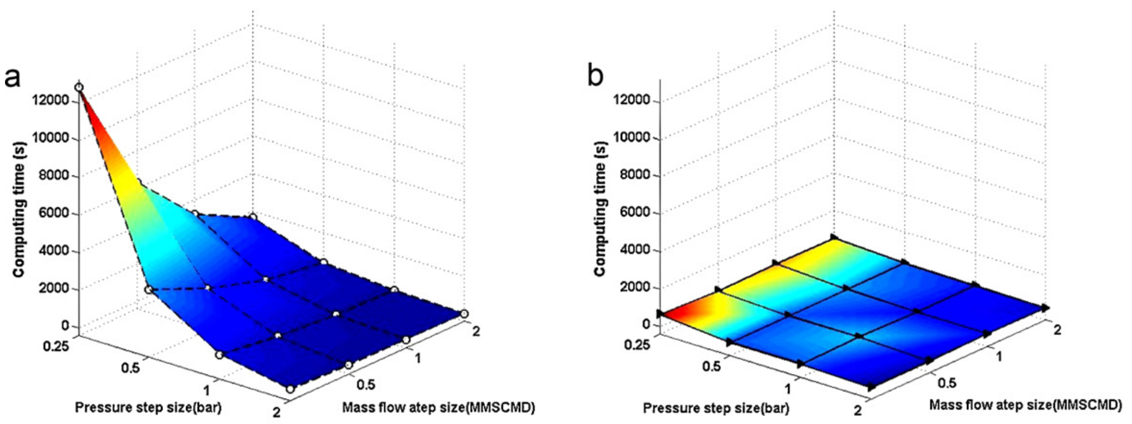

Fig. 16. Comparison of computing times required for NDP and GA methods in the third case study. (a) Computing time required for NDP method and (b) Computing time required for GA method.

6. Conclusions

The NDP method guarantees to find the optimal solution in the minimization of fuel consumption of natural gas networks, however, its application is limited to those cases at which the flow rate values in the network are known in priori. Therefore, in cyclic networks where the flow variables in the cycles are not uniquely specified in priori, NDP cannot be applied without appropriate modification.

Therefore the procedure of applying NDP for cyclic structures was modified in this paper. However, the modified procedure, except for a very few number of cycles or for coarse step sizes, requires an impractical computation time. Furthermore, GA was applied in this paper as an alternative optimization method for all network configurations including the cyclic one. The results obtained by GA and NDP methods for linear, branched and cyclic network structures were compared. The results showed that the GA method can be applied as an appropriate alternative method for both cyclic and non-cyclic natural gas network structures successfully. Furthermore, it was observed that while the computing time required by the NDP method exponentially depended on pressure and flow rate step sizes, the GA computing time did not show such a dependency on these parameters (however its results depended on the GA tuning parameters)

References

[1] Riva A, Angelosante SD, Trebeschi C. Natural gas and the environmental results of lifecycle assessment. Energy 2006;31:138–48.
[2] Borraz-Sanchez C, Rios-Mercado RZ. Improving the operation of pipeline systems on cyclic structures by tabu search. Comput Chem Eng 2009;33:58–64.
[3] Wu S, Rios-Mercado RZ, Boyd EA, Scott LR. Model relaxations for the fuel cost minimization of steady-state gas pipeline networks. Math Comput Model 2000;31:197–220.
[4] Carter RG. Pipeline optimization: dynamic programming after 30 years. In: 30th annual meeting pipeline simulation interest group (PSIG); 1998.
[5] Bellman R. Dynamic programming. New Jersey: Princeton University Press; 1957.
[6] Wong PJ, Larson RE. Optimization of natural gas pipeline systems via dynamic programming. IEEE Trans Autom Control 1968;AC-13(5):475–81.
[7] Wong PJ, Larson RE. Optimization of tree-structured natural-gas transmission networks. J Math Anal Appl 1968;24(3):613–26.
[8] Zimmer HI. Calculating optimum pipeline operations. Presented at the AGA transmission conference; 1975.
[9] Lall HS, Percell PB. A dynamic programming based gas pipeline optimizer. In: Bensoussan A, Lions JL, editors. Analysis and optimization of systems Lecture notes in control and information sciences, vol. 144. Springer: Berlin; 1990 p. 123–32.
[10] Chebouba A, Yalaoui F, Smati A, Amodeo L, Younsi K, Tairi A. Optimization of natural gas pipeline transportation using ant colony optimization. Comput Oper Res 2009;36:1916–23.
[11] Mercado RZ, Wu S, Scott LR, Boyd EA. A reduction technique for natural gas transmission network optimization problems. Ann Oper Res 2002;117(1-4):217–34.
[12] Luongo CA, Gilmour BJ, Schroeder DW. Optimization in natural gas transmission networks: a tool to improve operational efficiency. Presented at the third SIAM conference on optimization, Boston; April 1989.
[13] Rios-Mercado RZ, Kim S, Boyd EA. Efficient operation of natural gas transmission systems: a network-based heuristic for cyclic structures. Comput Oper Res 2006;33(8):2323–51.
[14] Nguyen HH, Uraikul V, Chan CW, Tontiwachwuthikul P. A comparison of automation techniques for optimization of compressor scheduling. Adv Eng Softw 2008;39:178–88.
[15] Nguyen HH, Chan CW. Applications of artificial intelligence for optimization of compressor scheduling. Eng Appl Artif Intel 2006;19:113–26.
[16] El-Mahdy OFM, Ahmed MEH, Metwalli S. Computer aided optimization of natural gas pipe networks using genetic algorithm. Appl Soft Comput 2010;10:1141–50.
[17] Katz DL, Cornel D, Kobayashi R, Poettmann FH, Vary JA, Elenbaas JR, et al. Handbook of natural gas engineering. New York: McGraw-Hill Book Company; 1959.
[18] Engineering data book, 10th ed., Tulsa, Oklahoma: GPSA Press; 1987.
[19] Mohitpour M, Golshan H, Murray A. Pipeline design and construction—a practical approach, 2nd ed., USA: ASME Press; 2003.
[20] Shashi Menon E. Gas pipeline hydraulics. Taylor & Francis; 2005.
[21] Andre J. Optimization of investments in gas networks. Thesis for the degree of doctor in economics, Université Lille Nord De France; September 2010.
[22] Babu BV, Angira R, Pallavi GC, Syed Mubeen JH. Optimal design of gas transmission network using differential evolution. http://citeseerx.ist.psu.edu/viewdoc/summary?doi=10.1.1.112.8986.
[23] Ehrhardt K, Steinbach MC. Nonlinear optimization in gas networks. ZIB-Report, 03-46; December 2003.
[24] Bales P, Geibler B, Kolb O, Lang J, Martin A, Morsi A. Comparison of nonlinear and linear optimization of transient gas networks. Darmstadt University of Technology, Department of Mathematics; 2008.
[25] Mohring J, Hoffmann J, Halfmann T, Zemitis A, Basso G. Automated model reduction of complex gas pipeline networks. In: 36th annual meeting pipeline simulation interest group (PSIG); 2004.
[26] Odom FM. Tutorial on modeling of gas turbine driven centrifugal compressors. In: 22nd annual meeting, pipeline simulation interest group (PSIG); 1990.
[27] Percell PB, Ryan MJ. Steady-state optimization of gas pipeline network operation. In: 19th annual meeting pipeline simulation interest group (PSIG); 1987.
[28] Cohen H, Rogers GFC, Saravanamuttoo HIH. Gas turbine theory, 3rd ed., Harlow: Longman Scientific & Technical; 1987.
[29] Kurz R, Ohanian S. Modeling turbomachinery in pipeline simulations. In: 35th annual meeting, pipeline simulation interest group (PSIG); 2003.
[30] Santos SPD, Ohanian S. Transient analysis a must in gas pipeline design. In: 29th Annual Meeting, Pipeline Simulation Interest Group (PSIG); 1997.
[31] Walsh PP, Fletcher P. Gas turbine performance, 2nd ed., Oxford: Blackwell Science; 2004.
[32] Kurz R, Brun K. Degradation of gas turbine performance in natural gas service. J Nat Gas Sci Eng 2009;1:95–102.
[33] Borraz-Sanchez C, Rios-Mercado RZ. A non-sequential dynamic programming approach for natural gas network optimization. WSEAS Trans Syst 2004;3(June (4)):1384–90.
[34] Hartmut P. Evolutionary algorithms: overview, methods and operators, Version 3.5a; July 2004. http://www.geatbx.com/.
[35] Genetic algorithm and direct search toolbox for use with MATLAB: User's Guide. MathWorks; 2005.
[36] Gen M, Cheng R. Genetic algorithms and engineering optimization. Ashikaga, Japan: Ashikaga Institute of Technology/John Wiley & Sons Inc.; 2000.
[37] Sanaye S, Mahmoudimehr J. Technical assessment of isothermal and non-isothermal modelings of natural gas pipeline operational conditions. Oil Gas Sci Technol, http://dx.doi.org/10.2516/ogst/2011117.