19 Optimization in Natural Gas Network Planning: a Survey

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Abstract In this paper, a survey on the role of optimization methods and operation research techniques in different fields of natural gas network planning, which have received more attentions from researchers due to their enormous effects on saving the related costs was made. The researches have been recognized by network topologies (cyclic or non-cyclic), system assumptions (steady state or non-steady), and the scope of defined problems in the network: design, flow, operation or expansion.

To make a good comparison between what has been done and what should be improved on in future, model characteristics and solution methods to optimize the problems have been discussed and the application of mathematical models regarding the most important problems of this field have been highlighted. To present the efficiency of developed models in the real world, two case studies are being presented. Eventually, the research is concluded by emphasis on the potential matters of modeling and solving the natural gas network planning problems.

Key Words: Optimization, Natural Gas Network, Survey

19.1 Introduction

19.1.1 Natural gas network modeling

A vast number of real world problems in various types of systems are presented using network modeling. A network model represents a powerful visual, which helps to present connections among the system's components used in different fields of science. Optimization of network design, network flow, and network operation has been considered as a fundamental issue in different research fields from technical or financial perspectives. Usually, network problems are known as complex structural problems, which in real cases search to find the optimum solutions. Considering the characteristics of networks optimization problems, a large number of algorithms have been developed in the literature and tried to solve them in reliable times to find the most suitable and optimum solutions.

Natural gas, as one of the most important sources of energy, which tries to satisfy many commercial and residential users' demands throughout the world, has a huge and complex network. Each day, a large amount of money is spent on the different stages of main processes of this network such as exploration, extraction, processing, production, transportation, storage, and distribution [1]. Since the 1960s, many different problems have been defined for the planning of design, flow, operation, and development of natural gas transmission and distribution networks. The natural gas network planning problems are multi-disciplinary, which many researchers in different fields have tried to solve them all over the world.

19.1.2 Natural gas network introducing

The aim of the natural gas network is satisfying consumers' demands efficiently in such a way that cost is set minimally. Therefore, some components should be used through the network properly and some processes should be planned exactly. In this section, main components of natural the gas network and after that its main processes are explained.

19.1.2.1 *Natural gas network components*

Natural gas network components like other networks can be presented as two physical entities: fixed entities and current entities:

- Fixed physical entities are included arcs, which correspond with pipelines, compressor stations, and valves and nodes that present physical interconnection points.
  - Arcs components:

- *Pipelines:* Two types of pipelines have been known to researchers: passive pipelines, which correspond with regular pipelines, and active pipelines, which are regular pipelines with compressors [2].
- *Compressor stations:* The transmission capacity of a gas pipeline is limited but can be arranged based on the supply and demand nodes by setting differences among input and output pressures of the pipeline. Compressors are located at suitable locations through the network to enlarge the pressures difference between two nodes of pipelines to increase the transmission capacity of the network [2].
- *Valves:* To make the flow of natural gas stop for a certain section of pipelines in situations such as maintenances or replacements, valves are used along the entire length of interstate pipelines.

o Nodes components: As in Figure 1 illustrated, components belonging to nodes in the natural gas network include:

- Supply nodes, which have only the output flow.
- Demand nodes, which have only the input flow.
- Intermediate nodes, which have input and output flows.

- Current physical entities can be classified as financial, informational, and physical flows. Usually flows in the natural gas network controlled by a dispatcher or in dispatching organizations. These organizations obtain information of the natural gas pressures and flows over the pipelines system and check the warning signals of companies via simulation systems.

Figure 1. Natural gas network components

19.1.2.2 *Natural gas network processes*

The natural gas is supplied through gas and oil wells and produced in refineries. In the natural gas network, some methods exist to move gas from producers to consumers in order to satisfy customers' demands but the pipeline system is the most cost effective way to transport gas over long distances. Consumers of the natural gas are divided into three main groups: domestic and commercial subscribers, industrial consumers, and exports. Usually the priority of natural gas networks is to serve domestic and commercial consumers.

Natural gas suppliers and natural gas consumers are connected through a complex and huge network in such a way that there is a long distance between them, and natural gas must be flowed by a suitable pressure. During the long transportation, pressures are lost due to the friction between the natural gas and the pipelines' inner walls. In addition, the natural gas volume is reduced because of transmitted heats of the environment. Therefore, compressor stations are installed to set the pressure to hold the natural gas continuity along with the network and determining the capacity of transmitted gas, periodically.

Generally, compressor stations are one of the most complex entities of natural gas network because they consist of several compressor units (typically 15 to 20 units) which have been connected in different configurations such as series, parallel or combination of both, and have different types [3]. Two of the main types of compressor units are centrifugal and reciprocating units, which the first one is more common in the natural gas industry and consequently in related researches assumptions [4]. Without considering which type of the compressor unit is used in the model, each unit has two options of turning on and turning off, which makes its behavior non-linear. When the demand of customer increases, the pressure of the pipeline drops. Therefore, at least one compressor should be opened until the gas pressure resumed an acceptable level [5].

The whole process of the natural gas network can be concluded in four main parts: supply, transportation, storage, and sale of the natural gas in the market places.

- Supply

Supply is usually started with development and exploration, and extracting the gas reserves and processing the extracted gas, which in practice usually is carried out by the same company in these three components. Exploration is concerned with locating natural gas and petroleum deposits. When a team of exploration geologists and geophysicists has located a potential natural gas deposit, in extraction phase, a team of drilling experts became ready to dig down to where the natural gas is assumed to be (www.naturalgas.org).

- Transportation

Natural gas transportation, which makes the most important segment in the natural gas industry, consists of a complex pipeline network to move the natural gas from a set of origins to the consumers in order to satisfy their demands. The natural gas transportation network is divided into two main networks namely transmission and distribution. Moving a large volume of gas at high pressure over long distances from a gas source to distribution centers is done in transmission networks; and routing gas to individual consumers is performed through distribution networks [6]. The efficiency of transportation is a suitable criterion to estimate the whole of natural gas system's performance. Considering the aim of the network and consequently the physical characteristics of its components, the natural gas network can be split into the transmission network and distribution networks, which the scope of both of them has been presented in Figure 2.

Figure 2. The process of transportation during the natural gas network

- *Transmission network*

Ríos-Mercado, et al.[6] presented how a gas transmission network, including pipelines, junction nodes, and compressor stations, is different from conventional networks by two special characteristics as follows:

- Beside the flow variables, which are considered for each arc to present the mass flow rates, at every node a pressure variable is defined.
- Unlike most of the networks, which consider only mass flows balances, in transmission networks two other types of constraints are taken into account as follows:
  - A connection exists between flow and pressure drop, which is represented by a nonlinear equality constraint on each pipe.
  - The feasible limits are available to operate pressures and flows inside each compressor, which are represented through a non-linear and non-convex set.

Usually the operating expenses of the natural gas transmission network are estimated through the operating costs of the compressor stations, which can be determined due to the fuel consumed at each compressor station. For example, Borraz-Sánchez and Ríos-Mercado [7] estimated that 3-5% of transmitted gas is consumed by compressor stations. Since a huge volume of natural gas is being transported through the network, about 25% -50% of the total operating budget of companies is spent on running the compressor stations. Therefore, minimizing the total fuel consumption of the compressor stations along the network is one of the main objectives for a transmission networking, because of its effects on overall costs of gas operations [6].

- *Distribution network*

Distribution networks are different from transmission networks in several perspectives. Distribution networks do not have valves, compressors, or nozzles and pipes act under fewer pressures. Therefore, the pipelines are smaller and the networks are simpler [8]. Most of the natural gas users, which are domestic and commercial consumers, receive the natural gas from local distribution companies. A smaller number of the natural gas users such as power generation companies receive the natural gas directly from high-capacity interstate and intrastate pipelines. In large municipal areas, local gas companies usually deliver gas to users through stations named "city gate". For the design of distribution networks, the network topology must firstly be defined by technical teams and secondly start a search for the features that its pipelines and pumps should possess in order to meet the flow of nodes and pressure requirements [8].

- Storage

Since the main process of the natural gas including exploration, production, and transportation are time-consuming and all of the produced natural gas is not always needed at destinations, a part of the extra gas is stored through injection to storages, which usually are located near market centers, and are usable for unlimited periods. Gas storage is one of the new and critical steps of the natural gas network process to respond to the variety of demand in different periods of the year. Traditionally, during the summer months, natural gas was stored to respond the increased demands of the cold months but nowadays, the demand of natural gas in summers also increases due to some special users such as power generation companies, where their production in summers is high because of the need to electricity for air conditioners. In addition, in some unexpected events such as natural disasters, which affect the processes of production or transportation, natural gas storages have a very critical role. In general, some of the main reasons behind using storage along the natural gas network are capability to respond cyclic fluctuations when the temperature is variable and consumption is high, improving the services to all customers, keeping market shares competitive with other sources of energy and achieving the operation with a high load factor. The natural gas storage can be done in different types which underground reservoirs are the most important way to gas storage. The storage deals with pipelines, local distribution companies, producers, and pipeline shippers (U.S. Department of Energy, U.S. Energy Information Agency, March 1995).

- Sale and marketing

Marketing in natural gas means the process of selling or organizing natural gas business from wells to the end users at various levels. In other words, at this stage all the required intermediate steps by a certain purchase such as transportation arrangements, storages, accountings, and especially sales can be considered. Marketers in the natural gas industry play a complex role and maybe joint with producers, pipelines, and local utilities or in the form of an independent group. They are concerned with selling natural gas to the retailers or end users. Natural gas usually has three to four owners before reaching the customers. Marketers utilize their skills to reduce their exposure to risks and increase the throughput by forecasting the behavior of the natural gas market. They should find natural gas buyers and secure ways to supply the natural gas in the marketplace to deliver the natural gas to the end users (http://www.naturalgas.org).

19.2 Natural gas network problems

19.2.1 Formulating

A number of notations and concepts are common in most of the developed models dealing with network optimization in the natural gas industry, which can be useful to researchers who want to start modeling in this scope. As mentioned before, the natural gas network is composed of pipelines and compressor stations as arcs and some nodes. Several general indices and parameters have been presented here and depicted in Figure 3. For more details, [3] can be seen.

Indices:

$i, j = index\ of\ nodes,\ i, j \in N = \{1,\ ...,\ |N|\}$
$k = index\ of\ compressor\ stations,\ k \in C = \{1,\ ...,\ |C|\}$
$m = index\ of\ units\ (turbocompressor)\ of\ compressor\ station,\ m \in U = \{1,\ ...,\ |U|\}$
$l = pipelines, l \in L = \{1,\ ...,\ |L|\}$
$n = nodes$
$c = compressor\ stations$
$l = pipelines$
$u = units$
$S_a = set\ of\ arcs$

Parameters:

P_{ij} : $accociate\ capacity\ between\ i-th\ node\ and\ j-th\ node$

C_{ij} : $transhipment\ \cos t\ for\ each\ unit\ of\ natural\ gas\ between\ i-th\ node\ and\ j-th\ node$

Variables:

$P_i = the\ gas\ pressure\ at\ node\ i$
$X_{ij} = the\ mass\ flow\ rate\ between\ node\ i\ and\ node\ j$
$\alpha_i = the\ net\ flow\ through\ the\ node\ i$

$$\mu_{mk} = \begin{cases} 1, & if\ unit\ m\ of\ compressor\ station\ k\ must\ be\ opened \\ 0, & if\ unit\ m\ of\ compressor\ station\ k\ must\ be\ \text{closed} \end{cases}$$

Considering the above notations, $S_a = L \cup C$ in a manner that $L \cap C = \varnothing$

Figure 3. Basic notations for planning models in natural gas networks

In some problem areas, the non-linear behaviors of compressor stations units (turbo compressors) are considered. Figure 4, presents the positions of turbo compressors in a compressor stations. Turning-on or turning-off of these turbo compressors is one of the main decisions, which has to be taken in the natural gas network [3].

Figure 4. The behavior of compressor station units

19.2.2 Optimization

In optimization problems, it is trying to find the optimum solution, which is done through iteratively transferring from the current solution to a new, hopefully better, solution. Optimization methods can usually overcome numerical simulation approaches because of two main limitations. First, there is no guarantee from simulation approaches that the achieved result is optimum (or cost is minimal). Second, determining of the pipes diameters merely depends on the experience of users. Therefore, for the same problem different users always take different decisions, which is not of interest [8].

A typical pipeline network for delivering natural gas requires a vast number of facilities and limitations, which should be considered. Due to the complex nature of the natural gas pipeline network, problems defined in this scope seek different aims and methods that certain requirements have to be considered in their optimization methodologies to achieve satisfactory and robust enough solutions to cover the most important aspects of the network. In such complex and huge networks, proper planning for transmission and distribution networks has a special importance because even a small reduction in the operation expenses and investment costs can include considerable amounts of money and improvements in the system utilization, which is more valuable in gas rich countries. Growing natural gas networks, make them more complex and from the optimization perspective, developing effective algorithms become more important.

19.2.2.1 *Network optimization*

Based on Osiadacz's definition [9], optimization means searching to find a certain objective function, which makes optimal design parameters, optimal structure for development or optimal parameters for networks operation. In the last two decades, so many researchers in the natural gas area have paid attention to optimization methods to find the optimum solution in various fields of the natural gas industry. Depending on which decisions are going to be made and what are the variables, which are sought to make optimum objective function, all optimization problems defined in this field can be decomposed into four groups: optimal design, optimal operation and flow, and optimal expansion.

- Network design

Network design decisions are key strategic decisions and consequences of making these decisions poorly are often severe [10]. The network design problem occurs in many diverse application areas including facility location, material handling systems, natural gas or electric power transportation, and telecommunications.

In the optimal design of a natural gas network, it is trying to provide the main design parameters of pipelines and compressor stations, which are the main components of the network, over a planning horizon to satisfy customers' requirements with the minimum annualized cost and subject to the network constraints [11, 12]. Outputs of the system will be the design characteristics of pipelines including diameters, pressures and flow rates, and the design parameters of compressor stations including location, suction pressure, pressure ratio, station throughput, fuel consumption, and station power consumption. Each of these parameters and characteristics influences the overall construction and operating cost in some degree [12]. Mohitpour, et al. [13] defined and explained the major influencing factors on the pipeline system design which contains properties of fluid, design conditions, magnitude or locations of demand and supply nodes, codes and standards, route, topography, and access, environmental impacts, financial matters, hydrological impacts, seismic and volcanic impacts, material, construction, operation, protection and long-term integrity.

- Network flow

The main objective of network flow optimization in the natural gas or other industries is minimizing costs and providing sufficient services to customers, which is very near to operational decisions. In this type of problem, decision variables are defined to determine the volume of flown gas through the network. Many of the network flow's problems such as minimum cost flow problems, shortest path problems, maximum flow problems, and the transmission networks planning can be modeled as different forms of mathematical programming with linear or non-linear functions and integer or mixed integer variables.

To date, many models have been developed to describe the gas flow though the network as well but there are some shortages to find the suitable solution for the developed models due to their non linear and non convex nature [3].

By making use of the introduced notations, the general form of network flow model, taken from Ahuja, et al. [14], can be presented as follows:

$$\text{Minimize} \sum_{(i,j)\in N} C_{ij} X_{ij} \tag{1}$$

Subject to:

$$\sum_{\{i:(i,j)\in N\}} X_{ij} - \sum_{\{j:(j,i)\in N\}} X_{ji} = \alpha_i \tag{2}$$

$$0 \le X_{ij} \le P_{ij} \quad \forall i,j \in N \tag{3}$$

Set of the first constraints [Eq. (2)] are mass balance constraints and set of the second constraints [Eq. (3)] presents the capacity boundaries for the flown gas between the i-th node and the j-th node.

- Network operation

Some operational decisions should be taken into account for the network to ensure that the demand of natural gas is met. At high pressures of natural gas, the operation cost of the network is determined based on the operation of compressors because of the significant percentage of running costs of the compressor stations in the total budget of companies. In low and medium pressures, an optimal operational cost is achieved through leakage reduction by optimizing the nodal pressures [9]. In general, the operating cost belonging to the natural gas network normally takes up more than 60% of the total cost of the pipeline [5]. Therefore, operational decisions have a significant effect on the network performances. Given the fact that in the pipeline system the amount of natural gas is set by the compressor stations and considering the cost associated with the operation of compressor stations, including their turning-on and turning-off, the most critical operational decision in natural gas network is the compressor selection. This important decision, which is influenced by the compressor's capacity and required energy to turn the compressor units on and off, makes a significant effect on the total natural gas operation cost. Another critical factor on the performance of the natural gas network is starting or stopping a compressor because of their different outputs [5]. Therefore, efficient operation of the complex networks of natural gas can substantially reduce airborne emissions, increase safety, and decrease the daily operating cost [3].

- Network expansion

In today's competitive markets, natural gas companies are interested in expanding the network and consequently serving potential customers because their market shares will be larger and the achieved profits will increase. Generally, in expansion of a network, the objective is scheduling the investments to supply an economic and reliable energy with minimal cost, which is not easy to achieve [15]. To make an optimum capacity expansion of natural gas network, several decisions regarding the time, size, and location of expansion should be determined [11]. The projects dealt with networks expansion have various steps, different from country to country and companies, based on the rules and governmental economic policies [16].

Referring to the literature, researchers mentioned different aspects to present difficulties of network development and expansion. Davidson, et al. [10] has indicated these difficulties from some angles: many existing options for expanding and generating a pre-specified layout, existing uncertainties in absorbing the customers and profits, difficulty in estimating construction cost because of difficulty in calculating the length and unit cost per length for new pipes to expand, and finally the dynamic nature of the problem. In this matter, Kabirian and Hemmati [16] have paid attentions regarding the presentation of an integrated strategic plan, which in long time horizon considers different aspects of the network development. They introduced the difficulties of this subject in various points including covering development and strategic planning in short and long run, identifying location and schedule of new compressor stations and pipelines in the network, determining the best type and routing of the pipelines, selecting the best combination of natural gas procurement from available sources, and providing the best operating conditions for compressor stations in long-run horizons [16]. By accuracy of inputted data for new substation availability, substation reinforcements, local generation, and future load location, which should be sent under dynamic or non-determined status, a robust decision-making is possible. These uncertainties can be presented in mathematical models as well, but the nature of the problem, which is non-convex and multi objective, makes it difficult to solve. However, it can be simplified through linearization of the objective functions and simplify the problem description [15].

Chung, et al. [17] focused on transmission networks planning through a mathematical model with three objectives including investment cost, reliability, and environmental impacts, which have been formulated using the approach of goal programming, and solved it by a GA. For analyzing the decisions, a fuzzy decision method was used to select the best scheme. In distribution networks, Carvalho and Ferreira [15] proposed an evolutionary algorithm for the stochastic planning of the large scale networks under uncertain conditions and introduced the difficulties of optimizing networks expansion including: multi-stage investment decisions, the large scale of distribution network, and a huge variety of operation policies, variable demands, investment cost, equipment variability, and location, that make the decision very insightful.

The reviewed papers in the scope of optimization in the natural gas industry based on the decisions tried to make, have been classified in Table 1.

Table 1. Classified papers based on their problem's objectives

19.2.3 Models characteristics

Each problem defined on natural gas networks has some assumptions, which has an effect on the formulation, defined equations as constraints, and complexity of the problem. In addition, sometimes researchers focus on a part of network, which has special characteristics. Usually, in this area some properties are determined as the problem statement first. Some of the most usual attributes of natural gas network problems have been illustrated in this section.

19.2.3.1 *Steady state or non-steady state*

The state of natural gas pipeline networks in different models is presented as two main categories: steady state and transient state, which are determined through considering or not considering a partial differential equation involving derivation with respect to the time [4]. In other words, this classification is dependent on how the gas flow changes in relation to time.

- Steady state: in a large number of previous researches with optimization problems in the field of natural gas network, the operation of systems is assumed in steady states. This can happen because of the no need to quick response to variation in demands and conditions in previous decades or to simplify the problem by dividing it into sub problems in steady states [20]. In a steady state system, the flow of gas is determined with some values which are independent from the time and constraints of the system, especially the ones describing the pipelines gas flow, are described by algebraic nonlinear equations [6] . In the steady-state assumptions, it is possible to work out the partial differential equation and reduce to a nonlinear equation with no derivatives, which from the optimization view makes the problem more tractable [4].

Since in the steady state problems, loads and supplies are not a function of the time, the structure of the network including the number of sources, compressor stations, valves and regulators , and the optimal parameters of operations including pressures and flows are determined once [9]. General equations for steady-state flows in natural gas networks have been collected in [22].

- Non-steady (transient) state: When load variations in a system are high, steady state operations of that system is not desirable or even possible to consider such as when factors like deregulation and peak shaving are being considered. Therefore, the need to efficient and responsive operations in dynamic status to respond rapid variation in demand and conditions is avoidable [20]. In a transient state system, the system variables such as mass flow rates through the pipelines and gas pressure levels at each node are defined as the functions of the time dynamically. Usually descriptive models are used to analyze transient states because of the highly intractable of them from the optimization perspective [7].

19.2.3.2 *Cyclic topology or non-cyclic topology*

Two fundamental types of network topologies are cyclic topology and non-cyclic topology:

- Cyclic: A cyclic topology is concerned with a network where at least one cycle is present including two or more compressor station arcs such as Figure 5. Practically, effective algorithms for cyclic topologies do not exist [7].

Figure 5. A cyclic topology

- Non- cyclic: Most of the pipeline systems have non-cyclic structures. A serial (or gun-barrel) structure is a special type of a non-cyclic network where the associated reduced network is a simple path [3]. Tree structures have also non-cyclic topologies, which involve multiple converging and diverging branches in such a way all nodes have in-degree equal to one except one of them which has in-degree equal to zero [3]. Figure 6 is a sample for a serial topology and Figure 7 presents a tree topology in natural gas networks.

Figure 6. A serial (gun-barrel) topology with three compressor stations

Figure 7. A tree topology

To recognize the natural gas network topologies, Borraz-Sánchez and Ríos-Mercado [7] explained a usual methodology as follows: first, removing the compressor arcs from a given network temporarily. Second, merging the remained connected components and eventually putting back the compressor arcs into their places. The obtained network is a reduced network. Three cases will occur from the reduced network. If it has a single path, the given network has a serial (gun-barrel) topology. If in the reduced network the compressors are arranged in branches, the topology is tree and if in the reduced network compressor stations are arranged to form cycle, the topology is cyclic.

19.2.4 Types of methods

After introducing the natural gas problems and their main characteristics to recognize them from each other, a basic classification is done regarding to methods to solve the natural gas pipeline networks. To find the best solution for the natural gas pipelines network problems, estimating the problem complexity is very important. It is quite clear that to scholars in this area that the problems with cyclic structure are more difficult to solve than problems with non-cyclic topology. In other words, the dimension of problems with cyclic topology is usually large and can not be reduced by removing or fixing some variables such as what happens in some non-cyclic topology problems. Optimization algorithms for non-cyclic gas network topologies, which the majority of them have been developed based on dynamic programming, are in a desired phase. Before explaining the suitability of methods to solve the planning problems of the natural gas network, Table 2 presents their priority regarding different topologies.

Table 2. Priority of methods to solve natural gas network problems

19.2.4.1 *Dynamic programming*

Since a few decades ago, Dynamic Programming (DP) has been utilized to optimally solve very large non-cyclic networks such as gun barrel and diverging branch tree systems, and some subclasses of cyclic networks. In general, to solve network problems with non-cyclic systems by DP, flow variables are determined in advance and pressure variables are kept. Therefore, by diverting a multi dimensional problem into the one-dimensional, the problem is simplified and is solved easily. In a diverging branch, the problem is decomposed into a sequence of several one-dimensional DP problems in such a way each of them deals with a single branch [6].

It follows that, the reasons behind using DP is its simplicity to satisfy the natural gas network constraints and overcome to non-convexity and non-linearity difficulties of feasible solutions but to apply for non-cyclic networks its computation difficulties increase with the dimension of problems exponentially [7]. Unlike non-cyclic topology, for cyclic topologies the applicability of DP is limited because the cycles break the linear structure of the network and the flow variables must be explicitly managed. In other words, the DP for cyclic networks will be multi-dimensional. The main limitation of DP regarding the cyclic topology is that to solve this type of the problem the flow variables must be fixed. Therefore, the achieved solution is optimal only with respect to a pre-specified set of flow variables [4]. As it would appear from the literature, by increasing consideration of cyclic topologies in the defined problems belonging to the natural gas system, the success of DP has been reduced.

19.2.4.2 *Gradient Search*

In 1987, Generalized Reduced Gradient (GRG) was introduced for the first time. GRG is based on a nonlinear optimization technique for non-cyclic structures. In comparison to DP approaches, in the dimensionality issue for cyclic topologies GRG acts well, but it does not guarantee a global optimal solution, especially in cases where decision variables are discrete [4, 7].

19.2.4.3 *Hierarchical control mechanisms*

In some transmission and distribution network problems, which are difficult to solve in an integrated way, other techniques such as hierarchical structures can be used in the process of solution to decompose the solution space to several levels. In the case of natural gas network hierarchical approaches, Ríos-Mercado, et al. [6] illustrated that the overall network is decomposed into two levels: the network state level as the highest level and the compressor station level as the lowest level.

19.2.4.4 *Mathematical programming*

In the natural gas network problems, DP approaches and gradient searches have not had a valuable success rate to overcome difficulties of cyclic topologies because of their limitations to avoid the trapping into the local optimality. Therefore, these methods are more useful for the problems, which have fixed the flow variables and consequently the optimality of the solution is only with respect to a pre-specified set of flow variables. Since more than half a century, mathematical programming approaches have been used in the variety parts of the natural gas industry. Because of non-convexity of feasible solutions and non-linearity and non-convexity of objective functions of natural gas optimization problems formulated by mathematical models, a large number of local optimum solutions exist where metaheuristic methods help to escape from the local optimality. Overall, a rapid improving in optimization algorithms is seen for solving complex mathematical models of natural gas networks, which has had a significant growth especially for cyclic topologies because of difficulties in solving the problems.

19.3 Survey on application of optimization

Many optimization approaches have been developed to make significant improvements in different fields of the natural gas networks with a number of general assumptions but still a tremendous potential exists in this field. By increasing the complexity of natural gas problems, more algorithms are being defined from the optimization perspective. Therefore, analyzing the previous researches can be helpful to scholars for future researches. Ríos-Mercado [3] focused on the reviewing of the fuel cost minimization, which is only one field of optimization on the natural gas industry. Considering the application of optimization methods in different fields of this critical industry and its importance in all fields, this section focuses on making a comprehensive survey on the most important optimization problems in the natural gas network and organizes the latest papers on this matter.

19.3.1 Sub-networks

19.3.1.1 *Transmission*

Some of the problems of the natural gas, which have been formulated mathematically, obey the general frame of mathematical models. This means each mathematical model includes an objective function, some constraints, and a number of variables in such a manner that the differences among these components, separate developed models.

Chung et al. [17] developed a fuzzy mathematical model considering more than one objective to solve the planning problem of transmission networks, which tries to optimize investment cost, reliability and environmental impacts. The developed model was solved through a genetic algorithm and efficient results were achieved. To minimize the operational cost of natural gas networks, one of the most critical operations, which have been studied by some researchers, is the compressor selection.

The importance of this problem is because this is associated with cost to turn on or turn off the compressors, which is considerable in the total operation cost. Uraikul, et al. [5] presented the compressor selection problem in the form of a mixed integer linear programming (MILP) and by considering three types of the cost including operating cost, start, and stop penalties. The mentioned penalties refer to cost of turning on and off the compressors in such a way that the energy used for starting or turning on a compressor is more than the required energy for stopping or turning off that compressor. To escape from being trapped in this model into non-linearity, some types of cost, which are based on time or have uncertainty such as the maintenance cost, have not been considered in the developed model. Among researchers, whom have focused on the natural gas network optimization, only a few have adopted the difficulties of cyclic topologies and not simplified the problem to the linear or tree structure. Research work of Borraz-Sánchez and Ríos-Mercado [23] is in this group. They presented the problem of optimal operation of natural gas pipeline system in cyclic topologies through combining a non-sequential DP approach within a Tabu Search (TS) technique through four main phases including preprocessing, finding an initial feasible flow, finding an optimal set of pressure values and flow modifications. What makes this work differently from non-cyclic researches is modifying the flows, which in non-cyclic approaches were done by determining a unique set of optimal flow values in the preprocessing phase, and what make this work a little far from reality are its steady state assumptions. Kabirian and Hemmati [16] developed a non-linear optimization mathematical model for formulating a strategic planning model to find the best feasible development plan for natural gas transmission networks. The objective of the developed model was determining the type, location, and installation schedule of pipelines and compressor stations over a long planning horizon with the least cost goal and considering network constraints. To achieve optimal or near optimal plans, an algorithm based on random searches was applied. Mahlke, et al. [24] formulated the problem of transient technical optimization in the form of mixed integer nonlinear problem with the aim of minimizing the fuel gas consumption but because of difficulty of time-dependent natural gas transmission network, they limited their work to achieve a good feasible solution in a suitable run time through a simulated annealing (SA).

19.3.1.2 *Distribution:*

In spite of simplicity of distribution network and low importance in comparison with transmission network from the design cost perspective, it is very critical to satisfy costumers. A considerable number of optimization methods in the form of mathematical models have been developed for distribution networking to find the best design and optimum operation along the pipelines. Among reviewed papers, Carvalho and Ferreira [15] tried to develop a mathematical model to large scale distribution networks to make robust expansion on variable conditions and under deregulation. Wu, et al. [8] considered a non-linear network and proposed the problem of minimizing the investment cost through the distribution network under steady state assumptions. They developed a model through introducing new variable and converting the primal problem to a non-convex constrained problem. Therefore, by escaping from the available difficulties in solving the primal non-smooth and non-convex problem of designing distributed layout, a global optimization approach was achieved. Moreover, Davidson et al. [10] focused on investment planning in the natural gas distribution networks too.

19.3.2 Main problems

19.3.2.1 *Fuel cost minimization*

Because of the long distances among the supply and consumption nodes in the natural gas network, many compressor stations are used along the route to set the natural gas pressure throughout the pipeline systems. By taking the tremendous amount of transported gas in pipelines per day into consideration, minimizing the consumed gas by compressors as the fuel has a critical importance. Global optimization can be lead to a 20% saving in fuels consumed by compressor stations [25]. To date, a great deal of research has been performed for developing new techniques to decrease the consumed fuel of consumption in compressor stations.

In the problem of fuel cost minimization, the decision variables are pressure dropped at each node of the network, flow rate at each pipeline, and the number of units operating within each compressor station [18]. In general, defined problems for the fuel cost minimization differ from each other due to some assumptions and methodologies applied by researchers to determine the value of variables in the optimum case. In a number of previous works, to avoid or decrease the non-linearity of the model, the number of compressor units in each of the compressor stations has been considered fixed. In addition, some of the developed models have been simplified by considering only one unit for each compressor station while compressor stations usually have multiple units. Balancing or not balancing the network is another matter. If the network is assumed balanced, in each node of the network the sum of all net flows will be equal to zero. This means there are no differences between the total output flows of supply nodes and input flows to demand nodes [26]. Other assumptions may be related to a steady state or a transient state of the model or topology of the networks, which are referred to the problem statement. In addition, regarding the methodology, in some researches if there is more than one variable, the values of variables are achieved simultaneously. In contrast, some of the researchers have proposed methodologies based on multi-stage iterative procedures. Ríos-Mercado, et al. [6] developed a two-stage procedure to optimize the fuel cost minimization in such a way that gas flow variables were fixed at the first stage and optimal pressure variables were found via DP. Then the pressure variables were considered fixed at the second stage and a set of flow variables were achieved taking the network topology into consideration to improve the objective function. Some authors relax the non-convex and non-linear models by relaxation techniques because generally such problems are very difficult to solve. For example, to fuel cost minimization, Wu, et al. [6] developed a mathematical model in the steady state assumptions and with a non-convex feasible domain, a non-linear, non-convex, and discontinuous fuel function, and a non-convex set of pipelines flow equations. To solve the developed model, it was relaxed in two ways; in the first way, the fuel cost objective function is relaxed and in the second way, non-convex and non-linear compressor domains are relaxed. In their procedure solution, the optimal solution of the original problems involves upper bound and the optimum solution of the relaxed problems is lower bound. The general formation of fuel cost minimization in the natural gas network considering to the most applied variables including flow rate and pressure, has been presented by Ríos-Mercado [3]. Another research which investigated the fuel cost minimization of compressor stations belongs to Mora and Ulieru [19], which focused on developing a new method to achieve a near optimal feasible solution in a shorter reasonable time for minimizing the amount of natural gas consumed by the compressor stations units.

Some of the latest papers regarding the minimizing of the fuel cost of compressor stations and types of their variables, which have been tried to get the optimum values, have been concluded in Table 3.

Table 3. Common decision variables in fuel cost minimization problems

19.3.2.2 ***Investment cost optimization***

Carvalho and Ferreira [15] presented the general form of optimizing investment policies, which have been adapted to the information structure of scenarios, based on the minimum cost as follows:

$$\begin{aligned} &\text{minimize } F(U) \\ &subjectto: \\ &\qquad U \in Y \\ &\qquad U(s) \in \Omega_s \\ &\qquad s \in S \end{aligned} \tag{4}$$

Where, F is a function of operational and investment costs, U presents a policy, Y is universe of not opposing decision policies, universe of admissible policies for the s-th scenario has been presented in Ω_s , and finally S illustrates a set of available scenarios.

To make an investment strategy for minimizing the risk and increasing profits, Davidson, et al. [10] developed a dynamic model integrated with a Geographical Information System (GIS), which among investment projects with sequencing, budget and timing limitations made a tradeoff to maximize the expected Net Present Value (NPV) and minimize the variance among NPVs. This model, which considers both the revenues and cost while selecting the best expansion project, with the aim of taking a decision support system to present the revenue of serving new customers and related costs to constructions besides considering the uncertainties was solved by rollout heuristic algorithms to improve the solution quality. In this case, GIS helps to identify opportunities in potential network expansion, data collection, and to identify perception made due to the developed model. Kabirian and Hemmati [16] developed a model with the aim of least discounted operating and capital cost to plan for the natural gas transmission network.

19.3.2.3 *Minimizing the cash-out penalties of the shipper*

In drawing a contract between the shipper and a pipeline company to deliver a certain volume of gas among several points, a problem may occur in marketing of natural gas because of the differences among the real mount of delivered gas and what originally had been agreed upon it along transmission networks. In those cases where imbalances took place, the pipeline company penalized the shipper by imposing a cash-out penalty policy, which is a function of daily imbalances. Therefore, the problem, which should be solved optimally, is making decisions for shippers to minimize their incurred penalty through carrying out their daily imbalances [28].

Table 4 presents a summary for some of the reviewed papers, which have focused on optimization problems in the natural gas industry.

Table 4. A summary of optimization problems in the natural gas industry

19.3.3 Mathematical Models Classifications

To the best of our knowledge, a vast number of works have been done on the optimization approaches and developing suitable algorithms to find the optimal solutions for the natural gas distribution and transmission networks, but in this part, the earliest papers, which have focused on the mathematical modeling, are classified in Table 5. Some general findings are obviously seen in this table. For example, the optimization has a more effective role in transmission network in comparison to distribution network because the natural gas spends more time under high pressures in transmission networks because of the long distances among producers and city gate stations and more instruments are used in the transmission network. Therefore, more problems have been defined in this segment. Another point which is clear from the table is whatever some data in the natural gas industry are not really deterministic, but because of other difficulties of this problem usually researchers avoid to consider variety in data in the form of fuzzy or statistics such as what [15] and Davidson, et al. [10] considered. Other issues such as type of problems, number of objectives and more useful solution methods regarding the optimization of mathematical models developed for natural gas network planning have been presented in the next sub-sections.

19.3.3.1 *Types of problem*

In general, optimization problems can be classified based on the type of variables (continuous, integer or mixed) and nature of functions used as the objective function and constraints. Considering these two factors, six types of problems are defined, which have been presented in Table 6. Most of the optimization problems of the natural gas network planning because of non-linearity behavior of the compressor station units and other factors and existence of mixed continuous and integer variables are put in the group of non-linear problems but depend on the defined variables they can be NLP, INLP, MNLP.

Table 5. Summary of developed mathematical models for natural gas networks

Table 6. Classification of optimization problems in mathematical models

19.3.3.2 *Number of objectives*

In practice, more than one objective should be considered in the optimal planning of natural gas networking that generally are conflicting such as minimizing the network flows or investment costs versus the maximum satisfaction of the customers.

In theory, if all objectives are seen in the solution methodology, problems become more difficult especially when a large number of objectives is considered. In a few cases, all objectives are transformed into a single objective but in the most of situations, it is not possible. Therefore, a number of researchers focus only on one objective and do not pay attention to others or relax them, and another group of researchers keeps the nature of problems and applies a multi-objective optimization method based on an approach. For example, if goals values of objective functions are known, the goal programming approach can be a suitable option.

19.3.3.3 *Solution methods*

The nature of natural gas network problems, which is categorized in the group of NP-hard problems, is non-convex and non-linear. Therefore, design and selection of proper solution methods are very critical in this field. Mallinson, et al. [29] described two general methods to optimize natural gas network problems; in the first method, numbers of optimization problem variables are reduced through eliminating the flow variables and in the second one, to achieve a better behavior, the optimization problem is solved without removing any variable. Although by reduction techniques the solution of the problem becomes easier, but finding the suitable algorithm has troubles because the selected algorithm could perhaps not find a feasible solution or in several cases it caused error messages. In addition, in some cases, however, a valid solution seems to be achieved but after inspecting the results, it is detected that some constraints have not been satisfied [29].

In general, three options exist to solve mathematical models, which researchers try to choose the best one based on the models complexity and solution time limitations. These options are exact methods, heuristic methods and metaheuristic methods.

- Exact Techniques

The problem featuring in the natural gas transmission and distribution networks because of its non-linear and non-convex nature can not be solved using classical techniques like exact methods from mathematical programming because these methods are usually time consuming and unable to solve NP-hard problems even on a small scale. A number of researchers have tried to solve the developed models by exact techniques but they had to oversimplify their approaches and compressor station models, which in practice may be inaccurate.

- Heuristic technique

The heuristic methods give the final solution in shorter time in comparision with exact methods but there is a risk of trapping in the first local optimality. Therefore, there is no guarantee to achieve a global optimum solution.

- Meta heuristic techniques

The best choice to solve NP-hard problems that their solution time is dependent on the problem-size exponentially, is metaheuristic methods, which guarantee finding the global optimum solution through decreasing the problem complexity without any limitations regarding the problem size. Some of the common effective methods, which researchers in different fields are interested in and in natural gas network planning also achieved many successes are Genetic Algorithm (GA), Simulated Annealing (SA), and Tabu Search (TS). Chung, et al. [17] used a GA for the problem of transmission networks planning to avoid arriving at local optimality and utilized a fuzzy decision analysis to select the best possible planning schem. Mahlke, et al. [24] exploited a SA to find a feasible solution in a reliable short time because of its simplicity to apply. TS allows designers to take advantages of the previous information in the selection of algorithms and sub algorithms. In optimization problems deal with natural gas networks, the high non-convexity of objective functions and the capability of TS to escape from local optimality have made it very efficient with an appropriate discrete solution space. Borraz-Sánchez and Ríos-Mercado [23] combined TS with non-sequential dynamic programming for the fuel cost minimization in the natural gas transmission network.

19.4 Case studies

Some researchers attended to apply developed optimization models to the natural gas industry in a number of special cases and achieved significant results. Two case studies have been described in this section to illustrate the real application of optimization models dealt with the natural gas industry.

Case 1: Optimization of planning in the natural gas supply chain

Hamedi, et al. [1] developed a mathematical model to optimize the flowing gas through the network along a six-level supply chain with the aim of minimizing direct or indirect distribution costs. The mathematical model is a mixed integer non-linear programming model converted to linear programming to solve and is limited to six groups of constraints including capacity, input and output balancing, demand satisfaction, network flow continuity, and relative constraints to the required binary variables. To reduce the model’s complexities for the large-size problems, it has been divided into two parts based on the relations among pipelines and solved hierarchically. In such a manner that in each step, one section of the problem is solved exactly through Lingo software and its outputs are passed to the next part as inputs. Therefore, by decreasing the computational complexity a near optimized solution is achieved.

The result of applying the developed model and hierarchical algorithm on the natural gas network of a gas-rich country presents the 19.839% improvements (near to 452754.222 cost unit with the given parameters) in the defined objective function in contrast with implemented plans in the reality. Even if assumed a part of this reduction is because of the simplicity in problem assumptions, the huge cost of transmission and distribution of natural gas to the consumers make this improvement valuable in the natural gas industry.

Case 2: optimization of a multi objective natural gas production planning

Barton and Selot [30] formulated a non-convex Mixed-Integer Non-Linear Programming (MINLP) model for the upstream natural gas production system, which has been considered from the wells to the LNG plants (excluding the plants). The upstream production-planning model involves two important components including the model of actual production facilities and networks (the infrastructure model) and the customer requirements (the contractual rule model). In this model, the natural gas network has been presented as a multi-product network with nonlinear pressure-flow rate relations in the wells and the trunk line network. Moreover, Production-Sharing Contracts (PSC) and operational rules have been considered. The developed model, which has been comprised from three objective functions, is a relatively large non-convex MINLP (several hundred continuous variables and tens of binary variables). Maximizing dry gas production to satisfy contractual demands, maximizing NGL production to increases revenue for the upstream operator, and prioritize production from certain fields are the objectives which the model seeks to achieve them. Since it was not possible obtaining global optimization approaches directly, the model has been reduced through a reduction heuristic and solved by a global branch-and-cut algorithm.

The result of applying the developed model in a real-world case study in Malaysia indicate its efficiency in increasing the secondary products production, achieving optimal long-term asset management beside satisfying contractual requirements of gas supply and customers in short-term.

19.5 Conclusions and directions for further research

Since using many instruments including pipelines, compressor stations, valves, and regulators in the long distances and utilizing a variety of network topologies and technologies, natural gas networks has been known as a complex and difficult problem to solve. Therefore, when this problem is mathematically modeled, the problem will be NP-hard, which can not be solved easily. On the other hand, gradient search and DP approaches have had limitations to consider real characteristics of network models because of their limitations to avoid trapping into a local optimality. From the optimization point of view, to solve planning models of natural gas networks, mathematical models and consequently metaheuristic algorithms seem the most desirable solution methods. This is more valuable when the problem is formulated based on transient assumptions and the cyclic topologies are considered. As it would appear from of reviewing papers implemented in real cases, by optimal design of natural gas networks, which is possible using mathematical models and solving with suitable algorithms to find the closest optimum solution, considerable improvements can be achieved. Due to the enormity of the problem, even a small improvement in a natural gas network could save a huge amount of money per day and the need to develop more models and algorithms is strongly felt among planners. Within optimization problems in natural gas networks, minimizing the fuel cost consumed by compressor stations has received more attentions among researchers while there is not many developed models to optimize expansion or investment costs. To date many optimization algorithms in different fields of natural gas networks planning have been introduced for all the steady state, the transient state, and different topologies, cyclic or non cyclic. However, a large number of successes have been achieved by applying them in the gas industry, it feels a comprehensive model, which can consider all conditions simultaneously and solve the problem completely, does not exist. Moreover, in the developed models transient systems have not been of interest to the researchers during the last decades to optimize because of increasing the difficulties. In addition, cyclic topologies have had a few successes in researches and implementations. Because of difficulties available in natural gas networks planning, researchers usually avoid consider varieties in demand data, production data, and others in the form of fuzzy or statistical data. Therefore, it can be a suitable point for future researches to develop new models. Furthermore, a number of technical perspectives can make scientific gaps for new researches in the planning of natural gas networks including considering the temperature as a new variable, considering various types of compressor station units, presenting the network in low or medium pressures instead of high pressures only.

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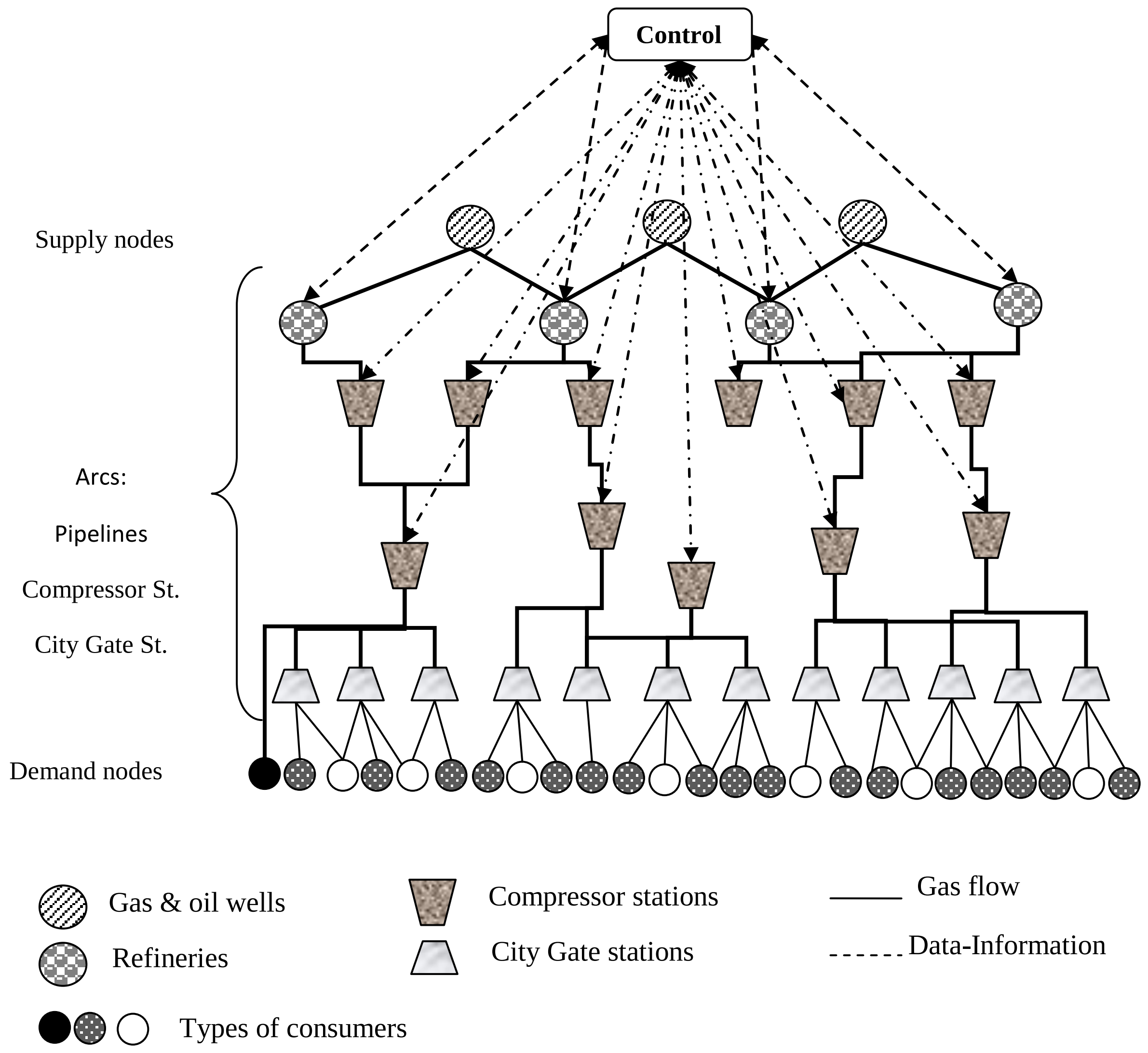

Figure 8. Natural gas network components

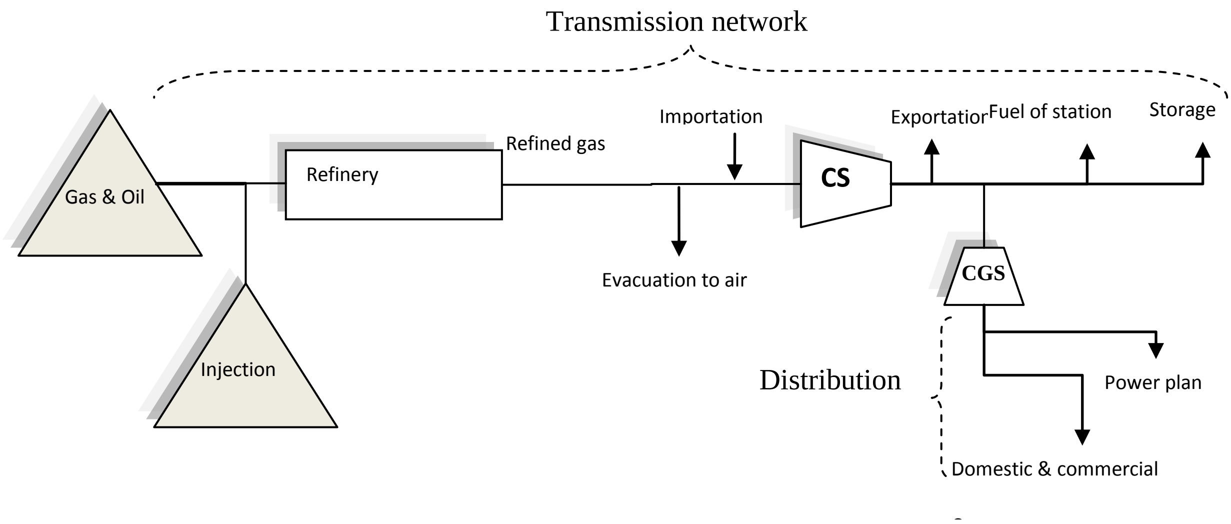

Figure 9. The process of transportation during a natural gas network

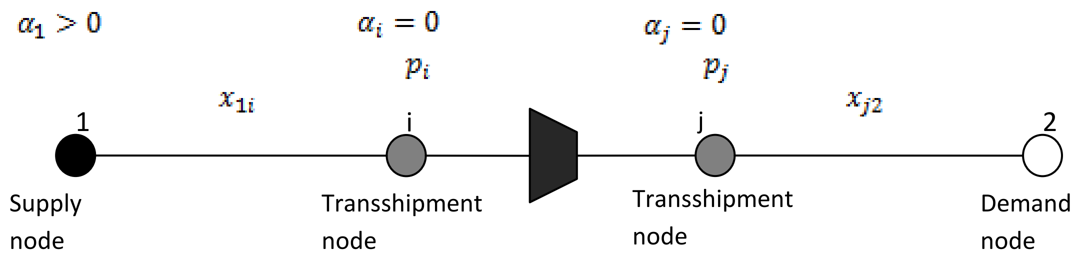

Figure 10. Basic notations for planning models in natural gas networks

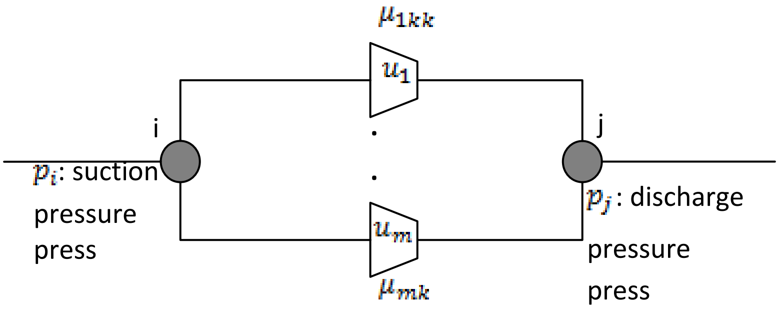

Figure 11. The behavior of compressor station units

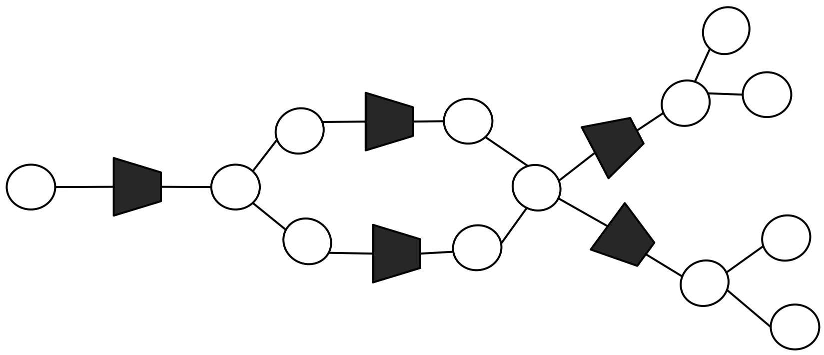

Figure 12. A cyclic topology

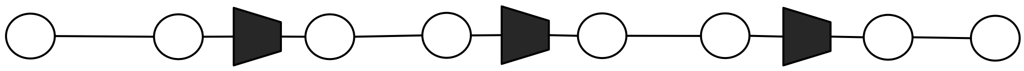

Figure 13. A serial (gun-barrel) topology with three compressor stations

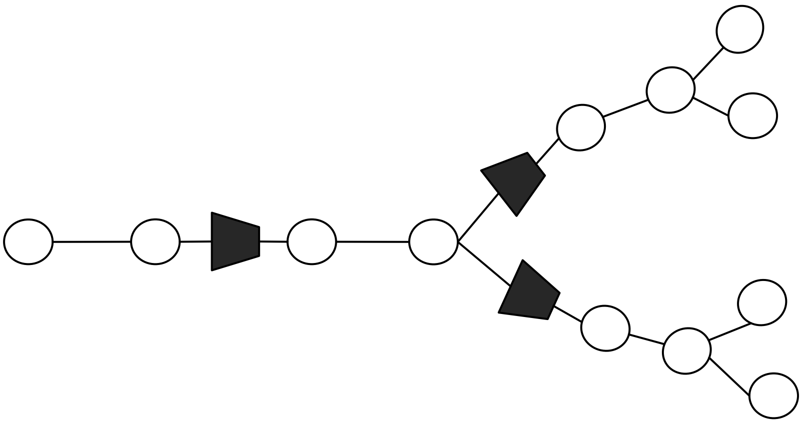

Figure 14. A tree topology

Table 7. Classified papers based on their problem's objectives

Author, Date	Type of network		Network design			Network flow and operation				Network expan-sion
	transmission	distribution	Investment Cost minimization	Maximizing NPV	Maximize customer satisfaction	Minimize transportation cost	Compressor Selection to minimize costs	Fuel cost minimization	Maximize customer satisfaction	Optimize scheduling of investments
(Carvalho and Ferreira 2000)	-	√	-	-	-	-	-	-	-	√
(Wu, Rios-Mercado et al. 2000)	√	-	-	-	-	-	-	√	-	-
(Ríos-Mercado 2002)	√	-	-	-	-	-	-	√	-	-
(Uraikul, Chan et al. 2003)	√	-	-	-	-	-	√	-	√	-
(Chung, Li et al. 2003)	√	-	-	-	-	-	-	-	-	√
(Borraz-Sánchez and Ríos-Mercado 2005)	√	-	-	-	-	-	-	√	-	-
(Mora and Ulieru 2005)	√	-	-	-	-	-	-	√	-	-
(Rachel A. Davidson, Jr. et al. 2006)	-	√	-	√	-	-	-	-	-	-
(Ríos-Mercado, Kim et al. 2006)	√	-	-	-	-	-	-	√	-	-
(Kabirian and Hemmati 2007)	√	-	√	-	-	-	-	-	-	-
(Abbaspour, Krishnaswami et al. 2007)	√	-	-	-	-	-	-	√	-	-
(Wu, Lai et al. 2007)	-	√	√	-	-	-	-	-	-	-
(Sadegheih and Drake 2008)	√	-	-	-	-	-	-	-	-	√
(Borraz-Sánchez and Ríos-Mercado 2009)	-	-	-	-	-	-	-	√	-	-
(Hamedi, Farahani et al. 2009)	√	√	-	-	-	√	-	-	-	-

Table 8. Priority of methods to solve natural gas network problems

Method \ Topology	Cyclic topology	Non-cyclic topology
Dynamic programming	4	1
Gradient search	2	2
Hierarchical programming	3	3
Mathematical Programming	1	4

Table 9. Common decision variables in fuel cost minimization problems

Author, Date	Mass flow rates through each arc	Suction pressure and discharge pressure)	Number of compressor units
(Wu, Rios-Mercado et al. 2000)	√	√	√
(Cobos-Zaleta and Ríos-Mercado 2002)	√	√	√
Borraz-Sánchez, C. and Ríos-Mercado, R. Z. (2005)	√	√	-
(Mora and Ulieru 2005)	√	√	-
(Ríos-Mercado, Kim et al. 2006)	√	√	-
(Abbaspour, Krishnaswami et al. 2007)	√	√	-
(Borraz-Sánchez and Ríos-Mercado 2009)	√	√	-
(Chebouba, F.Yalaoui et al. 2009)	-	√	√

Table 10. A summary of optimization problems in the natural gas industry

Author, Date	Transmission (T) or distribution (D)	**Type of optimization**			**System**		**Topology**			**Solution method**			
								Non-cyclic					
		Fuel cost minimization	Investment cost minimization	Cash-out penalties of the shipper	Steady state	Transient	Cyclic	Gun barrel	Tree	Dynamic programming	Gradient research	Hierarchical	Mathematical progming
(Wu, Rios-Mercado et al. 2000)	T	√	-	-	√	-	-	-	-	-	-	-	√
(Carvalho and Ferreira 2000)	D	-	√	-	√	-	-	-	-	-	-	-	√
(Chung, Li et al. 2003)	T	-	-	-	-	-	-	-	-	-	-	-	√
(Uraikul, Chan et al. 2003)	T	-	√	-	√	-	-	-	-	-	-	-	√
(Borraz-Sánchez and Ríos-Mercado 2005)	T	√	-	-	-	-	√	-	-	-	-	-	√
(Rachel A. Davidson, Jr. et al. 2006)	D	-	√	-	-	√	-	-	-	-	-	-	√
(Ríos-Mercado, Kim et al. 2006)	T	√	-	-	-		√	-	-	√	-	-	-
(Abbaspour, Krishnaswami et al. 2007)	T	√	-	-	-	√	-	-	-	-	-	-	-
(Wu, Lai et al. 2007)	D	-	√	-	√	-	-	-	√	-	-	-	√
(Mahlke, Martin et al. 2007)	T	√	-	-	-	√	_	_	_	-	-	-	√
(Chebouba, F.Yalaoui et al. 2009)	T	√	-	-	√	-	-	-	√	-	-	-	√
(Borraz-Sánchez and Ríos-Mercado 2009)	T	√	-	-	√	-	√	-	-	-	-	-	√

Table 11. Summary of developed mathematical models for natural gas

Author, Date	Network		Number of objectives		Type of model	Variables	Type of data			Solution method		
	Distribution	Transmission	Single	Multi			Deterministic	Fuzzy	Stochastic	Exact	Heuristic	Metaheuristic
(Carvalho and Ferreira 2000)	√	-	-	√	MILP	expected value of the scenario	-	-	√	-	√	-
(Chung, Li et al. 2003)	-	√	-	√	NLP	number of possible circuit additions in each path	-	-	-	-	-	GA
(Uraikul, Chan et al. 2003)	-	√	√	-	MILP	on -off the compressor units (compressor selection)	√	-	-	√	-	-
(Borraz-Sánchez and Ríos-Mercado 2005)	-	√	√	-	NLP	mass flow rates through each arc, gas pressure level at each node	√	-	-	-	-	TS
(Rachel A. Davidson, Jr. et al. 2006)	√	-	-	√	INLP	each phase of each project in each year is done or not	-	-	√	-	√	-
(Wu, Lai et al. 2007)	√	-	√	-	NLP	length of the pipes' diameter, pressure drops at each node of the network, and mass flow rate at each pipeline	√	-	-	√	-	-
(Kabirian and Hemmati 2007)	-	√	√	-	NLP	type, location, and installation schedule of pipeline and compressor s	√	-	-	-	√	-
(Mahlke, Martin et al. 2007)	-	√	√	-	MINLP	gas flow in valves and compressors, gas pressure at beginning and end of pipeline, fuel gas consumption, on-off compressor or valves	√	-	-	-	-	SA
(Borraz-Sánchez and Ríos-Mercado 2009)	-	√	√	-	NLP	mass flow rate in each arc, gas pressure in each node	√	-	-	-	-	TS
(Hamedi, Farahani et al. 2009)	-	√	-	√	NLP	transported gas volume, shortage volume, on-off the compressor units	√	-	-	√	√	-

Table 12: Classification of optimization problems in mathematical models

Variables / functions	Continuous	Integer	Mixed
Linear	LP	ILP	MILP
Nonlinear	NLP	INLP	MINLP