A bounded dynamic programming algorithm for the MMSP-W considering workstation dependencies and unrestricted interruption of the operations

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***Abstract*— In this paper, we propose a procedure based on Bounded Dynamic Programming (*BDP*) to solve the Mixed-Model Sequencing Problem with Workload Minimisation (*MMSP-W*), with serial workstations and unrestricted (or free) interruption of the operations. We performed a computational experiment with 225 instances from the literature. The results of our proposal are compared with those obtained through the CPLEX solver.**

Scheduling; sequencing; work overload; Dynamic Programming, linear programming

I. INTRODUCTION

In mixed-model manufacturing lines, which are common in Just-in-time (*JIT*) and Douki Seisan (*DS*) ideologies, several variants of one or more products can be handled. This flexibility determines the order in which the units are treated to drastically reduce intermediate stocks and to capitalise on the time available for manufacturing.

In these settings, we can choose two basic categories of objectives [1].

A. Work overload or lost work. The greatest possible reduction of work overload that may appear due to production programs with mixed products.
B. *JIT*. The greatest possible reduction of the levels of stocks in the system.

For category A, in addition to a relative focus on maximising the total amount of work completed [2], the excess effort that must be applied over time for certain operations can be modulated [3].

Using this perspective, sequencing problems can be grouped into three categories: (1) *Mixed-model sequencing*; (2) *Car sequencing*; and (3) *Level scheduling.*

Within this framework, [1] provided an up-to-date review of the literature.

The present study can be placed in category A.1., and it takes the minimisation of the total work overload as its optimisation criteria.

Overload, or excess effort, is a measurement, in units of time, of work that cannot be completed at the standard rhythm of an established activity, within the time granted to the workstations (cycle). This overload may arise when the processing time of a unit at a workstation is greater than the cycle time [2], although there may be a certain amount of play associated with extended cycles, which is called the length of the workstation or the time window.

When faced with a foreseeable workstation overload, at least three types of measures can be taken:

I. Stop the line and complete the pending work using reinforcements [4, 5].
II. Let the unit pass and finish the pending work in a final line at a later time. [2, 6, 7, 8].
III. Increase productive activity above the standard, using the assistance of reinforcement operators or previously programmed robotised systems.

The present study considered measures in categories II and III for handling work overloads.

Mixed-Model Sequencing Problem with Workload Minimisation (*MMSP-W*) is an NP-hard problem [2], for which several alternative solutions have been proposed. These include exact procedures based on branch and bound [9], dynamic programming [2, 10], heuristic procedures based on local search [6, 11], greedy algorithms with priority rules [6, 12], meta-heuristics [13, 14] and beam search [15].

For this paper, we used a procedure based on Bounded Dynamic Programming (*BDP*). This procedure combines features of dynamic programming with features of branch and bound algorithms. The principles of *BDP* have been described by [16] and [17]. Previous work on similar approaches has been done by [10] and [18]. This procedure is computationally competitive with the integer linear programming. Our proposal contains the following:

1. A model for the *MMSP-W* that combines properties from Yano and Scholl models.
2. A procedure based on dynamic programming to solve this problem.
3. A computational experiment with reference instances from the literature to compare the results offered by *BDP* with those offered by integer linear programming (CPLEX solver).

This paper is organised as follows. Section II presents a model for the *MMSP-W* with serial workstations and with unrestricted interruption of the operations. Section III shows an illustrative example. Section IV describes the basic elements and the application of the proposed *BDP* procedure. Section V describes the computational experiments with reference instances from the literature. Finally, Section VI shows the conclusions of the study.

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II. MODEL FOR MMSP-W WITH SERIAL WORKSTATIONS AND UNRESTRICTED INTERRUPTION OF THE OPERATIONS

For the *MMSP-W* with serial workstations, unrestricted interruption of the operations and work overload minimisation, we take as reference the model *M4*, proposed by [10].

The extended model *M4'* is focused on minimising the total overload (i.e., maximising the total work performed) and uses relative start instants of the units and considers more than one homogeneous processor at each workstation. The parameters and variables of this model are presented below.

Parameters

K	Set of workstations ($k = 1,\ldots,\|K\|$)
b_k	Number of homogeneous processors at workstation k
I	Set of product types ($i = 1,\ldots,\|I\|$)
d_i	Programmed demand of product type i
$p_{i,k}$	Processing time required by a unit of type i at workstation k for each homogeneous processor (at normal activity)
T	Total demand; obviously, $\sum_{i=1}^{\|I\|} d_i = T$
t	Position index in the sequence ($t = 1,\ldots,T$)
c	Cycle time, the standard time assigned to workstations to process any product unit
l_k	Time window, the maximum time that the workstation k is allowed to work on any product unit, where $l_k - c > 0$ is the maximum time that the work in process is held at workstation k

Variables

$x_{i,t}$	Binary variable equal to 1 if a product unit i ($i = 1,\ldots,\|I\|$) is assigned to the position t ($t = 1,\ldots,T$) of the sequence, and to 0 otherwise
$s_{k,t}$	Start instant of the operation in t^{th} unit of the sequence of products at workstation k ($k = 1,\ldots,\|K\|$)
$w_{k,t}$	Overload generated for the t^{th} unit of the product sequence at workstation k for each homogeneous processor (at normal activity); measured in time.
$\hat{s}_{k,t}$	Positive difference between the start instant and the minimum start instant of the t^{th} operation at workstation k. $\hat{s}_{k,t} = \left[s_{k,t} - (t-1)c\right]^+$ (with $[x]^+ = \max\{0,x\}$).
$\rho_{k,t}$	Processing time required by the t^{th} unit of the sequence of products at workstation k

Model *M4'*:

$$Min \quad W = \sum_{k=1}^{|K|}\left(b_k \sum_{t=1}^{T} w_{k,t} \right) \tag{1}$$

Subject to:

$$\sum_{t=1}^{T} x_{i,t} = d_i \qquad \forall i = 1,\ldots,|I| \tag{2}$$

$$\sum_{i=1}^{|I|} x_{i,t} = 1 \qquad \forall t = 1,\ldots,T \tag{3}$$

$$\rho_{k,t} = \sum_{i=1}^{|I|} p_{i,k}\, x_{i,t} \qquad \forall k = 1,\ldots,|K|;\ \forall t = 1,\ldots,T \tag{4}$$

$$\rho_{k,t} - w_{k,t} \geq 0 \qquad \forall k = 1,\ldots,|K|;\ \forall t = 1,\ldots,T \tag{5}$$

$$\hat{s}_{k,t} \geq \hat{s}_{k,t-1} + \rho_{k,t-1} - w_{k,t-1} - c \qquad \forall k = 1,\ldots,|K|;\ \forall t = 2,\ldots,T \tag{6}$$

$$\hat{s}_{k,t} \geq \hat{s}_{k-1,t} + \rho_{k-1,t} - w_{k-1,t} - c \qquad \forall k = 2,\ldots,|K|;\ \forall t = 1,\ldots,T \tag{7}$$

$$\hat{s}_{k,t} + \rho_{k,t} - w_{k,t} \leq l_k \qquad \forall k = 1,\ldots,|K|;\ \forall t = 1,\ldots,T \tag{8}$$

$$\hat{s}_{k,t} \geq 0 \qquad \forall k = 1,\ldots,|K|;\ \forall t = 1,\ldots,T \tag{9}$$

$$w_{k,t} \geq 0 \qquad \forall k = 1,\ldots,|K|;\ \forall t = 1,\ldots,T \tag{10}$$

$$x_{i,t} \in \{0,1\} \qquad \forall i = 1,\ldots,|I|;\ \forall t = 1,\ldots,T \tag{11}$$

$$\hat{s}_{11} = 0 \tag{12}$$

In the model, the equivalent objective function (1) is represented by the total overload (W). Constraint (2) requires that the programmed demand be satisfied. Constraint (3) indicates that only one product unit can be assigned to each position of the sequence. Constraint (4) links the processing times required by the types of product with the processing times required by the units in the sequence. Constraint (5) establishes upper bounds for the overload values through the required processing times by the units on the sequence. Constraints (6)-(9) constitute the set of relative start instants of the operations at each workstation and the processing times applied to the products. Constraint (10) indicates that the generated overloads are not negative. Constraint (11) requires the assigned variables to be binary. Finally, constraint (12) fixes the start of operations.

III. AN ILLUSTRATIVE EXAMPLE

To illustrate the model formulated above, we present the following example.

There are six units of product (T=6), of which three are type *A*, one is type *B* and two are type *C*, with a total work required V_0=104. The units are processed at three workstations ($|K|$=3) with different numbers of processors (b_k); the processing times for each processor (at normal activity) for each type of unit i (*A*, *B*, *C*) at each workstation k (m_1, m_2, m_3) are listed in Table I.

TABLE I. PROCESSING TIMES ($p_{i,k}$), NUMBER OF HOMOGENEOUS PROCESSORS (b_k) AND TOTAL WORK (V_0) REQUIRED BY EACH TYPE OF UNIT AT EACH WORKSTATION

	***A* (d_A=3)**	***B* (d_B=1)**	***C* (d_C=2)**	b_k
m_1	5	4	3	1
m_2	5	4	4	2
m_3	4	3	5	1
Total	*19* ($V_0(A)$ = 57)	*15* ($V_0(B)$ = 15)	*16* ($V_0(C)$ = 32)	V_0 = *104*

Furthermore, c = 4 (cycle time) and $l_k = 6$ for $k = 1,...,3$ (length of the workstation or time window).

Fig. 1 shows a Gantt diagram of the optimal solution offered by model *M4'*. The sequence of products that presents the minimum total overload is *C-B-A-C-A-A*. The total work performed is V=101, and the overload, which is concentrated between workstations m_1 and m_2, is W=3 (the grey area in Fig.1).

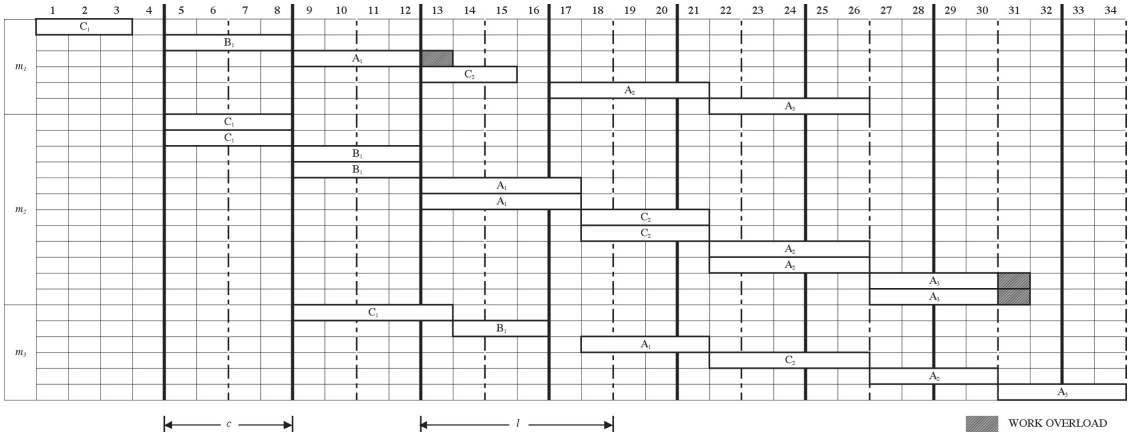

Figure 1. Gantt chart for the optimum solution for the example provided by *M4'*.

IV. BDP FOR THE MMSP-W

This section presents the basic elements of the *BDP* procedure applied to the resolution of *MMSP-W* with serial workstations and unrestricted interruption of the operations.

A. Global and partial Bounds

Given a vertex of the stage *t*, reached through a partial sequence $\pi(t)=\{\pi_1,\pi_2,...,\pi_t\}$, the overall bound for *W* and a partial bound for the complement $R(\pi(t))$ associated to the sequence or segment $\pi(t)$ can be determined according to the schema presented in Fig. 2.

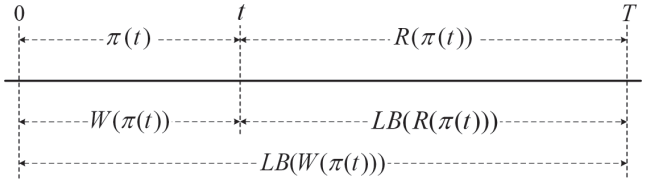

Figure 2. Bound scheme for a partial sequence $\pi(t)$

To obtain the bounds of the overloads associated to $\pi(t)$ and $R(\pi(t))$, we impose the following conditions to *M4'*:

- The values of the variables $x_{i,\tau}$ ($i=1,\ldots,|I|;\tau=1,\ldots,t$) are fixed by $\pi(t)$:

$$x_{i,\tau}=\begin{cases}1, if\ \pi_\tau=i\\0, if\ \pi_\tau\neq i\end{cases}\Leftrightarrow\begin{cases}x_{\pi_\tau,\tau}=1\\x_{i,\tau}=0, if\ i\neq\pi_\tau\end{cases}\quad(13)$$

- The binary condition is relaxed for the variables $x_{i,\tau}$ ($i=1,\ldots,|I|;\tau=t+1,\ldots,T$):

$$0\le x_{i,\tau}\le 1\qquad i=1,\ldots,|I|;\tau=t+1,\ldots,T\quad(14)$$

The result is the following linear program (*LB_M4'*):

$$Min\quad LB(W(\pi(t)))=\sum_{k=1}^{|K|}\left(b_k\sum_{t=1}^{T}w_{k,t}\right)\quad(15)$$

Subject to:
(2)-(10) and (12) from *M4'*

$$x_{\pi_\tau,\tau}=1\qquad\forall\tau=1,\ldots,t\quad(16)$$

$$0\le x_{i,\tau}\le 1\qquad\forall i=1,\ldots,|I|;\forall\tau=t+1,\ldots,T\quad(17)$$

The previous linear program provides an overall bound of the total overload ($LB(W(\pi(t)))$), the value of the overload associated to the segment $\pi(t)$ ($W(\pi(t))$) and a bound of the overload associated to the complement $R(\pi(t))$ ($LB(R(\pi(t)))$). These values can be calculated as follows:

$$W(\pi(t))=\sum_{k=1}^{|K|}\left(b_k\sum_{\tau=1}^{t}w_{k,\tau}\right)\quad(18)$$

$$LB(R(\pi(t)))=\sum_{k=1}^{|K|}\left(b_k\sum_{\tau=t+1}^{T}w_{k,\tau}\right)\quad(19)$$

The relative completion instants ($\hat{e}_{k,t}$) of the last operation of the partial sequence $\pi(t)$, in each workstations, can be obtained from *LB_M4'* as follows:

$$\hat{e}_{k,t}=\hat{s}_{k,t}+\rho_{k,t}-w_{k,t}\qquad\forall k=1,\ldots,|K|\quad(20)$$

B. Graph associated with the problem

Similar to [10], we can build a linked graph without loops or direct cycles of *T* + 1 stages. The set of vertices in level *t* (*t* = 0,. . . ,*T*) will be noted as *J*(*t*). *J*(*t*, *j*) (*j* = 1,..., |*J*(*t*)|) being a vertex of level *t*, which is defined by the tuple $\left(\vec{q}(t,j),\vec{e}(t,j),\ \pi(t,j),LB\left(W\left(\pi(t,j)\right)\right)\right)$, where:

- $\vec{q}(t,j)=\left(q_1(t,j),\ldots,q_{|I|}(t,j)\right)$ represents the vector of demand satisfied.
- $\vec{e}(t,j)=\left(e_1(t,j),\ldots,e_{|K|}(t,j)\right)$ is the vector of absolute completion instants of the operations at the workstations.
- $\pi(t,j)$ represents the sequence of *t* units of product associated to the vertex.
- $LB\left(W\left(\pi(t,j)\right)\right)$ is the bound of the overall overload, considering the sequence $\pi(t,j)$, obtained through the linear program *LB_M4'*.

The vertex *J*(*t*, *j*) has the following properties:

$$\sum_{i=1}^{|I|}q_i(t,j)=t\quad(21)$$

$$0\le q_i(t,j)\le d_i\qquad i=1,\ldots,|I|\quad(22)$$

$$e_k^c(t,j)=\max\{(t+k-2)c+\hat{e}_{k,t},(t+k-1)c\}\quad k=1,\ldots,|K|\quad(23)$$

Note that $\vec{e}^{\,c}(t,j)$ is the vector of corrected completion instants in accordance with the cycle time.

In short, a vertex *J*(*t*, *j*) will be represented as follows:

$$J(t,j)=\left\{(t,j),\vec{q}(t,j),\pi(t,j),LB\left(W\left(\pi(t,j)\right)\right),\vec{e}(t,j),\vec{e}^{\,c}(t,j)\right\}\quad(24)$$

At level 0 of the graph, there is only one $J(0)$ vertex. Initially, we may consider that at level t, $J(t)$ contains the vertices associated to all of the sub-sequences that can be built with t products that satisfy properties (21), (22) and (23). However, it is easy to reduce the cardinal that $J(t)$ may present a priori, establishing the following definition of pseudo-dominance: given the sequences $\pi(t,j_1)$ and $\pi(t,j_2)$ associated to the vertices $J(t,j_1)$ and $J(t,j_2)$, then $\pi(t,j_1)$ pseudo-dominates $\pi(t,j_2)$ if:

$$\pi(t,j_1) \prec \pi(t,j_2) \Leftrightarrow \left\{ \begin{array}{l} \left[\vec{q}(t,j_1) = \vec{q}(t,j_2)\right] \text{ and} \\ \left[LB(W(\pi(t,j_1))) \le LB(W(\pi(t,j_2)))\right] \text{ and} \\ \left[W(\pi(t,j_1)) \le W(\pi(t,j_2))\right] \text{ and} \\ \left[\vec{e}^{\,c}(t,j_1) \le \vec{e}^{\,c}(t,j_2)\right] \end{array} \right\} \quad (25)$$

The reduction of $J(t)$ through the pseudo-dominances defined in (25) cannot guarantee the optimality of the solutions.

C. The use of BDP

For this study, we used a procedure based on *BDP*. This procedure combines features of dynamic programming (determination of extreme paths in graphs) with features of branch and bound algorithms. The principles of *BDP* have been described by [16, 17]. The procedure is described below (see details on [10]):

BDP – MMSPW

Input: $T, |I|, |K|, d_i(\forall i), l_k, b_k(\forall k), p_{ik}(\forall i, \forall k), c, Z_0, H$
Output: list of sequences obtained by *BDP*
Initialization: $t = 0$; $LBZ_{\min} = \infty$
While ($t < T$) *do*
  $t = t+1$
  While (list of consolidated vertices in stage *t-1* **not empty**) *do*
    Select_vertex (t)
    Develop_vertex (t)
    Filter_vertices (Z_0, H, $LBZ_{\min}$)
  end while
  End_stage ()
end while
end BDP - MMSPW

In the procedure appear the following functions:

- Select_vertex (t): selects, following a nondecreasing ordering of the $LB(W(\pi(t-1,j)))$ values, one of the vertices consolidated in stage *t-1*.
- Develop_vertex (t): develops the selected vertex in previous function adding a new product unit with pending demand.
- Filter_vertices ($Z_0, H, LBZ_{\min}$): chooses, from all the vertices developed in the previous function, a maximum number H of the most promising vertices (according to the lowest values of the lower bound $LB(W(\pi(t,j)))$), and removing those vertices in which their lower bound is greater than Z_0 (known initial solution)
- End_stage (): consolidates the most promising vertices in stage t (H vertices as maximum).

V. COMPUTATIONAL EXPERIMENT

For the test operations of *M4'* and the *BDP* procedure, 225 instances from the literature were used. These instances were built from 45 production programs and 5 processing times structures, composed by four product types ($|I| = 4$) and four workstations ($|K| = 4$). These instances can be found in [6, 10].

The solutions for the 225 instances were obtained using the solver CPLEX v11.0 (single-processor license). Those solutions were compared with the solutions offered by the *BDP* procedure proposed, under the following conditions and features:

1. *BDP* procedure programmed in C++, using gcc v4.2.1, running on an Apple Macintosh iMac computer with an Intel Core i7 2.93 GHz processor and 8 GB RAM using MAC OS X 10.6.7 (not using any type of parallel code; therefore, the computer can be considered as a single 2.93 GHz processor).
2. Six windows width (H) were used, with values 1, 4, 16, 64, 256 and 1024.
3. The initial solution Z_0 for each window width was the solution obtained by *BDP* with the previous window width, except in the case $H = 1$, where Z_0 was established as ∞.
4. To calculate the lower bounds, $LB(W(\pi(t,j)))$, of the overload associated to each vertex in the *BDP* procedure, the solver Gurobi v4.5.0 was used, solving the linear program associated to *LB_M4'*.

Tables II to VI show the results obtained by *BDP* and *CPLEX* for the 225 instances, grouped by processing times structures (1 to 5).

TABLE II. RESULTS FOR STRUCTURE 1

Ins.	Opt CPLEX	H=1 W	H=4 W	H=16 W	H=64 W	H=256 W	H=1024 W	Best found	Best RPD
1/1	40	46	42	40	-	-	-	40	0.00
2/1	100	107	102	100	-	-	-	100	0.00
3/1	81	88	86	81	-	-	-	81	0.00
4/1	143	143	-	-	-	-	-	143	0.00
5/1	46	49	49	46	-	-	-	46	0.00
6/1	46	49	47	47	46	-	-	46	0.00
7/1	14	17	16	14	-	-	-	14	0.00
8/1	61	63	63	61	-	-	-	61	0.00
9/1	81	87	81	-	-	-	-	81	0.00
10/1	68	76	69	68	-	-	-	68	0.00
11/1	37	48	43	40	39	37	-	37	0.00
12/1	37	48	44	38	37	-	-	37	0.00
13/1	29	38	34	30	30	29	-	29	0.00
14/1	45	50	48	47	46	45	-	45	0.00
15/1	40	51	42	41	41	40	-	40	0.00
16/1	32	40	36	34	34	32	-	32	0.00
17/1	37	48	42	39	38	37	-	37	0.00
18/1	45	55	53	47	46	45	-	45	0.00
19/1	33	38	35	35	33	-	-	33	0.00
20/1	31	38	31	-	-	-	-	31	0.00
21/1	56	72	57	57	56	-	-	56	0.00
22/1	63	96	79	63	-	-	-	63	0.00
23/1	55	72	64	60	55	-	-	55	0.00
24/1	67	90	78	67	-	-	-	67	0.00
25/1	56	67	60	58	56	-	-	56	0.00
26/1	63	74	69	63	-	-	-	63	0.00
27/1	58	63	63	59	58	-	-	58	0.00
28/1	46	57	52	46	-	-	-	46	0.00
29/1	48	57	53	50	48	-	-	48	0.00
30/1	55	68	55	-	-	-	-	55	0.00
31/1	52	58	55	53	52	-	-	52	0.00
32/1	53	62	53	-	-	-	-	53	0.00
33/1	53	63	57	53	-	-	-	53	0.00
34/1	25	35	30	25	-	-	-	25	0.00
35/1	41	46	42	41	-	-	-	41	0.00
36/1	29	37	35	29	-	-	-	29	0.00
37/1	48	58	52	49	48	-	-	48	0.00
38/1	43	50	45	45	43	-	-	43	0.00
39/1	47	58	49	48	47	-	-	47	0.00
40/1	22	30	23	23	22	-	-	22	0.00
41/1	34	39	35	35	34	-	-	34	0.00
42/1	22	30	24	24	23	22	-	22	0.00
43/1	44	52	49	45	45	44	-	44	0.00
44/1	34	39	37	35	34	-	-	34	0.00
45/1	43	55	49	45	44	43	-	43	0.00

TABLE III. RESULTS FOR STRUCTURE 2

Ins.	Opt CPLEX	H=1 W	H=4 W	H=16 W	H=64 W	H=256 W	H=1024 W	Best found	Best RPD
1/2	230	230	-	-	-	-	-	230	0.00
2/2	282	296	296	282	-	-	-	282	0.00
3/2	270	271	271	270	-	-	-	270	0.00
4/2	324	328	324	-	-	-	-	324	0.00
5/2	200	227	203	200	-	-	-	200	0.00
6/2	100	114	108	100	-	-	-	100	0.00
7/2	151	160	155	152	151	-	-	151	0.00
8/2	199	218	205	203	201	199	-	199	0.00
9/2	218	238	224	218	-	-	-	218	0.00
10/2	225	234	225	-	-	-	-	225	0.00
11/2	132	162	139	136	132	-	-	132	0.00
12/2	112	129	123	113	112	-	-	112	0.00
13/2	122	145	122	-	-	-	-	122	0.00
14/2	153	169	163	153	-	-	-	153	0.00
15/2	147	161	153	147	-	-	-	147	0.00
16/2	117	132	123	117	-	-	-	117	0.00
17/2	133	151	140	133	-	-	-	133	0.00
18/2	136	160	138	138	138	137	136	136	0.00
19/2	150	179	162	150	-	-	-	150	0.00
20/2	114	131	119	115	114	-	-	114	0.00
21/2	193	196	195	193	-	-	-	193	0.00
22/2	213	227	218	213	-	-	-	213	0.00
23/2	197	213	203	197	-	-	-	197	0.00
24/2	205	210	210	208	208	205	-	205	0.00
25/2	190	196	196	190	-	-	-	190	0.00
26/2	206	214	214	208	206	-	-	206	0.00
27/2	200	211	202	201	200	-	-	200	0.00
28/2	175	189	189	175	-	-	-	175	0.00
29/2	158	194	178	158	-	-	-	158	0.00
30/2	178	186	183	179	179	178	-	178	0.00
31/2	153	184	159	159	155	154	153	153	0.00
32/2	187	204	202	188	188	187	-	187	0.00
33/2	174	194	177	174	-	-	-	174	0.00
34/2	141	155	146	141	-	-	-	141	0.00
35/2	117	145	126	121	117	-	-	117	0.00
36/2	152	156	152	-	-	-	-	152	0.00
37/2	116	144	124	122	116	-	-	116	0.00
38/2	179	209	185	179	-	-	-	179	0.00
39/2	168	190	168	-	-	-	-	168	0.00
40/2	104	119	111	104	-	-	-	104	0.00
41/2	92	106	101	92	-	-	-	92	0.00
42/2	137	157	148	140	137	-	-	137	0.00
43/2	116	135	120	116	-	-	-	116	0.00
44/2	163	185	163	-	-	-	-	163	0.00
45/2	149	161	149	-	-	-	-	149	0.00

TABLE IV. RESULTS FOR STRUCTURE 3

Ins.	Opt CPLEX	H=1 W	H=4 W	H=16 W	H=64 W	H=256 W	H=1024 W	Best found	Best RPD
1/3	369	399	386	369	-	-	-	369	0.00
2/3	339	373	356	341	339	-	-	339	0.00
3/3	421	444	434	421	-	-	-	421	0.00
4/3	313	329	321	314	313	-	-	313	0.00
5/3	345	397	364	349	345	-	-	345	0.00
6/3	227	244	227	-	-	-	-	227	0.00
7/3	185	211	199	185	-	-	-	185	0.00
8/3	279	290	285	279	-	-	-	279	0.00
9/3	232	243	241	238	232	-	-	232	0.00
10/3	367	383	381	373	367	-	-	367	0.00
11/3	215	276	246	231	224	215	-	215	0.00
12/3	221	243	234	223	221	-	-	221	0.00
13/3	207	240	221	212	210	207	-	207	0.00
14/3	231	260	252	244	240	237	237	237	2.60
15/3	239	285	268	267	249	239	-	239	0.00
16/3	196	222	210	210	200	196	-	196	0.00
17/3	219	236	236	229	225	221	221	221	0.91
18/3	229	262	242	238	229	-	-	229	0.00
19/3	217	284	259	226	219	217	-	217	0.00
20/3	221	253	245	236	221	-	-	221	0.00
21/3	277	303	303	290	281	277	-	277	0.00
22/3	311	336	332	325	314	311	-	311	0.00
23/3	329	354	350	343	330	329	-	329	0.00
24/3	259	284	276	269	263	259	-	259	0.00
25/3	295	321	314	307	299	295	-	295	0.00
26/3	243	263	256	248	245	245	245	245	0.82
27/3	261	272	272	261	-	-	-	261	0.00
28/3	265	309	296	296	273	265	-	265	0.00
29/3	283	327	321	310	299	283	-	283	0.00
30/3	214	239	225	217	214	-	-	214	0.00
31/3	245	267	267	257	247	247	247	247	0.82
32/3	235	281	259	248	235	-	-	235	0.00
33/3	243	273	258	243	-	-	-	243	0.00
34/3	208	235	222	208	-	-	-	208	0.00
35/3	235	266	257	251	235	-	-	235	0.00
36/3	196	229	215	199	196	-	-	196	0.00
37/3	235	253	241	237	235	-	-	235	0.00
38/3	263	333	306	287	265	263	-	263	0.00
39/3	257	326	294	274	259	257	-	257	0.00
40/3	199	222	213	199	-	-	-	199	0.00
41/3	213	230	217	213	-	-	-	213	0.00
42/3	211	292	242	233	211	-	-	211	0.00
43/3	232	257	248	235	232	-	-	232	0.00
44/3	271	341	314	281	271	-	-	271	0.00
45/3	265	334	306	285	270	265	-	265	0.00

TABLE V. RESULTS FOR STRUCTURE 4

Ins.	Opt CPLEX	H=1 W	H=4 W	H=16 W	H=64 W	H=256 W	H=1024 W	Best found	Best RPD
1/4	710	771	771	730	710	-	-	710	0.00
2/4	638	699	681	662	638	-	-	638	0.00
3/4	80	80	-	-	-	-	-	80	0.00
4/4	80	80	-	-	-	-	-	80	0.00
5/4	674	735	735	735	682	682	682	682	1.19
6/4	338	338	-	-	-	-	-	338	0.00
7/4	338	338	-	-	-	-	-	338	0.00
8/4	302	307	302	-	-	-	-	302	0.00
9/4	302	302	-	-	-	-	-	302	0.00
10/4	80	80	-	-	-	-	-	80	0.00
11/4	415	430	425	416	415	-	-	415	0.00
12/4	326	326	-	-	-	-	-	326	0.00
13/4	326	326	-	-	-	-	-	326	0.00
14/4	314	319	314	-	-	-	-	314	0.00
15/4	240	240	-	-	-	-	-	240	0.00
16/4	240	240	-	-	-	-	-	240	0.00
17/4	320	320	-	-	-	-	-	320	0.00
18/4	415	415	-	-	-	-	-	415	0.00
19/4	415	425	425	417	415	-	-	415	0.00
20/4	252	252	-	-	-	-	-	252	0.00
21/4	228	233	228	-	-	-	-	228	0.00
22/4	154	154	-	-	-	-	-	154	0.00
23/4	154	154	-	-	-	-	-	154	0.00
24/4	228	228	-	-	-	-	-	228	0.00
25/4	228	233	228	-	-	-	-	228	0.00
26/4	302	307	302	-	-	-	-	302	0.00
27/4	302	307	302	-	-	-	-	302	0.00
28/4	166	166	-	-	-	-	-	166	0.00
29/4	166	166	-	-	-	-	-	166	0.00
30/4	314	314	-	-	-	-	-	314	0.00
31/4	314	319	314	-	-	-	-	314	0.00
32/4	403	413	413	403	-	-	-	403	0.00
33/4	403	403	-	-	-	-	-	403	0.00
34/4	252	252	-	-	-	-	-	252	0.00
35/4	252	252	-	-	-	-	-	252	0.00
36/4	326	326	-	-	-	-	-	326	0.00
37/4	326	326	-	-	-	-	-	326	0.00
38/4	510	564	564	564	541	516	516	516	1.18
39/4	509	564	564	554	524	511	511	511	0.39
40/4	338	338	-	-	-	-	-	338	0.00
41/4	338	338	-	-	-	-	-	338	0.00
42/4	427	433	433	433	427	-	-	427	0.00
43/4	427	429	428	428	427	-	-	427	0.00
44/4	522	576	576	576	538	531	531	531	1.72
45/4	521	576	566	566	542	536	521	521	0.00

TABLE VI. RESULTS FOR STRUCTURE 5

Ins.	Opt CPLEX	H=1 W	H=4 W	H=16 W	H=64 W	H=256 W	H=1024 W	Best found	Best RPD
1/5	211	228	221	211	-	-	-	211	0.00
2/5	276	285	285	276	-	-	-	276	0.00
3/5	166	167	166	-	-	-	-	166	0.00
4/5	212	232	212	-	-	-	-	212	0.00
5/5	222	237	227	227	227	223	222	222	0.00
6/5	136	144	137	136	-	-	-	136	0.00
7/5	143	163	145	143	-	-	-	143	0.00
8/5	177	223	178	178	177	-	-	177	0.00
9/5	175	196	177	177	176	176	176	176	0.57
10/5	42	47	42	-	-	-	-	42	0.00
11/5	121	136	131	121	-	-	-	121	0.00
12/5	95	112	95	-	-	-	-	95	0.00
13/5	95	107	96	96	95	-	-	95	0.00
14/5	103	114	108	103	-	-	-	103	0.00
15/5	70	75	70	-	-	-	-	70	0.00
16/5	65	70	65	-	-	-	-	65	0.00
17/5	84	111	89	89	84	-	-	84	0.00
18/5	158	166	166	159	159	158	-	158	0.00
19/5	151	186	154	151	-	-	-	151	0.00
20/5	68	96	79	68	-	-	-	68	0.00
21/5	106	108	107	106	-	-	-	106	0.00
22/5	82	104	83	83	82	-	-	82	0.00
23/5	67	93	77	67	-	-	-	67	0.00
24/5	128	145	129	129	128	-	-	128	0.00
25/5	103	132	114	103	-	-	-	103	0.00
26/5	152	165	154	154	153	152	-	152	0.00
27/5	137	150	141	141	137	-	-	137	0.00
28/5	64	89	65	64	-	-	-	64	0.00
29/5	56	70	56	-	-	-	-	56	0.00
30/5	155	168	156	155	-	-	-	155	0.00
31/5	148	162	155	149	148	-	-	148	0.00
32/5	176	196	181	177	176	-	-	176	0.00
33/5	171	176	176	172	172	171	-	171	0.00
34/5	91	128	92	91	-	-	-	91	0.00
35/5	86	121	92	86	-	-	-	86	0.00
36/5	146	163	146	-	-	-	-	146	0.00
37/5	132	132	-	-	-	-	-	132	0.00
38/5	185	206	188	188	188	186	185	185	0.00
39/5	194	205	198	196	196	196	196	196	1.03
40/5	103	119	104	103	-	-	-	103	0.00
41/5	107	130	117	107	-	-	-	107	0.00
42/5	142	155	145	142	-	-	-	142	0.00
43/5	151	152	152	152	152	152	152	152	0.66
44/5	169	182	171	169	-	-	-	169	0.00
45/5	182	189	187	183	183	183	183	183	0.55

For each structure (Tables II to VI) are shown: (1) the code of the instance (production program/structure) in the column "*Ins*."; (2) the value of the total overload *W* of the optimal solution for each instance obtained by CPLEX ("*Opt CPLEX*"); (3) the values of the overloads *W* obtained by the *BDP* algorithm using different windows width *H* (*H*=1,…,1024); (4) the best value of *W*, for each instance, found by *BDP* ("*Best found*"); and (5) the best value, for each instance, of the relative percentage deviation (*RPD*) obtained as $RPD = ((Best\ found - Opt\ CPLEX) / Opt\ CPLEX) * 100$ ("*Best RPD*").

The *BDP* procedure, with the incorporation of the pseudo-dominances defined in (25), reaches the optimal result for 213 of the 225 instances. The instances in which the optimum was not reached are focused in structure 3 (production programs 14, 17, 26 and 31), structure 4 (production programs 5, 38, 39 and 44) and structure 5 (production programs 9, 39, 43 and 45) are shown in Tables IV, V and VI, respectively. The average values of the *RPD* for the instances of these structures are 0.11%, 0.10% and 0.06%.

In order to compare the computational efficiency, in CPU times, of *BDP* against the exact procedure in solver CPLEX, we have summarized the CPU times taken to obtain the best solutions of the 225 instances. Table VII show the minimum, maximum, and average CPU times, employed by both procedures to solve the set of instances grouped by production programs and processing times structures.

TABLE VII. CPU TIMES BY BLOCKS AND STRUCTURES

	M_4' - CPLEX			*BDP*		
	CPU min	*CPU max*	*Average*	*CPU min*	*CPU max*	*Average*
Block 1	0.0	1.5	0.4	0.0	1.3	0.5
Block 2	0.0	173.9	23.3	0.0	9.5	1.7
Block 3	5.1	2533.2	335.7	0.1	22.6	6.4
Block 4	6.6	111.5	35.2	0.1	10.5	3.7
Block 5	0.1	697.7	43.5	0.1	14.2	3.2
Structure 1	0.0	301.3	46.9	0.0	16.6	4.3
Structure 2	0.1	129.7	16.5	0.1	7.2	2.3
Structure 3	0.1	180.3	32.2	0.3	22.6	5.9
Structure 4	0.0	2533.2	288.5	0.0	7.2	1.2
Structure 5	0.2	126.4	24.4	0.1	14.2	2.8

In all cases (except *Block* 1), the average CPU times used by CPLEX are larger than the corresponding to the *BDP*, being the Structure 4 the extreme case, where the *BDP* was 239 times faster than CPLEX, and on average 54 times faster.

VI. CONCLUSIONS

We have proposed a heuristic procedure based on *BDP* to solve a variant of the *MMSP-W*, specifically the variant with serial workstations and unrestricted interruption of the operations.

The proposed procedure was computationally competitive against CPLEX. The *BDP* procedure obtained 213 of 225 optimums in the instances used; in the remaining 12 instances, *BDP* obtained solutions, on average, far less than 0.1% over the optimal value. The CPU times shows that *BDP* was 54 times faster than CPLEX.

In future research, our work will be focused on exploiting and improving the procedure in an industrial environment.

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