RESEARCH ARTICLE

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Optimization of the Shell-and-Tube Heat Exchanger by Simplified Conjugated Gradient Method

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This study is to obtain the optimal design of the heat removal of the 3D shell-and-tube heat exchanger (STHE) by integrating multiphysics script based on the finite element method (FEM) with the simplified conjugate-gradient method (SCGM). A numerical investigation of the performance of a STHE considered the tube distance and array type is conducted. The objective function is to obtain the maximum quantities of temperature difference between the inlet and outlet of the heat exchanger. After determining the optimal shape size, this study evaluates the thermal performance of the heat exchanger by varying the quantities of heat removal and efficiency. In addition, we obtain the synthesis for all the advantages of this proposed method.

Keywords: Optimization Design, Shell-and-Tube Heat Exchanger, SCGM, FEM.

1. INTRODUCTION

Heat exchangers are used extensively and regularly in process industries and thus are very important during plant design and operation. In general, the shell-and-tube heat exchangers (STHE) are probably the most common type of heat exchangers applicable for a wide range of operating temperatures and pressure. However, several issues will affect the performance of the STHE, such as the non-uniformity of the pressure and flow distributions between the channels inside the heat exchanger area. It decreases the thermal performance and increases the pressure drop across the heat exchanger, reducing the overall performance.[1] The heat transfer of the working fluids is an essential part of most chemical, petrochemical, food and energy industrial processes. To carry out such heat transfer process, the STHE are widely used because they are robust and can work in a wide range of pressures, flows and temperatures.[2]

The traditional design approach of the STHE involves rating a large number of different exchanger geometries to identify those that satisfy a given heat duty and a set of geometric and operational constraints.[3] Jegede and Polley[4] report a design approach based on the simplified equations that related the pressure drop, the surface area and the heat transfer coefficient of the heat exchanger; their model is based on the Dittus–Boelter correlation for the tube-side flow, and on the Kern correlations for the shell-side flow.[5] The combination of the pressure drop relationships with the basic exchanger design equation gives rise to a simple design algorithm that avoids the iterative procedure required to examine the different geometries.

It is noted from the paper review cited above, despite its practical importance, that studies of the optimization of a set of design parameters has not received sufficient attention. This motivates the present investigation to obtain the multi-variable optimal design of the STHE. An optimization algorithm is then applied to establish the optimal geometry parameters of each of the two STHE configurations.

Generally speaking, optimization methods can be broadly classified as either gradient-based techniques, e.g., the gradient search method (GSM)[6] and the conjugate gradient method (CGM),[7] or evolutionary-based techniques, e.g., genetic algorithms (GAs)[8] and simulated annealing (SA).[9] Methods of the former type generate a local or global solution given a set of initial values, and have the advantage of a rapid convergence time, while methods of the latter type obtain the globally optimal solution, but are computationally intensive.

The simplified conjugate gradient method proposed by Cheng and Chang[10] provides a simple means of optimizing the specified objective function and converges more rapidly than the traditional conjugate gradient method. In SCGM, the sensitivity of the objective function to changes in the design variables is evaluated initially and the optimal design is then iteratively derived using an appropriate step size. In a previous study,[11] the present group used the SCGM method to optimize the thermal management performance of a high power LED array and obtain the best thermal performance. In the present study, SCGM is integrated with COMSOL finite element analysis software to optimize the geometry parameters of the STHE in the two kinds of tube array, i.e., triangular and rectangular array. The optimal objective function, J, is to find the maximum temperature difference between the domain inlet and outlet of the heat exchanger. This study addresses the effects of the shape design parameters, including the tube distances and the array type on the thermal performance of the STHE. This research would be able to improve the temperature difference and increase the heat removal of the heat exchanger in the optimal design process.

2. HEAT EXCHANGER OPTIMIZATION APPROACH

2.1. Model and Analysis

The STHE model is built from the COMSOL module. Table I shows the original design parameters and operating conditions of the STHE. The working fluid inside the STHE is assumed as the three-dimensional, incompressible and turbulent flow. The cooling water flows through the pipes and enters from the side, and the hot air enters from above inlet. The tubes are assumed to be made of stainless steel.

The heat transfer equation is listed as below,

Fluid part

$$\nabla\cdot(-(k+k_t)\nabla T_f) = Q + q_s T_f - \rho C_p \vec{u}\cdot\nabla T_f \tag{1}$$

where $k_t = C_p\eta_t/\mathrm{Pr}_t$ is the turbulent conductivity, k is the fluid's physical thermal conductivity, η_t denotes the turbulent dynamic viscosity and C_p is the specific heat.

Shell tube part

$$\nabla\cdot(-k\nabla T_s) = Q + q_s T_s \tag{2}$$

The Reynolds averaged Navier-stokes equations and a Wilcox revised $k-\omega$ turbulence model is

$$\rho(\vec{u}\cdot\nabla)\vec{u} = \nabla\cdot(-p\vec{I} + (\eta+\eta_t)(\nabla\vec{u} + (\nabla\vec{u})^T - (2/3)(\nabla\cdot\vec{u})\vec{I}) - (2/3)\rho k\vec{I}) + \vec{F} \tag{3}$$

$$\nabla\cdot(\rho\vec{u}) = 0, \quad \rho = \rho(p, T) \tag{4}$$

$$\rho\vec{u}\cdot\nabla k = \nabla\cdot((\eta+\sigma_k\eta_t)\nabla k) + \eta_t P(\vec{u}) - (2\rho k/3)\nabla\cdot\vec{u} - \beta_k\rho k\omega \tag{5}$$

$$\rho\vec{u}\cdot\nabla\omega = \nabla\cdot((\eta+\sigma_\omega\eta_t)\nabla\omega) + (\alpha\omega/k)(\eta_t P(\vec{u}) - (2\rho k/3)\nabla\cdot\vec{u}) - \beta\rho\omega^2 \tag{6}$$

where

$$P(\vec{u}) = \nabla\vec{u}\cdot(\nabla\vec{u} + (\nabla\vec{u})^T - (2/3)(\nabla\cdot\vec{u})^2)$$

The boundary conditions of the heat transport equations are 50 °C at the inlet, convection-dominated transport at the outlet, thermal insulation at the region borders, thermal wall function at the pipe/water interfaces and fixed temperature at the inside of the heat pipes.

The periodicity of the flow is important as modeling a part of the heat exchanger where the flow is fully developed. An initial calculation with constant inlet velocity and fixed outlet pressure is first performed.

Table I. The design parameters of the initial STHE.

	Triangular array	Rectangular array
Inlet temperature	323 [K]	323 [K]
The temperature of tube side	273 [K]	273 [K]
Inlet temperature	−0.5 [m/s]	−0.5 [m/s]
The density of the working fluid	988 [kg/m^3]	988 [kg/m^3]
The width of the inlet	0.025 [m]	0.045 [m]

2.2. Optimization Approach

In optimizing the geometry parameters of the STHE under the triangular and rectangular array, the objective function J is specified as the temperature difference between the cooling fluid entrance and exit of the heat exchanger unit shown in Figure 1, i.e.,

$$J = \Delta T \tag{7}$$

As shown in Figure 1, each configuration is assigned two undetermined coefficients (dimensional parameters) to be optimized in the iterative process. The optimal values of these coefficients (i.e., the heat exchanger dimensions which maximize the value of Eq. (7)) are determined using an iterative scheme based on the

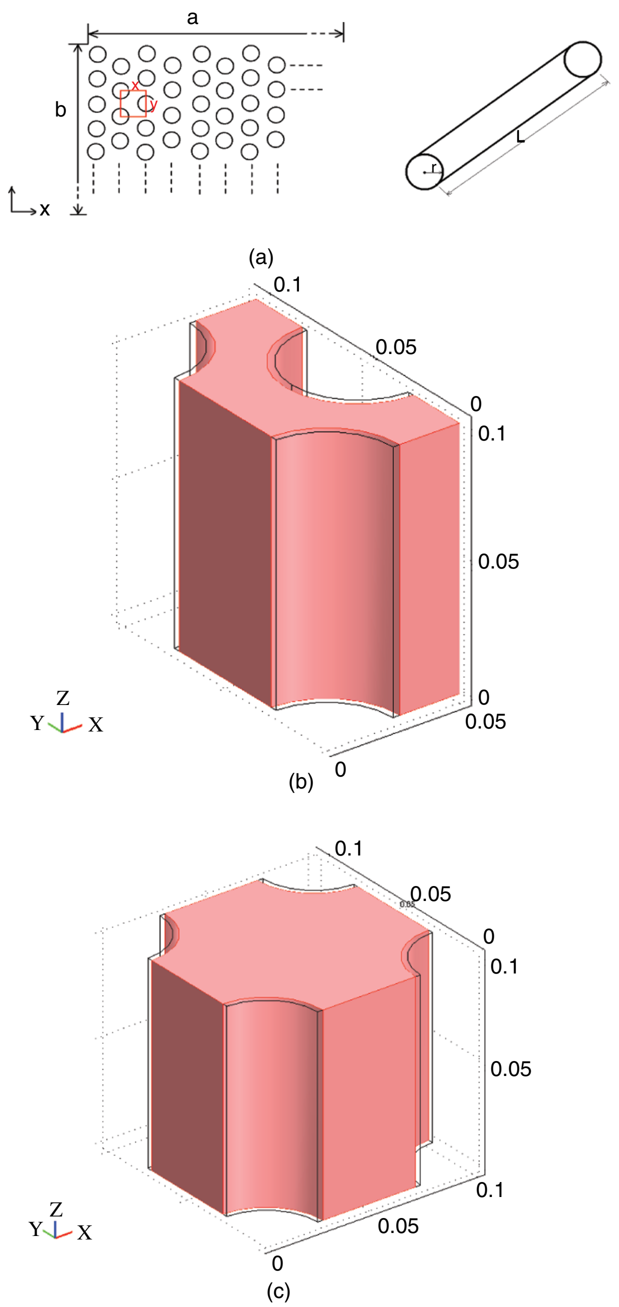

Fig. 1. The schematic diagrams of the STHE (a) tube arrangement, (b) triangular type, (c) rectangular type.

SCGM optimization method and COMSOL finite element analysis software. In performing the optimization process, the SCGM method is used to evaluate the gradient functions of the objective function and to set up a new conjugate direction for the updated undetermined coefficients with the assistance of a direct numerical sensitivity analysis. Meanwhile, the COMSOL package is used to solve the thermal profile associated with each set of undetermined coefficients (X and Y) considered in the iterative SCGM procedure.

We perform the direct numerical sensitivity analysis to determine the gradient functions $(\partial J/\partial a_i)^n, (i = 1, 2, \ldots, l)$. First, give a perturbation Δa_i to each of the undetermined coefficients, $\{a_i, i = 1, 2, \ldots, l\}$, and then find the change in the objective function (ΔJ) caused by Δa_i. The gradient function with respect to each of the undetermined coefficients can be calculated by the direct numerical differentiation as

$$\frac{\partial J}{\partial a_i} = \frac{\Delta J}{\Delta a_i} \tag{8}$$

Then, we can calculate the conjugate gradient coefficients, γ_i^n, and the search directions, π_i^{n+1}, for each of the undetermined coefficients with

$$\gamma_i^n = \left[\frac{(\partial J/\partial a_i)^n}{(\partial J/\partial a_i)^{n-1}}\right]^2, \quad i = 1, 2, \ldots, l \tag{9}$$

$$\pi_i^{n+1} = \left(\frac{\partial J}{\partial a_i}\right)^n + \gamma_i^n \pi_i^n, \quad i = 1, 2, \ldots, l \tag{10}$$

The step sizes $\{\beta_i, i = 1, 2, \ldots, l\}$ will be assigned for all the undetermined coefficients and leave it unchanged during the iteration.

$$a_i^{n+1} = a_i^n - \beta_i \pi_i^{n+1}, \ i = 1, 2, \ldots, l \tag{11}$$

Figure 2 presents a flow chart of the optimization process. In implementing the optimization process, the step size of the SCGM search procedure is determined on a trial-and-error basis, and was assigned a value of 1.0×10^{-4} s.

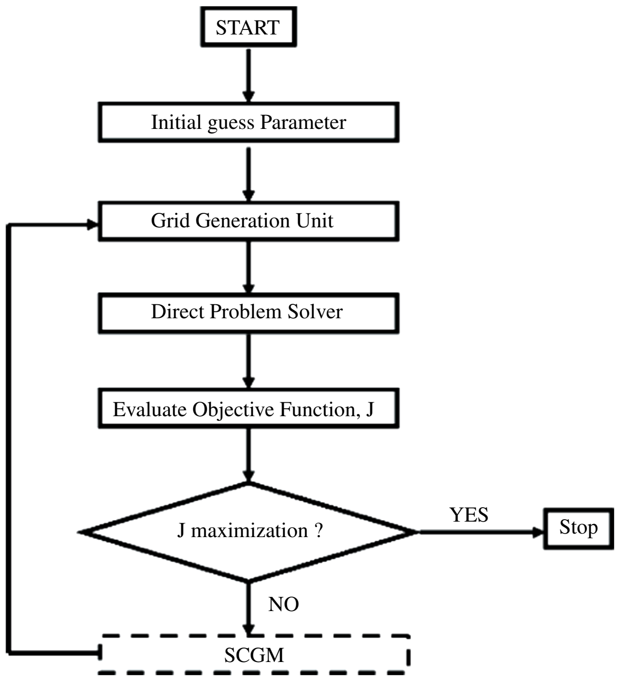

Fig. 2. The flow chart of the optimization process.

The parameters of a, b, L, r is the width, height, length and radius of the tube of the exchanger shown in Figure 1, separately, and the width and height of the unit region is X and Y. The ratio of unit region divided by the whole heat exchanger is XY/ab.

The quantity of heat removal can be obtained as:

$$\dot{q}' = \dot{m}c_p\Delta T = \rho v \cdot A \cdot c_p \Delta T = \rho v \cdot c_p \cdot L(X - r)\Delta T \tag{12}$$

Consequently, the total quantity of heat removal can be obtained as:

$$\dot{Q} = \rho v \cdot c_p \cdot L(X - r)\Delta T \cdot \frac{ab}{XY} \tag{13}$$

The variables of optimal design are X and Y, so the sample region scale of the objective function after optimal design is $X'Y'/ab$. The maximum quantity of heat removal $\dot{Q}'$ (or the maximum improve quantity $\dot{Q}' - \dot{Q}$) can be obtained as:

$$\dot{Q}' = \rho v \cdot c_p \cdot L(X' - r)\Delta T' \cdot \frac{ab}{X'Y'} \tag{14}$$

The efficiency of this optimal design can be obtained as:

$$\eta = \frac{\dot{Q}' - \dot{Q}}{\dot{Q}} \times 100\% = \frac{XY(X' - r)\Delta T'}{X'Y'(X - r)\Delta T}\% - 100\% \tag{15}$$

3. RESULTS AND DISCUSSION

In this study, the triangular and rectangular array of the STHE are studied and optimized, separately. The initial design variable X and Y in the different model are listed as below,

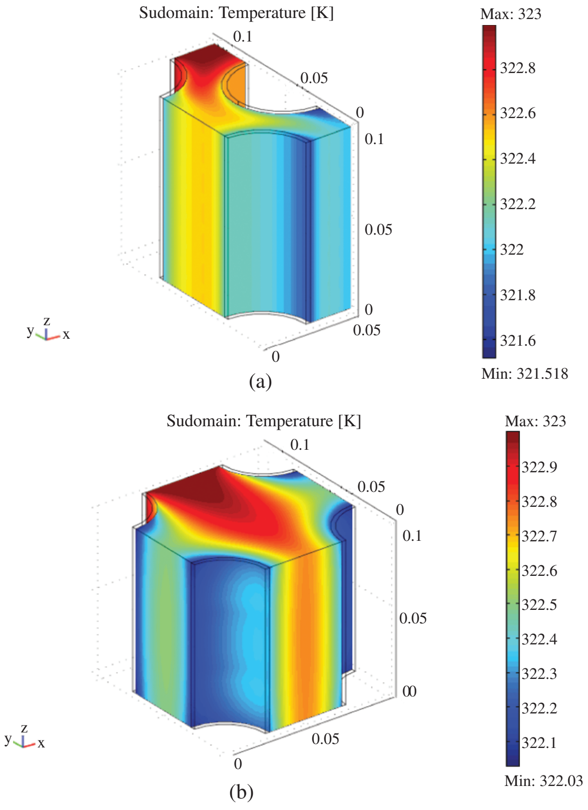

Fig. 3. The initial temperature contour of the STHE (a) triangular type, (b) rectangular type.

Case 1: The triangular array, X is 0.05 m and Y is 0.1 m.
Case 2: The rectangular array, X is 0.1 m, and Y is 0.1 m.

The temperature contours of the initial model are shown in Figure 3. Throughout these figures, the temperature distributions of the STHE can be observed clearly. The temperature difference between the entrance and exit is 0.63 K and 0.25 K in the triangular and rectangular array model, separately. The temperature difference of the initial arrangement of the tube array is small. It results in the low efficiency of the heat removal. Therefore, the proposed optimal method is employed to maximum the temperature difference for optimizing the heat removal of the STHE.

In the Figure 4, the results of the optimal process are shown. The temperature differences of the optimal process are displayed for understanding the variation of the objective function through the optimal process. As you can see in the Figure 4(a), the temperature difference of the triangular array of the STHE increases parabolic as the iteration increases. The optimal process terminates at the 111th iteration for the reason of the bound of the design variable. The temperature difference increases from 0.63 K to 1.74 K about 73.8%. The reason is that the total surface of the heat removal increases as the gap among the tubes of this unit decreases. The similar result of the rectangular array can be observed in Figure 4(b). The optimal process terminates at the 446th iteration and the objective function increase apparently, from 0.25 K to 4.36 K. The increasing of the temperature difference on the rectangular array is very large extremely. The simulated optimal result will provide the suitable trend for the product designing. The realistic performance still depends on the experiment.

The relationships between the objective function and the design variables are shown in Figure 5. In the model of the triangular array, the variable X decreases from 0.05 m to 0.039 m, the variable Y from 0.1 m to 0.089 m. In addition, the variable X decreases from 0.1 m to 0.0555 m, the variable Y from 0.1 m to 0.073 m in the model of the rectangular array. We notice that the decreasing of the temperature difference of the rectangular type is very large as the X is small than 0.06 m and Y is small than 0.08 m. It is interesting and valuable to study the reason in future.

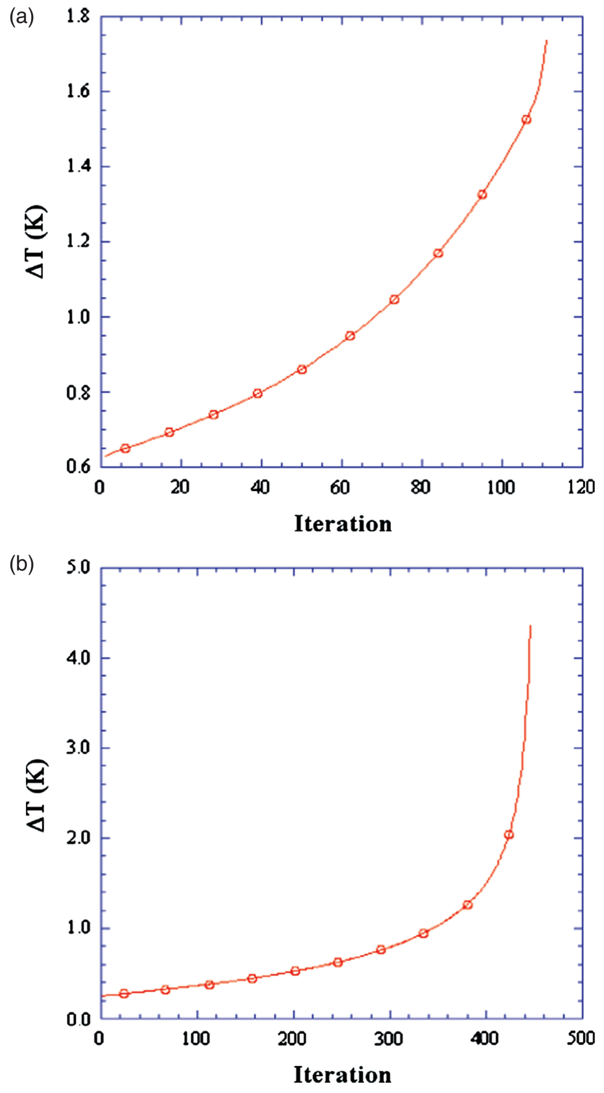

Fig. 4. The objective function through the optimal process (a) triangular type, (b) rectangular type.

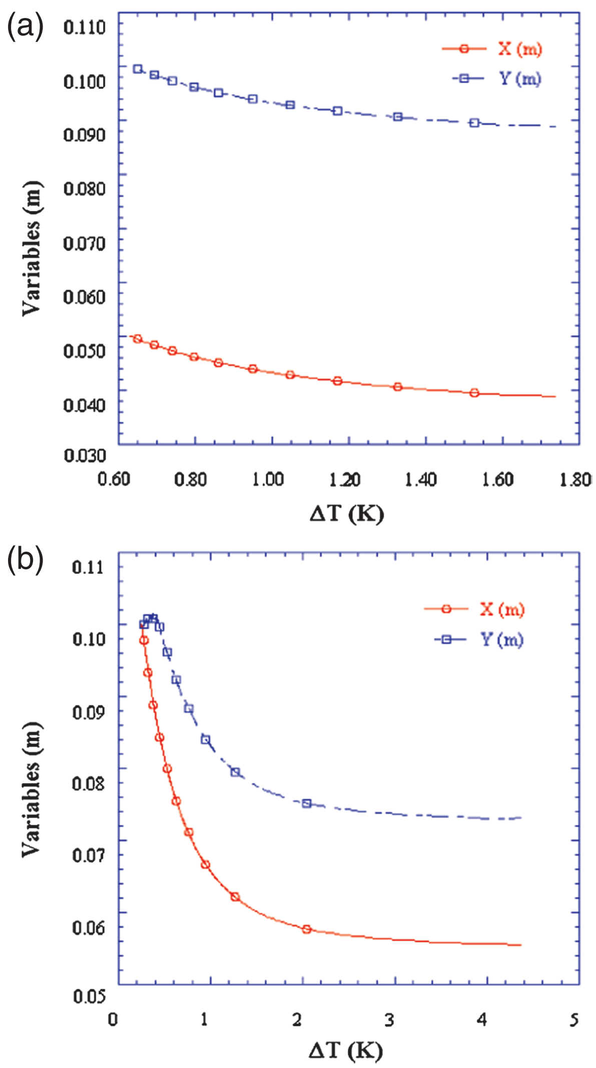

Fig. 5. The relationship of the design variables and the objective function (a) triangular type, (b) rectangular type.

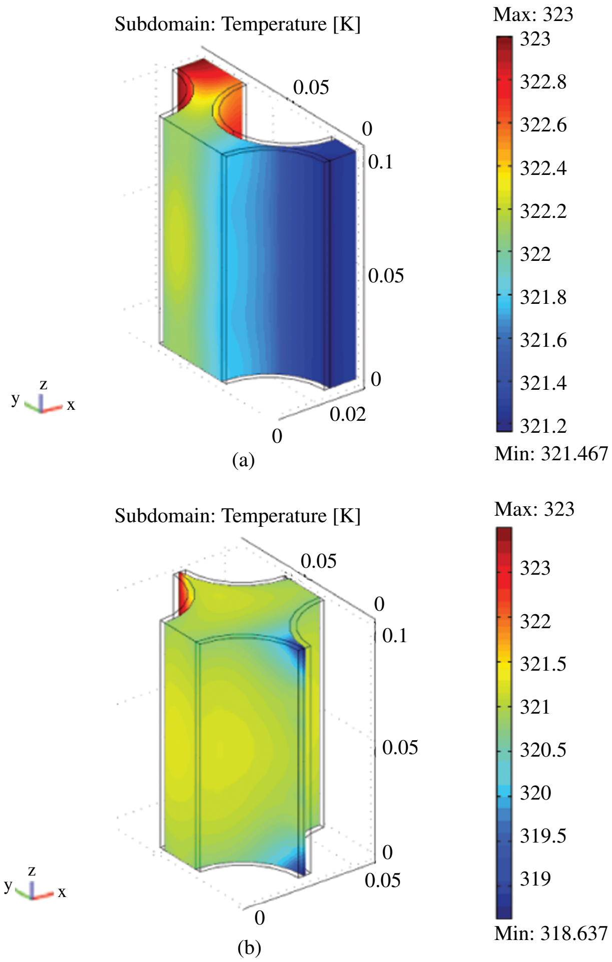

Fig. 6. The optimal temperature contour of the STHE (a) triangular type, (b) rectangular type.

The temperature contours of the optimal model are shown in Figure 6. The temperature difference is 1.74 K and 4.36 K of the triangular and rectangular type, separately. It observes clearly that the stop criteria of the optimal process are the limits of the design variables. Finally, the improved efficiency of the optimal process of the triangular array can be obtained from Eq. (15). The value of optimal efficiency is 237%. This means the heat removal increases 2.37 times through the optimal process.

4. CONCLUSIONS

This study is to obtain the optimal design of the heat removal of the 3D shell-and-tube heat exchanger by integrating multiphysics script based on the finite element method with the simplified conjugate-gradient method. A commercial COMSOL code validated is used for the calculation of temperature difference and heat removal of the STHE in this study. We discuss the heat removal of the 3D STHE optimal design problem. The SCGM is used as an optimal method to search the best arrangement in the whole domain of heat exchanger which to make the maximum temperature difference between the domain inlet and outlet of the heat exchanger. The results are presented in terms of heat removal and efficiency.

During the 3D optimal process, the temperature difference of case 1, triangular array, increases from 0.63 K to 1.74 K by using the optimal method. In addition, the temperature of case 2, rectangular array, also increases from 0.25 K to 4.36 K. The heat removal of the optimal STHE increases 2.37 times through the optimal process. In addition, the synthesis is obtained for all the advantages of this proposed method.

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