ISME2010-3300

# Application of Fuzzy and Predictive Algorithms in Modeling and Optimization of Natural Gas Pipeline Systems

D. Mohammadshahi<sup>1</sup>, A. Yousefi-Koma<sup>2</sup>

<sup>1</sup> PhD. Candidate, School of Mechanical Engineering, College of Engineering, University of Tehran; mohammadshahi@me.ut.ac.ir

<sup>2</sup> Associate Professor, School of Mechanical Engineering, College of Engineering, University of Tehran;

aykoma@ut.ac.ir

# Abstract

This paper consists of three parts which aim at developing an optimal dispatching algorithm for natural gas pipeline systems. First of all, a proficient model is proposed to determine the pressure level and the horsepower requirement in the network. This system constructs an input-output mapping based on human knowledge and input-output data pairs in the form of fuzzy if-then rules. In the second part, the problem of minimizing the energy consumption by compressor stations in a transmission network is addressed that is able to find an optimal solution using the model-based predictive scheme. The last part of the paper concentrates on dynamic modeling of gas distribution pipeline system; and deriving a simplified model from the set of PDE's governing the dynamics of the system.

**Keywords:** Gas Pipeline System, Modeling, Optimization, Fuzzy, Model-based Predictive Scheme

# Introduction

Pipeline networks are the most effective and common ways for transporting natural gas. According to the National Iranian Gas Company (NIGC), a huge amount of 5 milliard m<sup>3</sup> of natural gas are transported daily in Iran over 30,000 km of pipeline system. The main purpose of this project is to construct an autonomous natural gas dispatching system to satisfy customer demand with minimal operating cost, and document the operating knowledge of senior dispatchers as well as using a proficient system as a training tool for training new dispatchers. A brief literature history on the object of the presented research is proposed in "Relevant Literature". "Proficient System" describes the main two tasks of the proficient system and its role in the presented problem. It is explained how this proficient system performs these two tasks using historical data as well as heuristic knowledge from human experts.

As the gas flows through the network, energy and pressure are lost due to both friction between the gas and the pipes' inner wall, and heat transfer between the gas and its environment. The lost energy of the gas is periodically restored at the compressor stations which are installed in the network. These compressor stations typically consume about 3-5% of the transported gas that leads to a significant amount of fuel cost. These facts make the problem of how to optimally operate the compressors driving the gas in a pipeline network

important. "Optimization of Gas Pipeline Operation" addresses the problem of real time optimization of the natural gas pipeline system. This section starts with presenting a dynamic receding horizon optimization problem. Later in this section a quadratic programming problem is proposed by using a quadratic cost function. The cost function is the energy consumption of the compressor stations driving the gas in a transmission network under unsteady conditions.

This paper continues in "Modeling of the Gas Pipeline Network" with a review of the set of PDEs (continuity and momentum equations) governing the dynamic of the gas flow through a pipeline; a simplified model is also derived from the original one.

# **Relevant Literature**

Different researchers have worked on issues relating to the problems in the dispatching system of natural gas pipeline network. C.K. Sun et al. (2000) have developed an integrated decision support system combining proficient systems and mathematical modeling in order to optimize natural gas pipeline operations. In this integrated approach, both proficient systems and operations research techniques are used to model the operations of the gas pipelines, [1]. A. Martin et al. (2005) developed a mixed integer model for the solution of the stationary gas optimization problem. They described techniques for a piece-wise linear approximation of the nonlinearities in this model resulting in a large mixed integer linear program, [2]. S. Wu et al. (1999) addressed the problem of minimizing the fuel cost incurred by the compressor stations driving the gas in a transmission network under steady state assumptions, [3]. Rios-Mercado et al. (2001) presented a reduction technique for natural gas transmission network optimization problems, [4].

A. Herran-Gonzalez et al. (2008) have worked on the dynamic modeling and simulation of a gas distribution pipeline with a special emphasis on gas ducts, [5]. M. Herty et al. (2008) derived a model for gas dynamics in pipe networks by asymptotic analysis. The model is a strong improvement of the quasi static model and the SIMONE–model commonly used in the engineering community, [6]. Ke and Ti (2000) analyzed isothermal transient gas flow in the pipeline networks using the electrical analogy, [7]. Martinez-Romero et al. (2002) described steady-state compressible flow through a pipeline using software package "Gas Net". They

presented a sensibility analysis for the most important flow equations defining the key parameters in the optimization process, [8]. The receding horizon optimization is tested in a simulation environment using predictive scheme by Hans Aelto (2005). A dynamic, receding horizon optimization problem is defined by means of predictive control algorithm [9]. Chapman et al. (2005) developed a Virtual Pipeline System Testbed (VPST) for natural gas transmission, [10]. Continioustime System Identification from Discrete-time Measurements with Application to Natural Gas Pipeline Modeling was also done by E. St. Patrick Walter (2002) [11].

## **Proficient System**

Proficient system is used in problems and process control engineering where no mathematical models can be formulated, the working knowledge of the system is nonlinear and incomplete, and the knowledge of an experienced human expert can give a satisfactory solution. The main tasks of the proficient system in our model are first to determine the level of the pressure in the pipeline and recommend the control commands to be issued, and second to evaluate the associated horsepower requirement. An overview of the role that the proficient system plays in the operation of the pipeline network is shown in "Figure 1", [1].



Figure 1: Proficient system role in pipeline network

The knowledge base of the proficient system is based on an analysis of historical data, heuristic knowledge from human experts in the natural gas transportation company, and a computer simulation program that belonged to the gas company.

# Determining the Level of the Pressure in a Pipeline and Recommend the Control Commands

Determining the level of the pressure in a pipeline is realized with an if-then decision algorithm. Both the conditional variables and the decision variable of this algorithm are listed in "Table 1".

Table 1: Decision algorithm for natural gas pipeline operation

Kind of Variables	Variables	Variable Region	
Conditional Variables	Change of pressure at the end points Bate of change of	€ {rapidly decrease, decrease, no change, increase, rapidly increase}	
(IF)	pressure at the end	€ {+,-}	
	Flow	€ {very high, high, medium, low, very low}	
	Current Pressure level	$\in$ {CLP1, CLP2, CLP3, CLP4, CLP5}	

Decision Variables (THEN)	State of the line pack	€ { low, high, enough} No Action No Extra Compression Extra Compression
---------------------------------	---------------------------	--

Total of 250 Rules are generated using the above ifthen algorithm that are presented in 10 decision table. "Table 2" is a sample of these tables.

Table 2: Sample Decision Table

	Table 2. Sample Decision Table							
	Current Line Pack = CLP1 + Rate of Change of Pressure at the End Point							
-	ΔP end Flow	Rapidly Decrease	Decrease	No change	Increase	Rapidly Increase		
	Very high	Enough	Enough	Enough	High	High		
	High	Enough	Enough	Enough	High	High		
	Medium	Enough	Enough	Enough	Enough	High		
	Low	Low	Enough	Enough	Enough	High		
	Very Low	Low	Low	Enough	Enough	Enough		

# **Evaluating the Horsepower Requirement**

The horsepower (HP) requirement can be derived from heuristic data points. x and y components of each point represent the volume of natural gas (Load) and the amount of HP requirement respectively. After clustering the data points, Takagi-Sugeno Kang (TSK) fuzzy model, [12], of the form "Figure 2" is considered to be used in order to develop the linear rule fitting the historical data points.





We assume the fuzzy inference system under consideration has one input x and one output y. Suppose that the rule base contains two fuzzy if-then rules of Takagi and Sugeno's type:

 $R_i$ : IF x is  $A_i$  THEN  $f_i = p_i x + r_i$ ; i=1,2

Calculating the rule firing strength by equation (1);

$$w_i = \mu_{A_i}(x)$$
; i=1,2 (1)

The overall output can be expressed as linear combinations of the consequent parameters.

$$\overline{w}_i = \frac{w_i}{w_1 + w_2}$$
; i=1,2 (2)

$$f = \overline{w}_1 f_1 + \overline{w}_2 f_2 \tag{3}$$

The simplified fuzzy if-then rules become of the following form in which the output z is described by a crisp value (or equivalently, a singular membership function).

 $R_i$ : IF x is  $A_i$  THEN z is f

Most of all, with this simplified fuzzy if-then rule, it is possible to prove that under certain circumstance, the resulting fuzzy inference system has unlimited approximation power to match any nonlinear functions arbitrarily well on a compact set [13]. We will precede this in a descriptive way by applying the Stone-Weierstrass theorem [14] stated below.

Weierstrass theorem: let domain D be a compact space of N dimensions, and let F be a set of continuous real-valued functions on D, satisfying the following criteria.

1. Identity function: The constant f(x)=1 is in F.

2. Separability: For any two points  $x_1 \neq x_2$  in D, there is an f in F such that  $f(x_1) \neq f(x_2)$ .

3. Algebraic closure: If f and g are any two functions in F, then fg and af + bg are in F for any two real numbers a and b.

Then F is dense in C(D), the set of continuous realvalued functions on D. In other words, for any  $\epsilon > 0$ , and any function g in C(D), there is a function f in F such that  $|g(x) - f(x)| < \epsilon$  for all  $x \in D$ .

In application of fuzzy inference systems, the domain in which we operate is almost always closed and bounded and therefore it is compact. For the first and second criteria, it is trivial to find simplified fuzzy inference systems that satisfy them. Now all we need to do is examine the algebraic closure under addition and multiplication. Suppose we have two fuzzy inference systems S and  $\tilde{S}$ ; each has two rules and the output of each system can be expressed by equations (4) and (5).

$$S: z = \frac{w_1 f_1 + w_2 f_2}{w_1 + w_2} \tag{4}$$

$$\tilde{S}: z = \frac{\widetilde{w}_1 \tilde{f}_1 + \widetilde{w}_2 \tilde{f}_2}{\widetilde{w}_1 + \widetilde{w}_2}$$
<sup>(5)</sup>

 $f_1, f_2, \tilde{f}_1 and \tilde{f}_2$  are constant outputs of each rule;

$$az + b\tilde{z} = a \frac{w_1 f_1 + w_2 f_2}{w_1 + w_2} + b \frac{\tilde{w}_1 \tilde{f}_1 + \tilde{w}_2 \tilde{f}_2}{\tilde{w}_1 + \tilde{w}_2} =$$
(6)  
$$\frac{w_1 \tilde{w}_1 (af_1 + b\tilde{f}_1) + w_1 \tilde{w}_2 (af_1 + b\tilde{f}_2)}{w_1 \tilde{w}_1 + w_1 \tilde{w}_2 + w_2 \tilde{w}_1 (af_2 + b\tilde{f}_1) + w_2 \tilde{w}_2 (af_2 + b\tilde{f}_2)} + \frac{w_2 \tilde{w}_1 (af_2 + b\tilde{f}_1) + w_2 \tilde{w}_2 (af_2 + b\tilde{f}_2)}{w_1 \tilde{w}_1 + w_1 \tilde{w}_2 + w_2 \tilde{w}_1 + w_2 \tilde{w}_2}$$

$$Z\tilde{Z} = \frac{w_1\tilde{w}_1f_1\tilde{f}_1 + w_1\tilde{w}_2f_1\tilde{f}_2 + w_2\tilde{w}_1f_2\tilde{f}_1 + w_2\tilde{w}_2f_2\tilde{f}_2}{w_1\tilde{w}_1 + w_1\tilde{w}_2 + w_2\tilde{w}_1 + w_2\tilde{w}_2}$$
(7)

These equations are of the same form as (4) and (5). Apparently the model architectures that compute  $az + b\tilde{z}$  and  $z\tilde{z}$  are of the same class of S and  $\tilde{S}$  if and only if the class of membership functions is invariant under multiplication. This is loosely true if the class of membership functions is the set of all bell-shaped and scaled Gaussian membership functions, as pointed out by Wang [12].

Therefore by choosing an appropriate class of membership functions, we can conclude that the TSK model with simplified fuzzy if-then rules satisfy the four Stone-Weierstrass criteria of the theorem. Consequently, for any given  $\epsilon > 0$ , and any real-valued function g, there is a fuzzy inference system S such that  $|g(x) - S(x)| < \epsilon$  for all x in the underlying compact set. Moreover, we can draw the conclusion that all the TSK model that is used to evaluate the horsepower required, have unlimited approximation power to match any given data set. However, caution has to be taken in accepting this claim; there is no mention about how to construct the model according to the given data set.

## **Optimization of Gas Pipeline Operation**

In this step a dynamic, receding horizon optimization problem is defined, where the free response prediction of the pipeline is to be obtained from a pipeline simulator and the optimal values of the decision variables are obtained solving a quadratic programming problem. Quadratic Programming problem set up by using linear models, linear constraints and quadratic approximations of a cost function. The cost function is the energy consumption of the compressor stations.

### **Model-based Predictive Scheme**

Model-based Predictive Control (MPC) algorithms are reported to be very versatile and robust in process control applications. They usually outperform PID controllers and are applicable to non-minimum phase, open-loop unstable, time delay, and multivariable processes [16].



Figure 3: The Concept of Model-based Predictive Control

The Concept of Model-based Predictive Control is shown in "Figure 2". Predictive Control strategies bases on the idea of determining the manipulated variable in every step by optimizing the projected future control policy using a suitable chosen cost function. Although the whole future control trajectory was calculated in the previous step, only its first element u(k) is actually applied to the process at the next sampling time the procedure is repeated. This is known as the Receding Horizon concept [17].

# **Receding Horizon Optimization**

A discrete-time receding horizon real-time optimization problem is defined as finding optimal sequence of system inputs u(k), u(k+1),...,u(k+M-1) at the discrete moment of time "k" based on historical values of measured system outputs ...,y(k-1), y(k) and inputs ..., u(k-2), u(k-1) as well as predicted values of output  $\hat{y}$  (k+1),  $\hat{y}$  (k+2), ...  $\hat{y}$  (k+P) over the prediction horizon P, Fig. 3. The past input and output values before the present time "k" are used to calculate the predicted outputs for time steps k+i, i=1,2,...,P in the future under the assumption, that there is no change in future input values from the latest implemented input value u(k-1). This is often referred to as the "free response". The final output response is a combination of the free response and the response to optimal future input variables u(k), u(k+1),..., u(k+M-1) to be determined by the optimizer or controller at time "k".



Figure 4: Discrete-time receding horizon real-time optimization problem

Optimal inputs are obtained by solving the following optimization problem, equation 8, in which c is the cost function [9].

$$\min_{\substack{u(k), u(k+1), \dots, u(k+M-1)}} J(k)$$
(8)  
=  $\sum_{i=1}^{P} c_i [\hat{y}(k+i), u(k+i-1)]$ 

This equation subjects to the constraints (9), (10), (11) and (12).

$$x(k+1) = F(x(k), u(k), d(k), v_1(k)) + v_2(k)$$
(9)

$$y(k) = G(x(k), v_3(k)) + v_4(k)$$
(10)

$$h(x(k), y(k), u(k)) \le 0$$
 (11)

$$u(k + M + i) = u(k + M - 1), i = 0, ., P - M - 1$$
(12)

This problem is considered to be linear if and only if the first three constrains are linear. In this case the related MPC problem is linear and called Linear Modelbased Predictive Control (LMPC) [16].

#### Linear State-Space Model-Based Predictors

In LMPC, the predicted system output  $\hat{y}$  (k) is written as a sum of the predicted free response at moment "k",  $\hat{y}$ (k-1) and the contribution of the future inputs using a step response matrix S as it is shown in equation (13).

$$\hat{Y}(k) = S\Delta\tilde{u}(k) + \hat{Y}(k-1)$$
(13)

Where  

$$\hat{Y}(k)$$
  
=  $[\hat{y}_1(k+1|k) \ \hat{y}_1(k+2|k) \dots \ \hat{y}_1(k+P|k) |$   
 $\hat{y}_2(k+1|k) \ \hat{y}_2(k+2|k) \dots \ \hat{y}_2(k+P|k) | \dots$   
 $\hat{y}_n(k+1|k) \ \hat{y}_n(k+2|k) \dots \ \hat{y}_n(k+P|k)]'$  (14)

$$\Delta \tilde{u}(k) = [\Delta u_1(k) \Delta u_1(k+1) \dots \Delta u_1(k+M-1) | \Delta u_2(k) \Delta u_2(k+1) \dots \Delta u_2(k+M-1) | \dots \Delta u_m(k) \Delta u_m(k+1) \dots \Delta u_m(k+M-1)]'$$
(15)

$$\begin{split} \tilde{u}(k) &= [u_1(k) \, u_1(k+1) \dots \, u_1(k+M-1) \, | \\ u_2(k) \, u_2(k+1) \dots \, u_2(k+M-1) | \dots \\ u_m(k) \, u_m(k+1) \dots \, u_m(k+M-1) ]' \end{split}$$
 (16)

The nP  $\times$  mM system step response matrix S has the structure S.

$$S = \begin{bmatrix} S_{11} & \cdots & S_{1m} \\ \vdots & \ddots & \vdots \\ S_{n1} & \cdots & S_{nm} \end{bmatrix}$$

Each  $P \times M$  matrix Sij contains the step response coefficients for input uj to output yi.

A non-minimal state-space model like equation (17) can be used as a basis for the estimator design required (Li et al., 1989).

$$X(k) = M_{ss}X(k-1) + S_1\Delta u(k-1)$$
(17)

Where  

$$X(k) = [\hat{y}_1(k|k-1) \dots \hat{y}_1(k+P|k-1)]$$

$$\hat{y}_2(k|k-1) \dots \hat{y}_2(k+P|k-1)] \dots$$

$$\hat{y}_n(k|k-1) \dots \hat{y}_n(k+P|k-1)]'$$
(18)

$$\begin{aligned} X(k-1) &= [\hat{y}_1(k-1|k-2) \dots \hat{y}_1(k+P-1|k-2)] \\ \hat{y}_2(k-1|k-2) \dots \hat{y}_2(k+P-1|k-2)| \dots \\ \hat{y}_n(k-1|k-2) \dots \hat{y}_n(k+P-1|k-2)]' \end{aligned}$$
(19)

Where  $n \times (P+1)$  State Vector X includes the current output value at time "k" and P predicted values. Block diagonal matrix Mss includes n shift matrices Ms, diag(Ms, Ms, ..., Ms), each of dimension (P+1) × (P+1). n × (P+1) matrix S1 account for the last implemented incremental inputs. S1 is obtained by expanding the step response matrix S by one sample interval to a n(P+1) × mM matrix and then, in each submatrix Sij, delete all columns except the left ones [9].

### **State Update Model for Free Response Prediction**

A final state update model for the free response prediction is obtained by using a Kalman filter. Equations (20), (21), and (22) show this model.

$$\bar{X}(k) = M_{ss}X(k-1) + S_1\Delta u(k-1)$$
(20)

$$X(k) = \bar{X}(k) + K\left(y_M(k) - T\bar{X}(k)\right)$$
(21)

$$\hat{Y}(k-1) = HX(k) \tag{22}$$

Where K is a  $n(P+1) \times n$  dimensional Kalman filter gain matrix. T is an  $n \times n(P+1)$  "output" matrix used to pick the predicted output values corresponding to the measured value  $y_M(k)$ : for i=1,2,...,n, Tij=1 if j=i(P+1)-P, otherwise Tij=0. H is an  $nP \times n(P+1)$  shift matrix modified from Mss so, that the last row of each Ms is left away.

# **A Quadratic Cost Function**

As it is shown in equation (23), for the non-linear cost function of the real-time optimization problem at hand, the energy consumption of the CSs over the prediction horizon P is chosen, [9].

$$J = \sum_{i=1}^{P} \left( \sum_{j=1}^{Nc} a_j F_j(k+i) \left( \left( \frac{P_{d,j}(k+i)}{P_{s,j}(k+i)} \right)^{\gamma} - 1 \right) + b_j \right)$$
(23)

Discharge pressure of CS j,  $P_{d,j}$  is replaced by  $u_j$  in the previous equations.

$$P_{d,j} = \left[ P_{d,j}(k+1), \dots, P_{d,j}(k+P) \right]^T \\ = S_0 \Delta u_j(k) + I_P u_{j(k-1)}$$
(24)

Where

$$u(k+i) = \sum_{i=0}^{i} \Delta u(k+j) + u(k-1)$$
(25)

$$I_P = [1 \ 1 \ 1 \ 1 \ \dots \ 1]^T \tag{26}$$

$$\Delta u_j(k) = [\Delta u_j(k) \Delta u_j(k+1) \dots \Delta u_j(k+M - 1)]^T$$
(27)

 $S_0$  is also a sub triangular matrix; all of its non zero elements are set to 1.

## Modeling of the Gas Pipeline Network

The set of Partial Differential Equations (PDEs) governing the one-dimensional gas flow dynamic through a gas pipeline, continuity, momentum and energy equations (28), (29) and (30), are obtained from Herran-Gonzalez [1] and Anderson [18].

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho v) = 0$$
<sup>(28)</sup>

$$\frac{\partial}{\partial t}(\rho vA) + \frac{\partial}{\partial x}(pA + \rho v^2 A) + |\tau|\pi D$$

$$+ \rho Ag \sin \theta = 0$$
(29)

$$\frac{\partial}{\partial t} \left[ \left( e + \frac{v^2}{2} \right) \rho A \right] + \frac{\partial}{\partial x} \left[ \left( h + \frac{v^2}{2} \right) \rho v A \right] - \Omega \qquad (30)$$
$$+ \rho A g v \sin \theta = 0$$

The state for gas is also defined with equation (31).

$$p = \rho \frac{ZR_u T}{M} = \rho ZR_g T \tag{31}$$

Two important cases usually are considered in literature to solve the above Equations:

(a) Isothermal flow (T = constant) corresponding to slow dynamic changes, in which the value of  $\Omega$  can be calculated through energy equation

(b) Adiabatic flow ( $\Omega$ = 0) corresponding to fast dynamic changes, that includes the particular isentropic flow case

Rewriting the equations (28), (29) and (30) in function of fanning friction factor, f, and mass flow, q, that are evaluated from equations (32) and (33) and assuming isothermal flow, the one-dimensional gas flow dynamics inside a gas pipeline is described by the set of PDE's shown in Equation (34).

$$f = \frac{|\tau|}{\frac{1}{2}\rho v^2} \tag{32}$$

$$q = \rho v A = \rho Q \tag{33}$$

$$\begin{cases} \frac{\partial \rho}{\partial t} + \frac{1}{A} \frac{\partial \rho}{\partial x} = 0\\ \frac{\partial q}{\partial t} + \frac{\partial}{\partial x} \left( Sp + \frac{q^2}{A\rho} \right) + \frac{2fq|q|}{DA\rho} + \rho Ag \sin \theta = 0 \end{cases}$$
(34)

Friction term turns the equations of the non viscous gas classic dynamics to viscous ones. Inertia term changes the creeping motion to undulatory propagation phenomenon [5].

As the first simplification the third term in equation (34) is neglected, making reference to horizontal pipeline. Then, the gas flow dynamics through a gas pipeline can be represented by the system of PDE's shown in Equation (35) in which it is considered to have isothermal process,  $p = a^2 \rho$ . Besides the relation  $q = qvS = constant = qQ = q_nQ_n$  is used to express the model in function of flow rate in normal conditions,  $Q_n(x, t)$  and pressure p(x, t), where the subscript n refers to quantities at standard conditions of pressure  $P_n=0.1$  MPa and temperature  $T_n=288K$ . This system was used by Herran-Gonzalez, Cruz, Andres-Toto, and Risco-Martin [6] for unsteady flow simulation.

$$\begin{cases}
\frac{\partial p}{\partial t} = -\frac{a^2 \rho_n}{A} \frac{\partial Q_n}{\partial x} \\
\frac{\partial Q_n}{\partial t} = -\frac{A}{\rho_n} \frac{\partial p}{\partial x} - \frac{2f a^2 \rho_n Q_n^2}{DAp}
\end{cases}$$
(35)

## **Results and Discussion**

As shown in "Table 1", determining the level of the pressure in a pipeline is realized with an if-then decision algorithm. Consequently 250 rules are generated using the above if-then algorithm presented in 10 decision table such as "Table 2". Clustering the heuristic data points that represent load and HP requirement, TSK fuzzy model is used to develop a linear rule fitting the data; this rule is used to calculate the HP requirement.

Moreover it was shown that adequate results can be obtained from real-time receding horizon optimization, based on free response predictions obtained from a pipeline system simulator, using linear control variable models and approximate quadratic cost function presented in equation (23).

The final dynamic equations governing the pressure and flow dynamics of the system is obtained moving the x-derivatives to the left had side the system of equations (35) as shown in equation (36).

$$\begin{cases} \frac{\partial Q_n}{\partial x} = -\frac{A}{a^2 \rho_n} \frac{\partial p}{\partial t} \\ \frac{\partial p}{\partial x} = -\frac{\rho_n}{A} \frac{\partial Q_n}{\partial t} - \frac{2f a^2 \rho_n Q_n |Q|}{DA^2} \end{cases}$$
(36)

# Conclusions

In order to generate an optimal dispatching system for natural gas pipeline system a proficient decision making system was proposed as well as a dynamic, receding horizon optimization problem. The proficient system is responsible for specifying the level of pressure in the pipeline in addition to finding out the needed horsepower of the costumers. In the receding horizon optimization problem, the optimal values of the decision variables are obtained solving a quadratic programming problem. In addition a quadratic cost function for approximating the energy consumption of the compressor stations was developed.

Furthermore, the dynamic modeling of a gas distribution pipeline network was demonstrated by means of equations governing the pressure and flow dynamics of the system.

# List of Symbols

- A gas pipeline cross-sectional area  $(m^2)$
- D gas pipeline diameter (m)
- e specific internal energy  $(J \text{ kg}^{-1})$
- h specific enthalpy  $(J kg^{-1})$
- L L gas pipeline length (m)
- M molecular mass of gas (kg mol<sup>-1</sup>)
- P gas pressure (bar)
- q mass flow rate (kg  $s^{-1}$ )
- Q Q volumetric flow rate  $(m^3 s^{-1})$
- $Q_n$  volumetric flow rate in normal condition(m<sup>3</sup> s<sup>-1</sup>)
- $R_u$  universal gas constant (J mol<sup>-1</sup>K<sup>-1</sup>)
- $R_g$  gas constant (J kg<sup>-1</sup>K<sup>-1</sup>)
- T temperature of gas (K)
- v velocity (m/s)
- x axial coordinate (m)
- Z compressibility factor
- $\Omega$  heat flow per unit length (J m<sup>-1</sup> s<sup>-1</sup>)
- $\rho$  gas density (kg m<sup>-3</sup>)
- $\rho_n$  gas density in normal conditions (kg m<sup>3</sup>)
- $\theta$  inclination angle of the pipeline to horizon (°)
- $\tau$  tangential stress (N)

#### References

- [1] C. K. Suna, V. Uraikula, C. W. Chanb, P. Tontiwachwuthikul, 2000. "An integrated expert system/operations research approach for the optimization of natural gas pipeline operations". *Engineering Application of Artifitial Intelligence*, 13, pp. 465-475.
- [2] A. Martin, M. Moller, S. Moritz, 2005. "Mixed Integer Models for the Stationary Case of Gas Network Optimization", *Elsevier Sience Ltd.*
- [3] S. Wu, R. Z. Rios-Mercado, E. A. Boyd, L. R. Scott, 1999. "Model Relaxations for the Fuel Cost Minimization of Steady State Gas Pipeline Networks", *Elsevier Science Ltd.*

- [4] Rios-Mercado, R.Z, et al, 2001. "A Reduction Technique for Natural Gas Transmission Network Optimization Problems", *Elsevier Science Ltd*.
- [5] A. Herran-Gonzalez, J.M. De La Cruz, B. De Andres-Toro, J.L. Risco-Martin, 2008. "Modeling and simulation of a gas distribution pipeline network", *Applied Mathematical Modeling*, *Elsevier Science Ltd.*, 33, pp 1584-1600.
- [6] M. Herty, J. Mohring, V. Sachers, 2008. "A New Model for Gas Flow in Pipe Networks", *Elsevier Science Ltd.*
- [7] S.L. Ke, H.C. Ti, 2000. "Transient Analysis of Isothermal Gas Flow in Pipeline Network", *Elsevier Science S.A. Chemical Engineering Journal*, 76, pp 169-177.
- [8] Martinz-romero, N, et al, 2002. "Natural Gas Network Optimization and Sensibility Analysis", *SPE International Petroleum Conference and Exhibition in Mexico*.
- [9] Hans Aalto, 2005. "Real-time Receding Horizon Optimization of Gas Pipeline Network". PhD Thesis, Helsinki University of Technology, April.
- [10] K. S. Chapman, P. Krishniswami, V. Wallentine, M. Abbaspour, 2005. "Virtual Pipeline System Testbed to Optimize the U.S. Natural Gas Transmission Pipeline System". Final technical report prepared for The U.S. Department of Energy Strategic Center for Natural Gas, The National Gas Machinery Laboratory Kansas State University, June, pp 13-32.
- [11] E. St. Patrick Walter, 2002. "Continuous-time System Identification from Discrete-time Measurements with Application to Natural Gas Pipeline Modeling", Dissertation for the degree of doctor of Science, Marquette University of Milwaukee Wisconsin, May.
- [12] Li-Xin Wang, 1977. A Course in Fuzzy Systems and Control. Prentice-Hall International Inc.
- [13] Jyh-Shin Roger Jang, 1993. "ANFIS: Adaptive-Network-Based Fuzzy Inference System", *IEEE Trans. On Systems, Man and Cybernetics*, 23(3), pp. 665-685.
- [14] L. V. Kantorovich and G. P. Akilov, 1982. *Functional analysis*. Pergamon, Oxford.
- [15] M. B. Menhaj, 2008, "IntelligentCrisis Management for gas transmission network", Report of research project funded by National Iranian Gas Company.
- [16] E.F. Camacho and C. Bordons, 2003. *Model Predictive Control*, Springer.
- [17] M. Morari, J. H. Lee, 1997. "Model Predictive Control, Past, Present, and Future", 6<sup>th</sup> International Symposium on Process System Engineering (PES97), pp 1-15.
- [18] J. D. Anderson, 2003. *Modern Compressible Flow*, McGraw-Hill Education.