An optimal natural-gas network using minimum spanning tree

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***Abstract*— We consider the design of an optimal natural-gas network. Our proposed network contains two echelons, Town Broad Stations (TBSs), and consumers (demand zones). Here, our aim is a two-stage cost minimization. We first determine locations of the TBS so that the location-allocation cost is minimized. Then, we show how to distribute the flow of gas among the TBS minimizing the flow cost by using Minimum Spanning Tree (MST). A case study in Mazandaran Gas Company in Iran is made to assess the validity and effectiveness of our proposed model.**

Keywords- Natural-gas network; Minimum Spanning Tree (MST);location-allocation; mathematical model

I. INTRODUCTION

An optimal natural-gas network is a multi echelon compound having transitioning gas flow. Design of such a network with the aim to optimize flow using compressor stations to satisfy customers' demands efficiently can be seen in Martin et al. [1]. In [1], the authors presented a mixed integer nonlinear model. Dynamic programming approaches to optimize natural gas pipeline systems have been considered in recent years [2, 3]. Several other algorithms have been proposed to solve such problems. Chebouba et al. [4] optimized power consumed by stations in the system using an ant colony optimization algorithm. Gas pressure reduction stations are important elements in a natural gas network. Gas pressure stations are classified in different types due to the amount of consumption zones and they are set up with various equipments. Assuming a natural gas distribution chain in a city, we need to define echelons exactly. The first echelon includes stations called City Gate Stations (CGSs). They are used as converters so that they provide reduced gas pressure for the next echelon (i.e., they reduce gas pressure from 1000 psi to 250 psi). For transmitting the gas flow to consumers from the CGS, another type of stations is required. Town Broad Station (TBS) is a station which converts the received gas pressure from CGS to the desired gas pressure based on consumer's viewpoint (i.e., it reduces gas pressure from 250 psi to 60 psi). The second echelon consists of the TBS. This way, the connections among CGS and the TBS are significant in a gas piping project considering huge amount of investments. Therefore, we should know locations of the TBS to minimize the cost between the CGS and the TBS. Our proposed approach consists of two steps. At the first step, we determine the TBS locations and decide how to distribute the gas flow among the TBS and consumers so that the location-allocation cost is minimized. Then, at the second step, distance costs among the TBS are minimized using the Minimum Spanning Tree (MST) technique. The network is defined as G = (V, E), a connected graph, where V is a set of nodes and E is a set of arcs. A spanning tree is a subgraph of G, having no cycles, spanning all nodes in the network [5, 6]. In our model, the nodes are the set of candidate geographical locations of the TBS. Also, arcs are the connecting links. In the network, assume there are N nodes and (N-1) arcs. Minimum spanning trees have various applications in transportation and communication network design, natural-gas system, and clustering.

Here, we propose a network considering multi TBSs and multi consumers having deterministic demands. A set of TBSs are selected to service the consumers. They receive the required gas flow from the CGS and transmit it to the consumers. There is a variety of TBSs in this network; the TBS can adopt different pipelines with different costs. Then, we are faced with a tradeoff between the number of covered consumers and piping costs. In each location, only one type of TBS can be adopted if it selected to service consumers. Also, each consumer is fed by only one TBS. Determining both the locations and the types of the TBS, we have a set of nodes including the TBS. In order to minimize both the connection cost among the TBS and the spanning of all the TBS, we use the minimum spanning tree (MST) technique. We thus obtain a route covering all the nodes in the network and minimizing the total cost. Section 2 describes the natural-gas network. The mathematical model is given in Section 3. The effectiveness of the model is shown by numerical results in Section 4. Section 5 gives the concluding remarks.

II. PROBLEM DESCRIPTION

The proposed problem considers multi consumers supplied by multi TBSs. The TBS reduce gas pressure received from the CGS. For our model, there are several potential locations for the TBS. Moreover, three types of TBS with different capacities exist in the network. A number of locations with known types are selected to supply the consumers. A configuration of the proposed network is shown in Fig. 1

III. THE MATHEMATICAL MODEL

The proposed approach consists of two parts. First, we minimize the location-allocation cost between the TBS and consumers, and second we minimize the flow cost among the TBS, using the Minimum Spanning Tree (MST). The model description follows here:

Part one:

Notations:

I= Set of candidate TBSs

T= Set of TBS types

Z= Set of consumer/demand zones

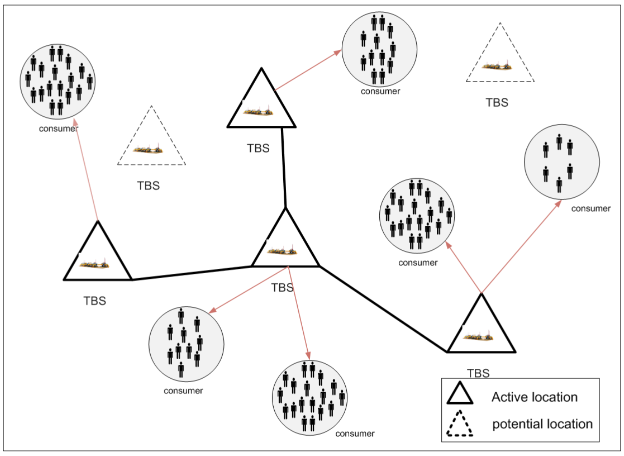

Figure 1. A configuration of the proposed natural-gas network

Parameters:

C = The average cost of piping per distance unit between the TBS and consumers

S_t = Establishing cost for TBS of type t

q_z = Demand of zone z

Q_{it} = Capacity of TBS i of type t

d_{iz} = Distance between TBS i and customer z

U = Maximum number of TBSs that can be selected

M = Large number

Decision variables:

$$x_i = \begin{cases} 1 & \text{if TBS } i \text{ is located} \\ 0 & \text{o.w.} \end{cases}$$

$$h_{it} = \begin{cases} 1 & \text{if TBS } i \text{ of type } t \text{ is selected} \\ 0 & \text{o.w.} \end{cases}$$

$$y_{itz} = \begin{cases} 1 & \text{if zone } z \text{ is connected to TBS } t \text{ of type } t \\ 0 & \text{o.w.} \end{cases}$$

Objective function:

$$\min f = f_1 + f_2$$

where,

$$f_1 = \sum_{i \in I} \sum_{t \in T} h_{it} S_t, \tag{1}$$

$$f_2 = \sum_{i \in I} \sum_{t \in T} \sum_{z \in Z} y_{itz} d_{iz} C, \tag{2}$$

Constraints:

$$\sum_{t \in T} h_{it} = x_i, \qquad \forall i \in I, \tag{3}$$

$$\sum_{i \in I} x_i \le U, \tag{4}$$

$$\sum_{z \in Z} y_{itz} \ge h_{it}, \qquad \forall i \in I, \forall t \in T, \tag{5}$$

$$\sum_{z\in Z} y_{itz} \le h_{it}M, \qquad \forall i\in I, \forall t\in T, \tag{6}$$

$$\sum_{i\in I}\sum_{t\in T} y_{itz} = 1, \qquad \forall z\in Z, \tag{7}$$

$$\sum_{z\in Z} y_{itz} q_z \le Q_{it}, \qquad \forall i\in I, \forall t\in T, \tag{8}$$

$$x_i, h_{it}, y_{itz} \in \{0,1\}, \qquad \forall i\in I, \forall t\in T, \forall z\in Z, \tag{9}$$

Formulas (1) and (2) are objective functions corresponding to the location-allocation costs, respectively. Constraints (3) show that each TBS can adopt only one type if it is selected to service consumers. Maximum number of TBSs that can be selected is expressed by constraint (4). Constraints (5) and (6) ensure that each TBS covers at least one consumer. Constraints (7) ensure that each consumer receives service from only one TBS. Capacity restriction is shown by constraints (8). Constraints (9) impose that the variables be binary.

Part two:

Notations:

I = Set of nodes corresponding to TBS

Parameters:

$N =$ Number of nodes

$d'_{ij} =$ The distance between node i and node j

$C =$ The average cost of piping per distance unit among the TBS

$M =$ Large number

Decision variables:

$$u_i = \begin{cases} 1 & \text{if node } i \text{ is a root} \\ 0 & \text{o.w.} \end{cases}$$

$$x_{ij} = \begin{cases} 1 & \text{if there is a directed link from node } i \text{ to node } j \\ 0 & \text{o.w.} \end{cases}$$

f_{ij} = Amount of flow from node i to node j

Objective function:

$$\min z = \sum_{i\in I}\sum_{j\in J} x_{ij} d'_{ij} C, \tag{10}$$

Constraints:

$$\sum_{i\in I} u_i = 1, \tag{11}$$

$$\sum_{i\in I} x_{ij} = (1-u_j), \qquad \forall j\in J, \tag{12}$$

$$x_{ij} \le f_{ij}, \qquad \forall i\in I, \forall j\in J, \tag{13}$$

$$x_{ij}(n-1) \ge f_{ij}, \qquad \forall i\in I, \forall j\in J, \tag{14}$$

$$\sum_{i\in I} f_{ij} - \sum_{i'\in I} f_{ji'} \ge (u_j(-M)) + 1, \qquad \forall j\in J, \tag{15}$$

$$\sum_{i\in I} f_{ij} - \sum_{i'\in I} f_{ji'} \le (u_j M) + 1, \qquad \forall j\in J, \tag{16}$$

$$x_{ij} \in \{0,1\}, \qquad \forall i\in I, \forall j\in J, \tag{17}$$

$$u_i \in \{0,1\}, \qquad \forall i\in I, \tag{18}$$

$$f_{ij} \ge 0, \qquad \forall i\in I, \forall j\in J, \tag{19}$$

Formula (10) gives the objective function minimizing total cost among the TBS. Constraints (11) ensure that there is only one node as the root in the network. Constraints (12) impose that each node receives only one link from other nodes if it is not the root node. The amount of flow between node i and node j is represented by constraints (13) and (14). The maximum accepting amount of flow for each link is set to be number of arcs. Constraints (15) and (16) guarantee that there is no closed loop in the network. Constraints (17) and (18) impose that the variables be binary. Non-negativity of the variables is shown by constraints (19).

IV. NUMERICAL ILLUSTRATION

Here, we present a numerical example to show the effectiveness of the proposed model. In this example, we suppose that there are 119 consumers having various demands. Equation (20) below gives the maximum number of TBSs that can be selected:

$$U = \left(\frac{\sum_{z \in Z} q_z}{\min_{i \in I, t \in T} Q_{it}} \right), \qquad (20)$$

where, q_z is demand of consumer z and Q_{it} is capacity of TBS i of type t. Therefore, having an upper bound for the TBS, our model determines the required TBSs to service consumers. There are three types of TBSs in this network. Table I shows the establishing costs and capacities for the different TBS. Table II represents consumption percent and piping cost per distance unit for different types of pipe among the TBS and consumers. Different pipes are characterized due to their diameters. Consumers' demands are given in Table III.

TABLE I. THE ESTABLISHING COST AND CAPACITY OF THE DIFFERENT TBS

TBS types	Capacity (m3/h)	Cost (unit)
TBS 1	5000	50000
TBS 2	10000	65000
TBS 3	20000	85000

TABLE II. CONSUMPTION PERCENT AND PIPING COST PER DISTANCE UNIT FOR DIFFERENT TYPES OF PIPE

Type of pipe (inch)	Consumption percent	Cost (unit)
2"	60%	17.3
4"	20%	28.5
6"	10%	38
8"	10%	53

TABLE III. CONSUMERS' DEMANDS

Con	D	Con	D	Con	D	Con	D
1	211.9	**31**	97.1	**61**	17.1	**91**	51.4
2	153.3	**32**	63.1	**62**	10.6	**92**	32.3
3	110.1	**33**	138.6	**63**	73.1	**93**	34.5
4	649	**34**	34.4	**64**	79.8	**94**	49.5
5	36.6	**35**	49.4	**65**	43.1	**95**	83.7
6	63.9	**36**	32.2	**66**	32	**96**	51.9
7	79.9	**37**	155.9	**67**	222.8	**97**	6.4
8	114.5	**38**	68.6	**68**	50.5	**98**	97
9	96.8	**39**	117.1	**69**	135.4	**99**	40.7
10	28	**40**	143.8	**70**	81.7	**100**	220
11	196.5	**41**	25.8	**71**	30.5	**101**	32.4
12	210	**42**	34.6	**72**	67.3	**102**	352
13	111	**43**	36.5	**73**	22.5	**103**	228.8
14	82	**44**	104.9	**74**	8.6	**104**	31.2
15	38.7	**45**	103.5	**75**	19.3	**105**	55.4
16	105.8	**46**	125	**76**	46.4	**106**	191
17	138	**47**	39.1	**77**	117.9	**107**	35.4
18	49.6	**48**	83.9	**78**	52.7	**108**	116.4
19	142.4	**49**	21.5	**79**	67.7	**109**	19.3
20	69	**50**	47.4	**80**	168.7	**110**	28
21	105.7	**51**	45.2	**81**	63.2	**111**	84.2
22	41.9	**52**	47.4	**82**	95	**112**	242.6
23	49.5	**53**	114.3	**83**	10.7	**113**	67
24	131.2	**54**	84.1	**84**	325.7	**114**	465.5
25	180.9	**55**	351	**85**	56	**115**	17.2
26	100	**56**	45.2	**86**	23.6	**116**	167.5
27	40.9	**57**	79.9	**87**	119.1	**117**	28
28	53.9	**58**	73.3	**88**	71.5	**118**	71.9
29	47.3	**59**	27.9	**89**	100.4	**119**	32.8
30	43.1	**60**	99.2	**90**	79.8		

The average cost of piping per distance unit among the TBS and consumers is calculated as follows:

$$\text{Average cost of piping among the TBS and consumers} = 60\%(17.3) + 20\%(28.5) + 10\%(38) + 10\%(53) = 25.18.$$

To facilitate computations in the proposed model, LINGO 9.0 software package is applied. Selected locations and types of the TBS are shown in Table IV. Fig. 2 shows allocations of consumers to the known TBS. This figure explains an optimal distribution chain of gas flow between the TBS and consumers. In this network, we assume that there are nine potential TBS locations and 119 consumers. Sum of consumers' demands and minimum capacity of the TBS are 11198 (m3/h) and 5000 (m3/h), respectively. Then, the maximum number of TBSs that can be selected (U) is:

$$U = \left(\frac{\sum_{z \in Z} q_z}{\min_{i \in I, t \in T} Q_{it}} \right) = \left(\frac{11198}{5000} \right) = 2.2396 \cong 3$$

The tradeoff between distributing and establishing costs makes our model select three TBSs of type one for satisfying consumers' demands. The average cost of piping per distance unit among the TBS is considered to be 38 units. Knowing the TBS locations, we construct the distance matrix of the TBS. Table V gives the matrix. Using Minimum Spanning Tree (MST) technique, we minimize the flow cost in the distribution chain among the TBS. Fig. 3 demonstrates this chain.

TABLE IV. LOCATION TYPES OF THE TBS

	Type	X	Y
TBS 1	1	684567	3999856
TBS 2	1	684884	3999525
TBS 3	1	685091	3999187

TABLE V. DISTANCE MATRIX

Distance matrix	TBS 1	TBS 2	TBS 3
TBS 1	0	458.31	849.79
TBS 2	458.31	0	396.35
TBS 3	849.79	396.35	0

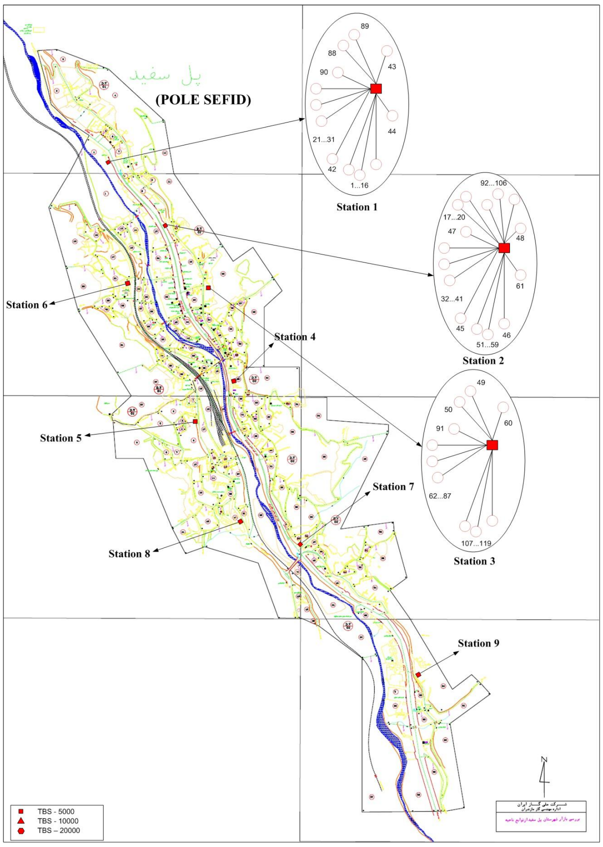

Figure 2. A configuration of location-allocation

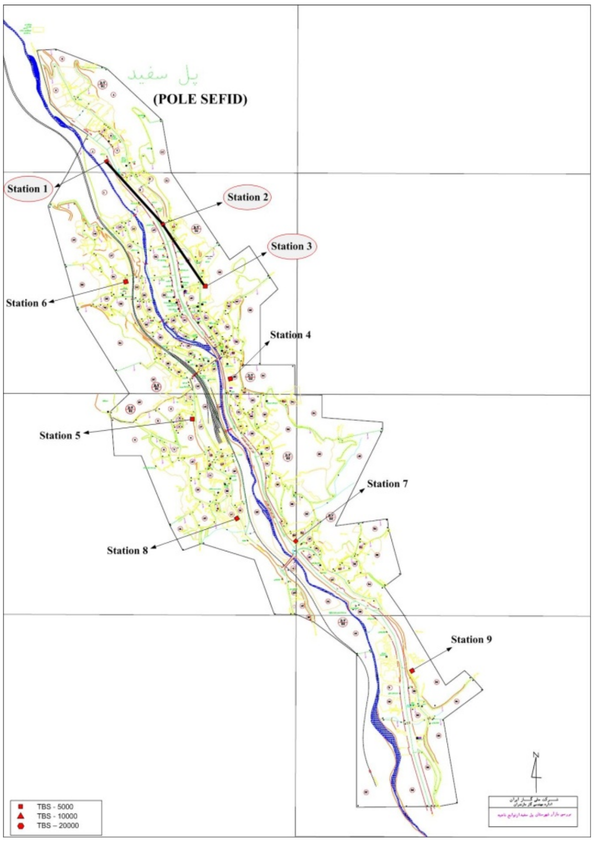

Figure 3. Distribution chain among the TBS using MST technique

V. CONCLUSIONS

We presented a mathematical model with the aim to service consumers optimally in a natural-gas environment. We designed an optimal network with two echelons. An illustrating example of the model was worked out by an LP solver.

REFERENCES

[1] Martin A, Moller M, Moritzs, P, "Mixed integer models for the stationary case of gas network optimization", Mathematical programming 105, 563–82, 2006.

[2] Ríos-Mercado, RZ, Kim, S, Boyd, EA, "Efficient operation of natural gas transmission systems: a network-based heuristic for cyclic structures", Computers & Operations Research 33, 2323–51, 2006.

[3] Wong, PJ, Larson, RE, "Optimization of natural gas pipeline system via dynamic Programming", *IEEE Transactions on Automatic Control* 5 (AC-13), 475–81, 1968.

[4] Chebouba, A, Yalaoui, F, Smati, A, Amodeo, L, Younsi, K, Tairi, A, "Optimization of natural gas pipeline transportation using antcolony optimization", *Computers & Operations Research* 36, 1916-1923, 2009.

[5] Akgun, I, Barbaros, Ç, Tansel, K, "Min-degree constrained minimum spanning tree problem: New formulation via Miller–Tucker–Zemlin constraints", *Computers & Operations Research* 37, 72-82, 2010.

[6] Fang-Guo, H, Qi, H, "The minimum spanning tree problems in stochastic networks", *Intelligent Information Management Systems and Technologies* 4, 16-22, 2008.